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# Wheeled Robot Formation Control under Uncertainty Including Adaptive Rejection of Harmonic Disturbances with Arbitrary Frequencies

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**Abstract**

*This work addresses an adaptive regulation problem for formation control of uncertain unicycle-like dynamical robots. The control goal is to drive a finite number of robots to a prescribed formation despite the presence of multi-sinusoidal input disturbances of unknown frequencies and despite uncertainties in the unicycle model, (mass and inertia's are unknown). Numerical Simulations were done to illustrate the viability of the suggested strategy. The only measurements available for control are the orientation of the unicycles and the relative positions and velocities according to a given connectivity graph.*

## 1. Introduction

Due to the numerous applications in robotics and vehicle platforms, formation control—, which entails achieving mandated shape and orientation of a network of multivalent systems—represents an intriguing problem. Formation control of wheeled robots has gained significant attention in recent years due to its potential applications in various fields such as surveillance, inspection, and transportation. In particular, sinusoidal disturbances with unknown frequencies can significantly affect the performance of the control system [1]. In the palette of available approaches to formation keeping, we would like to mention passivity-based techniques and stabilization of planar collective motion via Lyapunov techniques [2]. In particular, formation control has long been studied for wheeled mobile robots for general no holonomic systems with uncertainty and for unicycles by using ensemble control [3,4]. A comprehensive introduction to the dynamics and control of no holonomic mobile robots is provided by [5]. What makes the dynamics of the unicycle robot interesting is that it does not satisfy Brockett's necessary condition for smooth state-feedback stabilization, and for this reason, one must resort either to discontinuous or time-varying control laws. However, smooth stabilization is still possible whenever the front end of the robot - instead of its geometric center - is required to be controlled [6].

A central issue arising in formation control consists in input disturbance rejection. Various control techniques such as decentralized, centralized, and leader-follower approaches have been proposed. However, most of these methods assume perfect knowledge of the system dynamics and disturbances. To address the uncertainties in the system, adaptive control techniques have been proposed. In particular, the adaptive rejection method has been widely used to reject sinusoidal disturbances with known frequencies. Several contributions have highlighted the importance of the internal model principle in addressing disturbance rejection in coordination control of nonlinear systems [7]. In particular, the problem of formation control and rejection of matched multiharmonic input disturbances for a network of no holonomic wheeled robots have been tackled and solved in by embedding the unicycle dynamics, the disturbance internal model, and the controller equations with a port Hamiltonian structure [8-10].

Multi-harmonic input disturbances model an interesting class of faults for mechanical systems such as acoustic disturbances and vibrations in rotating equipment, and they can even provide a way to tackle unmolded flexible dynamics. In fact, flexible appendages usually induce sinusoidal velocity ripples on the main body of the mechanical system in consideration but their modeling typically requires involved analytical considerations and computations, which are not always feasible to perform. Within this context, only an educated guess can be made on the frequencies of the multi-harmonic disturbances, which thus need to be estimated in an adaptive fashion. Recently, the objectives of adaptive trajectory tracking and rejection of multi-harmonic disturbances with unknown frequencies have been achieved for a spacecraft system with uncertain inertia matrix [11]. Building upon the authors in have achieved the very same objectives for the class of fully actuated uncertain mechanical systems [12,13].

One may wonder whether it is possible to achieve formation control of a network of unicycle-like dynamical robots whose masses and inertias are uncertain, in the presence of multi-harmonic matched input disturbances whose frequencies are unknown. This work tackles and solves this very same problem by bridging the contributions of for the objective of formation control and of for the objective of online estimation of the disturbance frequencies, and of the masses and inertia's of the unicycle models. The proposed controller is distributed and consists of adaptive internal model units - one for each robot -, which estimate the aforementioned unknown quantities in a joint fashion. The proposed control scheme is based on an adaptive rejection mechanism that estimates the frequency of the disturbances and updates the control inputs accordingly. Furthermore, we bring to the attention of the reader that our peculiar formulation of an error system for the overall dynamics plays a crucial role in solving the adaptive regulation problem in consideration [14-17].

Outline of the paper. Section II introduces the reader to the problem in consideration and related control goals. Section III lists the linear characterizations to deal with both model and frequency uncertainties, and addresses the definition of our error system. Regressor matrix, controller equations and our main stability result are presented in Section IV. The effectiveness of the proposed control strategy is assessed by mans of simulations in Section V. Concluding remarks follow.

*Notation.* Given two matrices $A,B,$ the symbol $A \otimes B$ denotes the Kronecker product. $A$ = block.diag $\{A_1,...,A_N\}$ represents the block diagonal matrix with $A_1,...,A_N$ being the diagonal blocks. Operator $\|\bullet\|$ simply represents the Euclidean norm. The $n$-dimensional identity matrix is denoted as $I_n$. We denote with $L^2$ the class of all squareintegral signals, namely all those signals $x(t)$ such that

$$\int_0^{+\infty} \|x(s)\| \, ds < +\infty.$$

## 2. Problem Formulation

In order to create the ideal configuration, we employ a leader-follower strategy. The following robots use a feedback control rule to monitor the leader robot as it travels along a predetermined path. For the follower robots to converge on the intended configuration, a feedback control law based on the Lyapunov stability theory is created. We provide an adaptive rejection technique to filter out the sinusoidal disturbances that evaluates the frequency of the disturbances and modifies the control inputs appropriately. Recursive least squares, which assesses the frequency and amplitude of the disturbances, is the foundation of the suggested process. The generated compensating signal eliminates the disturbances using the calculated frequency. In order to maintain reliable formation control in the presence of sinusoidal disturbances with unknown frequencies, the compensating signal is supplied to the follower robots' control inputs.

### A. Wheeled Robot Dynamics

Consider a wheeled robot i with heading $\phi_i$ and let $(x_{A,i}, y_{A,i})$ and $(x_{C,i}, y_{C,i})$, respectively denote the center of the wheel axle and a point at the front end of the robot as depicted in [16, Figure 1]. Furthermore, let $d_{AC,i}$ denote the distance between these points.

For robot $i$ define the position $q_i := (x_{A,i}, y_{A,i}, \phi_i)^\top \in SE(2)$ and velocity $v_i := (v_{f,i}, v_{a,i})^\top \in \mathrm{R}^2$ where $v_{f,i}$ and $v_{a,i}$ respectively denotes the forward and angular velocity. For robot $i$ define the mass matrix $M_i :=$ diag $\{m_i, I_i\}$, where $m_i$ is the mass of the robot and $I_i$ is its inertia. Define the matrix $S_i : SE(2) \to \mathrm{R}^{3\times 2}$ as:

$$S_i(q_i) := \begin{pmatrix} \cos\phi_i & 0 \\ \sin\phi_i & 0 \\ 0 & 1 \end{pmatrix}. \quad (1)$$

The dynamical model of robot $i$ affected by matched input disturbance $d_i$ is then given as follows:

$q^{\cdot}_i = S_i(q_i)v_i$

$$\begin{aligned} \dot{q}_i &= S_i(q_i)\,\nu_i \\ M_i\,\dot{\nu}_i &= u_i + d_i, \end{aligned} \quad (2)$$

with $u_i, d_i \in \mathrm{R}^2$ control inputs and matched input disturbances respectively.

### B. Matched Input Disturbances

Each disturbance $d_i$ is generated by exosystem $i$:

$$\begin{aligned} \dot{\theta}_i &= T_i^{\sigma_i}\Phi_i^{\sigma_i}(T_i^{\sigma_i})^{-1}\theta_i \\ d_i &= -\Psi_i(T_i^{\sigma_i})^{-1}\theta_i, \end{aligned} \quad (3)$$

where $T_i^{\sigma_i} \in \mathbb{R}^{(2k_i+1)\times(2k_i+1)}$ is a nonsingular matrix and $\Phi_i^{\sigma_i} \in \mathbb{R}^{(2k_i+1)\times(2k_i+1)}$ is a matrix having all eigenvalues on the imaginary axis. Both $\Phi_i^{\sigma_i}$ and $T_i^{\sigma_i}$ depend on a vector of unknown frequencies $f_i + \sigma_i \in R^{n\sigma}{}_i$ where $f_i = (f_{1,i},...,f_{ki,i})^\top$ represents the nominal frequencies and $\sigma_i = (\sigma_{1,i},...,\sigma_{ki,i})^\top$ represents the mismatches between nominal and real frequencies. Thus, matrix $\Phi^{\sigma i}$ can be defined by

$$\Phi_i^{\sigma_i} := \text{diag } \{\Phi_{0,i}, \Phi_{1,i}, \ldots, \Phi_{k,i}\} \text{ with } \Phi_{0,i} := 0 \text{ and}$$

$$\Phi_{h,i} := \Phi_{h,i}^0 + \Phi_{h,i}^1 = \begin{bmatrix} 0 & f_{h,i} \\ -f_{h,i} & 0 \end{bmatrix} + \begin{bmatrix} 0 & \sigma_{h,i} \\ -\sigma_{h,i} & 0 \end{bmatrix},$$

$$f_{h,i} > 0, h = 1, \ldots, k_i.$$

### C. Formation Control Goals

In this work, we consider a network of $N$ wheeled robots of the form (2). This network is modeled as a connected undirected graph $G$ (V, E) where the node-set V corresponds to N robots and the edge-set E = V × V corresponds to
$M$ virtual couplings. We assign a positive/negative label to each of the nodes connected by an edge. The labeling of the nodes can be done in an arbitrary manner, and it does not have any effect on the results. Label one end of each edge in E with a positive sign and the other end with a negative sign. The incidence matrix $B \in \{-1,0,1\}^{N\times M}$ associated to $G$(V,E) describes which nodes are coupled by an edge, and is defined as

$$b_{il} := \begin{cases} +1 & \text{if node } i \text{ is at the positive side of edge } l, \\ -1 & \text{if node } i \text{ is at the negative side of edge } l, \\ 0 & \text{otherwise.} \end{cases}$$

The relative position $z_k$ between agent $i$ and $j$ is then defined for all edge $k$ as follows:

$$\begin{aligned} z_{x,k} &:= x_{C,i} - x_{C,j} \\ z_{y,k} &:= y_{C,i} - y_{C,j}. \end{aligned}$$

We can define $z_k := [z_{x,k}, z_{y,k}]^\top$ and, consequently, $\underline{z} := [z_1^\top, \dots, z_M^\top]^\top$. In a similar way, we can define $r_i :=$ $[x_{C,i}, y_{C,i}]^\top$ and $r := [r_1^\top, \dots, r_N^\top]^\top$. Note that each ri can be computed from the state of robot $i$ as

$$r_i = \begin{pmatrix} x_{A,i} \\ y_{A,i} \end{pmatrix} + d_{AC,i} \begin{pmatrix} \cos\phi_i \\ \sin\phi_i^\top \end{pmatrix}. \tag{4}$$

The relation between $r$ and $z$ is then given as:

$$z = (B \otimes I_2)^\top r. \tag{5}$$

Given a desired relative $z_k^*$ position for each edge in E, we can define the error variables $\tilde{z}_k := z_k - z_k^*$, and stack

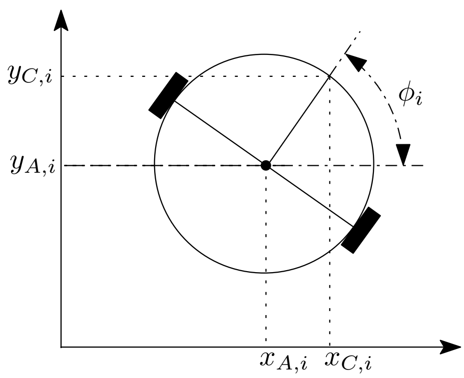


**Figure 1**: Unicycle robot $i$.

them into the vector $\tilde{z} := [\tilde{z}_1^\top, \dots, \tilde{z}_M^\top]^\top$. By introducing the matrix $G_i : S \to \mathrm{R}^{2\times 2}$ as

$$G_i(\phi_i) := \begin{pmatrix} \cos\phi_i & \sin\phi_i \\ -d_{AC,i}\sin\phi_i & d_{AC,i}\cos\phi_i \end{pmatrix}, \tag{6}$$

it is straightforward to obtain from (2) and (4) that

$$\dot{r}_i = G_i(\phi_i)^\top \nu_i. \tag{7}$$

We can now formally state the control goal as formation control plus disturbance rejection, namely:

$$\lim_{t\to+\infty} \tilde{z}_i = 0 \text{ for } 1, \dots, M, \tag{8}$$

in presence of matched input disturbances $d_i$, generated by exosystems (3).

## 3. Preliminary Manipulations

### A. Robot Dynamics of the Overall Network

We first write the dynamics of the overall network of N robots in compact form. To this end, we introduce the following notation:

$$\begin{aligned} q &:= (q_1^\top, \dots, q_N^\top)^\top, \ \nu := (\nu_1^\top, \dots, \nu_N^\top)^\top, \\ u &:= (u_1^\top, \dots, u_N^\top)^\top, \ d := (d_1^\top, \dots, d_N^\top)^\top, \\ S(q) &:= \text{block.diag } \{S_1(q_1), \dots, S_N(q_N)\}, \\ M &:= \text{block.diag } \{M_1, \dots, M_N\}. \end{aligned}$$

We can now write (2) as follows:

$$\begin{aligned} \dot{q} &= S(q)\,\nu_i \\ M\,\dot{\nu} &= u + d, \end{aligned} \tag{9}$$

### B. Exosystem dynamics in compact form

With the aim of formulating the overall dynamics of the N exosystems in compact form, we introduce the following notation:

$$\begin{aligned} \theta &:= (\theta_1^\top, \dots, \theta_N^\top)^\top, \ f := (f_1^\top, \dots, f_N^\top)^\top, \\ \sigma &:= (\sigma_1^\top, \dots, \sigma_N^\top)^\top, \\ T^\sigma &:= \text{block.diag } \{T_1^{\sigma_1}, \dots, T_N^{\sigma_N}\}, \\ \Phi^\sigma &:= \text{block.diag } \{\Phi_1^{\sigma_1}, \dots, \Phi_N^{\sigma_N}\}, \\ \Psi &:= \text{block.diag } \{\Psi_1, \dots, \Psi_N\}. \end{aligned}$$

We can now write (3) as follows:

$$\begin{aligned} \dot{\theta} &= T^\sigma \Phi^\sigma (T^\sigma)^{-1}\theta \\ d &= -\Psi (T^\sigma)^{-1}\theta. \end{aligned} \tag{10}$$

Note that $\theta \in R^k$, where $k := k_1, \dots, k_N$, i.e. the sum of the dimension of the state of each exosystem (3).

### C. Linear Parametrization for Model Mismatches

We assume that the uncertainties in model (9) are structured. Let $\delta_i \in R^{n\delta,i}$ denote the linear mismatches between the nominal and real matrix $M_i$. In order to fully characterize $\delta_i$, we introduce the nominal mass matrix $M_i^0 = \text{diag}\,\{m_i^0, \mathcal{I}_i^0\}$, with $m_i^0$ and $I_i^0$ respectively denoting the nominal mass and inertia of robot $i$. Furthermore, we define the matrix function $M_i^1 :\in \mathrm{R}^2 \to R^{N\times n\delta i}$ in such a way that the following linear parametrization for robot $i = 1, \dots N$ holds true for all vectors $s \in \mathrm{R}^2$:

$$M_i s =: M_i^0 s + M_i^1(s)\,\delta_i, \tag{11}$$

We can then define $M^0 := \text{block.diag}\,\{M_1^0, \dots, M_N^0\}$, $M^1 := \text{block.diag}\,\{M_1^1, \dots, M_N^1\}$, and $\delta = (\delta_1^\top, \dots, \delta_N^\top)$ so that the following linear parametrization holds true for the overall network:

$$Ms =: M^0 s + M^1(s)\,\delta. \tag{12}$$

for all $s \in R^{2N}$.

### D. Error System Definition

In the spirit of [7] and [4], an error system is built from the overall network dynamics (9) in order to embed the disturbance rejection and formation control goals (8). To this end, we first recall the control law used in [8], [17], [16] to achieve formation control without disturbance rejection, then we define our error system according to such control law. By defining

$$\phi := (\phi_1^\top, \dots, \phi_N^\top)^\top,$$

$G(\phi) :=$ block.diag $\{G_1(\phi_1), \dots, G_N(\phi_N)\}$, it is then immediate to check from (5) and (7) that

$$\dot{\tilde{z}} = (B \otimes I_2)^\top G(\phi)^\top \nu. \tag{13}$$

The control law $u = u^f$ used in [8], [17], [16] to achieve formation control, namely

$$u_f = -G(\phi)\,(B \otimes I_2)\Big(K_p\,\tilde{z} + K_d\,(B \otimes I_2)^\top\, G(\phi)^\top \nu\Big), \tag{14}$$

Where $K_p, K_v$ are diagonal positive definite matrices of appropriate dimensions. Based upon (14), we define the following error variables:

$$Z := \nu + A(\phi)\tilde{z} \tag{15}$$

$$V := -A(\phi)\tilde{z}, \tag{16}$$

Where $A(\phi) := (M^0)^{-1}G(\phi)(B \otimes I_2)\lambda_{\tilde{z}}$ with $\lambda_{\tilde{z}} > 0$ being a tunable scalar controller gain. Observe that $Z + V = v$, and thus it follows from the second equation of (9) that:

$$M\dot{Z} = u + d + F_0(\dot{V}) + F_1(\dot{V})\delta \qquad (17)$$

With $F_0(V^\cdot) := -M^0V^\cdot$ and $F_1(V^\cdot) := -M_1(V^\cdot)$. Therefore the control input $u^- := u + F_0(V^\cdot)$ applied to (17) yields the error system:

$$M\dot{Z} = F_1(\dot{V})\delta + \bar{u} + d. \qquad (18)$$

The following Lemma motivates the choice of Z in the definition of error system (18).

*Lemma 1:* If $Z(t)$ is a $\mathcal{L}^2$ signal, then $\lim_{t\to+\infty} \tilde{z}(t) = 0$ and limt→+∞ $\tilde{z}^\cdot(t) = 0$ for any arbitrary continuous signal $\phi(t)$.
*Proof:* Pre-multiply the definition of $Z(t)$ in (15) by $(B^\top \otimes I_2) G(\phi(t))^\top$ so as to obtain:

$$\begin{aligned}\dot{\tilde{z}}(t) &= (B^\top \otimes I_2)G(\phi(t))^\top Z(t)\\ &- (B^\top \otimes I_2)G(\phi(t))^\top (M^0)^{-1}G(\phi(t))(B \otimes I_2)\lambda_{\tilde{z}}\ \tilde{z}(t). \quad (19)\end{aligned}$$

where we made use of equality (13). Note that $G(\phi)$ is nonsingular for all $\phi \in [0,\pi)^N$. It follows that $G(\phi)^\top(M^0)^{-1}G(\phi)$ is symmetric positive definite for all $\phi \in [0,\pi)$ , and thus there exists a matrix $\bar{M} \succ 0$ such that $G(\phi)^\top(M^0)^{-1}G(\phi) \geq \bar{M}$ for all $\phi \in [0,\pi)^N$. Consider the following Lyapunov function $V := \frac{1}{2}\tilde{z}^\top\tilde{z}$ Note that $V$ is radially unbounded. Let η(t) := $(B \otimes I_2)\tilde{z}(t)$ and $e(t) := G(\phi(t)) Z(t)$. Then, taking the time derivative of $V$ along the trajectories of (19) yields:

$$\begin{aligned}\dot{V} &= -\eta^\top G(\phi)^\top(M^0)^{-1}G(\phi)\eta + \eta^\top G(\phi)Z\\ &\leq -\eta^\top\bar{M}\eta + \eta^\top e. \qquad (20)\end{aligned}$$

Dissipation (20) shows that system (19) is strictly output passive wrt output $\eta(t)$ and input $e(t)$. Since $Z(t)$ is a $\mathcal{L}^2$ signal and $\|G(\phi)\| \leq b < +\infty$ for any $\phi \in [0,\pi)$, it follows that $e(t)$ is also a $\mathcal{L}^2$ signal.

System (19) is zero-state detectable. Indeed, if $\eta \equiv 0$ and $e \equiv 0$, then $\tilde{z} \in \mathcal{N}(B \otimes I_2) = \mathcal{R}(B^\top \otimes I_2)^\perp$, with N and $\mathcal{R}$ denoting null- and range-space respectively. Relation (5) proves that $\tilde{z} = \mathcal{R}(B^\top \otimes I_2)$, and we thus conclude that $\tilde{z} = 0$.

Given strict output passivity and zero-state detectability of the system, and due to $e(t)$ being a $\mathcal{L}^2$ signal and V being a radially unbounded storage function, we conclude by virtue of [9, Theorem III.1] that $\tilde{z} \to 0$ as $t \to +\infty$. By virtue of Barbalat's Lemma, it follows $\tilde{z}^\cdot \to 0$ as $t \to +\infty$.

## E. Linear Parametrization for Frequency Mismatches

As in standard model-based approaches, an internal model of exosystem (10) has to be incorporated in the dynamic controller. According to the procedure proposed in [12], let H $\in \mathbb{R}^{(2k+1)\times(2k+1)}$ and $N \in \mathbb{R}^{(2k+1)\times n}$ be matrices satisfying the following set of assumptions:

- (A0) H is Hurwitz;
- (A1) the couple $(H,N)$ is controllable;
- (A2) $H,\Phi^\sigma$ have disjoint spectra. Since (A2) holds, Sylvester equation

$$T^\sigma\Phi^\sigma - HT^\sigma = N\Psi \qquad (21)$$

has a unique nonsingular solution $T^\sigma$. The dynamics of the internal model unit are then defined as:

$$\dot{\eta} = H\eta + N\bar{u} + P_0(Z),$$

where $\eta \in \mathbb{R}^{(2k+1)}$ is the internal state and $P_0$ will be defined in section IV. In order to make the internal model unit also adaptive, additional states taking the form of a matrix $\zeta \in \mathbb{R}^{(2k+1)\times n\delta}$ are imported in the dynamics and the following linear parametrization is introduced:

$$\begin{aligned}l_{11}(\zeta)\delta + l_{12}(\zeta)Q_1(\delta,\sigma) &:= \Psi(T^\sigma)^{-1}\zeta\delta\\ l_{21}(q,Z)\delta + l_{22}(q,Z)Q_2(\delta,\sigma) + l_{23}(q,Z) &:= \Psi(T^\sigma)^{-1}NMZ\\ l_{31}(\eta)Q_3(\sigma) + l_{32}(\eta) &:= -\Psi(T^\sigma)^{-1}\eta,\end{aligned} \qquad (22)$$

with functions $Q_1(\cdot,\cdot), Q_2(\cdot,\cdot), Q_3(\cdot)$ vanishing at $\sigma = 0$. Note that if $T^0$ denotes the solution of Sylvester equation (21) for the nominal case ($\sigma = 0$), then we have $l_{32}(\eta) = -\Psi(T^0)^{-1}\eta$. We are now ready to state our main result.

## 4. Main Result and Proof's Discussion

Let $\xi$ and $\rho$ respectively denote the vector of unknown parameters and regressor matrix, as follows:

$$\xi := \begin{bmatrix}\delta\\ Q_1(\delta,\sigma)\\ Q_2(\delta,\sigma)\\ Q_3(\sigma)\end{bmatrix},$$

$$\rho(q,\dot{q},v,\dot{v},z,\eta,\zeta) := \begin{bmatrix}\left(F_1(\dot{V}) + l_{11}(\zeta) + l_{21}(q,Z)\right)^\top\\ l_{12}(\zeta)^\top\\ l_{22}(q,Z)^\top\\ l_{31}(\eta)^\top\end{bmatrix}^\top.$$

Let $\tilde{\xi} := \hat{\xi} - \xi$ denote the parameter estimation error. The adaptive controller is then defined as:

$$\begin{cases}\dot{\zeta} &= H\zeta + P_1(\dot{V},Z)\\ \dot{\eta} &= H\eta + N\bar{u} + P_0(Z)\\ \dot{\hat{\xi}} &= \Gamma_{\hat{\xi}}\rho^\top Z\\ \bar{u} &= \Psi(T^0)^{-1}\eta - l_{23}(Z) - K_Z Z - \rho\hat{\xi},\end{cases} \qquad (23)$$

where $K_Z$, $\Gamma_{\hat{\xi}}$ are positive-definite controller gains and $P_0$, $P_1$ are given as:

$$\begin{aligned}P_0(Z) &:= -HNM^0Z\\ P_1(\dot{V},Z) &:= HNM^1(Z) - NF_1(\dot{V}).\end{aligned}$$

*Theorem 1: (Convergence of proposed controller)* Consider system (10) and (18). Controller (23) achieves the disturbance rejection and formation control goal (8), for any initial condition of the closed-loop system.
*Proof:* Consider the change of coordinates:

$$\begin{cases}\tilde{\xi} &= \hat{\xi} - \xi\\ \tilde{\eta} &= \eta - \theta - NMZ.\end{cases}$$

Taking the time derivative of $\tilde{\eta}$ yields:

$$\begin{aligned}\dot{\tilde{\eta}} &= \underbrace{H\eta + N\bar{u} + P_0(Z)}_{\dot{\eta}} - T^\sigma\Phi^\sigma(T^\sigma)^{-1}\theta \\ &\quad -\underbrace{N\left[\bar{u} + F_1(\dot{V})\delta - \Psi(T^\sigma)^{-1}\theta\right]}_{M\dot{Z}} \\ &= \underbrace{H\eta - \underbrace{\left[T^\sigma\Phi^\sigma(T^\sigma)^{-1} - N\Psi(T^\sigma)^{-1}\right]}_{H\text{ (by assumption A3)}}\theta - HNMZ}_{H\tilde{\eta}} \\ &\quad +HNMZ - HNM^0(Z) - NF_1(\dot{V})\delta \\ &= H\tilde{\eta} + P_1(\dot{V}, Z)\delta. \end{aligned} \tag{24}$$

In the new coordinates, equation of motion (18) reads as:

$$\begin{aligned} M\dot{Z} &= F_1(\dot{V})\,\delta + \underbrace{\Psi(T^0)^{-1}\eta}_{-l_{32}(\eta)} - l_{23}(Z) - K_Z Z - \rho\,\hat{\xi} \underbrace{-\Psi(T^\sigma)^{-1}\theta}_{d} \\ &= F_1(\dot{V})\,\delta \underbrace{-l_{32}(\eta) - l_{31}(\eta)Q_3(\sigma)}_{\Psi(T^\sigma)^{-1}\eta} + l_{31}(\eta)Q_3(\sigma) \\ &\quad \underbrace{-l_{23}(Z) - l_{22}(Z)Q_2(\delta,\sigma) - l_{21}(z)\delta}_{-\Psi(T^\sigma)^{-1}NMZ} \\ &\quad + l_{21}(Z)\delta + l_{22}(Z)Q_2(\delta,\sigma) - K_Z Z - \rho\,\hat{\xi} - \Psi(T^\sigma)^{-1}\theta = \\ &= F_1(\dot{V})\delta + \Psi(T^\sigma)^{-1}\tilde{\eta} - K_z z \\ &\quad - \rho\,\hat{\xi} + \begin{bmatrix} l_{21} & l_{22} & l_{23}\end{bmatrix}\begin{bmatrix}\delta \\ Q_2 \\ Q_3\end{bmatrix}. \end{aligned} \tag{25}$$

A further change of coordinates is introduced to deal with model uncertainty $\delta$, namely:

$$\hat{\eta} := \tilde{\eta} - \zeta\delta. \tag{26}$$

Thanks to the change of coordinates (26), model uncertainty $\delta$ is embedded in the internal model of the system. In fact, the time derivative of $\hat{\eta}$ now reads as:

$$\dot{\hat{\eta}} = \dot{\tilde{\eta}} - \dot{\zeta}\delta = H\tilde{\eta} + P_1\delta - (H\zeta + P_1)\delta = H\hat{\eta}. \tag{27}$$

In the new coordinates, equation (25) reads as:

$$\begin{aligned} M\dot{Z} &= F_1(\dot{V})\delta + \Psi(T^\sigma)^{-1}\hat{\eta} - K_Z Z - \rho\,\hat{\xi} \\ &\quad + \underbrace{\Psi(T^\sigma)^{-1}\zeta\delta}_{l_{11}(\zeta)\delta + l_{12}(\zeta)Q_1(\delta,\sigma)} + \begin{bmatrix} l_{21} & l_{22} & l_{23}\end{bmatrix}\begin{bmatrix}\delta \\ Q_2 \\ Q_3\end{bmatrix} \\ &= \Psi(T^\sigma)^{-1}\hat{\eta} - K_Z Z - \rho\,\tilde{\xi}. \end{aligned}$$

Therefore the final form of the closed-loop system is:

$$\begin{cases} M\dot{Z} &= \Psi(T^\sigma)^{-1}\hat{\eta} - K_Z Z - \rho\,\tilde{\xi} \\ \dot{\zeta} &= H\zeta + P_1(\dot{V}, Z) \\ \dot{\hat{\eta}} &= H\hat{\eta} \\ \dot{\tilde{\xi}} &= \dot{\hat{\xi}} = \Gamma_{\tilde{\xi}}\,\rho^\top Z. \end{cases} \tag{28}$$

Let $Q$ be a positive-definite matrix such that $H^\top Q + QH = -I$ and $\epsilon > 0$ a positive real constant. Let $V(\hat{\eta}, Z, \tilde{\xi})$ be the following candidate Lyapunov function:

$$V(\hat{\eta}, Z, \tilde{\xi}) = \epsilon\hat{\eta}^\top Q\hat{\eta} + \frac{1}{2}Z^\top MZ + \frac{1}{2}\tilde{\xi}^\top\Gamma_{\tilde{\xi}}^{-1}\,\tilde{\xi}. \tag{29}$$

Then, taking the time derivative of (29) along the trajectories of (28) yields:

$$\begin{aligned} \frac{dV(\hat{\eta}, Z, \tilde{\xi})}{dt} &= -\epsilon\,\hat{\eta}^\top\left(H^\top Q + QH\right)\hat{\eta} + Z^\top M\dot{Z} + \tilde{\xi}^\top\Gamma_{\tilde{\xi}}^{-1}\dot{\tilde{\xi}} \\ &= -\epsilon\,\|\hat{\eta}\|^2 + -Z^\top K_z Z \\ &\quad + Z^\top\left[\Psi(T^\sigma)^{-1}\hat{\eta} - \rho\,\tilde{\xi}\right] + \tilde{\xi}^\top\Gamma_{\tilde{\xi}}^{-1}\Gamma_{\tilde{\xi}}\,\rho^\top Z \\ &= -\epsilon\,\|\hat{\eta}\|^2 - Z^\top K_Z Z + Z^\top\Psi(T^\sigma)^{-1}\hat{\eta} \\ &\leq -\frac{1}{2}\epsilon\,\|\hat{\eta}\|^2 - \frac{1}{2}Z^\top K_Z Z, \end{aligned} \tag{30}$$

where last inequality stems from a Young's inequality argument and by setting $\epsilon$ such that $\epsilon \geq \Psi(T^\sigma)^{-1}/k_{z,min}$ with $k_{z,min}$ being the smallest eigenvalue of $K_Z$. Since inequality (30) holds, states $\hat{\eta}$, $z$, $\tilde{\xi}$ are bounded. In particular, integration of (30) yields:

$$c_0\int_0^t \|Z(s)\|\,ds \leq \int_0^t \frac{1}{2}Z(s)^\top K_Z Z(s)\,ds \leq V(0),$$

for all $t \geq 0$ and for some positive constant $c_0$, and thus $Z(s)$ is a $\mathcal{L}^2$ signal. By virtue of Lemma 1, it holds that $\tilde{z}(t)$, $\dot{\tilde{z}}(t) \to 0$ as $t \to +\infty$, and we have thus proved the formation control goal (8).

## 5. Simulation

We do numerical simulations on a fleet of wheeled robots to illustrate the viability of the suggested strategy. The simulation results demonstrate that even in the face of sinusoidal disturbances with unknown frequencies, the suggested technique may provide reliable formation control. The robots can maintain the proper configuration even in the face of disturbances, and the formation error is greatly decreased.

To the end of showing the effectiveness of controller (23), we simulate a network of 4 unicycle robots. For ease of presentation, we assumed same nominal mass, nominal inertia, front-end distance, and same scalar mismatch $\delta = 0.2$ (with according units) for each robot $i \in \{1, \dots, 4\}$, namely $m_i = m^0_i + \delta$, $I_i = I^0_i + \frac{1}{2}\delta$ with $m^0_i = 1$ kg, $I^0_i = 0.1$ kgm², and $d_{AC,i} = 0.2$m. For this robot network, we have considered 6 virtual springs, thus a connectivity graph described by the following incidence matrix:

$$B = \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{pmatrix}.$$

Our goal is to drive the 4 robots into a formation described by the following desired relative positions:

$$z_1^\star = \begin{pmatrix} 8 \\ 0\end{pmatrix},\; z_2^\star = \begin{pmatrix} -4 \\ 8\end{pmatrix},\; z_3^\star = \begin{pmatrix} 0 \\ -4\end{pmatrix}$$
$$z_4^\star = \begin{pmatrix} -4 \\ -4\end{pmatrix},\; z_5^\star = \begin{pmatrix} 4 \\ -4\end{pmatrix},\; z_5^\star = \begin{pmatrix} 4 \\ -4\end{pmatrix}. \tag{31}$$

A common incoming input disturbance of frequency $f_0 + \sigma$ with $f_0 = 6$ [rad/s] is generated by exosystem (10). Given the nominal disturbance frequency $f_0$, we have selected

$$\Phi_i^0 = \begin{pmatrix} 0 & -f_0 \\ f_0 & 0\end{pmatrix},\; \Psi_i = \begin{pmatrix} 2 & 1 \\ 0 & 1\end{pmatrix}.$$

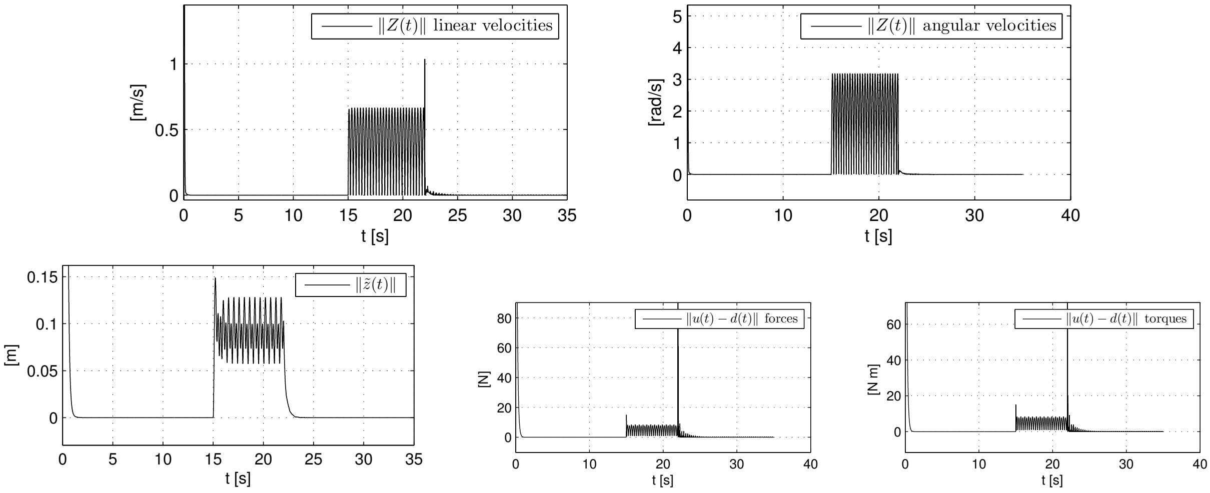


**Figure 2:** Simulation of controller (23) for multi-harmonic matched input disturbances: time evolution of variable $\|Z(t)\|$ for linear and angular velocities, relative position error $\|\tilde{z}\|$, and control input $\|u(t)-d(t)\|$ for forces and torques.

for $i = 1, \ldots, 4$.
Controller parameters are set as follows:

$$H = \text{block.diag}\ \{H_1, \ldots, H_4\}$$
$$N = \text{block.diag}\ \{N_1, \ldots, N_4\}$$
$$K_Z = 50M_0,\ \lambda_{\tilde{z}} = 2.$$

with

$$H_i = \begin{pmatrix} -80 & 10 \\ 20 & -100 \end{pmatrix},\ N_i = \begin{pmatrix} 20 & 8 \\ 10 & 15 \end{pmatrix}$$

for $i = 1, \ldots, 4$. Due to space reasons, we omit the presentation of linear parametrization (22). However, due to the selected controller parameters and due to the structure of linear parametrizations (22), we select the vector of parameter estimates as:

$$\hat{\xi} = l(\sigma) \begin{pmatrix} \delta & \sigma & \delta\sigma & \sigma^2 & \delta\sigma^2 & \sigma^3 & \delta\sigma^3 \end{pmatrix},$$

with $l(\sigma) := (203283290940 + 739810970\sigma + 25611080\sigma^2)^{-1}$, and consequently we select the controller gain $\Gamma_{\xi}$ as:

$$\Gamma_{\xi} = \text{diag}\ \{0.1, 0.3, 1, 10, 10, 100, 20\}\ .$$

Initial positions of the robots are set as:

$$q_1 = \begin{pmatrix} 0 \\ 0 \\ \pi/2 \end{pmatrix},\ q_2 = \begin{pmatrix} 5 \\ 5 \\ \pi/3 \end{pmatrix},\ q_3 = \begin{pmatrix} -1 \\ 3 \\ \pi/2 \end{pmatrix},\ q_4 = \begin{pmatrix} 4 \\ 3 \\ -\pi \end{pmatrix},$$

while $v_i = 0$ for all $i = 1,\ldots,4$. We set $\eta(0) = \zeta(0) = 0_{8\times1}$ and $\hat{\xi}(0) = 0_{7\times1}$. In order to assess the validity of the adaptive strategy in consideration, the simulation is carried out as follows. First, the controller is initialized on the nominal values of the frequency, mass and inertia of each robot $i = 1,\ldots,4$, namely $\delta = \sigma = 0$. Second, at $t = 15$s, uncertainty in the model and in the disturbing frequency is introduced, namely $\sigma = 6$ [rad/s] and $\delta = 0.2$ (according units), while no adaptation of the controller follows the parameters change, and thus the formation control and disturbance rejection objectives are not achieved anymore. Third, at $t = 22$s, the adaptive strategy is turned on, hence the parameter estimates converge to the real estimates and formation control and disturbance rejection are restored. Simulation results are depicted in Figure 2.

## 6. Conclusion

Combining their contributions of [8], [17], [16] and [7], [5], in this work we have proposed a distributed adaptive controller which achieves formation control of a network of unicycle-like robots despite the presence multi-harmonic input disturbances of unknown frequencies and despite uncertainty in the unicycle models (mass and inertia's are unknown). The internal model units of the adaptive controller asymptotically estimate the frequency of the unknowns and the model uncertainties. In order to deal with such uncertainties, certain linear characterizations and a peculiar definition of an error system have been introduced.

We believe that the current research is pertinent to all applications that deal with formation control in the presence of frequent multi-harmonic disturbances, or even in the case of unsettled flexible or spinning equipment integrated on the unicycles.

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