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Research Article

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Variable gravity in an expanding universe: Kepler's problem with cosmic relevance!

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Abstract

Most recent observations from the James Webb space telescope (JWST) do obviously show by highly resolved infrared observations of highest sensitivity that structure formation in the universe into forms of early galaxies, star forming regions and planetary systems has already taken place at cosmic times less than half a Gigayear after the Big-Bang. This is taken as a big surprise by the whole astronomic community, though it nevertheless might have been predictable from some basic theoretical considerations concerning the basic structure of the universe and its forminvariant laws. A little bit an ironic question could perhaps be asked: Would Isaac Newton when being knocked by an apple falling down on him from his nearby apple tree have invented the same gravitational law, when this would have happened one Megayear before, or one Megayear after, Newton's real historic time? In other words, the Keplerian laws, derivable with the help of Newton's law, would they reflect the changes in the cosmic times? And if yes, - how would they do it? In this article we conclude that in fact Newton's pendulum would represent a cosmic clock, unless Newton's gravitational constant varies with the scale of the universe.

Why at all should collapses of cosmic masses happen in expanding universes?

In principle it is a problem hardly understandable that cosmic matter with, - as generally assumed -, an initially perfectly uniform distribution in space may at all have undergone structure formation by means of local, gravitationally induced collapses into large local mass units? In an expanding universe the initially widely and uniformly distributed cosmic matter must only be subject to the expansion into a permanently growing cosmic space connected with permanently decreasing cosmic mass densities. The opposite can only be possible, if the velocity of a gravitationally induced local structuring process is larger than the general expansion velocity. The problem thus evidently is and must be connected with the specific form of the actual expansion dynamics of the whole universe. To say it in simple words: If the universe is expanding too fast, then this should not allow for any structure formation (see Fahr and Heyl, 2022)!

Therefore a serious study of this problem certainly is and must be based on a well founded theory of the cosmic expansion. In a static universe structure formation runs along the lines that astronomers have developed since long ago for the static space (e.g. Jeans, 1909, 1929, or later see e.g. Fahr and Willerding, 1998). Processes of structure formation of course are very much different in the expanding universe, because then structure formation definitely will depend on the specific form of the prevailing cosmic expansion (e.g. decelerated, accelerated or coasting expansion etc.).

To best explain the SN 1a luminosities Perlmutter et al. (1998), Schmidt et al.(1998), or Riess et al. (1998) have prefered an accelerated expansion of the universe connected with the action of a constant vacuum energy density (Einstein, 1917, Peebles and Ratra, 2003, Fahr, 2004, Kragh and Overduin, 2014, Fahr and Heyl, 2014), however, there are more recent attempts by Casado (2011) and Casado and Jou (2013) showing that a "coasting, non-accelerated universe" can equally well explain these supernovae luminosities. In our following considerations we shall consider first here - mainly for the simplifying mathematical reasons - the case of a "coasting expansion" (see e.g. Kolb, 1989, Gehlaut et al., 2003, Dev et al., 2001, Fahr and Heyl, 2020). This case in fact must be expected to prevail, if the universe expands under the form of thermodynamic and gravodynamic action of vacuum pressure, as shown by Fahr (2022).

If then as our working basis such a "coasting universe" may be assumed, like given in the case when $\rho_\Lambda \sim R^{-2}$ (ϱ_Λ denoting the mass density equivalent of the vacuum energy, R denoting the scale of the universe , see e.g. Fahr, 2022) and when vacuum energy in the later phases of cosmic expansion has become the dominant ingredient to the cosmic mass density $\rho_A \gg \rho_b, \rho_d, \rho_v$ (indices *b, d, v* standing for baryons, dark matter, and photons, respectively) and to the relativistic energy-momentum tensor, then one unavoidably finds:

$$\dot{R} = \frac{dR}{dt} = const \qquad \#$$

which in fact because of $\ddot{R} = 0$ means and necessarily implies: a "coasting expansion" of the universe! Then consequently a Hubble parameter must be expected that falls off with the cosmic scale *R* like:

$$H(R) = \frac{\dot{R}}{R} = H_0 \cdot (\frac{R_0}{R}) \qquad \#$$

The interesting point thus is that the Hubble parameter in course of the coasting cosmic expansion permanently decreases like $H \sim R^{-1}$, and consequently the inverse of it, the expansion time period $\tau_{ex} = 1/H(R)$, permanently grows proportional to *R*!

Creation of gravitational collapse centers during the cosmic expansion

Let us ask here under which conditions stars like our Sun can have formed over the epochs of cosmic expansion, in order to clarify whether or not solar systems over the cosmic epochs have had different parameters and consequently have looked different. And let us start assuming a specific cosmic expansion state characterized by the actual cosmic scale $R = R_0$ and the prevailing homogeneous cosmic mass density $\varrho = \varrho(R_0) = \varrho_0$ of this epoch. Let us further assume that in this cosmic phase by a locally induced gravitational collapse process a mass center with a central mass *M*, just equal to the solar mass $M_\circ$, is formed from all the matter originally uniformly distributed inside the originating vacuole of a linear dimension $D = D(R)$, i.e. we obtain the following request:

$$\frac{4\pi}{3} D(R)^3 \varrho(R) = \frac{4\pi}{3} D(R)^3 \varrho(R_0) (\frac{R_0}{R})^3 = M_\circ$$

This makes evident that the actual linear dimension $D = D(R)$ forming one solar mass unit $M = M_\circ$ in the expanding universe is given by:

$$D_\circ(R) = R \cdot [\frac{M_\circ}{\frac{4\pi}{3} R_0^3 \varrho_0}]^{1/3}$$

which tells us that the characteristic "solar mass"- collaps dimension $D_\circ(R)$ is just proportional to the cosmic scale *R*, i.e. increases linearily with the scale of the universe. Hereby it has tacitly been assumed that the universe has a Euclidean geometry with a curvature parameter of $k = 0$.

Now let us further assume for reasons given in detail by Fahr and Heyl (2022) but also in the beginning of this article, to have a coasting expansion of the universe with the property as explained that the Hubble constant is given by $H(R) = \dot{R}/R = H_0 \,.\, (R_0/R)$. Then producing a mass unit of one solar mass $M_\circ$ in the center of a sphere with radius $D(R)$ might mean that any piece of matter *m* at the outer surface of this sphere now is attracted in Newton's sense by the gravitational field of the central mass $M_\circ$, but at the same time is subject to the general coasting expansion dynamics leading to its differential Hubble drift of $v_H = D(R) \,.\, H(R)$ with respect to the mass center.

Looking now A) for the kinetic energy E_{kin} with respect to the mass center, and B) for the gravitational binding energy E_{bind} of this mass *m* to the central mass $M_\circ$ one finds:

A)
$$E_{kin} = \frac{1}{2} m \cdot (D(R) \cdot H(R))^2$$

and

B)
$$E_{bind} = \frac{GmM_\circ}{D(R)}$$

with *G* denoting Newton's gravitational constant. One could then conclude that over all the periods of the whole cosmic expansion, i.e. over all cosmic eons, the same Kepler problem (i.e. motion of a planet around the Sun) would be appearing, "if!" the ratio ϵ of kinetic over binding energy would have turned out from this consideration as a constant, i.e. if one would find ϵ = const!, instead of what one in fact numerically obtains:

$$\epsilon = \frac{\frac{1}{2} m \cdot (D(R) \cdot H(R))^2}{\frac{GmM_\circ}{D(R)}} = \frac{\frac{1}{2}(D(R)^2 \cdot H(R)^2}{\frac{GM_\circ}{D(R)}} = \frac{D(R)^3 H(R)^2}{2GM_\circ} =$$

$$\frac{R^3 \cdot [\frac{M_\circ}{\frac{4\pi}{3} R_0^3 \varrho_0}] H_0^2 \cdot (R_0/R)^2}{2GM_\circ} = \frac{R \cdot [\frac{H_0^2}{\frac{4\pi}{3} R_0 \varrho_0}]}{2G}$$

As one can see the ratio ϵ obtained above turns out to be a linear function of the scale R of the universe meaning that the Kepler problem all the time in the universe would change its character with the cosmic scale *R*, making for instance the "Kepler pendulum" (with the specific acceleration $g(R) = G \,.\, M_\circ/D^2(R)$ at a distance $D(R)$ from a solar mass $M_\circ$) something like "a cosmic clock" with a cosmic oscillation period of

$$\tau(R) = 2\pi\sqrt{L(R)/g(R)} = 2\pi\sqrt{D(R)/g(R)} = 2\pi\sqrt{D^3(R)/GM_\circ} =$$

$$2\pi\sqrt{R^3[\frac{M_\circ}{\frac{4\pi}{3} R_0^3 \varrho_0}]/GM_\circ} = 2\pi R\sqrt{\frac{R}{GM_\circ}}$$

i.e. delivering a real "linear" cosmic clock $\tau(R) \sim R$ with $G = G(R) = G_0 \,.\, (R/R_0)$.

The more interesting point, however, finally is that this above ratio ϵ would in fact be! A cosmologic constant

$$\epsilon_0 = \frac{R_0 \cdot [\frac{H_0^2}{\frac{4\pi}{3} R_0 \varrho_0}]}{2G_0}$$

if the Newton gravitational coupling coefficient G seen over the cosmic eons would not be a constant, but instead would scale with R according to the formula $G = G(R) = G_0 \,.\, (R/R_0)$! Then Newton's dream induced by the falling apple hitting him could simply not be seen as a casual event that happened just at Newton's epoch (~1660nC.), but as an event with a deep, fundamental cosmologic truth of enduring validity, - and, all the more, Kepler's laws would attain the rank of "cosmologically relevant laws"!!!

Conclusions

On the other hand at the end of this article it must perhaps appear as a highly provocative assumption that Newton's gravitational constant depends on the scale $R = R(t)$ of the universe and thereby on the cosmic time t. Actually this statement reminds one to similar Mach´ ian requests that inertial masses m of all massive elementary particles are not genuinely fixed as "nature's preselected constants", but also vary with the constellation of cosmic space hosting them, and thus with the scale $R = R(t)$ of the universe (for this idea see: Mach, 1883, Thirring, 1918, Sciama, 1953, Barbour and Pfister, 1995, Fahr, 2012).

A surprisingly nice support for Mach's idea was published by Thirring (1918) who compared the effects of centrifugal forces of the rotating Earth on the ocean water level treated in two physically identical views - A) the Earth rotating with respect to the universe at rest, and B) the Earth at rest with a solidly counter-rotating universe around. As he could demonstrate with general relativistic approaches the centrifugal forces in these two cases are related by the following equation:

$$K_A = X \circ K_B$$

where the factor X is calculated to be equal to:

$$X = \frac{4GM_U}{3c^2R}$$

with M_U and R denoting the total mass and the scale of the universe. Requiring now physical identity between the two cases A) and B) would enforce the value X to be equal to 1, with the important implication:

$$1 = \frac{4GM_U}{3c^2R}!$$

This request, however, seems really to require a Machìan kind of behaviour of the total mass of the universe M_U with the scale R, since only when M_U would be proportional to R, the above relation could be numerically fulfilled.

On the other hand it is interesting to recognize that we would not need a Machìan behaviour of the mass of the universe, if the already above discussed requirement of $G \sim R$ would be fulfilled. The question therefore is, do we prefer to rely on Mach's principle, or do we prefer to accept the linear increase of Newton's coupling constant G?

In fact, to mention the truth, the concept behind the quantity " M_U" , i.e. the mass of the universe" , is controversial and must be precisely defined as "the simultaneous mass of the universe" according to a definition already worked out by Tolman (1934), but never recognizable by any physical detector. Not including the mass equivalents of the thermal motions it has been shown by Fahr and Heyl (2006) that Tolman's expression can be evaluated into the following form:

$$M_U(t)c^2 = 4\pi\varrho_0(t)c^2 \int_0^{R_U} \frac{\exp[\lambda(r)/2)r^2dr}{\sqrt{1-(\frac{Hr}{c})^2}}$$

where the function $exp[\lambda(r)]$ is given by;

$$exp[\lambda(r)] = 1 - \frac{8\pi G}{rc^2}\varrho_0 \int_0^r \frac{x^2dx}{\sqrt{1-(Hr/c)^2}}$$

In the present, prefered view of a coasting universe with $H = H_{o\,o}(R_o/R)$ and $(HR/c)^2 \ll c^2$ one thus can evaluate the upper expression by (see Fahr and Heyl, 2006):

$$M_U(t) = \frac{c^2}{G}R_u$$

which looks as if a Machìan universe would be requested with $M_U \sim R_U$, but with the already above discussed scale-dependence of the gravitational constant by $G = G_{o\,o}(R/R_o)$ this would in fact bring back a universe with a constant total mass M_U as always expected by most cosmologists. This may perhaps give some support for the, in a first view - provocative assumption - of a scale-variable G.

References

1. Barbour, J. B., & Pfister, H. (Eds.). (1995). Mach's principle: from Newton's bucket to quantum gravity (Vol. 6). Springer Science & Business Media.
2. Bennet, C.I., Halpern, M., Hinshaw, G. et al. (2003). First year of Wilkinson Anisotropy Probe (WMAP) observations, Astrophys. Journal Supplements, 148(1), 97-117.
3. Casado, J., & Jou, D. (2013). Steady Flow cosmological model. Astrophysics and Space Science, 344(2), 513-520.
4. Casado, J. (2020). Linear expansion models vs. standard cosmologies: a critical and historical overview. Astrophysics and Space Science, 365(1), 1-14.
5. Dev, A., Sethi, M., & Lohiya, D. (2001). Linear coasting in cosmology and SNe Ia. Physics Letters B, 504(3), 207-212.
6. Einstein, A. (1917). Kosmologische Betrachtungen zur Allgemeinen Relativitätstheorie, Sitzungsberichte der K.P.Akademie der Wissenschaften, Phys.Math. Klasse, 142-152.
7. Fahr, H. J. (2004). The cosmology of empty space: How heavy is the vacuum? What we know enforces our belief. SCHRIFTENREIHE-WITTGENSTEIN GESELLSCHAFT, 33, 339-352.
8. Fahr, H. J. (2012). "Was ist Trägheit?", in "Wie gewiss ist unser Wissen?, Ed. H.Müller, Frank & Timme Verlag, Berlin, S.208-226.
9. Fahr, H. J. (2021). The thermodynamics of cosmic gases in expanding universes based on Vlasow-theoretical grounds, Advances Theoret. Computational Physics, 4(2), 129-133.
10. Fahr, H. J. (2021). The baryon distribution function in the expanding universe after the recombination era, Phys. § Astron. Internat. Journal, 5(2), 37-41.
11. Fahr, H. J. (2022). How much could gravitational binding energy act as hidden cosmic vacuum energy?, Advances Theoret. Computational Physics, 5(2), 449-457.
12. Fahr, H. J., & Willerding, E. (1998). Die Entstehung von Sonnensystemen: eine Einführung in das Problem der Planetenentstehung. Spektrum, Akad. Verlag.
13. Fahr, H. J., & Heyl, M. (2006). Concerning the instantaneous mass and the extent of an expanding universe. Astronomische Nachrichten: Astronomical Notes, 327(7), 733-736.
14. Fahr, H. J., & Heyl, M. (2014). The thermodynamics of a

gravitating vacuum, Astron. Nachr./AN , 999(88) 789-793.
15. Fahr, H. J., & Heyl, M. (2019). Stellar matter distribution with scale-invariant hierarchical structuring. Physics & Astronomy International Journal, 3(4), 146-159.
16. Fahr, H. J., & Heyl, M. (2020). A universe with a constant expansion rate, Physics & Astronomy Internat. J., 4(4), 156-163.
17. Fahr, H. J., & Heyl, M. (2021). Structure formation after the era of cosmic recombination, Advances Theoret. & Computational Physics, 4(3), 253-258.
18. Fahr, H. J., & Heyl, M. (2022). Evolution of cosmic structures in the expanding universe: Could not one have known it all before?, Advances Theoret.& Computational Physics, 5(3), 524-528.
19. Gehlaut, S., Kumar, P., & Lohiya, D. (2003). A Concordant" Freely Coasting Cosmology". arXiv preprint astro-ph/0306448.
20. Jeans, J. (1929). Phil. Transactions Royal Society, 199A, 42, 1902, or: Astronomy and Cosmogony, Cambridge University Press (Reprinted by Dover Publications, INC, New York, 1940).
21. Kolb, E. W. (1989). A coasting cosmology. The Astrophysical Journal, 344, 543-550.
22. Mach, E. (1883). "Die Mechanik in ihrer Entwicklung; historisch, kritisch dargestellt", Wien, 1883
23. Perlmutter, S. (2003). Supernovae, dark energy, and the accelerating universe. Physics today, 56(4), 53-62.
24. Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., ... & Supernova Cosmology Project. (1999). Measurements of Ω and Λ from 42 high-redshift supernovae. The Astrophysical Journal, 517(2), 565.
25. Kragh, H.S., and Overduin, J.M. (2014). The weight of the vacuum, Springer Briefs in Physics.
26. Peebles, P. J. E., & Ratra, B. (2003). The cosmological constant and dark energy. Reviews of modern physics, 75(2), 559.
27. Riess, A. G., Filippenko, A. V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P. M., ... & Tonry, J. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. The Astronomical Journal, 116(3), 1009.
28. Schmidt, B. P., Suntzeff, N. B., Phillips, M. M., Schommer, R. A., Clocchiatti, A., Kirshner, R. P., ... & Ciardullo, R. (1998). The high-Z supernova search: measuring cosmic deceleration and global curvature of the universe using type Ia supernovae. The Astrophysical Journal, 507(1), 46.
29. Sciama, D. W. (1953). On the origin of inertia, Mon.Not. Roy. Astron. Soc. 113, 34-42.
30. Thirring, H. (1918). Über die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie. Physikalische Zeitschrift, 19, 33.
31. Tolman, R. C. (1987). Relativity, thermodynamics, and cosmology. Courier Corporation.