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Research Article

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The Maxwell-Cassano Equations of an Electromagnetic-nuclear Field Yields the Fermion & Hadron Architecture

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Abstract

The Helmholtzian operator and factorization, via the Maxwell-Cassano equations yields a fermion architecture table equivalent to that of the standard model and lead to linear transformation groups of the mesons and baryons, respectively; plus a straightforward elementary description of quark colour based on integral indices: 1,0,1, rather than the subjective, correlative explanation using: {***R,G,B;Y***} *indexes.*

keywords: Helmholtzian, Klein-Gordon equation, electromagnetism, preon, elementary particles, standard model, elementary particle, particle interactions, fermions, leptons, quarks, hadrons, mesons, baryons, Pauli matrices, Gell-Mann matrices, Hermitian matrices, Lie algebra, su(2), Yukawa potential

Introduction

Using the principles of the analysis of a linear function of a linear variable [1], constuctive algebras developed using the weighted matrix product [2] leads to the d'Alembertian operator and it's factorization [3], and a space with all smooth functions satisfying Maxwell's equations [4] [5] [6] [7]. This leads to the Helmholtzian operator and factorization[8], and a space in which all smooth functions satisfy the Maxwell-Cassano equations[9] (which generalizes both Maxwell's equations and the Dirac equation [10] [11] [12]) - a linearization of the Klein-Gordon equations [13] [14] [12]. These insights lead to a fermion architecture providing a firm mathematical foundation of the hadrons (mesons [15] and baryons[16])

Analysis Details and Results

Merely a cursory look demonstrates that the Helmholtzian operator and factorization is a generalization of the d'Alembertian operator and it's factorization [17].

Recalling the Helmholtzian operator matrix product from [8]:

$$\mathbf{J} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} (\Box - |m|^2)f^1 \\ (\Box - |m|^2)f^2 \\ (\Box - |m|^2)f^3 \\ (\Box - |m|^2)f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} =$$

$$= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} \quad (1)$$

where:

$$D_i^{+} = (\partial_i + m_i) \;,\; D_i^{-} = (\partial_i - m_i) \;,\; \partial_i = \frac{\partial}{\partial x^i} \;,\; m_i \text{ constants} \quad (2)$$

$$D_i = \begin{pmatrix} D_i^{+} & 0 \\ 0 & D_i^{-} \end{pmatrix} , D_i^{\Updownarrow} = \begin{pmatrix} D_i^{-} & 0 \\ 0 & D_i^{+} \end{pmatrix} , D_i^{\Leftrightarrow} = \begin{pmatrix} 0 & D_i^{-} \\ D_i^{+} & 0 \end{pmatrix} , D_i^{\Leftrightarrow\Updownarrow} = \begin{pmatrix} 0 & D_i^{+} \\ D_i^{-} & 0 \end{pmatrix} \quad (3)$$

Similarly, mass-generalized electric and magnetic potentials for the Helmholtzian operator factorization :

$$\mathbf{E} = \mathbf{w}^{4;1}\left(-D_0^{\Updownarrow}f^1 - D_1 f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Updownarrow}f^2 - D_2 f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Updownarrow}f^3 - D_3 f^0\right)$$

$$\mathbf{B} = \mathbf{w}^{4;1}(D_2 f^3 - D_3 f^2) + \mathbf{w}^{4;2}(-D_1 f^3 + D_3 f^1) + \mathbf{w}^{4;3}(D_1 f^2 - D_2 f^1)$$

$$\mathbf{E}_{\Updownarrow} = \mathbf{w}^{4;1}\left(-D_0^{\Leftrightarrow\Updownarrow}f^1 - D_1^{\Leftrightarrow}f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Leftrightarrow\Updownarrow}f^2 - D_2^{\Leftrightarrow}f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Leftrightarrow\Updownarrow}f^3 - D_3^{\Leftrightarrow}f^0\right)$$

$$\mathbf{B}_{\Updownarrow} = \mathbf{w}^{4;1}(D_2^{\Leftrightarrow}f^3 - D_3^{\Leftrightarrow}f^2) + \mathbf{w}^{4;2}(-D_1^{\Leftrightarrow}f^3 + D_3^{\Leftrightarrow}f^1) + \mathbf{w}^{4;3}(D_1^{\Leftrightarrow}f^2 - D_2^{\Leftrightarrow}f^1) \quad (4)$$

Where:

$\mathbf{f} \equiv \mathbf{w}^{4;1}f^\mu$	$f^\mu \equiv \begin{pmatrix} f_+^\mu \\ f_-^\mu \end{pmatrix}$

$$\mathbf{J} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} = \begin{pmatrix} (\Box - |m|^2)f^1 \\ (\Box - |m|^2)f^2 \\ (\Box - |m|^2)f^3 \\ (\Box - |m|^2)f^0 \end{pmatrix} =$$

$$
= \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} B_{\Updownarrow}^1 + E^1 \\ B_{\Updownarrow}^2 + E^2 \\ B_{\Updownarrow}^3 + E^3 \\ -\overset{m*}{\nabla}_{\Updownarrow} \cdot \mathbf{f} \end{pmatrix} = \tag{5a}
$$

$$
= \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} B_{\Updownarrow}^1 - E^1 \\ B_{\Updownarrow}^2 - E^2 \\ B_{\Updownarrow}^3 - E^3 \\ \overset{m}{\nabla}_{\Updownarrow} \cdot \mathbf{f}^* \end{pmatrix} \tag{5b}
$$

These mass-generalized Maxwell's equations may be simpy written:

$\mathbf{0} = (\partial_0 - m_0)\vec{\mathbf{B}} + (\vec{\nabla} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}}$;	$0 = (\vec{\nabla} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}}$	; Homgeneous
$\vec{\mathbf{J}} = (\partial_0 + m_0)\vec{\mathbf{E}} - (\vec{\nabla} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}}$;	$\rho = (\vec{\nabla} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{E}}$	; Inhomgeneous

as the **Maxwell-Cassano equations** of an electromagnetic-nuclear field[9].

It is easy to demonstrate that in free space, the thus defined E and B (generalizations of the electric and magnetic field strengths) also satisfy the Klein-Gordon equations, so have a particle-nature. (Also, the potential of the time-independent Klein-Gordon equations is the Yukawa potential[18].)

Identifying a particle-nature member **R** as either an **E** or a **B** , and $\overline{\mathbf{R}}_+$ as either an $\mathbf{E}_+$ or a $\mathbf{B}_+$, then a notation consistent with common usage would denote it's particle-nature anti-member $\overline{\mathbf{R}}_+$ as the corresponding $\mathbf{E}_+$ or a $\mathbf{B}_+$ (and correspondingly for $\overline{\mathbf{R}}$_, $\overline{\mathbf{E}}$, & $\overline{\mathbf{B}}$_). And, of course, the particle-nature anti-member components correspond in the same way. Each of these members satisfies the Klein-Gordon equation, but only really do so as three-vectors with three components or triplets. And, each bag of triplets must be triplets or triplets of triplets or triplets of triplets of triplets, and so on (i.e.: 3^n of triplets).

The simplest , and thus, most fundamental members are triplets.
The next most fundamental is triplets of triplets.
These will be considered, here.

Denoting a triplet of triplets by: $\mathbf{S_R} \equiv \mathbf{R}_1,\mathbf{R}_2,\mathbf{R}_3$) $\mathbf{R}_{1+},\mathbf{R}_{2+},\mathbf{R}_{3+}$) + ($\mathbf{R}_{1-},\mathbf{R}_{2-},\mathbf{R}_{3-}$), is a 3 X 3 matrix.

Then we can write: $\mathbf{S_E} \equiv \mathbf{E}_1,\mathbf{E}_2,\mathbf{E}_3$, and $S_B \equiv (\mathbf{B}_1,\mathbf{B}_2,\mathbf{B}_3)$.

The components of each vector written vertically:

$$
\mathbf{S_R} = \left(\begin{pmatrix} R^1 \\ R^2 \\ R^3 \end{pmatrix}_1, \begin{pmatrix} R^1 \\ R^2 \\ R^3 \end{pmatrix}_2, \begin{pmatrix} R^1 \\ R^2 \\ R^3 \end{pmatrix}_3 \right) = \begin{pmatrix} R_1^1 & R_2^1 & R_3^1 \\ R_1^2 & R_2^2 & R_3^2 \\ R_1^3 & R_2^3 & R_3^3 \end{pmatrix} \tag{6}
$$

Define the first fundamental objects as follows:

$Li \equiv \mathbf{S}_{\mathbf{E}i}$, $\Lambda_i \equiv \mathbf{S}_{\mathbf{B}i}$

[where: $\mathbf{S}_{\mathbf{R}1} \equiv (\mathbf{R}_1,\mathbf{0},\mathbf{0})$, $\mathbf{S}_{\mathbf{R}2} \equiv (\mathbf{0},\mathbf{R}_2,\mathbf{0})$, $\mathbf{S}_{\mathbf{R}3} \equiv (\mathbf{0},\mathbf{0},\mathbf{R}_3)$]

and define in-line notation: $\mathbf{S}_{\mathbf{R}i} \equiv (\mathbf{R}^1, \mathbf{R}^2, \mathbf{R}^3)$

So, there are 3 pair of L, Λ :

$$
L_1 \equiv (\mathbf{E}_1,\mathbf{0},\mathbf{0}) = \left(\begin{pmatrix} E^1 \\ E^2 \\ E^3 \end{pmatrix}_1, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_2, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_3 \right) = \begin{pmatrix} E_1^1 & 0 & 0 \\ E_1^2 & 0 & 0 \\ E_1^3 & 0 & 0 \end{pmatrix} \equiv
$$

$$
L_2 \equiv (\mathbf{0},\mathbf{E}_2,\mathbf{0}) = \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_1, \begin{pmatrix} E^1 \\ E^2 \\ E^3 \end{pmatrix}_2, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_3 \right) = \begin{pmatrix} 0 & E_2^1 & 0 \\ 0 & E_2^2 & 0 \\ 0 & E_2^3 & 0 \end{pmatrix} \equiv
$$

$$
L_3 \equiv (\mathbf{0},\mathbf{0},\mathbf{E}_3) = \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_1, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_2, \begin{pmatrix} E^1 \\ E^2 \\ E^3 \end{pmatrix}_3 \right) = \begin{pmatrix} 0 & 0 & E_3^1 \\ 0 & 0 & E_3^2 \\ 0 & 0 & E_3^3 \end{pmatrix} \equiv
$$

$$
\Lambda_1 \equiv (\mathbf{B}_1,\mathbf{0},\mathbf{0}) = \left(\begin{pmatrix} B^1 \\ B^2 \\ B^3 \end{pmatrix}_1, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_2, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_3 \right) = \begin{pmatrix} B_1^1 & 0 & 0 \\ B_1^2 & 0 & 0 \\ B_1^3 & 0 & 0 \end{pmatrix} \equiv
$$

$$
\Lambda_2 \equiv (\mathbf{0},\mathbf{B}_2,\mathbf{0}) = \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_1, \begin{pmatrix} B^1 \\ B^2 \\ B^3 \end{pmatrix}_2, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_3 \right) = \begin{pmatrix} 0 & B_2^1 & 0 \\ 0 & B_2^2 & 0 \\ 0 & B_2^3 & 0 \end{pmatrix} \equiv
$$

$$
\Lambda_3 \equiv (\mathbf{0},\mathbf{0},\mathbf{B}_3) = \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_1, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_2, \begin{pmatrix} B^1 \\ B^2 \\ B^3 \end{pmatrix}_3 \right) = \begin{pmatrix} 0 & 0 & B_3^1 \\ 0 & 0 & B_3^2 \\ 0 & 0 & B_3^3 \end{pmatrix} \equiv
$$

And, let:

$$
\eta_j(R_k^h) \equiv \begin{cases} R_k^h & , \ j \neq 0 \\ E_k^h & , \ j = 0 \ , \ \mathbf{R} = \mathbf{B} \\ B_k^h & , \ j = 0 \ , \ \mathbf{R} = \mathbf{E} \end{cases} \quad , \ \sigma_j(\mathbf{R}_k) \equiv \begin{pmatrix} \eta_{j-1}(R_k^1) \\ \eta_{j-2}(R_k^2) \\ \eta_{j-3}(R_k^3) \end{pmatrix}
$$

$$
\widehat{\mathbf{S}_{\mathbf{R}_{ij}}} \equiv (\delta_1^i \sigma_j(\mathbf{R}_1), \delta_2^i \sigma_j(\mathbf{R}_2), \delta_3^i \sigma_j(\mathbf{R}_3))
$$

Corresponding to these fundamental objects, define these second order objects as 3 pair of triples, as follows:

$$
Q_{jh}^l = \widehat{\mathbf{S}_{\mathbf{E}_{hj}}} \quad , \quad Q_{jh}^{\Lambda} = \widehat{\mathbf{S}_{\mathbf{B}_{hj}}} \tag{11}
$$

with in-line notation: $\widehat{\mathbf{S}_{\mathbf{R}_{hj}}} \equiv (\eta_{j-1}(R^1), \eta_{j-2}(R^2), \eta_{j-3}(R^3))_h$ (11a)

$$
Q_{11}^L = (B^1,E^2,E^3)_1 \ , \ Q_{11}^{\Lambda} = (E^1,B^2,B^3)_1 \tag{12.1a}
$$
$$
Q_{21}^L = (E^1,B^2,E^3)_1 \ , \ Q_{21}^{\Lambda} = (B^1,E^2,B^3)_1 \tag{12.1b}
$$
$$
Q_{31}^L = (E^1,E^2,B^3)_1 \ , \ Q_{31}^{\Lambda} = (B^1,B^2,E^3)_1 \tag{12.1c}
$$

(which are, of course, merely a swapping of one component between the pair)
(Note: any number of swappings between the pair results in a member of these 9 matrices)
(- i.e. it is a group transformation, so it is sufficient to consider a single swapping)
(Note also that including the originals, there are 8 members)
The other two pair of triples are:

$$
Q_{12}^L = (B^1,E^2,E^3)_2 \ , \ Q_{12}^{\Lambda} = (E^1,B^2,B^3)_2 \tag{12.2a}
$$
$$
Q_{22}^L = (E^1,B^2,E^3)_2 \ , \ Q_{22}^{\Lambda} = (B^1,E^2,B^3)_2 \tag{12.2b}
$$
$$
Q_{32}^L = (E^1,E^2,B^3)_2 \ , \ Q_{32}^{\Lambda} = (B^1,B^2,E^3)_2 \tag{12.2c}
$$

$$
Q_{13}^L = (B^1,E^2,E^3)_3 \ , \ Q_{13}^{\Lambda} = (E^1,B^2,B^3)_3 \tag{12.3a}
$$
$$
Q_{23}^L = (E^1,B^2,E^3)_3 \ , \ Q_{23}^{\Lambda} = (B^1,E^2,B^3)_3 \tag{12.3b}
$$
$$
Q_{33}^L = (E^1,E^2,B^3)_3 \ , \ Q_{33}^{\Lambda} = (B^1,B^2,E^3)_3 \tag{12.3c}
$$

The following assignments/definitions:

$$e^+ \equiv \overline{L_1} \quad , \quad \nu_e \equiv \Lambda_1 \qquad (14.1a)$$
$$\mu^+ \equiv \overline{L_2} \quad , \quad \nu_\mu \equiv \Lambda_2 \qquad (14.1b)$$
$$\tau^+ \equiv \overline{L_3} \quad , \quad \nu_\tau \equiv \Lambda_3 \qquad (14.1c)$$

correspond to the leptons.
And the following:

$$u_R \equiv Q^l_{11} \quad , \quad d_R \equiv \overline{Q^\Lambda_{11}} \qquad (14.2a)$$
$$u_G \equiv Q^l_{21} \quad , \quad d_G \equiv \overline{Q^\Lambda_{21}} \qquad (14.2b)$$
$$u_B \equiv Q^l_{31} \quad , \quad d_B \equiv \overline{Q^\Lambda_{31}} \qquad (14.2c)$$

$$c_R \equiv Q^l_{12} \quad , \quad s_R \equiv \overline{Q^\Lambda_{12}} \qquad (14.3a)$$
$$c_G \equiv Q^l_{22} \quad , \quad s_G \equiv \overline{Q^\Lambda_{22}} \qquad (14.3b)$$
$$c_B \equiv Q^l_{32} \quad , \quad s_B \equiv \overline{Q^\Lambda_{32}} \qquad (14.3c)$$

$$t_R \equiv Q^l_{13} \quad , \quad b_R \equiv \overline{Q^\Lambda_{13}} \qquad (14.4a)$$
$$t_G \equiv Q^l_{23} \quad , \quad b_G \equiv \overline{Q^\Lambda_{23}} \qquad (14.4b)$$
$$t_B \equiv Q^l_{33} \quad , \quad b_B \equiv \overline{Q^\Lambda_{33}} \qquad (14.4c)$$

correspond to all colors and flavors and generations of the quarks. From this point on, represent the generations of the most fundamental objects by:

$$e(i) \equiv \overline{L_i} = \overline{(E^1,E^2,E^3)_i}$$
$$\nu(i) \equiv \Lambda_i = (B^1,B^2,B^3)_i$$

$$, \; u_j(i) \equiv Q^L_{ji} = (\eta_{j-1}(E^1),\eta_{j-2}(E^2),\eta_{j-3}(E^3))_i \qquad (14.5a)$$
$$, \; d_j(i) \equiv \overline{Q^\Lambda_{ji}} = \overline{(\eta_{j-1}(B^1),\eta_{j-2}(B^1),\eta_{j-3}(B^1))_i} \qquad (14.5b)$$

(i denoting column/generation, j denoting row/color)
So, in particular:

$e = e(1) = \overline{(E^1, E^2, E^3)}_1$	$\mu = e(2) = (E^1, E^2, E^3)_2$	$\tau = e(3) = (E^1, E^2, E^3)_3$
$\nu_e = \nu(1) = (B^1, B^2, B^3)_1$	$\nu_\mu = \nu(2)\ (B^1,B^2,B^3)_2$	$\nu_t = \nu(3) = (B^1, B^2, B^3)_3$
$u_R = u_1(1) = B^1, E^2, E^3)_1$	$c_R\ u_1(2) = (B^1,E^2,E^3)_2$	$t_R = u_1(3) = (B^1, E^2, E^3)_3$
$u_G\ u_2(1) = (E^1, B^2, E^3)_1$	$c_G = u_2(2) = (E^1, B^2, E^3)_2$	$t_G = u_2(3) = (E^1, B^2, E^3)_3$
$u_B = u_3(1) = (E^1,E^2,B^3)_1$	$c_B = u_3(2) = (E^1, E^2, B^3)_2$	$t_B = u_3(3) = (E^1,E^2,B^3)_3$
$d_R = d_1(1) = \overline{(E^1,B^2,B^3)}_1$	$s_R = d_1(2) = \overline{E^1, B^2, B^3)}_2$	$b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$
$d_G = d_2(1) = \overline{B^1,E^2,B^3)}_1$	$sG = d_2(2) = \overline{(B^1, E^2, B^3)}_2$	$b_G = d_2(3) = \overline{B^1, E^2, B^3)}_3$
$d_B = d_3(1) = \overline{(B^1, B^2,E^3)}_1$	$s_B = d_3(2) = \overline{B^1, B^2, E^3)}_2$	$b_B = d_3(3) = \overline{(B^1,B^2,E^3)}_3$

(15)

Examples of hadrons (second order compositions):
mesons:

$$u_R : \overline{d_R} = (B^1,E^2,E^3)_1 : (E^1,B^2,B^3)_1 = \pi^+ .$$
$$d_R : \overline{u_R} = (E^1,B^2,B^3)_1 : \overline{(B^1,E^2,E^3)_1} = \pi^- .$$
$$c_R : \overline{c_R} = (B^1,E^2,E^3)_2 : \overline{(B^1,E^2,E^3)_2} = \eta_c .$$
$$u_R : \overline{s_R} = (B^1,E^2,E^3)_1 : (E^1,B^2,B^3)_2 = K^+ .$$
$$d_R : \overline{s_R} = \overline{(E^1,B^2,B^3)_1} : (E^1,B^2,B^3)_2 = K^0 .$$
$$c_R : \overline{d_R} = (B^1,E^2,E^3)_2 : (E^1,B^2,B^3)_1 = D^+ .$$
$$u_R : \overline{b_R} = (B^1,E^2,E^3)_1 : (E^1,B^2,B^3)_3 = B^+ .$$
$$d_R : \overline{b_R} = \overline{(E^1,B^2,B^3)_1} : (E^1,B^2,B^3)_3 = B^0 .$$

These aren't all the mesons, but illustrates that they are of two families:
1) all the are matched: R^h_j with $\eta_0(R^h_m)$. (the charged ones)
2) all the are matched: R^h_j with $\overline{R^h_m}$. (the uncharged ones)

baryons:
a baryon is a quark triplet each quark of a different color.

$$u_R : u_G : d_B = (B^1,E^2,E^3)_1 : (B^1,E^2,E^3)_2 : \overline{(E^1,B^2,B^3)_3} = p^+ .$$
$$u_R : u_B : d_G = (B^1,E^2,E^3)_1 : (E^1,B^2,E^3)_3 : \overline{(E^1,B^2,B^3)_2} = p^+ .$$
$$d_R : u_G : d_B = \overline{(E^1,B^2,B^3)_1} : (B^1,E^2,E^3)_2 : \overline{(E^1,B^2,B^3)_3} = n^0 .$$
$$d_R : u_B : d_G = \overline{(E^1,B^2,B^3)_1} : (E^1,E^2,B^3)_3 : \overline{(E^1,B^2,B^3)_2} = n^0 .$$

As an $\mathbf{S_R}$ matrix, the proton and neutron incarnations are all the same. except for one swapped pair of elements.

Just as coordinates may be used to describe phenomena, chosen to facilitate analysis (whether rectangular cartesian, spherical, cylindrical, paraboloidal, ellipsoidal, etc.); so, too may a vector basis be chosen to consider generators of the vector space as a group and it's structure.

U(1) is the multiplicative group of all complex numbers with absolute value 1; that is, the unit circle in the complex plane [19]. The unitary group U(n) is a real Lie group of dimension n^2. (complex n^2 , real $2n^2$) [20].

The unitary group U(n) is endowed with the relative topology as a subset of M(n, C), the set of all n x n complex matrices, which is itself homeomorphic to a $2n^2$-dimensional Euclidean space [20].

The dimension of the group SU(n) is n^2 -1 [21].

The orthogonal group in dimension n, denoted O(n), is the group of distance-preserving transformations of a Euclidean space of dimension n that preserve a fixed point, where the group operation is given by composing transformations. Equivalently, it is the group of n x n orthogonal matrices, where the group operation is given by matrix multiplication [22].

(an orthogonal matrix is a real matrix whose inverse equals its transpose) [23]

Over the field ℝ of real numbers, the orthogonal group O(n, ℝ) and the special orthogonal group SO(n, ℝ) are often simply denoted by O(n) and SO(n) if no confusion is possible. SO(n) forms the real compact Lie groups of dimension $n(n - 1)/2$ [22].

⇒ SO(2) is of dimension 1 , SO(3) is of dimension 3 , SO(4) is of dimension 6 , ...

All the possible quark doublets are given by:

$u_R(h) : u_R(j)$	$u_G(h) : u_G(j)$	$u_B(h) : u_B(j)$
$d_R(h) : d_R(j)$	$d_G(h) : d_G(j)$	$d_B(h) : d_B(j)$

(16)

(Notice that casual appearance suggests a 6-dimensional double-cover)

A qXj : $(R^1, R^2, R^3)_j$ is an n-tuple; but consider a transformation/ mapping:

$$(R^1,R^2,R^3)_j \mapsto \sum_{h=1}^{3}\left(E^h\delta_{R^h}^{E^h} + iB^h\delta_{R^h}^{B^h}\right)_j \;\&\; K_j \equiv \sum_{h=1}^{3}\left(E^h + iB^h\right)_j$$

$$\Rightarrow \eta_0(q_Xj) \equiv K_j - \sum_{h=1}^{3}\left(E^h\delta_{R^h}^{E^h} + iB^h\delta_{R^h}^{B^h}\right)_j$$

so: :

$$\overline{(R^1,R^2,R^3)_j} \mapsto \sum_{h=1}^{3}\left(E^h\delta_{R^h}^{E^h} - iB^h\delta_{R^h}^{B^h}\right)_j \text{ and: } \overline{\eta_0(q_Xj)} \equiv$$

$$\overline{K_j - \sum_{h=1}^{3}\left(E^h\delta_{R^h}^{E^h} + iB^h\delta_{R^h}^{B^h}\right)_j}$$

$$K_0 \equiv (E^1+B^1, E^2+B^2, E^3+B^3) \Rightarrow K_0 - u_X = \overline{d_X} \Leftrightarrow \overline{K_0 - u_X}$$

$$= d_X \Leftrightarrow K_0 - \overline{d_X} = u_X$$

In other words, the n-tuple $(R^1, R^2, R^3)_j$: qXj represent the coordinates of a fermion complex four-vector space.

Hadrons, mesons & baryons are the major objects in this fermion complex four-vector space, the mathematics of which follows.

Let: $\rho,\sigma \in \{u,d\}$ & $\Pi,\Phi \in \{\mathbf{R},\mathbf{G},\mathbf{B}\}$ & $m,n,r,s \in \{1,2,3\}$

then each member of this meson vector space

may be written as a 2 x 1 column vector: $\begin{pmatrix}\rho_\Pi \\ \overline{\rho}_\Pi\end{pmatrix}$

So:

$$T\begin{pmatrix}\rho_\Pi \\ \overline{\rho}_\Pi\end{pmatrix} = \begin{pmatrix}\sigma_\Phi \\ \overline{\sigma}_\Phi\end{pmatrix}$$

is a operation transforming one meson into another meson.
If it is linear, T is a 2 x 2 matrix.
So, if the field of the vector space and transformation is ℂ then:

$$\begin{pmatrix} w \\ \overline{w}\end{pmatrix} = \begin{pmatrix} a & b \\ c & d\end{pmatrix}\begin{pmatrix} v \\ \overline{v}\end{pmatrix} = \begin{pmatrix} a_R + ia_I & b_R + ib_I \\ c_R + ic_I & d_R + id_I\end{pmatrix}\begin{pmatrix} v \\ \overline{v}\end{pmatrix}$$
$$= \begin{pmatrix}(a_R + ia_I)v + (b_R + ib_I)\overline{v} \\ (c_R + ic_I)v + (d_R + id_I)\overline{v}\end{pmatrix}$$

And, if the meson vectors may be represented by complex variables: (with the property that the anti-object is the complex conjugate of the corresponding object)

$$\begin{pmatrix} w_R + iw_I \\ w_R - iw_I\end{pmatrix} = \begin{pmatrix}(a_R + ia_I)(v_R + iv_I) + (b_R + ib_I)(v_R - iv_I) \\ (c_R + ic_I)(v_R + iv_I) + (d_R + id_I)(v_R - iv_I)\end{pmatrix}$$

$$= \begin{pmatrix}(a_R + ia_I)v_R + (a_R + ia_I)iv_I + (b_R + ib_I)v_R - (b_R + ib_I)iv_I \\ (c_R + ic_I)v_R + (c_R + ic_I)iv_I + (d_R + id_I)v_R - (d_R + id_I)iv_I\end{pmatrix}$$

$$= \begin{pmatrix}a_Rv_R + ia_Iv_R + a_Riv_I + ia_Iiv_I + b_Rv_R + ib_Iv_R - b_Riv_I - ib_Iiv_I \\ c_Rv_R + ic_Iv_R + c_Riv_I + ic_Iiv_I + d_Rv_R + id_Iv_R - d_Riv_I - id_Iiv_I\end{pmatrix}$$

$$= \begin{pmatrix}a_Rv_R + ia_Iv_R + ia_Rv_I - a_Iv_I + b_Rv_R + ib_Iv_R - ib_Rv_I + b_Iv_I \\ c_Rv_R + ic_Iv_R + ic_Rv_I - c_Iv_I + d_Rv_R + id_Iv_R - id_Rv_I + d_Iv_I\end{pmatrix}$$

$$= \begin{pmatrix}a_Rv_R - a_Iv_I + b_Rv_R + b_Iv_I + ia_Iv_R + ia_Rv_I + ib_Iv_R - ib_Rv_I \\ c_Rv_R - c_Iv_I + d_Rv_R + d_Iv_I + ic_Iv_R + ic_Rv_I + id_Iv_R - id_Rv_I\end{pmatrix}$$

$$= \begin{pmatrix}[a_Rv_R - a_Iv_I + b_Rv_R + b_Iv_I] + i[a_Iv_R + a_Rv_I + b_Iv_R - b_Rv_I] \\ [c_Rv_R - c_Iv_I + d_Rv_R + d_Iv_I] + i[c_Iv_R + c_Rv_I + d_Iv_R - d_Rv_I]\end{pmatrix}$$

$$= \begin{pmatrix}[a_Rv_R + b_Rv_R - a_Iv_I + b_Iv_I] + i[a_Iv_R + b_Iv_R + a_Rv_I - b_Rv_I] \\ [c_Rv_R + d_Rv_R - c_Iv_I + d_Iv_I] + i[c_Iv_R + d_Iv_R + c_Rv_I - d_Rv_I]\end{pmatrix}$$

$$= \begin{pmatrix}[(a_R + b_R)v_R + (-a_I + b_I)v_I] + i[(a_I + b_I)v_R + (a_R - b_R)v_I] \\ [(c_R + d_R)v_R + (-c_I + d_I)v_I] + i[(c_I + d_I)v_R + (c_R - d_R)v_I]\end{pmatrix}$$

$$\Rightarrow \left\{\begin{array}{l|l} w_R = (a_R + b_R)v_R + (-a_I + b_I)v_I & w_I = (a_I + b_I)v_R + (a_R - b_R)v_I \\ w_R = (c_R + d_R)v_R + (-c_I + d_I)v_I & -w_I = (c_I + d_I)v_R + (c_R - d_R)v_I \end{array}\right.$$

for arbitrary v_R, v_I :

$$\Rightarrow \left\{\begin{array}{l|l} c_R + d_R = a_R + b_R & -c_I - d_I = a_I + b_I \\ -a_I + b_I = -c_I + d_I & a_R - b_R = -c_R + d_R \end{array}\right.$$

$$\Rightarrow \left\{\begin{array}{l|l} a_I + b_I = -c_I - d_I & a_R + b_R = c_R + d_R \\ -a_I + b_I = -c_I + d_I & a_R - b_R = -c_R + d_R \end{array}\right.$$

$$\Rightarrow \left\{\begin{array}{l|l} 2b_I = -2c_I & 2a_R = 2d_R \\ 2a_I = -2d_I & 2b_R = 2c_R \end{array}\right. \Rightarrow \left\{\begin{array}{l|l} c_I = -b_I & d_R = a_R \\ d_I = -a_I & c_R = b_R \end{array}\right.$$

$$\Rightarrow T = \begin{pmatrix} a & b \\ c & d\end{pmatrix} = \begin{pmatrix} a_R + ia_I & b_R + ib_I \\ b_R - ib_I & a_R - ia_I\end{pmatrix} = \begin{pmatrix} a & b \\ b^* & a^*\end{pmatrix} \quad (17)$$

$$= \begin{pmatrix} a_R & b_R \\ b_R & a_R\end{pmatrix} + \begin{pmatrix} ia_I & ib_I \\ -ib_I & -ia_I\end{pmatrix}$$

$$= a_R\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix} + b_R\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix} + ia_I\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix} + ib_I\begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix}$$

$$= a_R\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix} + b_R\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix} + ia_I\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix} - b_I\begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix} \quad (18)$$

for: $(w,v) = (\sigma_\Phi, \rho_\Pi)$:

$$\begin{pmatrix}\sigma_\Phi \\ \overline{\sigma_\Phi}\end{pmatrix} = \begin{pmatrix} a & b \\ b^* & a^*\end{pmatrix}\begin{pmatrix}\rho_\Pi \\ \overline{\rho_\Pi}\end{pmatrix}$$

Let: $a_R = 0$, this is the root of the basis for the 6-dimensionl transformation: the Pauli matrices

The complexified Lie algebra su(2) + $i$su(2) = sl(2;ℂ). [24]

$$\mathrm{SU}(2) = \left\{\begin{pmatrix}\alpha & -\overline{\beta} \\ \beta & \overline{\alpha}\end{pmatrix}; \forall\alpha,\beta \in \mathbb{C} : |\alpha|^2 + |\beta|^2 = 1\right\}$$

i.e.: the Lie algebra su(2) consists of 2 × 2 skew-Hermitian matrices with trace zero:

$$\mathrm{su}(2) = \left\{\begin{pmatrix} ia & -\overline{z} \\ z & -ia\end{pmatrix}; \forall a \in \mathbb{R}, z \in \mathbb{C} : |\alpha|^2 + |\beta|^2 = 1\right\}$$

using the above basis modified: $i\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}, i\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}, -i\begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}$

The group:

$$\mathrm{SU}(1,1) = \left\{\begin{pmatrix} u & v \\ v^* & u^*\end{pmatrix}; \forall u,v \in \mathbb{C} : uu^* - uv^* = 1\right\}$$

is isomorphic to SO(2,1) and SL(2,ℝ). [24]
(The Lie algebra su(2) may be constructed from this group similarly to the above.)
(which is just a convenient representation system like a convenient coordinate system)
Now, instead of writing the meson as a vector, if it is written as a 2x 2 -square matrix, as:

$$\begin{pmatrix} u \\ \overline{w} \end{pmatrix} = \begin{pmatrix} u+iv \\ w-iz \end{pmatrix} \Rightarrow \begin{pmatrix} u+iv & 0 \\ 0 & w-iz \end{pmatrix}$$

$$T : \begin{pmatrix} u \\ \overline{w} \end{pmatrix} \mapsto \begin{pmatrix} u+iv & 0 \\ 0 & w-iz \end{pmatrix}$$

then under ordinary matrix multilication:

$$\begin{pmatrix} u_1+iv_1 & 0 \\ 0 & w_1-iz_1 \end{pmatrix}\begin{pmatrix} u_2+iv_2 & 0 \\ 0 & w_2-iz_2 \end{pmatrix} =$$

$$\begin{pmatrix} (u_1u_2 - v_1v_2) + i(u_1v_2 + v_1u_2) & 0 \\ 0 & (w_1w_2 - z_1z_2) - i(w_1z_2 + z_1w_2) \end{pmatrix}$$

()

spans; is even commutative; and for such non-singular matrices forms a group, and:

$$\begin{pmatrix} u_1+iv_1 & 0 \\ 0 & w_1-iz_1 \end{pmatrix}\begin{pmatrix} u_1+iv_1 & 0 \\ 0 & w_1-iz_1 \end{pmatrix}^* =$$

$$\begin{pmatrix} u_1+iv_1 & 0 \\ 0 & w_1-iz_1 \end{pmatrix}\begin{pmatrix} u_1-iv_1 & 0 \\ 0 & w_1+iz_1 \end{pmatrix}$$

$$= \begin{pmatrix} (u_1^2+v_1^2) & 0 \\ 0 & (w_1^2+z_1^2) \end{pmatrix}$$

So, for unit vecors u1 + iv1,w1 - iz1 :

$$\begin{pmatrix} u_1+iv_1 & 0 \\ 0 & w_1-iz_1 \end{pmatrix} \text{ is unitary.}$$

Using equation (), quark and meson characteristics, such as mass, may be determined.
For example, if the mass constituents of a meson are

$$u_1, v_1, w_1, z_1 \Rightarrow |m|^2 = u_1^2 + v_1^2 + w_1^2 + z_1^2 \ .$$

A quark mass:

$$|m_u| = \sqrt{2(u_1^2+v_1^2)} \quad \text{for a meson: } u\bar{u} \ .$$

(In the Helmholtzian: $u^2_1 = |m_1|^2 + |m_2|^2 \ |m_3|^2$, $v^2_1 = |m_0|^2$; similarly for $w1$, $z1$).

Noting how the meson color indices cancel, and looking forward to the baryons an appropriate **RGB** notation

transformation would be: (**RGB**) → (- 1,0,1).

Note that the sum of the indices is 0 (as is the meson's X + X) (Just as particle-anti-particle pairs color indices add up to color indices of light = the color indices of empty space = 0 ,
the color indices of all composite-fermion particles must always sum to empty space = 0).
The following scheme has been conjectured for meson transformations without violating empty space, via 'virtual' particles.
i.e.: at every point there exists two "virual" particle (particle-anti-particle) pairs equivalent to empty space except for
total-energy (like pushing onto he stack and poping it off).

NOTE: $\rho_\Phi(m) : \overline{p}_\Pi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Phi(s) = \rho_\Phi(m) : \overline{\sigma}_\Phi(s) + \sigma_\Pi(r) : \overline{\overline{p}}_\Pi(n)$

$$\Rightarrow \rho_\Pi(m) : \overline{p}_\Pi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Pi(s) = \rho_\Pi(m) : \overline{\sigma}_\Pi(s) + \sigma_\Pi(r) : \overline{p}_\Pi(n)$$
$$\text{where: } \rho, \sigma \in \{u, d\} \ \& \ \Pi, \Phi \in \{1, 0, -1\}$$

Since the color force is much stronger than the electromagnetic at short range, the color/anti-color pairs dominate & requires another color force to uncouple.

$\rho_\Phi(m) : \overline{p}_\Pi(n) \ \& \ \sigma_\Pi(r) : \overline{\sigma}_\Phi(s)$, are electromagnetically bonded = 'weak' bond
$\rho_\Phi(m) : \overline{\sigma}_\Phi(s) \ \& \ \sigma_\Pi(r) : \overline{p}_\Pi(n)$, are color bonded = 'strong' bond

Note how meson color pairs tend to couple together:

$$\rho_\Pi(m) : \overline{\sigma}_\Pi(n) + \overline{p}_\Pi(r) : \sigma_\Phi(s) + \sigma_\Pi(h) : \overline{p}_\Phi(j)$$
$$\Downarrow$$
$$\rho_\Pi(h) : \overline{p}_\Pi(r) + \overline{\sigma}_\Pi(n) : \sigma_\Phi(s) + \sigma_\Pi(h) : \overline{p}_\Phi(j)$$
$$\Downarrow$$
$$\rho_\Pi(h) : \overline{p}_\Pi(r) + \overline{\sigma}_\Pi(n) : \sigma_\Pi(h) + \sigma_\Phi(s) : \overline{p}_\Phi(j)$$

where: $\rho, \sigma \in \{u, d\} \ \& \ \Pi, \Phi \in \{1, 0, -1\} \ \& \ m, n, r, s, h, j \in \{1, 2, 3\}$

This pairing is clearly the stronger bonding, since it is both color and electromagnetic attraction.

Meson $\rho_\Pi(h) : \overline{\sigma}_\Pi(j) \rightarrow \rho_\Phi(m) : \overline{\sigma}_\Phi(n)$ color & flavour transformations:

$$\rho_\Pi(h) : \overline{\sigma}_\Pi(j) + [\rho_\Phi(m) : \overline{p}_\Phi(r) + \sigma_\Phi(s) : \overline{\sigma}_\Phi(n)]$$
$$\Downarrow$$
$$\rho_\Phi(m) : \overline{\sigma}_\Pi(j) + [\rho_\Pi(h) : \overline{p}_\Phi(r) + \sigma_\Phi(s) : \overline{\sigma}_\Phi(n)]$$
$$\Downarrow$$
$$\rho_\Phi(m) : \overline{\sigma}_\Phi(n) + [\rho_\Pi(h) : \overline{p}_\Phi(r) + \sigma_\Phi(s) : \overline{\sigma}_\Pi(j)]$$
$$\Downarrow$$
$$\rho_\Phi(m) : \overline{\sigma}_\Phi(n) + [\rho_\Pi(h) : \overline{\sigma}_\Pi(j) + \sigma_\Phi(s) : \overline{p}_\Phi(r)]$$

where: $\rho, \sigma, \theta \in \{u, d\} \ \& \ \Pi, \Phi, \Psi \in \{1, 0, -1 \mid \Pi \neq \Phi, \Pi \neq \Psi, \Phi \neq \Psi\} \ \& \ h, j, k \in \{1, 2, 3\}$

(the terms in brackets are the virtual, (appear & disappear))

However, it is an open question as to whether or not the above transformations are consistent with these processes.
All the possible quark triplets are given by:

$u_R(h) : u_G(j) : u_B(k)$	$u_R(h) : u_G(j) : d_B(k)$	$u_R(h) : d_G(j) : u_B(k)$	$u_R(h) : d_G(j) : d_B(k)$
$d_R(h) : d_G(j) : d_B(k)$	$d_R(h) : d_G(j) : u_B(k)$	$d_R(h) : u_G(j) : d_B(k)$	$d_R(h) : u_G(j) : u_B(k)$

(19)

(Notice that casual appearance suggests a 8-dimensional double-cover)

Because u's have 2 E^h's & 1 B^h & d's have 1 E^h & 2 B^h's the 2 x 3 & 2 x 4 symmetries are not perfect.

Similarly to how the mesons were analyzed, a baryon vector transformation would look like:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}\begin{pmatrix} r \\ p \\ q \end{pmatrix}$$

For a point in the complex plane: x+ iy at an angle θ from the origin, it's complex conjugte x - iy is at an angle -θ from the origin.

On the unit circle: $x+iy$ $\cos\theta + i \sin\theta$ and: $x - iy = \cos\theta - i \sin\theta$

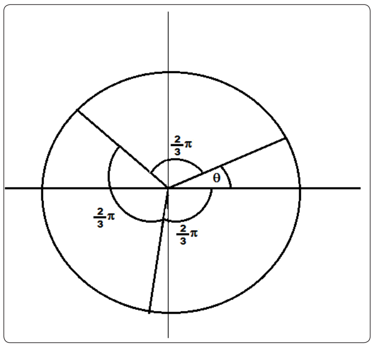
If there are 2n such angles dividing the circle:

$$2n\theta = 2\pi \Rightarrow \theta = \frac{1}{n}\pi \Rightarrow 2\theta = \frac{2}{n}\pi$$

So, for $n = 3 : \frac{2}{3}\pi = 120°$

$$\Rightarrow x+iy=\cos 2\theta+i\sin 2\theta=-\frac{1}{2}+i\frac{\sqrt{3}}{2}\ ;\ x-iy=\cos\theta-i\sin\theta=-\frac{1}{2}-i\frac{\sqrt{3}}{2}\ ;\ \theta=0: x+iy=x-iy=1$$

This figure shows how *u*, *v*,*w* would be situated with respect to each other, everything being equal:



points on an arc of a circle equidistant in thirds may be formulated as follows:

$$\rho(\cos\theta+i\sin\theta)\ ;\ \rho\left(\cos\left(\frac{2}{3}\pi+\theta\right)+i\sin\left(\frac{2}{3}\pi+\theta\right)\right)\ ;\ \rho\left(\cos\left(\frac{4}{3}\pi+\theta\right)+i\sin\left(\frac{4}{3}\pi+\theta\right)\right)$$

$$\rho\left(\cos\left(\frac{2}{3}\pi+\theta\right)+i\sin\left(\frac{2}{3}\pi+\theta\right)\right)=$$

$$\cos\frac{2}{3}\pi\cos\theta-\sin\frac{2}{3}\pi\sin\theta+i\left(\sin\frac{2}{3}\pi\cos\theta+\cos\frac{2}{3}\pi\sin\theta\right)$$
$$=\rho\left(\left(-\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right)+i\left(\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)\right)$$
$$=\rho\left(-\frac{1}{2}(\cos\theta+i\sin\theta)+\frac{\sqrt{3}}{2}(i\cos\theta-\sin\theta)\right)$$
$$=\left(-1+i\sqrt{3}\right)\frac{1}{2}\rho(\cos\theta+i\sin\theta)$$
$$=\left(-1+i\sqrt{3}\right)(u+iv)$$
$$=-\left(u+\sqrt{3}v\right)+i\left(\sqrt{3}u-v\right)$$

$$\rho\left(\cos\left(\frac{4}{3}\pi+\theta\right)+i\sin\left(\frac{4}{3}\pi+\theta\right)\right)=\cos\frac{4}{3}\pi\cos\theta-\sin\frac{4}{3}\pi\sin\theta+i\left(\sin\frac{4}{3}\pi\cos\theta+\cos\frac{4}{3}\pi\sin\theta\right)$$
$$=\rho\left(\left(-\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta\right)+i\left(-\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta\right)\right)$$
$$=\rho\left(-\frac{1}{2}(\cos\theta+i\sin\theta)-\frac{\sqrt{3}}{2}(-\sin\theta+i\cos\theta)\right)$$
$$=\left(-1-i\sqrt{3}\right)\frac{1}{2}\rho(\cos\theta+i\sin\theta)$$
$$=\left(-1-i\sqrt{3}\right)(u+iv)$$
$$=-\left(u-\sqrt{3}v\right)-i\left(\sqrt{3}u+v\right)$$

So, for any angle any triple-thirds may be expressed:

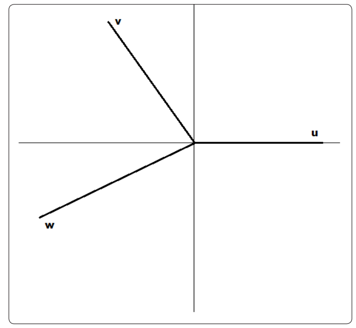
$$2z,\left(-1+i\sqrt{3}\right)z,\left(-1-i\sqrt{3}\right)z$$

Normal vectors parallel to these are: $1,\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right),\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$

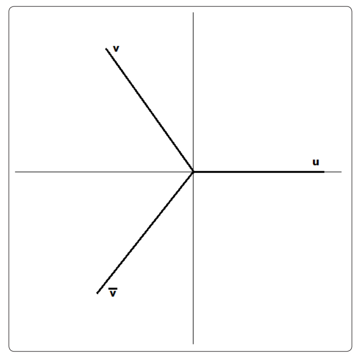
Assuming a 3-type quark triple with magnitudes u, v, w, ϵ ℝ oriented by these normal vectors, they may be identified by:

$$u,\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)v,\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)w$$

Each quark may be considered a triplet; but, generally, the angles need not all be equal; but even so there may always be an orientation where one "anchor" vector may be real-only, and the other two fully complex (with non-zero imaginary parts, and not necessarily complex conjugates - only so, when the angles are both $\frac{2\pi}{3}$



If the quarks are symmetric (conjugate) with respect to the 'anchor', then they may be represented as *v* & *v* :



As with the mesons, above, instead of writing a baryon as a column vector; if it is written as a 3 3 square matrix, as:

$$q=\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}u_R+iu_I\\v_R+iv_I\\w_R+iw_I\end{pmatrix}\Rightarrow\begin{pmatrix}u_R+iu_I&0&0\\0&v_R+iv_I&0\\0&0&w_R+iw_I\end{pmatrix}$$

(for the general case)

$$T:\begin{pmatrix}u\\v\\w\end{pmatrix}\mapsto\begin{pmatrix}u&0&0\\0&v&0\\0&0&w\end{pmatrix}=\begin{pmatrix}u_R+iu_I&0&0\\0&v_R+iv_I&0\\0&0&w_R+iw_I\end{pmatrix}$$

then under ordinary matrix multilication:

$$\begin{pmatrix}u_{1R}+iu_{1I}&0&0\\0&v_{1R}+iv_{1I}&0\\0&0&w_{1R}+iw_{1I}\end{pmatrix}\begin{pmatrix}u_{2R}+iu_{2I}&0&0\\0&v_{2R}+iv_{2I}&0\\0&0&w_{2R}+iw_{2I}\end{pmatrix}=$$
$$=\begin{pmatrix}(u_{1R}u_{2R}-u_{1I}u_{2I})+i(u_{1R}u_{2I}-u_{1I}u_{2R})&0&0\\0&(v_{1R}v_{2R}-v_{1I}v_{2I})+i(v_{1R}v_{2I}-v_{1I}v_{2R})&0\\0&0&(w_{1R}w_{2R}-w_{1I}w_{2I})+i(w_{1R}w_{2I}-w_{1I}w_{2R})\end{pmatrix}$$

spans; is even commutative; and for such non-singular matrices forms a group, and:

$$\begin{pmatrix}u_{1R}+iu_{1I}&0&0\\0&v_{1R}+iv_{1I}&0\\0&0&w_{1R}+iw_{1I}\end{pmatrix}\begin{pmatrix}u_{1R}+iu_{1I}&0&0\\0&v_{1R}+iv_{1I}&0\\0&0&w_{1R}+iw_{1I}\end{pmatrix}^*=$$
$$=\begin{pmatrix}(u_{1R}^2+u_{1I}^2)&0&0\\0&(v_{1R}^2+v_{1I}^2)&0\\0&0&(w_{1R}^2+w_{1I}^2)\end{pmatrix}$$

For the symmetric-conjugate 'anchor'-type case, each quark may be expressed as a column vector:

$$q=\begin{pmatrix}u\\ \gamma\\ \bar{u}\end{pmatrix}=\begin{pmatrix}u_R+iu_I\\ \gamma\\ u_R-iu_I\end{pmatrix}\Rightarrow\begin{pmatrix}u_R+iu_I&0&0\\0&\gamma&0\\0&0&u_R-iu_I\end{pmatrix}$$

$$T:\begin{pmatrix}u\\ \gamma\\ \bar{u}\end{pmatrix}\mapsto\begin{pmatrix}u&0&0\\0&\gamma&0\\0&0&\bar{u}\end{pmatrix}=\begin{pmatrix}u_R+iu_I&0&0\\0&\gamma&0\\0&0&u_R-iu_I\end{pmatrix}$$

And, these transformations are:

$$\begin{pmatrix}v_R+iv_I\\ \Gamma\\ v_R-iv_I\end{pmatrix}=\begin{pmatrix}a_R+ia_I&b_R+ib_I&c_R+ic_I\\d_R+id_I&e_R+ie_I&f_R+if_I\\g_R+ig_I&h_R+ih_I&k_R+ik_I\end{pmatrix}\begin{pmatrix}u_R+iu_I\\ \gamma\\ u_R-iu_I\end{pmatrix}$$

$$=\begin{pmatrix}(a_R+ia_I)(u_R+iu_I)+(b_R+ib_I)\gamma+(c_R+ic_I)(u_R-iu_I)\\(d_R+id_I)(u_R+iu_I)+(e_R+ie_I)\gamma+(f_R+if_I)(u_R-iu_I)\\(g_R+ig_I)(u_R+iu_I)+(h_R+ih_I)\gamma+(k_R+ik_I)(u_R-iu_I)\end{pmatrix}$$

$$=\begin{pmatrix}a_R(u_R+iu_I)+ia_I(u_R+iu_I)+b_R\gamma+ib_I\gamma+c_R(u_R-iu_I)+ic_I(u_R-iu_I)\\d_R(u_R+iu_I)+id_I(u_R+iu_I)+e_R\gamma+ie_I\gamma+f_R(u_R-iu_I)+if_I(u_R-iu_I)\\g_R(u_R+iu_I)+ig_I(u_R+iu_I)+h_R\gamma+ih_I\gamma+k_R(u_R-iu_I)+ik_I(u_R-iu_I)\end{pmatrix}$$

$$=\begin{pmatrix}a_Ru_R+ia_Ru_I+ia_Iu_R-a_Iu_I+b_R\gamma+ib_I\gamma+c_Ru_R-ic_Ru_I+ic_Iu_R+c_Iu_I\\d_Ru_R+id_Ru_I+id_Iu_R-d_Iu_I+e_R\gamma+ie_I\gamma+f_Ru_R-if_Ru_I+if_Iu_R+f_Iu_I\\g_Ru_R+ig_Ru_I+ig_Iu_R-g_Iu_I+h_R\gamma+ih_I\gamma+k_Ru_R-ik_Ru_I+ik_Iu_R+k_Iu_I\end{pmatrix}$$

$$=\begin{pmatrix}[a_Ru_R-a_Iu_I+b_R\gamma+c_Ru_R+c_Iu_I]+i[a_Ru_I+a_Iu_R+b_I\gamma-c_Ru_I+c_Iu_R]\\ [d_Ru_R-d_Iu_I+e_R\gamma+f_Ru_R+f_Iu_I]+i[d_Ru_I+d_Iu_R+e_I\gamma-f_Ru_I+f_Iu_R]\\ [g_Ru_R-g_Iu_I+h_R\gamma+k_Ru_R+k_Iu_I]+i[g_Ru_I+g_Iu_R+h_I\gamma-k_Ru_I+k_Iu_R]\end{pmatrix}$$

$$\begin{pmatrix}v_R+iv_I\\ \Gamma\\ v_R-iv_I\end{pmatrix}=\begin{pmatrix}[(a_R+c_R)u_R+(-a_I+c_I)u_I+b_R\gamma]+i[(a_I+c_I)u_R+(a_R-c_R)u_I+b_I\gamma]\\ [(d_R+f_R)u_R+(-d_I+f_I)u_I+e_R\gamma]+i[(d_I+f_I)u_R+(d_R-f_R)u_I+e_I\gamma]\\ [(g_R+k_R)u_R+(-g_I+k_I)u_I+h_R\gamma]+i[(g_I+k_I)u_R+(g_R-k_R)u_I+h_I\gamma]\end{pmatrix}$$

$$\Rightarrow\left\{\begin{array}{|l|l|}\hline v_R=(a_R+c_R)u_R+(-a_I+c_I)u_I+b_R\gamma & v_I=(a_I+c_I)u_R+(a_R-c_R)u_I+b_I\gamma\\ \hline \Gamma=(d_R+f_R)u_R+(-d_I+f_I)u_I+e_R\gamma & 0=(d_I+f_I)u_R+(d_R-f_R)u_I+e_I\gamma\\ \hline v_R=(g_R+k_R)u_R+(-g_I+k_I)u_I+h_R\gamma & -v_I=(g_I+k_I)u_R+(g_R-k_R)u_I+h_I\gamma\\ \hline\end{array}\right.$$

for arbitrary *uR*, *uI*, γ:

$$\Rightarrow\left\{\begin{array}{|l|l|}\hline g_R+k_R=a_R+c_R & -g_I-k_I=a_I+c_I\\ \hline -g_I+k_I=-a_I+c_I & -g_R+k_R=a_R-c_R\\ \hline h_R=b_R & h_I=-b_I\\ \hline f_R=d_R & f_I=-d_I\\ \hline 0=e_I & \\ \hline\end{array}\right\}\Rightarrow\left\{\begin{array}{|l|l|}\hline k_R=a_R & g_I=-c_I\\ \hline k_I=-a_I & g_R=c_R\\ \hline h_R=b_R & h_I=-b_I\\ \hline f_R=d_R & f_I=-d_I\\ \hline 0=e_I & \\ \hline\end{array}\right.$$

(since Γ is an unknown/undetermined function of *uR*, *uI*, γ:

$$\Rightarrow\begin{pmatrix}a_R+ia_I&b_R+ib_I&c_R+ic_I\\d_R+id_I&e_R+ie_I&f_R+if_I\\g_R+ig_I&h_R+ih_I&k_R+ik_I\end{pmatrix}=\begin{pmatrix}a_R+ia_I&b_R+ib_I&c_R+ic_I\\d_R+id_I&e_R&d_R-id_I\\c_R-ic_I&b_R-ib_I&a_R-ia_I\end{pmatrix}=\begin{pmatrix}a&b&c\\d&e_R&d^*\\c^*&b^*&a^*\end{pmatrix}$$

$$\begin{pmatrix}v_R+iv_I\\ \Gamma\\ v_R-iv_I\end{pmatrix}=\begin{pmatrix}a_R+ia_I&b_R+ib_I&c_R+ic_I\\d_R+id_I&e_R+ie_I&f_R+if_I\\g_R+ig_I&h_R+ih_I&k_R+ik_I\end{pmatrix}\begin{pmatrix}u_R-iu_I\\u_R+iu_I\\ \gamma\end{pmatrix}$$

$$=\begin{pmatrix}(a_R+ia_I)(u_R-iu_I)+(b_R+ib_I)(u_R+iu_I)+(c_R+ic_I)\gamma\\(d_R+id_I)(u_R-iu_I)+(e_R+ie_I)(u_R+iu_I)+(f_R+if_I)\gamma\\(g_R+ig_I)(u_R-iu_I)+(h_R+ih_I)(u_R+iu_I)+(k_R+ik_I)\gamma\end{pmatrix}$$

$$=\begin{pmatrix}(a_R+ia_I)u_R-i(a_R+ia_I)u_I+(b_R+ib_I)u_R+i(b_R+ib_I)u_I+c_R\gamma+ic_I\gamma\\(d_R+id_I)u_R-i(d_R+id_I)u_I+(e_R+ie_I)u_R+i(e_R+ie_I)u_I+f_R\gamma+if_I\gamma\\(g_R+ig_I)u_R-i(g_R+ig_I)u_I+(h_R+ih_I)u_R+i(h_R+ih_I)u_I+k_R\gamma+ik_I\gamma\end{pmatrix}$$

$$=\begin{pmatrix}a_Ru_R+ia_Iu_R-ia_Ru_I+a_Iu_I+b_Ru_R+ib_Iu_R+ib_Ru_I-b_Iu_I+c_R\gamma+ic_I\gamma\\d_Ru_R+id_Iu_R-id_Ru_I+d_Iu_I+e_Ru_R+ie_Iu_R+ie_Ru_I-e_Iu_I+f_R\gamma+if_I\gamma\\g_Ru_R+ig_Iu_R-ig_Ru_I+g_Iu_I+h_Ru_R+ih_Iu_R+ih_Ru_I-h_Iu_I+k_R\gamma+ik_I\gamma\end{pmatrix}$$

$$=\begin{pmatrix}a_Ru_R+a_Iu_I+b_Ru_R-b_Iu_I+c_R\gamma+ia_Iu_R-ia_Ru_I+ib_Iu_R+ib_Ru_I+ic_I\gamma\\d_Ru_R+d_Iu_I+e_Ru_R-e_Iu_I+f_R\gamma+id_Iu_R-id_Ru_I+ie_Iu_R+ie_Ru_I+if_I\gamma\\g_Ru_R+g_Iu_I+h_Ru_R-h_Iu_I+k_R\gamma+ig_Iu_R-ig_Ru_I+ih_Iu_R+ih_Ru_I+ik_I\gamma\end{pmatrix}$$

$$=\begin{pmatrix}[a_Ru_R+a_Iu_I+b_Ru_R-b_Iu_I+c_R\gamma]+i[a_Iu_R-a_Ru_I+b_Iu_R+b_Ru_I+c_I\gamma]\\ [d_Ru_R+d_Iu_I+e_Ru_R-e_Iu_I+f_R\gamma]+i[d_Iu_R-d_Ru_I+e_Iu_R+e_Ru_I+f_I\gamma]\\ [g_Ru_R+g_Iu_I+h_Ru_R-h_Iu_I+k_R\gamma]+i[g_Iu_R-g_Ru_I+h_Iu_R+h_Ru_I+k_I\gamma]\end{pmatrix}$$

$$=\begin{pmatrix}a_Ru_R+a_Iu_I+b_Ru_R-b_Iu_I+c_R\gamma+ia_Iu_R-ia_Ru_I+ib_Iu_R+ib_Ru_I+ic_I\gamma\\d_Ru_R+d_Iu_I+e_Ru_R-e_Iu_I+f_R\gamma+id_Iu_R-id_Ru_I+ie_Iu_R+ie_Ru_I+if_I\gamma\\g_Ru_R+g_Iu_I+h_Ru_R-h_Iu_I+k_R\gamma+ig_Iu_R-ig_Ru_I+ih_Iu_R+ih_Ru_I+ik_I\gamma\end{pmatrix}$$

$$=\begin{pmatrix}[a_Ru_R+a_Iu_I+b_Ru_R-b_Iu_I+c_R\gamma]+i[a_Iu_R-a_Ru_I+b_Iu_R+b_Ru_I+c_I\gamma]\\ [d_Ru_R+d_Iu_I+e_Ru_R-e_Iu_I+f_R\gamma]+i[d_Iu_R-d_Ru_I+e_Iu_R+e_Ru_I+f_I\gamma]\\ [g_Ru_R+g_Iu_I+h_Ru_R-h_Iu_I+k_R\gamma]+i[g_Iu_R-g_Ru_I+h_Iu_R+h_Ru_I+k_I\gamma]\end{pmatrix}$$

$$\begin{pmatrix}v_R+iv_I\\ \Gamma\\ v_R-iv_I\end{pmatrix}=\begin{pmatrix}[(a_R+b_R)u_R+(a_I-b_I)u_I+c_R\gamma]+i[(a_I+b_I)u_R+(-a_R+b_R)u_I+c_I\gamma]\\ [(d_R+e_R)u_R+(d_I-e_I)u_I+f_R\gamma]+i[(d_I+e_I)u_R+(-d_R+e_R)u_I+f_I\gamma]\\ [(g_R+h_R)u_R+(g_I-h_I)u_I+k_R\gamma]+i[(g_I+h_I)u_R+(-g_R+h_R)u_I+k_I\gamma]\end{pmatrix}$$

$$\Rightarrow\left\{\begin{array}{|l|l|}\hline v_R=(a_R+b_R)u_R+(a_I-b_I)u_I+c_R\gamma & v_I=(a_I+b_I)u_R+(-a_R+b_R)u_I+c_I\gamma\\ \hline \Gamma=(d_R+e_R)u_R+(d_I-e_I)u_I+f_R\gamma & 0=(d_I+e_I)u_R+(-d_R+e_R)u_I+f_I\gamma\\ \hline v_R=(g_R+h_R)u_R+(g_I-h_I)u_I+k_R\gamma & -v_I=(g_I+h_I)u_R+(-g_R+h_R)u_I+k_I\gamma\\ \hline\end{array}\right.$$

for arbitrary *uR*,*uI*, γ :

$$\Rightarrow\left\{\begin{array}{|l|l|}\hline g_R+h_R=a_R+b_R & g_I-h_I=a_I-b_I\\ \hline -g_I-h_I=a_I+b_I & g_R-h_R=-a_R+b_R\\ \hline k_R=c_R & k_I=-c_I\\ \hline e_R=d_R & e_I=-d_I\\ \hline f_I=0 & \\ \hline\end{array}\right\}\Rightarrow\left\{\begin{array}{|l|l|}\hline h_R=a_R & g_I=-b_I\\ \hline h_I=-a_I & g_R=b_R\\ \hline k_R=c_R & k_I=-c_I\\ \hline e_R=d_R & e_I=-d_I\\ \hline f_I=0 & \\ \hline\end{array}\right.$$

(since Γ is an unknown/undetermined function of *uR*,*uI*, *γ*)

$$\Rightarrow\begin{pmatrix}a_R+ia_I&b_R+ib_I&c_R+ic_I\\d_R+id_I&e_R+ie_I&f_R+if_I\\g_R+ig_I&h_R+ih_I&k_R+ik_I\end{pmatrix}=\begin{pmatrix}a_R+ia_I&b_R+ib_I&c_R+ic_I\\d_R+id_I&d_R+id_I&f_R\\b_R-ib_I&a_R-ia_I&c_R-ic_I\end{pmatrix}$$

$$=\begin{pmatrix}a&b&c\\d&d^*&f_R\\b^*&a^*&c^*\end{pmatrix}$$

These are both 9 dimensional.

And, for the general, 'anchor'-type case, each quark may be expressed as a column vector:

$$q=\begin{pmatrix}u\\v\\ \bar{u}\end{pmatrix}=\begin{pmatrix}u_R+iu_I\\v_R+iv_I\\u_R-iu_I\end{pmatrix}\Rightarrow\begin{pmatrix}u_R+iu_I&0&0\\0&v_R+iv_I&0\\0&0&u_R-iu_I\end{pmatrix}$$

$$T:\begin{pmatrix}u\\v\\ \bar{u}\end{pmatrix}\mapsto\begin{pmatrix}u&0&0\\0&v&0\\0&0&\bar{u}\end{pmatrix}=\begin{pmatrix}u_R+iu_I&0&0\\0&v_R+iv_I&0\\0&0&u_R-iu_I\end{pmatrix}$$

And, these transformations are:

$$\begin{pmatrix}v_R+iv_I\\w_R+iw_I\\v_R-iv_I\end{pmatrix}=\begin{pmatrix}a_R+ia_I&b_R+ib_I&c_R+ic_I\\d_R+id_I&e_R+ie_I&f_R+if_I\\g_R+ig_I&h_R+ih_I&k_R+ik_I\end{pmatrix}\begin{pmatrix}u_R+iu_I\\z_R+iz_I\\u_R-iu_I\end{pmatrix}$$

$$=\begin{pmatrix}(a_R+ia_I)(u_R+iu_I)+(b_R+ib_I)(z_R+iz_I)+(c_R+ic_I)(u_R-iu_I)\\(d_R+id_I)(u_R+iu_I)+(e_R+ie_I)(z_R+iz_I)+(f_R+if_I)(u_R-iu_I)\\(g_R+ig_I)(u_R+iu_I)+(h_R+ih_I)(z_R+iz_I)+(k_R+ik_I)(u_R-iu_I)\end{pmatrix}$$

$$= \begin{pmatrix} a_R(u_R+iu_I)+ia_I(u_R+iu_I)+b_R(z_R+iz_I)+ib_I(z_R+iz_I)+c_R(u_R-iu_I)+ic_I(u_R-iu_I) \\ d_R(u_R+iu_I)+id_I(u_R+iu_I)+e_R(z_R+iz_I)+ie_I(z_R+iz_I)+f_R(u_R-iu_I)+if_I(u_R-iu_I) \\ g_R(u_R+iu_I)+ig_I(u_R+iu_I)+h_R(z_R+iz_I)+ih_I(z_R+iz_I)+k_R(u_R-iu_I)+ik_I(u_R-iu_I) \end{pmatrix}$$

$$= \begin{pmatrix} a_Ru_R+ia_Ru_I+ia_Iu_R-a_Iu_I+b_Rz_R+ib_Rz_I+ib_Iz_R-b_Iz_I+c_Ru_R-ic_Ru_I+ic_Iu_R+c_Iu_I \\ d_Ru_R+id_Ru_I+id_Iu_R-d_Iu_I+e_Rz_R+ie_Rz_I+ie_Iz_R-e_Iz_I+f_Ru_R-if_Ru_I+if_Iu_R+f_Iu_I \\ g_Ru_R+ig_Ru_I+ig_Iu_R-g_Iu_I+h_Rz_R+ih_Rz_I+ih_Iz_R-h_Iz_I+k_Ru_R-ik_Ru_I+ik_Iu_R+k_Iu_I \end{pmatrix}$$

$$= \begin{pmatrix} [a_Ru_R-a_Iu_I+b_Rz_R-b_Iz_I+c_Ru_R+c_Iu_I]+i[a_Ru_I+a_Iu_R+b_Rz_I+b_Iz_R-c_Ru_I+c_Iu_R] \\ [d_Ru_R-d_Iu_I+e_Rz_R-e_Iz_I+f_Ru_R+f_Iu_I]+i[d_Ru_I+d_Iu_R+e_Rz_I+e_Iz_R-f_Ru_I+f_Iu_R] \\ [g_Ru_R-g_Iu_I+h_Rz_R-h_Iz_I+k_Ru_R+k_Iu_I]+i[g_Ru_I+g_Iu_R+h_Rz_I+h_Iz_R-k_Ru_I+k_Iu_R] \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} v_R+iv_I \\ w_R+iw_I \\ v_R-iv_I \end{pmatrix} = \begin{pmatrix} [(a_R+c_R)u_R+(-a_I+c_I)u_I+b_Rz_R-b_Iz_I]+i[(a_I+c_I)u_R+(a_R-c_R)u_I+b_Iz_R+b_Rz_I] \\ [(d_R+f_R)u_R+(-d_I+f_I)u_I+e_Rz_R-e_Iz_I]+i[(d_I+f_I)u_R+(d_R-f_R)u_I+e_Iz_R+e_Rz_I] \\ [(g_R+k_R)u_R+(-g_I+k_I)u_I+h_Rz_R-h_Iz_I]+i[(g_I+k_I)u_R+(g_R-k_R)u_I+h_Iz_R+h_Rz_I] \end{pmatrix}$$

$$\Rightarrow \begin{cases} v_R=(a_R+c_R)u_R+(-a_I+c_I)u_I+b_Rz_R-b_Iz_I & v_I=(a_I+c_I)u_R+(a_R-c_R)u_I+b_Iz_R+b_Rz_I \\ w_R=(d_R+f_R)u_R+(-d_I+f_I)u_I+e_Rz_R-e_Iz_I & w_I=(d_I+f_I)u_R+(d_R-f_R)u_I+e_Iz_R+e_Rz_I \\ v_R=(g_R+k_R)u_R+(-g_I+k_I)u_I+h_Rz_R-h_Iz_I & -v_I=(g_I+k_I)u_R+(g_R-k_R)u_I+h_Iz_R+h_Rz_I \end{cases}$$

$$\Rightarrow \begin{cases} v_R=(a_R+c_R)u_R+(-a_I+c_I)u_I+b_Rz_R-b_Iz_I & v_I=(a_I+c_I)u_R+(a_R-c_R)u_I+b_Iz_R+b_Rz_I \\ v_R=(g_R+k_R)u_R+(-g_I+k_I)u_I+h_Rz_R-h_Iz_I & v_I=(-g_I-k_I)u_R+(-g_R+k_R)u_I-h_Iz_R-h_Rz_I \\ w_R=(d_R+f_R)u_R+(-d_I+f_I)u_I+e_Rz_R-e_Iz_I & w_I=(d_I+f_I)u_R+(d_R-f_R)u_I+e_Iz_R+e_Rz_I \end{cases}$$

for arbitrary *uR*, *uI*, *zR*, *zI* :

$$\Rightarrow \begin{cases} (g_R+k_R)=(a_R+c_R) & (-g_I+k_I)=(-a_I+c_I) & h_R=b_R & -h_I=b_I \\ (-g_I-k_I)=(a_I+c_I) & (-g_R+k_R)=(a_R-c_R) & -h_I=-b_I & -h_R=b_R \end{cases}$$

$$\Rightarrow \begin{cases} g_R+k_R=a_R+c_R & -g_I+k_I=-a_I+c_I & h_R=b_R=0 \\ -g_R+k_R=a_R-c_R & -g_I-k_I=a_I+c_I & h_I=b_I=0 \end{cases}$$

$$\Rightarrow \begin{cases} k_R=a_R & g_I=-c_I & h_R=b_R=0 \\ g_R=c_R & k_I=-a_I & h_I=b_I=0 \end{cases}$$

$$\Rightarrow \begin{pmatrix} a_R+ia_I & 0 & c_R+ic_I \\ d_R+id_I & e_R+ie_I & f_R+if_I \\ c_R-ic_I & 0 & a_R-ia_I \end{pmatrix} = \begin{pmatrix} a & 0 & c \\ d & e & f \\ c^* & 0 & a^* \end{pmatrix}$$

$$\begin{pmatrix} v_R+iv_I \\ w_R+iw_I \\ v_R-iv_I \end{pmatrix} = \begin{pmatrix} a_R+ia_I & b_R+ib_I & c_R+ic_I \\ d_R+id_I & e_R+ie_I & f_R+if_I \\ g_R+ig_I & h_R+ih_I & k_R+ik_I \end{pmatrix} \begin{pmatrix} u_R-iu_I \\ u_R+iu_I \\ z_R+iz_I \end{pmatrix}$$

$$= \begin{pmatrix} (a_R+ia_I)(u_R-iu_I)+(b_R+ib_I)(u_R+iu_I)+(c_R+ic_I)(z_R+iz_I) \\ (d_R+id_I)(u_R-iu_I)+(e_R+ie_I)(u_R+iu_I)+(f_R+if_I)(z_R+iz_I) \\ (g_R+ig_I)(u_R-iu_I)+(h_R+ih_I)(u_R+iu_I)+(k_R+ik_I)(z_R+iz_I) \end{pmatrix}$$

$$= \begin{pmatrix} a_R(u_R-iu_I)+ia_I(u_R-iu_I)+b_R(u_R+iu_I)+ib_I(u_R+iu_I)+c_R(z_R+iz_I)+ic_I(z_R+iz_I) \\ d_R(u_R-iu_I)+id_I(u_R-iu_I)+e_R(u_R+iu_I)+ie_I(u_R+iu_I)+f_R(z_R+iz_I)+if_I(z_R+iz_I) \\ g_R(u_R-iu_I)+ig_I(u_R-iu_I)+h_R(u_R+iu_I)+ih_I(u_R+iu_I)+k_R(z_R+iz_I)+ik_I(z_R+iz_I) \end{pmatrix}$$

$$= \begin{pmatrix} a_Ru_R-ia_Ru_I+ia_Iu_R+a_Iu_I+b_Ru_R+ib_Ru_I+ib_Iu_R-b_Iu_I+c_Rz_R+ic_Rz_I+ic_Iz_R-c_Iz_I \\ d_Ru_R-id_Ru_I+id_Iu_R+d_Iu_I+e_Ru_R+ie_Ru_I+ie_Iu_R-e_Iu_I+f_Rz_R+if_Rz_I+if_Iz_R-f_Iz_I \\ g_Ru_R-ig_Ru_I+ig_Iu_R+g_Iu_I+h_Ru_R+ih_Ru_I+ih_Iu_R-h_Iu_I+k_Rz_R+ik_Rz_I+ik_Iz_R-k_Iz_I \end{pmatrix}$$

$$= \begin{pmatrix} [a_Ru_R+a_Iu_I+b_Ru_R-b_Iu_I+c_Rz_R-c_Iz_I]+i[-a_Ru_I+a_Iu_R+b_Ru_I+b_Iu_R+c_Rz_I+c_Iz_R] \\ [d_Ru_R+d_Iu_I+e_Ru_R-e_Iu_I+f_Rz_R-f_Iz_I]+i[-d_Ru_I+d_Iu_R+e_Ru_I+e_Iu_R+f_Rz_I+f_Iz_R] \\ [g_Ru_R+g_Iu_I+h_Ru_R-h_Iu_I+k_Rz_R-k_Iz_I]+i[-g_Ru_I+g_Iu_R+h_Ru_I+h_Iu_R+k_Rz_I+k_Iz_R] \end{pmatrix}$$

$$= \begin{pmatrix} [a_Ru_R+b_Ru_R+a_Iu_I-b_Iu_I+c_Rz_R-c_Iz_I]+i[a_Iu_R+b_Iu_R+-a_Ru_I+b_Ru_I+c_Iz_R+c_Rz_I] \\ [d_Ru_R+e_Ru_R+d_Iu_I-e_Iu_I+f_Rz_R-f_Iz_I]+i[d_Iu_R+e_Iu_R+-d_Ru_I+e_Ru_I+f_Iz_R+f_Rz_I] \\ [g_Ru_R+h_Ru_R+g_Iu_I-h_Iu_I+k_Rz_R-k_Iz_I]+i[g_Iu_R+h_Iu_R+-g_Ru_I+h_Ru_I+k_Iz_R+k_Rz_I] \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} v_R+iv_I \\ w_R+iw_I \\ v_R-iv_I \end{pmatrix} = \begin{pmatrix} [(a_R+b_R)u_R+(a_I-b_I)u_I+c_Rz_R-c_Iz_I]+i[(a_I+b_I)u_R+(-a_R+b_R)u_I+c_Iz_R+c_Rz_I] \\ [(d_R+e_R)u_R+(d_I-e_I)u_I+f_Rz_R-f_Iz_I]+i[(d_I+e_I)u_R+(-d_R+e_R)u_I+f_Iz_R+f_Rz_I] \\ [(g_R+h_R)u_R+(g_I-h_I)u_I+k_Rz_R-k_Iz_I]+i[(g_I+h_I)u_R+(-g_R+h_R)u_I+k_Iz_R+k_Rz_I] \end{pmatrix}$$

$$\Rightarrow \begin{cases} v_R=[(a_R+b_R)u_R+(a_I-b_I)u_I+c_Rz_R-c_Iz_I] & v_I=[(a_I+b_I)u_R+(-a_R+b_R)u_I+c_Iz_R+c_Rz_I] \\ w_R=[(d_R+e_R)u_R+(d_I-e_I)u_I+f_Rz_R-f_Iz_I] & w_I=[(d_I+e_I)u_R+(-d_R+e_R)u_I+f_Iz_R+f_Rz_I] \\ v_R=[(g_R+h_R)u_R+(g_I-h_I)u_I+k_Rz_R-k_Iz_I] & -v_I=[(g_I+h_I)u_R+(-g_R+h_R)u_I+k_Iz_R+k_Rz_I] \end{cases}$$

$$\Rightarrow \begin{cases} v_R=(a_R+b_R)u_R+(a_I-b_I)u_I+c_Rz_R-c_Iz_I & v_I=(a_I+b_I)u_R+(-a_R+b_R)u_I+c_Iz_R+c_Rz_I \\ v_R=(g_R+h_R)u_R+(g_I-h_I)u_I+k_Rz_R-k_Iz_I & v_I=(-g_I-h_I)u_R+(g_R-h_R)u_I-k_Iz_R-k_Rz_I \\ w_R=(d_R+e_R)u_R+(d_I-e_I)u_I+f_Rz_R-f_Iz_I & w_I=(d_I+e_I)u_R+(-d_R+e_R)u_I+f_Iz_R+f_Rz_I \end{cases}$$

$$\Rightarrow \begin{cases} v_R=[(a_R+b_R)u_R+(a_I-b_I)u_I+c_Rz_R-c_Iz_I] & v_I=[(a_I+b_I)u_R+(-a_R+b_R)u_I+c_Iz_R+c_Rz_I] \\ w_R=[(d_R+e_R)u_R+(d_I-e_I)u_I+f_Rz_R-f_Iz_I] & w_I=[(d_I+e_I)u_R+(-d_R+e_R)u_I+f_Iz_R+f_Rz_I] \\ v_R=[(g_R+h_R)u_R+(g_I-h_I)u_I+k_Rz_R-k_Iz_I] & -v_I=[(g_I+h_I)u_R+(-g_R+h_R)u_I+k_Iz_R+k_Rz_I] \end{cases}$$

$$\Rightarrow \begin{cases} v_R=(a_R+b_R)u_R+(a_I-b_I)u_I+c_Rz_R-c_Iz_I & v_I=(a_I+b_I)u_R+(-a_R+b_R)u_I+c_Iz_R+c_Rz_I \\ v_R=(g_R+h_R)u_R+(g_I-h_I)u_I+k_Rz_R-k_Iz_I & v_I=(-g_I-h_I)u_R+(g_R-h_R)u_I-k_Iz_R-k_Rz_I \\ w_R=(d_R+e_R)u_R+(d_I-e_I)u_I+f_Rz_R-f_Iz_I & w_I=(d_I+e_I)u_R+(-d_R+e_R)u_I+f_Iz_R+f_Rz_I \end{cases}$$

for arbitrary u_R, u_I, z_R, z_I :

$$\Rightarrow \begin{cases} (g_R+h_R)=(a_R+b_R) & (g_I-h_I)=(a_I-b_I) & k_R=c_R & -k_I=-c_I \\ (-g_I-h_I)=(a_I+b_I) & (g_R-h_R)=(-a_R+b_R) & -k_I=c_I & -k_R=c_R \end{cases}$$

(since w_R, w_I are unknown/undetermined functions of u_R, u_I, z_R, z_I)

$$\Rightarrow \begin{cases} g_R+h_R=a_R+b_R & g_I-h_I=a_I-b_I & k_R=c_R=0 \\ g_R-h_R=-a_R+b_R & -g_I-h_I=a_I+b_I & k_I=c_I=0 \end{cases}$$

$$\Rightarrow \begin{cases} g_R=b_R & g_I=-b_I & k_R=c_R=0 \\ h_R=a_R & h_I=-a_I & k_I=c_I=0 \end{cases}$$

$$\Rightarrow \begin{pmatrix} a_R+ia_I & b_R+ib_I & 0 \\ d_R+id_I & e_R+ie_I & f_R+if_I \\ b_R-ib_I & a_R-ia_I & 0 \end{pmatrix} = \begin{pmatrix} a & b & 0 \\ d & e & f \\ b^* & a^* & 0 \end{pmatrix}$$

These are 10-dimensional.
Thus, the two anchors *w* & *z* may not be considered indpendent.
Thus, the picture of the above equivalent symmetric-anchor types is sufficient.
So, using transformation matrix:

$$\begin{pmatrix} a & b & c \\ d & e_R & d^* \\ c^* & b^* & a^* \end{pmatrix} = \begin{pmatrix} a_R+ia_I & b_R+ib_I & c_R+ic_I \\ d_R+id_I & e_R & d_R-id_I \\ c_R-ic_I & b_R-ib_I & a_R-ia_I \end{pmatrix}$$

$$= a_R\begin{pmatrix} 1&0&0\\0&0&0\\0&0&1 \end{pmatrix} + b_R\begin{pmatrix} 0&1&0\\0&0&0\\0&1&0 \end{pmatrix} + c_R\begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix} + d_R\begin{pmatrix} 0&0&0\\1&0&1\\0&0&0 \end{pmatrix} + e_R\begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix} +$$
$$+ ia_I\begin{pmatrix} 1&0&0\\0&0&0\\0&0&-1 \end{pmatrix} + ib_I\begin{pmatrix} 0&1&0\\0&0&0\\0&-1&0 \end{pmatrix} + ic_I\begin{pmatrix} 0&0&1\\0&0&0\\-1&0&0 \end{pmatrix} + id_I\begin{pmatrix} 0&0&0\\1&0&-1\\0&0&0 \end{pmatrix}$$

$$= a_R\begin{pmatrix} 1&0&0\\0&1&0\\0&0&1 \end{pmatrix} +$$
$$+ b_R\begin{pmatrix} 0&1&0\\0&0&0\\0&1&0 \end{pmatrix} + c_R\begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix} + d_R\begin{pmatrix} 0&0&0\\1&0&1\\0&0&0 \end{pmatrix} + (e_R-a_R)\begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix} +$$
$$+ ia_I\begin{pmatrix} 1&0&0\\0&0&0\\0&0&-1 \end{pmatrix} + ib_I\begin{pmatrix} 0&1&0\\0&0&0\\0&-1&0 \end{pmatrix} + ic_I\begin{pmatrix} 0&0&1\\0&0&0\\-1&0&0 \end{pmatrix} + id_I\begin{pmatrix} 0&0&0\\1&0&-1\\0&0&0 \end{pmatrix} \quad (21)$$

8-dimensional basis.
With: d = b* & *eR* = - 2*aR* :

$$\begin{pmatrix} a & b & c \\ d & e_R & d^* \\ c^* & b^* & a^* \end{pmatrix} = \begin{pmatrix} a & b & c \\ b^* & -2a_R & b \\ c^* & b^* & a^* \end{pmatrix} \quad (21a)$$

makes this traceless equivalent to the Gell-Mann matrices

$$\begin{pmatrix} \lambda_9 & \lambda_1^* & \lambda_4^* \\ \lambda_1 & \lambda_{10} & \lambda_6^* \\ \lambda_4 & \lambda_6 & -\lambda_9-\lambda_{10} \end{pmatrix} = \begin{pmatrix} \lambda_3+\lambda_8 & \lambda_1-i\lambda_2 & \lambda_4-i\lambda_5 \\ \lambda_1+i\lambda_2 & -\lambda_3+\lambda_8 & \lambda_6-i\lambda_7 \\ \lambda_4+i\lambda_5 & \lambda_6+i\lambda_7 & -2\lambda_8 \end{pmatrix} =$$

$$= \lambda_1\begin{pmatrix} 0&1&0\\1&0&0\\0&0&0 \end{pmatrix} + \lambda_2\begin{pmatrix} 0&-i&0\\i&0&0\\0&0&0 \end{pmatrix} + \lambda_3\begin{pmatrix} 1&0&0\\0&-1&0\\0&0&0 \end{pmatrix} +$$
$$+ \lambda_4\begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix} + \lambda_5\begin{pmatrix} 0&0&-i\\0&0&0\\i&0&0 \end{pmatrix} +$$
$$+ \lambda_6\begin{pmatrix} 0&0&0\\0&0&1\\0&1&0 \end{pmatrix} + \lambda_7\begin{pmatrix} 0&0&0\\0&0&-i\\0&i&0 \end{pmatrix} + \lambda_8\begin{pmatrix} 1&0&0\\0&1&0\\0&0&-2 \end{pmatrix}$$

$$= \begin{pmatrix} \lambda_3+\lambda_8 & \lambda_1-i\lambda_2 & \lambda_4-i\lambda_5 \\ \lambda_1+i\lambda_2 & -\lambda_3+\lambda_8 & \lambda_6-i\lambda_7 \\ \lambda_4+i\lambda_5 & \lambda_6+i\lambda_7 & -2\lambda_8 \end{pmatrix}$$

The non-singular linear transformation of the quarks to one another establishes a group[25].
Though it may be a subgroup including quark-quark-anti-quark and quark-anti-quark-anti-quark, there is not
evience supporting this[16].

Using the fermion triple table, above, mesons & baryons may be noted as:

$$u_R : \overline{u_R} \sim w(0,1,1) : \overline{w(0,1,1)}$$
$$u_R : u_G : d_B \sim w(0,1,1) : w(1,0,1) : \overline{w(0,0,1)}$$

For the mesons, the transformation is:

$$\rho_X(h) : \overline{\sigma_X(j)} \Rightarrow \eta_0(\rho_X(m)) : \eta_0\left(\overline{\sigma_X(n)}\right)$$

(this is equivalent to: $\rho_X(h) : \overline{\sigma_X(j)} \Rightarrow \overline{\rho_X(m)} : \sigma_X(n)$)

For the baryons, the transformation is:

$$\rho_R(h) : \sigma_G(j) : \sigma_B(k) \Rightarrow \eta_\alpha(\rho_R(m)) : \eta_\beta(\rho_G(m)) : \eta_\gamma(\sigma_B(n))$$

(where any pair of α, β, γ are 0 and the third is NOT,

(because it is an ordinary fermion interaction netween two ingredients)
(a chain of any single transformation is sufficient for any transformation)
For example:

$$\begin{aligned}
& d_R : u_G : d_B \sim \overline{w(1,0,0)} : w(1,0,1) : \overline{w(0,0,1)} \\
& \Rightarrow \eta_0(\overline{d_R}) : \overline{\eta_0(u_G)} : d_B \sim w(0,1,1) : \overline{w(0,1,0)} : \overline{w(0,0,1)} = u_R : d_G : d_B \\
& \Rightarrow d_R : \overline{\eta_0(u_G)} : \eta_0(\overline{d_B}) \sim \overline{w(1,0,0)} : \overline{w(0,1,0)} : w(1,1,0) = d_R : d_G : u_B \\
& \Rightarrow \eta_0(\overline{d_R}) : u_G : \eta_0(\overline{d_B}) \sim w(0,1,1) : w(1,0,1) : w(1,1,0) = u_R : u_G : u_B \\
& d_R : d_G : d_B \sim \overline{w(1,0,0)} : \overline{w(0,1,0)} : \overline{w(0,0,1)} \\
& \Rightarrow \eta_0(\overline{d_R}) : \eta_0(\overline{d_G}) : d_B \sim w(0,1,1) : w(1,0,1) : \overline{w(0,0,1)} = u_R : u_G : d_B \\
& \Rightarrow d_R : \eta_0(\overline{d_G}) : \eta_0(\overline{d_B}) \sim \overline{w(1,0,0)} : w(1,0,1) : w(1,1,0) = d_R : u_G : u_B \\
& \Rightarrow \eta_0(\overline{d_R}) : d_G : \eta_0(\overline{d_B}) \sim w(0,1,1) : \overline{w(0,1,0)} : w(1,1,0) = u_R : d_G : u_B
\end{aligned}$$

As noted above, all the possible quark triplets are given by:

$u_R(h) : u_G(j) : u_B(k)$	$u_R(h) : u_G(j) : d_B(k)$	$u_R(h) : d_G(j) : u_B(k)$	$u_R(h) : d_G(j) : d_B(k)$
$d_R(h) : d_G(j) : d_B(k)$	$d_R(h) : d_G(j) : u_B(k)$	$d_R(h) : u_G(j) : d_B(k)$	$d_R(h) : u_G(j) : u_B(k)$

(Notice that casual appearance suggests a 8-dimensional double-cover)

All baryon quark-triples seem to be of the form quark-quark-quark (or anti-quark-anti-quark-anti-quark); and not including quark-quark-ant-quark or quark-anti-quark-anti-quark[].

There are six elements in the **RGB** color triplet set (baryon):
{(**R**,**G**,**B**, (**G**,**R**,**B**), (**G**,**B**,**R**), (**R**,**B**,**G**), (**B**,**R**,**G**), (**B**,**G**,**R**)}

As with the mesons, the following scheme has been conjectured for baryon transformations without violating empty space, via 'virtual' particles.

i.e.: at every point there exists two "virual" particle (particle-anti-particle) pairs equivalent to empty space except for total-energy (like pushing onto he stack and poping it off).

The color triplet permutation operation on the three quarks enables continued 'existence'.

Baryon $\rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k)$ flavour transformations:

$$\begin{aligned}
& \rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(j) : \overline{\rho}_\Pi(h) + \sigma_\Phi(h) : \overline{\sigma}_\Phi(j)] \\
& \Downarrow \\
& \rho_\Pi(j) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\sigma}_\Phi(j) + \overline{\rho}_\Pi(h) : \sigma_\Phi(h)] \\
& \Downarrow \\
& \rho_\Pi(j) : \sigma_\Phi(h) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Pi(h) + \overline{\sigma}_\Phi(j) : \sigma_\Phi(j)]
\end{aligned}$$

and:

$$\begin{aligned}
& \rho_\Pi(j) : \sigma_\Phi(h) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Pi(j) + \overline{\sigma}_\Phi(h) : \sigma_\Phi(j)] \\
& \Downarrow \\
& \rho_\Pi(h) : \sigma_\Phi(h) : \theta_\Psi(k) + [\rho_\Pi(j) : \overline{\rho}_\Pi(h) + \sigma_\Phi(h) : \overline{\sigma}_\Phi(j)] \\
& \Downarrow \\
& \rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(j) : \overline{\rho}_\Pi(h) + \sigma_\Phi(h) : \overline{\sigma}_\Phi(j)]
\end{aligned}$$

$$\rho, \sigma, \theta \in \{u, d\} \ \& \ \Pi, \Phi, \Psi \in \{1, 0, -1 \mid \Pi \neq \Phi, \Pi \neq \Psi, \Phi \neq \Psi\}$$

$$\& \ h, j, k \in \{1, 2, 3\}$$

These transformations are sufficient to describe all permutations (simply change designations as necessary).

$$\text{(i.e.: } (\Pi, \Phi, \Psi) \Rightarrow (\Phi, \Pi, \Psi) \Rightarrow (\Phi, \Psi, \Pi) \Rightarrow (\Pi, \Psi, \Phi)$$

$$\Rightarrow (\Psi, \Pi, \Phi) \Rightarrow (\Psi, \Phi, \Pi))$$

Baryon $\rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k)$ color & flavour transformations:

$$\begin{aligned}
& \rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Phi(m) : \overline{\sigma}_\Phi(s) + \sigma_\Pi(r) : \overline{\rho}_\Pi(n)] \\
& \Downarrow \\
& \rho_\Phi(m) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\sigma}_\Phi(s) + \sigma_\Pi(r) : \overline{\rho}_\Pi(n)] \\
& \Downarrow \\
& \rho_\Phi(m) : \sigma_\Pi(r) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\sigma}_\Phi(s) + \sigma_\Phi(j) : \overline{\rho}_\Pi(n)] \\
& \| \\
& \rho_\Phi(m) : \sigma_\Pi(r) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Pi(n) + \sigma_\Phi(j) : \overline{\sigma}_\Phi(s)]
\end{aligned}$$

and:

$$\begin{aligned}
& \rho_\Phi(m) : \sigma_\Pi(r) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Pi(n) + \sigma_\Phi(j) : \overline{\sigma}_\Phi(s)] \\
& \Downarrow \\
& \rho_\Phi(m) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Pi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Phi(s)] \\
& \Downarrow \\
& \rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Phi(m) : \overline{\rho}_\Pi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Phi(s)] \\
& \|
\end{aligned}$$

and:

$$\begin{aligned}
& \rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Phi(m) : \overline{\rho}_\Phi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Pi(s)] \\
& \Downarrow \\
& \rho_\Phi(m) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Phi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Pi(s)] \\
& \Downarrow \\
& \rho_\Phi(m) : \sigma_\Pi(r) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Phi(n) + \sigma_\Phi(j) : \overline{\sigma}_\Pi(s)] \\
& \| \\
& \rho_\Phi(m) : \sigma_\Pi(r) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\sigma}_\Pi(s) + \sigma_\Phi(j) : \overline{\rho}_\Phi(n)]
\end{aligned}$$

and:

$$\begin{aligned}
& \rho_\Phi(m) : \sigma_\Pi(r) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\sigma}_\Pi(s) + \sigma_\Phi(j) : \overline{\rho}_\Phi(n)] \\
& \Downarrow \\
& \rho_\Phi(m) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Phi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Pi(s)] \\
& \Downarrow \\
& \rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Phi(m) : \overline{\rho}_\Phi(n) + \sigma_\Pi(r) : \overline{\sigma}_\Pi(s)]
\end{aligned}$$

where: $\rho, \sigma, \theta \in \{u, d\} \ \& \ \Pi, \Phi, \Psi \in \{1, 0, -1 \mid$

$$\Pi \neq \Phi, \Pi \neq \Psi, \Phi \neq \Psi\} \ \& \ h, j, k \in \{1, 2, 3\}$$

Again, these transformations are sufficient to describe all permutations
(simply change designations as necessary).

$$(\text{i.e.: } (\Pi,\Phi,\Psi) \Rightarrow (\Phi,\Pi,\Psi) \Rightarrow (\Phi,\Psi,\Pi) \Rightarrow (\Pi,\Psi,\Phi)$$
$$\Rightarrow (\Psi,\Pi,\Phi) \Rightarrow (\Psi,\Phi,\Pi))$$

All the permutations are handled by this operation (perhaps randomly, not necessarily in any order)
A pair of virtual weak/strong mesons combines in and a pair of virtual strong/weak mesons uncombines out.
(with charge & color conservation).

Baryon $\rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k)$ color transformations:

$(\Pi,\Phi,\Psi) \Rightarrow (\Phi,\Pi,\Psi)$:

$$\rho_\Pi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Phi(h) : \overline{\rho}_\Phi(n) + \sigma_\Pi(j) : \overline{\sigma}_\Pi(s)]$$
$$\Downarrow$$
$$\rho_\Phi(h) : \sigma_\Phi(j) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\rho}_\Phi(n) + \sigma_\Pi(j) : \overline{\sigma}_\Pi(s)]$$
$$\Downarrow$$
$$\rho_\Phi(h) : \sigma_\Pi(j) : \theta_\Psi(k) + [\rho_\Pi(h) : \overline{\sigma}_\Pi(s) + \sigma_\Phi(j) : \overline{\rho}_\Phi(n)]$$

$(\Phi,\Pi,\Psi) \Rightarrow (\Phi,\Psi,\Pi)$:

$$\rho_\Phi(h) : \sigma_\Pi(j) : \theta_\Psi(k) + [\sigma_\Psi(j) : \overline{\sigma}_\Psi(r) + \theta_\Pi(k) : \overline{\theta}_\Pi(s)]$$
$$\Downarrow$$
$$\rho_\Phi(h) : \sigma_\Psi(j) : \theta_\Psi(k) + [\sigma_\Pi(j) : \overline{\sigma}_\Psi(r) + \theta_\Pi(k) : \overline{\theta}_\Pi(s)]$$
$$\Downarrow$$
$$\rho_\Phi(h) : \sigma_\Psi(j) : \theta_\Pi(k) + [\sigma_\Pi(j) : \overline{\theta}_\Pi(s) + \theta_\Psi(k) : \overline{\sigma}_\Psi(r)]$$

$(\Phi,\Psi,\Pi) \Rightarrow (\Pi,\Psi,\Phi)$:

$$\rho_\Phi(h) : \sigma_\Psi(j) : \theta_\Pi(k) + [\rho_\Pi(h) : \overline{\rho}_\Pi(r) + \theta_\Phi(k) : \overline{\theta}_\Phi(s)]$$
$$\Downarrow$$
$$\rho_\Pi(h) : \sigma_\Psi(j) : \theta_\Pi(k) + [\rho_\Phi(h) : \overline{\rho}_\Pi(r) + \theta_\Phi(k) : \overline{\theta}_\Phi(s)]$$
$$\Downarrow$$
$$\rho_\Pi(h) : \sigma_\Psi(j) : \theta_\Phi(k) + [\theta_\Pi(k) : \overline{\rho}_\Pi(r) + \rho_\Phi(h) : \overline{\theta}_\Phi(s)]$$

$(\Pi,\Psi,\Phi) \Rightarrow (\Psi,\Pi,\Phi)$:

$$\rho_\Pi(h) : \sigma_\Psi(j) : \theta_\Phi(k) + [\rho_\Psi(h) : \overline{\rho}_\Psi(r) + \sigma_\Pi(j) : \overline{\sigma}_\Pi(s)]$$
$$\Downarrow$$
$$\rho_\Psi(h) : \sigma_\Psi(j) : \theta_\Phi(k) + [\rho_\Pi(h) : \overline{\rho}_\Psi(r) + \sigma_\Pi(j) : \overline{\sigma}_\Pi(s)]$$
$$\Downarrow$$
$$\rho_\Psi(h) : \sigma_\Pi(j) : \theta_\Phi(k) + [\sigma_\Psi(j) : \overline{\rho}_\Psi(r) + \rho_\Pi(h) : \overline{\sigma}_\Pi(s)]$$

$(\Psi,\Pi,\Phi) \Rightarrow (\Psi,\Phi,\Pi)$:

$$\rho_\Psi(h) : \sigma_\Pi(j) : \theta_\Phi(k) + \left[\sigma_\Phi\left(j\right) : \overline{\sigma}_\Phi(r) + \theta_\Pi(k) : \overline{\theta}_\Pi(s)\right]$$
$$\Downarrow$$
$$\rho_\Psi(h) : \sigma_\Phi\left(j\right) : \theta_\Phi(k) + [\sigma_\Pi(j) : \overline{\sigma}_\Phi(r) + \theta_\Pi(k) : \overline{\theta}_\Pi(s)]$$
$$\Downarrow$$
$$\rho_\Psi(h) : \sigma_\Phi\left(j\right) : \theta_\Pi(k) + \left[\theta_\Phi(k)\,\overline{\sigma}_\Phi(r) + \sigma_\Pi(j) : \overline{\theta}_\Pi(s)\right]$$

where:

$$\rho,\sigma,\theta \in \{u,d\} \ \& \ \Pi,\Phi,\Psi \in \{1,0,-1 \mid \Pi \neq \Phi, \Pi \neq \Psi, \Phi \neq \Psi\}$$
$$\& \ h,j,k \in \{1,2,3\}$$

Just as with the mesons the property that the anti-object is the complex conjugate of the corresponding object was the only fundamental principle required for the analysis; for the baryons two facts are fundamental:
1) the order of the triplet is immaterial to it's description (also for mesons via the anti-meson complex conjugate)

$$\text{i.e: } \rho_\Psi(h) : \sigma_\Phi\left(j\right) : \theta_\Pi(k) = \left(\rho_\Psi(h), \sigma_\Phi\left(j\right), \theta_\Pi(k)\right)$$

is an equivalence class

2) the set of the colors of the triplet is {**R,G,B**}; i.e.

$$\rho(h) : \sigma\left(j\right) : \theta(k) = \rho_\Psi(h) : \sigma_\Phi\left(j\right) : \theta_\Pi(k)$$
$$= \sigma_\Phi\left(j\right) : \rho_\Psi(h) : \theta_\Pi(k)$$
$$= \sigma_\Phi\left(j\right) : \theta_\Pi(k) : \rho_\Psi(h) = \rho_\Psi(h) : \theta_\Pi(k) :$$
$$\sigma_\Phi\left(j\right) = \theta_\Pi(k) : \rho_\Psi(h) : \sigma_\Phi\left(j\right)$$

$$\text{where: } \rho,\sigma,\theta \in \{u,d\} \ \& \ \Pi,\Phi,\Psi \in \{1,0,-1 \mid$$
$$\Pi \neq \Phi, \Pi \neq \Psi, \Phi \neq \Psi\} \ \& \ h,j,k \in \{1,2,3\}$$

3) the transformation is accomplished by swapping the color of any two objects

However, as with the mesons, it is an open question as to whether or not these baryon processes are consistent with the baryon transformations.

However, how is it that *p0q0* is not a meson, and *p0q0r0* not a baryon?
Quarks are fermions, just as electrons are, satisfying Bose–Einstein statisticso a collection of non-interacting indistinguishable particles may not occupy a set of available discrete energy states. i.e. quarks of the same color index cannot simultaneously occupy the same place. So, since there are only quarks in the triplets, and there are no color duplications, there must be one-and-only-one of each color type, and thus, the sum of the color indices: -1+ 0+ 1 = 0.

Conclusion

Thus, the insights provided by the constuctive algebras developable from the weighted matrix product leading to the d'Alembertian operator and it's factorization, leading further to the Helmholtzian operator and factorization, from which the Maxwell-Cassano Equations arise generating the fermion architecture provides firm mathematical foundation of the hadrons (masons and baryons) as just demonstrated. As shown above, the color notion is better understood using integral indices; so, up to this point, the fermion architecture is clearly described via table:

$e = e\,(1) = \overline{(E^1, E^2, E^3)}_{\,1}$	$\mu = e(2) = \overline{(E^1, E^2, E^3)}_2$	$\tau = e(3) = \overline{(E^1, E^2, E^3)}_3$
$v_e = v\,(1) = (B^1, B^2, B^3)_{\,1}$	$v_\mu = v\,(2)\,(B^1,B^2,B^3)_{\,2}$	$v_t = v(3) = (B^1, B^2, B^3)_3$
$u_R = u_1(1) = B^1, E^2, E^3)_{\,1}$	$c_R\ u_1(2) = (B^1,E^2,E^3)_{\,2}$	$t_R = u_1\,(3) = (B^1, E^2, E^3)_3$
$u_G\ u_2\,(1) = (E^1, B^2, E^3)_{\,1}$	$c_G = u_0(2) = (E^1, B^2, E^3)_{\,2}$	$t_G = u_0\,(3) = (E^1, B^2, E^3)_3$
$u_B = u_3\,(1) = (E^1,E^2,B^3)_{\,1}$	$c_B = u_{-1}\,(2) = (E^1, E^2, B^3)_2$	$t_B = u_{-1}(3) = (E^1,E^2,B^3)_{\,3}$
$d_R = d_1\,(1) = \overline{(E^1,B^2,B^3)}_{\,1}$	$s_R = d_1(2) = \overline{E^1, B^2, B^3)}_{\,2}$	$b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$
$d_G = d_2\,(1) = \overline{B^1,E^2,B^3)}_{\,1}$	$sG = d_0\,(2) = \overline{(B^1, E^2, B^3)}_2$	$b_G = d_0(3) = \overline{B^1, E^2, B^3)}_3$
$d_B = d_3\,(1) = \overline{(B^1, B^2,E^3)}_{\,1}$	$s_B = d_{-1}\,(2) = \overline{B^1, B^2, E^3)}_2$	$b_B = d_{-1}\,(3) = \overline{(B^1,B^2,E^3)}_{\,3}$

(22)

(Obviously, the original designer did not go through the RGB designations, but began with the numerical indices.)
(-the RGB indexes have been used above to allow quick and easy correspondence and transition to this fundamental description)

And, of course, the up/down concept yields to the simple η_0 transformation on each one's constituents-triplet (the same η_0

transformation generating the fermion interactions).

So, just as the fool on the hill sees the sun going down and the eyes in his head see the world spinning 'round, the fermions may be viewed as constituents-triplets.

I hope I have shined a light to see through the fog.

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