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Quantum Elliptic Curve

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Abstract

The present paper summarizes the author's work on Elliptic Curve and its Rational Points. We derive *new Quantum Mechanical Equation* from the Quantum Mechanics first principles.

We then describe **the Quantum Entanglement Mechanism,** based on the link between the **two Quantum Elliptic Curves** author first derived in the original version of the present paper in October 2020. The two Quantum Elliptic Curves represent states of the two entangled particles forming an EPR-pair (aka Bell state).

Also we present two methods of manipulation of state of an EPR-pair. Together they provide algorithm for the **Quantum Teleportation.**

Keywords and Phrases: Hydrogen Atom Energy Levels, Quantum Zeeman and Stark effects, Elliptic Curve Triangulation, Rational Points degeneracy, Elliptic Curve ABC ansatz and algorithm, Quantum Elliptic Curve, Quantum Entanglement Mechanism, Motley String Theory, Theory of Everything, ToE.

1. Elliptic Curve Triangulation

It has been demonstrated by author in [18] that any nonsingular Elliptic Curve *E*

$$y^2 = x(x-r_1)(x-r_2) \tag{1}$$

can be parametrized using ancient formula (generally attributed to Heron) for area *S* of a triangle with sides (*a, b, c*) given by the equation:

$$S = \sqrt{p(p-a)(p-b)(p-c)} \tag{2}$$

where *p* is semi-perimeter:

$$p = \frac{a+b+c}{2} \tag{3}$$

Introducing parameter t:

$$t = \frac{p-a}{p} \tag{4}$$

and then dividing S^2 by p^2, Elliptic curve *E* may be given new form:

$$y^2 = x[x^2 + Ax + B] = x(x-r_1)(x-r_2) \tag{5}$$

Where we have for coordinates (*x, y*) of a point on Elliptic curve:

$$\begin{cases} x = tbc = \frac{p-a}{p}bc \\ y = \frac{Sabc}{p^2} \end{cases} \tag{6}$$

Elliptic curve roots, coefficients and triangle sides are related via:

$$\begin{cases} r_1 r_2 = B = (a-b)(a-c) \\ r_1 + r_2 = -A = 2bc - a(b+c) \end{cases} \tag{7}$$

Which means that as soon as we find a triangle △(a,b,c) with rational sides and rational area *S*, we shall automatically get rational point (*x, y*) on Elliptic Curve (5) with coordinates given by (6).

We begin by expressing sides b and c as functions of curve's coefficients *A* and *B*, keeping side a of triangle △(a,b,c) free parameter.

From the first equation for roots and curve coefficients in (7) we have:

$$\begin{cases} b = \frac{A-ac}{a-2c} \\ b + c = \frac{A-2c^2}{a-2c} \end{cases} \tag{8}$$

Inserting this into the second equation of (7) we get the equation for c:

$$ac^2 + c(A - 2(a^2 - B)) - a(A + B) = 0 \quad (9)$$

For discriminant D of this equation we have:

$$D = (A + 2B)^2 + (2a)^2(1 - 2A - 3B) \quad (10)$$

And therefore for c:

$$c = \frac{2a^2 - 2B - A \pm \sqrt{D}}{2a} = f(a, b, c) \quad (11)$$

And for side b:

$$b = \frac{A - ac}{a - 2c} = g(a, b, c) \quad (12)$$

We can also express A and B in terms of triangle sides (*a, b, c*) and obtain the following for the discriminant D:

$$D = 4a^3(b + c) - 8a^4 + 7a^2(b^2 + c^2) - 20a^2bc + 4a^2 - 12a(b + c) + 16bc \quad (13)$$

Given side a is rational and D is a square of integer or rational number, sides c and b will be rational too, therefore giving us triangle $\triangle(a,b,c)$ with rational sides.

Numerical results available in the *TEC* paper made it clear that majority of integer points on elliptic curve are produced by several (*a, b, c*) triplets and thus were called *degenerate*. This property has all important implications for the Quantum Physics, as we shall see below.

This is the basics of Elliptic Curve Triangulation method. For more details with numerical examples and interesting formula for the j – *invariant* see my original paper "Elliptic Curve Triangulation" [18].

At this point we have (at least) two different options to chose from, leading to two different algorithms: *GYM* algorithm for *search* of rational points devised in 2015 and *ABC* algorithm for *generation* of rational points devised in 2017.

2. Brahmagupta formula and *GYM* algorithm

One can start from triangles with integer sides and integer area originating from the explicit parametrization found by Brahmagupta [5]:

$$\begin{cases} a = n(m^2 + k^2) \\ b = m(k^2 + n^2) \\ c = (m + n)(mn - k^2) \\ S = kmn(m + n)(mn - k^2) \\ where\ \mathrm{k, m, n} \in \mathbb{N}, with\ k^2 < mn \end{cases} \quad (14)$$

Given a triangle $\triangle$(a,b,c) with Integer sides and Integer area *S*, one can always produce new triangle with Rational sides $\triangle$(a/d,b/d,c/d) and Rational area S_r using integer d as "tuning parameter":

$$S_r = \frac{S}{d^2} \quad (15)$$

GYM algorithm is about finding a triangle with rational sides, rational area and $D = r^2$ for some rational r. It requires exhaustive search among all possible triplets $\triangle$(*a, b, c*) and therefore implies multithreaded application and considerable computing power.

3. Elliptic Curve *ABC* ansatz and algorithm

Much more effective algorithm for Rational Points *generation* on Elliptic Curve, which I discovered in the spring of 2017, is based on simple ansatz $A \cdot B = C$, see [21] for details and numerical results.

We begin by attempting to find a solution to Elliptic Curve equation (5) using simple *ansatz*:

$$y^2 = x[x^2 + Ax + B] = x(x - r_1)(x - r_2) = abc = (ab)c = c^2, \quad (16)$$

i.e. we are looking for a solution to the following system

$$\begin{cases} x = a \\ x - r_1 = b \\ x - r_2 = c \end{cases} \quad (17)$$

where

$$x(x - r_1) = x^2 - xr_1 = a \cdot b = x - r_2 = c \quad (18)$$

and roots r_1 and r_2 of elliptic curve are related to the curves coefficients via

$$\begin{cases} A_2 = -(r_1 + r_2) \\ A_4 = r_1 \cdot r_2 \end{cases} \quad (19)$$

and thus we can express roots via coefficients

$$\begin{cases} r_1 = \frac{-A_2 \pm \sqrt{A_2^2 - 4A_4}}{2} \\ r_2 = \frac{A_4}{r_1} \end{cases} \quad (20)$$

From the equation (18) we see that

$$x^2 - x(r_1 + 1) + r_2 = 0 \quad (21)$$

which is solvable if there exists k, such that

$$D_1 = (r_1 + 1)^2 - 4r_2 = r_1^2 + 2r_1 + 1 - 4r_2 = k^2,\ for\ some\ k. \quad (22)$$

Given such k, we obtain from ansatz (16) for x and y coordinates

$$\begin{cases} x_{1,2} = \frac{(r_1+1)\pm k}{2} \\ y_{1,2} = ab = x(x-r_1) = c = x_{1,2} - r_2 \end{cases} \quad (23)$$

Solution to (22) exists if we can find *l*, such that

$$D_2 = 4 - 4(1 - 4r_2 - k^2) = 16r_2 + 4k^2 = l^2, \quad for\ some\ l. \quad (24)$$

Given such *l*, we can express the root r_2 via

$$r_2 = \frac{(l^2 - (2k)^2)}{16} \quad (25)$$

On the other hand solution of (21), which links the roots, implies that we find *m*, such that

$$D_3 = (r_1 + 1)^2 - 4r_2 = m^2, \ for\ some\ m. \quad (26)$$

Given such *m*, we can express root r_1 as

$$r_1 + 1 = \frac{\sqrt{4m^2 + (l^2 - (2k)^2)}}{2} \quad (27)$$

This means that in order to find a solution to our $A \cdot B = C$ ansatz (16) we need to find (*k, l, m*) triplet that will satisfy the following system related to Elliptic Curve with roots 0, r_1 and r_2:

$$\begin{cases} D_2 = 16r_2 + 4k^2 = l^2 \\ D_3 = (r_1 + 1)^2 - 4r_2 = m^2 \end{cases} \quad (28)$$

And then for the same *x* coordinate as in equation (23) we'll have

$$x_{1,2} = \frac{(r_1 + 1) \pm \sqrt{D_3}}{2} = \frac{(r_1 + 1) \pm m}{2} \quad (29)$$

Comparing equations (23) and (29) for $x_{1,2}$ we see that *k=m*.

As a result r_1+1 in (27) simplifies to

$$r_1 + 1 = \frac{l}{2} \quad (30)$$

Basically this means that we need to find a triplet (*k, l, m*), where *k* and *l* and *m* are parts of **two Pythagorean triplets** "*chained*" to each other via corresponding D_2 and D_3 in (28) and linked to the Elliptic Curve (5) via the two equations (20) for roots r_1 and r_2, to be able to solve our ansatz and find a Rational point on Elliptic Curve.

Simple constructive examples available in the "Elliptic Curve ABC" paper [21] demonstrate how the *ABC* algorithm works in practice.

4. Hydrogen Atom and Elliptic Curve

In his *Hydrogen Aton and Elliptic Curve, aka HEC* paper, delivered on "Quantum Mechanics and Nuclear Engineering" conference in Paris in September 2019, author established the *correspondence* between Rational Points on Elliptic Curve and Hydrogen Atom energy levels (aka bound states), using his two methods described above - *GYM* and *ABC*. See [22] for details.

The numerical results present in author's first paper on Elliptic Curve - *TEC* paper [18] made it clear that majority of integer points are generated by more than one triplet (*a, b, c*) and thus were called "*degenerate*".

Papers on Rational Points [20] and [21], as well as authors pet project *Elliptic Curve Applet* [19] confirmed this result.

The *manifestation of degeneracy* of Rational Points on Elliptic Curve is well known in Quantum Physics: Zeeman effect in magnetic field and Stark effect in electric field.

At this point it became clear to author, that there must exist certain very Special Elliptic Curve with information about All energy levels of hydrogenic atoms in the Universe (from H to *QDOTs*), which author called *God's Curve*, and it was time to find it.

5. The Special Elliptic Curve

To find energy levels of Hydrogen Atom one can use *either* Schrödinger equation analysis (like author did in his HEC paper [22]) or one can choose Hamiltonian approach.

Major advantage of the Hamiltonian approach is that one is free to use whatever coordinate system one feels comfortable with and is the most suitable for the task at hand [6, 13, 15, 16]. This is why we use this approach here.

We aim to determine the Hydrogen Atom energy levels in electric field like Schwarzschild and Epstein did independently and almost simultaneously in 1916 when analyzing the Stark effect ([3, 4]). Here we follow closely the brilliant treatment of this essential subject provided by Arthur E.Ruark and Harold C.Urey in their well known to specialists but very rare book [7].

Let us say that strong (100k Volt/cm field was used in 1913 by Stark) and uniform electric field *F* is directed along the *Z* axis. The potential energy E_{pot} of charge e in the field is *eF z*. We begin by expressing the orthogonal coordinates (*x, y, z*) via *parabolic* coordinates *ξ*, *η* and *ϕ* as:

$$\begin{cases} x = \xi\eta \cdot cos\phi \\ y = \xi\eta \cdot sin\phi \\ z = \frac{\xi^2 - \eta^2}{2} \end{cases} \quad (31)$$

This means that *ϕ* is the polar angle in the (x,y) plane.

But we also can express the standard *polar* coordinates (*ρ, r*) in the same orthogonal coordinates (*x, y, z*) like

$$\rho^2 = x^2 + y^2 \quad \text{and} \quad r^2 = x^2 + y^2 + z^2$$

And thus we re-write the parabolic coordinates *ξ* and *η* as:

$$\begin{cases} \rho^2 = \xi^2\eta^2 \\ \xi^2 = r + z \\ \eta^2 = r - z \\ r = \frac{\xi^2+\eta^2}{2} \end{cases} \quad (32)$$

From the above we see that surfaces ξ = const and η = const are two paraboloids that cut (y, z) plane in two parabolas:

$$\begin{cases} y_1^2 = -2\xi^2(z - \xi^2/2) \\ y_2^2 = 2\eta^2(z - \eta^2/2) \end{cases} \quad (33)$$

Since we are interested in knowing the energy of electron of mass m_e in this field using the Hamilton-Jacobi equation, we need to find the element ds of the arc, the two parabolas cut. In *cylindrical* coordinates the arc is given by the standard expression:

$$ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2 \quad (34)$$

Therefore, the kinetic energy T_{kin} of electron is given by the following expression:

$$T_{kin} = \frac{m_e}{2}\{(\xi^2+\eta^2)(\dot{\xi}^2+\dot{\eta}^2) + \xi^2\eta^2\dot{\phi}^2\}, \quad (35)$$

from which we obtain the momentum components:

$$\begin{cases} p_\xi = m_e(\xi^2+\eta^2)\dot{\xi} \\ p_\eta = m_e(\xi^2+\eta^2)\dot{\eta} \\ p_\phi = m_e\dot{\xi}^2\eta^2\dot{\phi} \end{cases} \quad (36)$$

It is clear from the above that p_ϕ is component of angular momentum around Z axis.

To understand the meaning of two other components of the momentum, we re-write them as

$$\begin{cases} p_\xi = m_e(\dot{r}+\dot{z})\frac{r}{\xi} \\ p_\eta = m_e(\dot{r}-\dot{z})\frac{r}{\eta} \end{cases} \quad (37)$$

Expressing energy T_{kin} in terms of the momentum components we obtain:

$$\frac{1}{2m_e(\xi^2+\eta^2)}\{p_\xi^2 + p_\eta^2 + (\frac{1}{\xi^2}+\frac{1}{\eta^2})p_\phi^2\} - \frac{2Ze^2}{\xi^2+\eta^2} + eF\frac{\xi^2-\eta^2}{2} = E \quad (38)$$

Because of the obvious rotational symmetry of the system, ϕ does not feature in the expression for energy above, and we realize that p_ϕ component of the momentum is constant.

Multiplication of the energy function by $2m_e(\xi^2+\eta^2)$ yields the following expression:

$$p_\xi^2 + p_\eta^2 + (\frac{1}{\xi^2}+\frac{1}{\eta^2})p_\phi^2 - 4m_eZe^2 + em_eF(\xi^4-\eta^4) = 2m_eE(\xi^2+\eta^2) \quad (39)$$

And this equation is clearly *separable* into ξ^2 and η^2 parts:

$$\begin{cases} p_\xi^2 = em_eF\xi^4 + 2m_eE\xi^2 + \alpha - \frac{p_\phi^2}{\xi^2} \\ p_\eta^2 = em_eF\eta^4 + 2m_eE\eta^2 + \beta - \frac{p_\phi^2}{\eta^2} \end{cases} \quad (40)$$

where we've introduced α and β such that:

$$\alpha + \beta = -4m_eZe^2 = const. \quad (41)$$

The simple quantum relation above provides the **All Important Link** between *the two parts* of the quantum system, which is the raison d'être for **the Quantum Entanglement** we discuss below.

The system of equations for p_ξ^2 and p_η^2 above is almost what we need from this analysis.

Multiplying both sides of the system of Elliptic Curves equations (40) respectively by ξ^2/em_e and η^2/em_e we obtain:

$$\begin{cases} p_\xi^2\xi^2/em_e = F\xi^6 + (2E/em_e)\xi^4 + \alpha\xi^2/em_e - p_\phi^2/em_e \\ p_\eta^2\eta^2/em_e = F\eta^6 + (2E/em_e)\eta^4 + \beta\eta^2/em_e - p_\phi^2/em_e \end{cases} \quad (42)$$

Introducing new variables x and y:

$$\begin{cases} x = \xi^2\sqrt[3]{F} \\ y = p_\xi\xi/\sqrt{em_e} \end{cases} \quad (43)$$

we return to the orthogonal (x, y) coordinates and finally arrive at the equation of Elliptic Curve related to the Quantum Stark effect:

$$y^2 = x^3 + (2E/em_eF^{2/3})x^2 + \alpha x/(em_e\sqrt[3]{F}) - p_\phi^2/em_e \quad (44)$$

with similar equation for η and p_η variables and β in place of α.

The quantized orbits of electron are obtained by applying the following conditions:

$$\begin{cases} \oint p_\xi d\xi = kh \\ \oint p_\eta d\eta = lh \\ \oint p_\phi d\phi = mh \end{cases} \quad (45)$$

where h is Planck's constant.

The rotational symmetry of the Stark experimental set up as well as the quantization conditions above imply that $p_\phi = \frac{mh}{2\pi} = m\hbar$, where m is called the **magnetic quantum number.**

Now we can recast our newly found **Quantum Elliptic Curve** into the familiar Number Theoretical form using the standard notations (see III.1 in [10] for details):

$$\begin{cases} a_1 = a_3 = 0 \\ a_2 = -(r_1 + r_2 + r_3) = 2E/em_eF^{2/3} \\ a_4 = r_1r_2 + r_1r_3 + r_2r_3 = \alpha/(em_e\sqrt[3]{F}) \\ a_6 = r_1r_2r_3 = -p_\phi^2/em_e \\ b_2 = 4a_2 = 8E/em_eF^{2/3} \\ b_4 = 2a_4 = 2\alpha/(em_e\sqrt[3]{F}) \\ b_6 = a_3^2 + 4a_6 = -4p_\phi^2/em_e \\ b_8 = a_1^2a_6 + 4a_2a_6 - a_1a_3a_4 + a_2a_3^2 - a_4^2 = -(8Ep_\phi^2 + \alpha^2)/(e^2m_e^2F^{2/3}) \end{cases} \tag{46}$$

and thus obtain the following for the Quantum Elliptic Curve c_4, the *discriminant* $\triangle$ and the *j-invariant*:

$$c_4 = b_2^2 - 24b_4 = \frac{64E^2 - 48em_eF\alpha}{e^2m_e^2F^{4/3}} \tag{47}$$

$$\triangle = -b_2^2b_8 - 8b_4^3 - 27b_6^2 + 9b_2b_4b_6 = 4\frac{128p_\phi^2E^3 + 16em_eF\alpha^3 + 27(em_eF)^2 + Eem_eF\alpha(12p_\phi)^2}{e^4m_e^4F^2} \tag{48}$$

$$j = c_4^3/\triangle = 2^{10}\frac{((2E)^2 - 3em_eF\alpha)^3}{(128p_\phi^2E^2 + 16em_eF\alpha^3 + 27(em_eF)^2 + Eem_eF\alpha(12p_\phi)^2)(em_eF)^2} \tag{49}$$

Keeping in mind the nature of the *eFz* as potential energy E_{pot} of electron in the electric field F along Z axis in the Stark Experiment, we can introduce new variable $U = eF \cdot 1m/1kg$ to be understood as energy of *unit mass* (Elliptic Curve is after all Mathematical object and cares not about kilograms or meters) and recast the formula for the Quantum Elliptic Curve j-invariant above in a more compact and physically meaningful form:

$$j = c_4^3/\triangle = 2^{10}\frac{((2E)^2 - 3U\alpha)^3}{(128p_\phi^2E^2 + 16U\alpha^3 + 27U^2 + EU\alpha(12p_\phi)^2)U^2} \tag{50}$$

The results of the calculations above make it 100% clear that there are **striking similarities** between the Number Theoretic approach author used in the *ABC* ansatz paper ([21]) and the Quantum Mechanics based analysis:

1. The resulting *two linked Quantum Elliptic Curve*s correspond to the *two "chained" Pythagorean triples (k, l,m)* we have seen in the *ABC* ansatz above.

2. The *link* between the two Quantum Elliptic Curves $\alpha + \beta = -4m_eZe^2$, where α and β belong to the p_ξ^2 and p_η^2 curves respectively, corresponds to the compatibility condition for the two discriminants D_2 and D_3 (28) in the *ABC* ansatz.

3. Most remarkably, the *structure* of the Rational Point coordinates obtained from the *ABC* ansatz **resembles** the structure of the Rational Points coordinates on the Quantum Elliptic Curves: in *both* cases y coordinate of a Rational Point contains part of x coordinate as a *hidden common factor!*

This fact may be used for development of *New Quantum Cryptography* systems.

All in all, the apparent similarity of the two results based on two totally different approaches seems to *validate both of them.*

Also we see that the free member (i.e. the product of the Quantum Elliptic Curve roots) in the equation (44) of the Quantum Elliptic Curve, $p_\phi^2/em_e = (m\hbar)^2/em_e$, is essentially square of the *magnetic quantum number* times constant parameter.

This suggests the *New Approach to Quantum Communication*, since direct manipulation of *Qbits* (see [17], or *qubits* elsewhere for details) via electric or magnetic fields may not require cryogenics and thus could be easier to achieve and may lead to *increased time of entanglement* of particles. The *decoherence of entangled objects* is the main stumbling block of all currently used approaches. Future experimental results may verify this idea.

Explicit derivation of the Quantum Elliptic Curve *coefficients* also hints at potentially more *direct and precise control* mechanisms of Quantum States, which may be important in development of *New Ultra Powerful Lasers.*

6. The Quantum Entanglement Mechanism

Looking at the new Quantum Elliptic Curve Equation (44) above and the relation (41) between α and β parameters of the **pair of equations,** we realize that there are two different methods that can be used to manipulate the state of the particle encoded in the Quantum Elliptic Curve:

1. via **free member** of the Quantum Elliptic Curve equation $p_\phi^2/em_e = (m\hbar)^2/em_e = r_1 \cdot r_2 \cdot r_3$, which will change the state of *only one particle* in the EPR-pair.

This is possible **if** roots r_i of *one* the curves changed via free member in such way that $\alpha = r_1 \cdot r_2 + r_1 \cdot r_3 + r_2 \cdot r_3$ does *not* change, which is always doable using one of the roots as tuning parameter.

The free member of each equation is a "private copy" of the Quantum Elliptic Curve Equation, corresponding to that *particular* particle. If α parameter of the Quantum Elliptic System does not change in such transformation, the second β part of the Entangled System will not change either.

Applying this mechanism one can change state of *one* of the particles forming an EPR-pair, *without* affecting the state of another particle. One can therefore "fine-tune" components of an EPR-pair without destroying the entangled system.

2. via **the Quantum Entanglement relation** (41) which links the two quantum particles forming an EPR-pair in a single quantum system. In that case changing *either* α or β parameter will result in change of an EPR-pair and *both* particles will be affected. And because this new Quantum Entanglement relation (41) does Not contain any time parameter we have to conclude that the change in the state of *one* particle will result in the **INSTANTANEOUS change** in the state of the *other* particle in the EPR-pair! No matter how far away from each other they are!

Which is what the Quantum Entanglement is all about!

Choice of manipulation technique used for changing the state of entangled particles depends on the nature of the quantum application and goal one aims to achieve.

One can chose, for example, to manipulate the state of one of the particles first (say, put one particle in a state "Alpha"), and then "translate" that state to the other *remote* entangled particle by manipulating the α or β parameters of the system as described above.

This is what may be called the **"Quantum Teleportation"** procedure.

7. Summary

In the present paper we demonstrated how method of *Triangulation of Elliptic Curve*, which plays central role in Elliptic Curve analysis in classical Number Theory, leads to deeper understanding of the foundations of the Quantum Mechanics, Hydrogen Atom energy levels, quantum Zeeman and Stark effects and ultimately to the new equation of the **Quantum Elliptic Curve.**

This allowed us to explain how and why two *linked Quantum Elliptic Curves* play central enabling role in the Quantum Entanglement.

About 370 years ago Messieur Pierre Fermat initiated the research of *Elliptic Curve* with his challenge to the English mathematicians regarding points (3,±5) on Elliptic Curve $y^2 = x^3 - 2$ (to which no reply came) and his *method of infinite descent* [9, 12].

And about 120 years ago Max Planck postulated the existence of *quanta*, which gave birth to the Quantum Physics [1, 2].

Today we have good reasons to believe that Elliptic Curve and Quantum Physics have Rational Points as their *common shared secret,* which paper *Quantum Elliptic Curve* version 1.0 [24] made public at Halloween 2020.

Version 3.0 of the Quantum Elliptic Curve paper (March 2022) described the enabling **Mechanism** required for the Quantum Entanglement to work.

Authors papers on Elliptic Curve and its relation to Hydrogen Atom bound states [22] and other Quantum Phenomena like Virasoro Algebra and String Theory [23] made it clear that to be fully understood Elliptic Curve needs to be treated as **Dynamic System.**

"Elliptic Curve Dynamics" program-paper [26] contains more technical details.

We have also described the **Quantum Entanglement Mechanism,** which may lead to new Practical and Exciting Applications - from Quantum Internet to Quantum Computing to *Teleportation*.

Motley String Theory (**MST**) [27] and Motley String based Quantum Mechanics (**MSQM**) [25] together with the Quantum Elliptic Curve (**QEC**) bring humanity closer to the final Theory of Everything (ToE).

Thanks God!

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Figure 1: Author with Mother and Grand Mother, photo by uncle Valery Ivanovich Yarushkin.

Dedicated to my Grandmother Ekaterina Tikhonovna Yarushkina née Ivinskaya, and to my Mother Zoya Ivanovna Matveeva née Yarushkina.