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On virtual scalar fields in a conformally flat FLRW spacetime

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Abstract

The case of a virtual bosonic scalar field $\phi(x)$ gravitationally coupled in a conformally flat spacetime is investigated. The action S is known to be Weyl invariant only for specific expressions of the potential $V(\phi)$. In the conformally flat FLRW case the harmonic angular frequency $\omega_k(\eta)$ becomes uniquely temporally independent. Therefore, any inertial observers embedded in a conformally flat FLRW spacetime all agree on the choice of virtual vacuum states. We postulate that the wavenumber k must be quantized in order to be able to regulate the vev by zeta regularization. Some fundamental implications of this result are derived, and likely conclusions drawn.

Key Words: QFT in Conformally Flat Spacetimes, Zeta Regularization, Holography, FLRW.

Background

Consider the case of a conformally flat spacetime connected by a conformal transformation to Minkowski space

$$g_{\mu\nu}(x) = \Omega^2(x)\eta_{\mu\nu} = e^{2\omega(x)}\eta_{\mu\nu} \tag{1}$$

For some Weyl transformation $\Omega(x)$ mapping one Riemannian manifold to another. Since it is possible to relate the quantization of the field $\phi(x)$ in a Weyl invariant theory to the known quantization in Minkowski space by conformal scaling, we require any action S to be invariant under Weyl transformations. The action S of a virtual bosonic scalar field $\phi(x)$ coupled to gravity R gives an additional, effective mass squared term in the potential $V(\phi)$ and is of the form [1]

$$S = \int d^dx\sqrt{|g|}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}(m^2+\xi R)\phi^2\right) \tag{2}$$

Which is Weyl invariant iff m = 0 and the scalar field is conformally coupled

$$\xi = \xi_d = \frac{d-2}{4(d-1)} \tag{3}$$

For the case of a conformally flat FLRW metric the EoM for an auxilliary field $\chi_k(\eta) = a(\eta)\phi_k(\eta)$ with scale factor a and conformal time η reduces to a harmonic oscillator

$$\chi_k^{''}(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0 \tag{4}$$

Where the effective, time-dependent, harmonic angular frequency $\omega_k(\eta)$ is

$$\omega_k^2(\eta) = k^2 + (m^2+\xi R)a^2 - \frac{a^{''}}{a} \tag{5}$$

Therefore, in general, two inertial observers do not agree on the choice of vacua. However, in the case of a massless, conformally coupled, four-dimensional scalar field $\phi_k(\eta)$, $\omega_k(\eta)$ becomes independent of η

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$$
$$\omega_k^2(\eta) = k^2 + \xi_4 R a^2 - \frac{a''}{a} = k^2 + \ddot{a}a + \dot{a}^2 - \frac{a''}{a} = k^2 + \ddot{a}a + \dot{a}^2 - \frac{2a\dot{a}^2}{a} = k^2 + \ddot{a}a - \dot{a}^2 = k^2 \tag{6}$$

Since

$$d\eta = \frac{dt}{a} \to a = \frac{dt}{d\eta} \to a' = \frac{da}{d\eta} = \frac{d}{d\eta}\frac{dt}{d\eta} = \frac{da}{dt}\frac{dt}{d\eta} = \dot{a}a$$
$$\frac{da}{dt} = \frac{d}{dt}\frac{dt}{d\eta} = \frac{d}{d\eta} \to a'' = \frac{d}{d\eta}(\dot{a}a) = a\frac{d^2a}{d\eta dt} + \frac{da}{dt}\frac{da}{d\eta} \tag{7}$$
$$= a\frac{d}{d\eta}\dot{a} + \dot{a}(\dot{a}a) = a\dot{a}\dot{a} + \dot{a}\dot{a}a = 2a\dot{a}^2 = a''$$

Where the last equality in (6) follows from the Friedmann equations [2].
Hence the angular frequency $\omega_k(\eta) = \omega_k = k$ becomes independent of the expansion of the universe. Therefore, any inertial observers all agree on the choice of virtual vacuum states in this specific instance.

Results

We postulate that the wavenumber $k = \omega_k$ must be quantized

$$k = \frac{2\pi n}{R_0} \tag{8}$$

For some lengthscale R_0, in order to be able to regularize the vev. Under the quantization postulate the vev as measured by any inertial observers in a Weyl invariant, four dimensional, flat FLRW spacetime becomes

$$\text{vev} = \langle 0_M | V(\phi) | 0_M \rangle = \langle 0_M | \int dV \xi_4 R \phi^2 / 2 | 0_M \rangle = \frac{1}{2}\omega_k(\eta) = \frac{1}{2}k = \frac{\pi n}{R_0}$$
$$= \frac{2\pi}{R_0}\sum_{n\in\mathbb{N}} n = -\frac{\pi}{6R_0} \tag{9}$$

Integrating over the spatial volume V for dimensional consistency and n, the radial quantum number being defined as positive. The analytic continuation of the Riemann zeta function $\zeta(s)$ evaluated at $s = -1$ is used to regularize the summation. The polarization DoF yields a factor of two.

Notice that this summability method only works for a conformally flat FLRW spacetime. Otherwise the angular frequency ω would not be time-independent. Which means that two inertial observers in general would not agree on the choice of vacua, and the vev would not be background independent and therefore not a Lagrange multiplier.

Therefore, the Lagrange multiplier Λ as measured by any inertial observers becomes regularizable in flat FLRW, even though the summability is unlikely covariant in general. Therefore, we will remind ourselves of this likely fact by adding a subscript Λ_{FLRW}, which within a spatial $V = R^3_0 = 1\ m^3$ unit in SI units becomes

$$\Lambda_{FLRW} = \text{vev}^2 = \frac{\pi^2}{36}\hbar^2 c^2 \tag{10}$$

Within an order of magnitude as compared with the Planck data [3]. The factor $\hbar^2 c^2$ accounts for the scaling required in the relevant flat FLRW case, since it is likely that different spacetimes will yield different Lagrange multipliers.

Implications

The following implications emerge from this picture: the energy density ρ_{FLRW} of an ideal fluid with EoS $w = -1$ yields a gravitational coupling $\kappa = \frac{8\pi G}{c^4}$

$$\Lambda = \kappa\rho \to \rho_{FLRW} = \frac{\pi}{288}\frac{\hbar^2 c^6}{G} \tag{11}$$

Under the plausible assumption that the cosmological event horizon is identical to a Schwarzschild horizon, the repulsive pressure does work and gives a counter effect to the evaporative Hawking radiation. The quadratic energy conservation with a holographic Bekenstein entropy yields a symmetry break with regards to the baryonic mass-energy and gives observable cosmological scales

$$\begin{aligned}
&dQ = TdS + p_{FLRW}dV = 0 = \frac{\hbar c^3}{8\pi GkM}\frac{2\pi kR}{l_p^2}dR - \frac{\pi}{288}\frac{\hbar^2c^6}{G}4\pi R^2dR \\
&\to \frac{\hbar c^3}{4GMl_p^2} = \frac{\pi^2}{72}\frac{\hbar^2c^6}{G}R \to \frac{18}{l_p^2} = \pi^2\hbar c^3RM \qquad (12)\\
&\xrightarrow{R=R_s} \frac{18c^3}{\hbar G} = \pi^2\hbar c^3\frac{2GM^2}{c^2} \to M^2 = \left(\frac{3}{\pi}\frac{c}{\hbar G}\right)^2
\end{aligned}$$

Therefore, we arrive at a quadratic symmetry break with respect to the baryonic mass-energy. It is reasonable to think that this is related to the baryon number asymmetry of the observable universe. Though the exact relation remains unclear.

Regardless, the cosmological event horizon being assumed identical to a holographic Schwarzschild horizon implies the observable cosmological scales

$$\begin{aligned}
R &= \frac{2GM}{c^2} = \frac{6}{\pi}\frac{1}{\hbar c} \\
M &= \frac{3}{\pi}\frac{c}{\hbar G} \qquad (13)\\
E &= Mc^2 = \frac{3}{\pi}\frac{R_0}{l_p^2}
\end{aligned}$$

With correct dimensions. Hence our plausible assumption of holography yields roughly empirically compatible cosmological scales. On the other hand, an extensive DoF remains empirically incompatible by the same methodology. In addition, the Bekenstein entropy implies the system considered must be expanding with time at all times.

Finally, the repulsive acceleration a_{FLRW} can be derived by the Friedmann equations [2] as

$$a_{FLRW} = \sqrt{\frac{\Lambda_{FLRW}}{3}}c^2 = \frac{\hbar c^3}{2\sqrt{27}} \qquad (14)$$

Which in the case of rotating galaxies gives a force balance

$$\begin{aligned}
&g = G\frac{M}{R^2} = a_{FLRW} = \frac{\hbar c^3}{2\sqrt{27}} = \frac{v^2}{R} \\
&\to R = \sqrt{\frac{GM}{a_{FLRW}}} \to v^2 = a_{FLRW}R \qquad (15)\\
&\to v^4 = a_{FLRW}GM = \frac{\hbar Gc^3}{2\sqrt{27}}M
\end{aligned}$$

Therefore, the baryonic Tully-Fisher relation [4] is of the form

$$v^4 = \frac{\hbar Gc^3}{2\sqrt{27}}M \qquad (16)$$

After which you may re-derive the cosmological scales independently of the assumption of holography by setting v = c, up to a geometrical proportionality constant.

It is also clear that the singularity theorems do not hold in this case, since there exists an EoS $w < -1/3$ which counteracts the gravitational collapse.

Conclusions

We have investigated the summability of virtual bosonic scalar fields $\phi(x)$ in a conformally flat FLRW spacetime, and realized that in the four dimensional, Weyl invariant case the angular frequency $\omega_k(\eta)$ becomes independent of the expansion of the universe. This allows any inertial observers at any time t in such a spacetime to agree on the choice of virtual vacuum states, without the need for Bogolyubov coefficients. We postulate a quantization of the wavenumber k in order to be able to regularize the vev by zeta function regularization. Under such a postulate the Lagrange multiplier as measured by any inertial observers at any time t, becomes within an order of magnitude of the Planck data. This strongly suggests that such a summability remains valid for the case investigated, though likely not covariant in general since different spacetimes likely yield different Lagrange multipliers.

Various implications emerge from this picture; it is likely that the cosmological event horizon is identical to a holographic Schwarzschild horizon and gives a mechanism for the breaking of baryon mass-symmetry by giving rise to a counter effect to the evaporative Hawking radiation. The baryonic Tully-Fisher relation for rotating galaxies emerge as well, where you may re-derive the cosmological scales independently of the assumption of holography up to an undetermined geometrical proportionality constant. It is also likely that this picture gives rise to a mechanism for halting gravitational collapse. In general, more work is needed in order to find all the implications of this emerging picture if they exist.

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