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On the Origin of 1/f Noise due to Generated Entropy

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Abstract

Noise measurements analysis in this paper is associated with degradation in materials. In particular, one type is called 1/f noise and is not fully understood. In the time domain, the signal has a random noise appearance. However, in the frequency domain, the spectrum goes as 1/f in intensity at low frequencies; noise issues, of course, occur at all frequencies. In reviewing the literature, we note that 1/f noise in particular seems to be strongly related to aspects in materials that can be interpreted in terms of aging degradation in materials (i.e. disorder). In this paper, some key aspects of 1/f noise found in the literature are described and discussed how these observations are related to generated entropy. We can conclude from the literature, that the 1/f noise region is of paramount importance to observing subtle aging degradation occurring in materials. A thermodynamic framework is then used to help interpret the entropy-noise view. A 1/f spectral region entropy model is provided. We suggest two types of analyses. Results help to provide a broader understanding of 1/f noise, identify the region of the spectrum related to the onset of degradation, and show how it can be used to do prognostics. Experiments are suggested to demonstrate how 1/f noise measurements can be used as a prognostic tool for reliability testing to identify and predict degradation over time.

1. Introduction

In this paper, we build a case that 1/f noise (also called flicker noise), is a sensitive measure of degradation. As well, we suggest experiments to investigate the possibility of using flicker noise as a prognostic tool for making reliability predictions of degradation occurring in materials. We will initially look at the literature and illustrate why it likely makes sense to explain flicker noise in terms of generated entropy.

The typical argument of entropy change (Δs) occurring due to current flow is

$$\Delta s = \Delta s_{resistor} + \Delta s_{Environment} \geq 0 \qquad (1)$$

Fundamentally, one can view this as a current flowing through a resistor to illustrate the entropy production and how it can create a noise current.

In detail, the entropy change to the environment is the heat dissipated

$$\Delta s_{Environment} = \frac{\Delta Q}{T} = \frac{i^2 R \Delta t}{T} \qquad (2)$$

The entropy change to the resistor is often considered negligible since the average current may be taken as constant (Δi=0) as is the average temperature. Therefore, no net change to heat ΔQ and $\Delta s_{resistor} = 0$.

Conversely, no process is truly reversible, a common thermodynamic argument. If a system process is in thermal equilibrium, then the process is reversible, but in thermal equilibrium, there is no measurement process! However, in noise measurements, higher resolution occurs, and we can observe small voltage noise fluctuations occurring across the resistor. Therefore, the current is not constant, a temperature gradient exists to dissipate heat and current fluctuation must generate complex entropy at the microscopic level. This is not a reversible process. We have the possibility that resistor entropy could increase $\Delta s_R \neq 0$, and the current itself becomes noisy and somewhat disorganized ($\Delta s_{Current} \neq 0$). A basic model for the perturbed entropy change is,

$$\Delta s = \Delta s_{resistor}(W, s_{resistor}) + \Delta s_{current}(s_{resistor}) + \Delta s_{Environment}(i^2 R \geq 0 \qquad (3)$$

The $\Delta s_{Environment}$ (with small s for entropy) in general represents the entropy flow of heat to the environment and is not of immediate interest. Here we can focus on the resistor and its internal disorder that may cause disorder in the measurement current which generates entropy in the current flow. Often in thermodynamics entropy flow (heat for example) is distinguished from generated entropy which causes permanent damage or disorder to the material from thermodynamic work W and in this case to the disruption of current flow. Note that generated entropy due to irreversible damage in the material is also termed in this paper, ***damage entropy.***

Many of the features of flicker noise in resistors are illustrated by the phenomenological equation due to Hooge [1].

$$S(f) = \gamma \frac{V_{DC}^2}{f} \qquad (4)$$

Here capital S is the noise spectral density, γ is the Hogg constant, and V_{DC} is the applied voltage. We see that noise power $S(f) \sim \langle V^2 \rangle = \langle (IR)^2 \rangle$, where I is the driving current and R is the sample resistance. In terms of an entropy interpretation, R is a direct measure of internal friction, which when interacting with current flow, generates entropy.

The concept that 1/f noise is related to internal friction is not new as it has been described in metals by Kogan and Nagaev (1982) [2]. They argued that 1/f low-frequency noise fluctuations could occur in mechanical strain and then electrical resistance would depend on the strain displacements. Their detailed model is a type of mechanical approach. Here we use an interpretation from an energy approach and provide different details.

2. Method and Data

In the entropy view, 1/f noise in particular seems to be strongly related to aspects in materials that can be interpreted in terms of aging degradation in materials (i.e. disorder). Noise issues are a function of the entropy state of the material and the method provided here depends on the material's resistive properties and internal fabrication stresses. We can note that for materials with less disorder such as wire wound resistors compared to say carbon (c) resistors, one would in the entropy view, expect less noise [3,4]. We can highlight a method related to this view. To do this, it is helpful to understand some background as well.

2.1 Entropy Due to Current Fluctuation

If we consider the random current fluctuations observable in sensitive *1/f* noise measurements, we can consider this as providing an observation related to the disorganization occurring in the measurement current as it passes through the material under test; it is then quantifiable in terms of generated entropy current change. One can assume this current disorganization is a function of the entropy state of a material related to its resistance. This is theoretically supported by the fact that if current instead was to flow through a material with zero resistance; the process would be reversible with no generated entropy.

2.2 Resistance Change Generates Entropy

Theoretically, any thermodynamic process creates entropy due to thermodynamic work. Thermodynamic stress in the material creates strain and this work is denoted as W in Eq. 3, likely created by current interactions in the material resulting in current fluctuations, other neighboring thermodynamic processes in the material may also occur. For example, the resistor may not be in complete thermodynamic equilibrium, even in the absence of measurement current flow! This is likely due to manufacturing stresses that occur in any fabricated material. The best likely way to observe Δs_R according to Eq. 3, is to turn off any active current in a 1/f noise measurement. Note a clear argument is that noise will depend on the state of the system's entropy. This argument comes about since system entropy itself can create stress due to a lack of structural internal integrity. This suggests that the entropy change in the material (the first term $\Delta s_{resistor}(W, s_{resistor})$ on the RHS of Eq. 3, goes as the system's entropy state in the material. Also, in the presence of a measurement current I, the disorder observed in the measurement current $\Delta s_{current}(s_{resistor})$ will go as the entropy state in the material

$$\frac{ds}{dt} = \lambda\, s(W)_R \qquad (5)$$

The entropy in the current increases as disorder increases in the material, yielding a higher level of measurement noise from the current entropy dissipating heat from i^2R losses. The physics argument can be used in the absence of a measurement current. Without a measurement current, a system may be in an unstable equilibrium state and may start to yield, generating entropy. Initially, the higher the instability, the larger the entropy change. Once equilibrium is reached, aging is maximized. The system is in a maximum entropy state with fabrication stresses released (ds/dt~0). Fabrication stress relaxation is one form of aging in the material.

Clarke and Voss found that 1/f noise was present even if there was no driving current at equilibrium. However, they could not guarantee true thermal equilibrium. In this view, non-thermal equilibrium would likely be due to the fabrication of internal stresses. Note that the thermodynamic work W in Eq. 3 is then defined to be due to the current workflow (from the measurement current) or any other neighboring thermodynamic work process (i.e., internal fabrication stresses).

At this point, we will need to look further into modeling to determine how an entropy approach leads to the 1/f noise dependence.

2.3 Wire Wound vs. Carbon Resistor Entropy Comparison

From the above discussion, we can assume current noise entropy is a function of the entropy state of the material like a resistor and depends on the resistance (Eq. 4). It also depends on the resistor type. For example, 1/f noise observations indicate that wire wound (w) resistors have less noise than carbon (c) resistors. In terms of entropy, a comparison of entropy created in the current i during a measurement would show

$$\Delta s_{i-c}(s_c) \geq \Delta s_{i-w}(s_w) \qquad (6)$$

Furthermore, a comparison of any damage entropy contribution in the two materials indicates

$$\Delta s_{R-c}(W_i) \geq \Delta s_{R-w}(W_i) \qquad (7)$$

so damage entropy in a carbon resistor of the same initial R-value over time would exceed that of the wire wound resistor, given the same amount of electrical work. Alternately, wire wound resistors are manufactured with higher order and are therefore more reliable so there is less internal degradation over time.

2.4 Oscillator Phase Noise Entropy

The phase noise of an oscillator is perhaps one of the most important parameters. Here *1/f* noise is known to dominate. Phase noise affects the purity of the carrier frequency in transmission. It is known that the unloaded Q in flicker noise goes as the inverse of Q to the fourth power observed [5-7]. Here again, damping (an inverse function of Q) is characteristic of another form of internal friction and strongly associated with entropy generation. A higher Q also indicates a more stable material and less susceptible to entropy damage.

2.5 MOS Observation and Entropy

Flicker models vary widely for MOSFET devices. One basic theory results due to fluctuations in bulk mobility based on Hooge's empirical relation is,

$$S(f) = \gamma \frac{I^2}{Nf} \qquad (8)$$

Here capital S is the noise spectral density, γ is the Hooge constant, I is the current, and N is the number of charge carriers. A simple entropy interpretation indicates that charge carriers reduce internal friction or alternately resistance in the bulk will vary inversely with N as it provides higher conductivity. The current flow I increases the amplitude of the flicker noise as it is perturbed in the channel and generates entropy (heat and damage entropy) [8].

2.6 Effect of Temperature

We note the spectral density in Hooge's Equation 8 is independent of temperature. This reinforces the fact that generated entropy damage compared with entropy flow (heat added) is the issue in 1/f noise. However, 1/f noise shows some atypical temperature-dependent characteristics (Eberhard and Horn, 1978) where they noted an ad-hoc function of γ(T). In our view, this would be due to damage created by heat as part of the aging thermodynamic entropy damage process [9].

2.7 Noise Measurement as a Tool to Observe Entropy Damage

Noise in operating systems has been linked to degradation [10,11]. An interesting example is congestive heart patients compared with healthy hearts had a distinctly different noise spectrum [12]. The entropy change to the resistor is often considered negligible since the average current may be taken as constant (Δi=0) as is the average temperature. Therefore, no net change to heat ΔQ and $\Delta s_{resistor} = 0$.

3. Results

Here we suggest two possible analytical approaches to illustrate how entropy can be treated in the case of *1/f* noise dependence. The first is a Gaussian model that is transformed to the frequency domain, leading to the *1/f* dependence. The second is based on Schottky's original model which is compared with Eq. 5 [13].

3.1 Gaussian Entropy Analysis

The definitions of entropy, s, for discrete and continuous random variable X, are [14,15].

Discrete case X, $p(x)$:

$$s(X) = -\sum_{x \in X} p(x) \log_2 P(x) \qquad (9)$$

Continuous case *X, f(x),*:

$$s(X) = -\int_X f(x) \log(f(x)\, dx \qquad (10)$$

The continuous case is termed the differential entropy. Here we are concerned with the continuous variations in time t distributed by *f*(*t*). Consider a Gaussian spectral density due to a process that generates entropy current fluctuations with distribution

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(t-\mu)^2}{2\sigma^2} \right) \qquad (11)$$

Gaussian spectra density for 1/f noise time-domain processes is often described as logical in the literature [such as 16]. Milotti summarized the question noting [16,17].

Voss produced experimental plots of the quantity *<V(t)|V0>/ V0* in several conductors and was able to show that the noise processes observed were reasonably Gaussian [18]. Further, it was noted that the superposition of many non-Gaussian microscopic processes can result in Gaussian form at the macroscopic level (demonstrated via the central limit theorem). J. B. Johnson in his 1925 experiment in vacuum tubes asserted that the spectral density characterizes a noise process completely only if the process is stationary, ergodic and Gaussian: does the observed *1/f* noise satisfy all constraints [19].

When Eq. 11 is inserted into the differential entropy Eq. 10, the result for a temporal process shows that for a Gaussian noise system, entropy (disorder s) is only a function of the variance [15].

$$s(t) = \frac{1}{2} \log(2\pi e \sigma(t)^2) \qquad (12)$$

Here the variance is a slowly increasing function due to aging which will be explained in an example. First note that for white noise, we note that the mean is zero (fluctuating around zero) but the variance is non-vanishing. As an example, consider a Macro system like an engine that exhibits a vibration power spectral density characteristic of $3G_{rms}$ content in a bandwidth from say 10 to 500 Hz. Let's say this is an average for this type of engine. Now compare this to a similar engine that is older and has aged showing PSD characteristics of $5G_{rms}$ content in this bandwidth. The system nose damage ratio can be of interest. In this case, we first note that the standard deviation equals the G_{rms} content for white noise having a zero mean. Then according to Eq. 12, the damage ratio is

$$\begin{aligned} &\Delta s_{Damage-noise-ratio}(t_2, t_1) \\ &= \log(2\pi e \sigma(t_2)^2) / \log(2\pi e \sigma(t_1)^2) \\ &= \log(2\pi e 5^2) / Log(2\pi e 3^2) = 1.2 \end{aligned} \qquad (13)$$

Here engine 2 has increased entropy damage by about 20% compared to engine 1 which is a measure of increased aging found by characterizing the engine's vibration noise instability through a comparison to the expected value for this type of engine. Such a system might indicate that if the noise level is 1.3, we might decide that this qualifies as a parametric threshold value for repair or failure.

Note that the entropy does not depend on the mean only on the variance, as this is a more sensitive measure. Solving Eq. 12 in terms of the variance and expanding terms by assuming a small change to the entropy and looking at the temporal part

$$\begin{aligned} \sigma(t)^2 &= \frac{1}{2\pi e} \exp\{4.6 s(t)\} \\ &\approx \frac{1}{2\pi e}(1 + 4.6 s(t)) \\ &\approx C + 4.6\{s(0) + s'(0)t + \ldots\} \end{aligned} \qquad (14)$$

A common noise measure related to the variance in the time domain is the Allan Variance given by

$$\sigma^2(\tau) = \frac{1}{2(n-1)} \sum_i \{\bar{y}(\tau)_{i+1} - \bar{y}(\tau)_i\}^2 \qquad (15)$$

The Allan variance frequency domain transforms are well established. For *stationary process* and the equivalent frequency domain PSD spectrum S is transformed for $\sigma^2(t) \propto t^0$ to the frequency domain giving $S(f) \propto 1/f$ and for a temporal dependence (non-stochastic process) where $\sigma^2(t) \propto t$, $S(f) \propto 1/f^2$. These terms are present in Eq. 12. Here frequency domain spectral density of Equation 14 (first terms on the RHS) is approximately [20,21].

$$S(f) = \frac{2.3}{2\pi \ln 2}\frac{1}{f}s(0) = \frac{1}{f}\frac{2.3}{2\pi \ln 2}\frac{1}{k}\frac{ds(0)}{dt} \tag{16}$$

We note that the noise *1/f* dependence is a function of the entropy and the RHS and we have included the concept of Eq. 5 where entropy changes are easier measured than entropy itself which brings in the constant k. Note the temporal term in the Taylor expansion in Eq. 14 leads to Brownian motion [17, 20].

$$S(f) \propto \frac{1}{f^2} \tag{17}$$

We note that the temporal model indicates that the variance and entropy rates change together. Therefore, we anticipate 1/f noise provides more fundamental significance to generated entropy damage sensitivity than say Brownian motion.

3.2 Schottky-Entropy Flicker Analysis

Equation 5 is similar to Schottky's (1926) original premise. In his analysis, contributions to the vacuum tube current were assumed as surface trapping sites that released electrons according to simple exponential relaxation.

$$N(t) = N_o \exp(-\lambda t) \tag{18}$$

In comparison, Equation 5 has an identical form in entropy terms with solution

$$s(W) = s_o \exp(\lambda t) \tag{19}$$

The results leads to Schottky's [13, 17] spectrum model which can then be put in terms of entropy

$$s(W) = \frac{N_o^2 n}{\lambda^2 + f^2} = \frac{s_o^2 n}{\lambda^2 + f^2} \tag{20}$$

In the Schottky model, n is an average pulse rate. Here it is the interactive stress such as the current. To be consistent with Eq. 16 we might let $N_o^2 \sim s_o^2$.

In terms of the Schottky model, Bernamont [16] pointed out that only a superposition of processes with a variety of relaxation rates λ would yield *1/f* noise for a reasonable range of frequencies. He showed that if λ is uniformly distributed between λ_1 and λ_2, and the amplitudes remain constant, the spectrum can be interpreted in the flicker *1/f* noise region

$$S(\omega) = \frac{N_o^2 n\pi}{2\omega(\lambda_2 - \lambda_1)}, \qquad \lambda_1 << \omega << \lambda_2 \tag{21}$$

and Brownian noise for example

$$S(\omega) = \frac{N_o^2 n}{\omega^2}, \qquad \lambda_1 << \lambda_2 << \omega \tag{22}$$

3.2.1 Flicker Amplification Effect

Using Eq. 16, the entropy flicker model with the aid of Equation 3 can be written

$$\begin{aligned} S_{Material-stress}(f) &= \frac{1}{f}\chi\, \Delta s_{Material-stress} \\ &= \frac{1}{f}\chi\, \{\Delta s_{Material}(W, s_{Material}) \\ &+ \Delta s_{stress}(s_{Material})\} \end{aligned} \tag{23}$$

Here χ is a calibration constant (similar to the Hooge constant) discussed in Eq. 4. Note in the absence of stress current, we can still observe the material flicker noise entropy as indicated in this equation and as mentioned earlier, has been observed experimentally, so Eq. 23 becomes

$$S_{Material-stress}(f) = \frac{1}{f}\chi\ \Delta s_{Material}(W, s_{Material}) \quad (24)$$

Now when say a measurement current flows, the current itself interacts with the internal frictional resistance which can create more entropy in the material (depending on the current density), but also amplifying the existing flicker "entropy damage" noise in the material. This notion is further supported by the observed flicker noise expressions in Eq. 4 and 8. Therefore, in this view, the origin or source of the flicker noise is initially due to entropy changes in the material, its entropy state, and interactions I2R stress with the measurement current.

4. Discussion

Several experimental methods can be performed to illustrate these results. Accelerated testing of materials and products is often done in industry. Since entropy increases with aging time, and we have illustrated how flicker noise is a likely sensitive measure of entropy change, then with standardized testing, degradation can be quantified through 1/f noise analysis. Below are some suggested experiments.

4.1 Suggested Flicker Aging Experiments

The noise spectral density can depend on aging test time when entropy (internal resistance) increases so that

$$\begin{aligned} &S_{Material-stress}(f, aging\ test\ time) \\ &= \frac{1}{f}\chi\ \Delta s_{Material-stress}(aging\ test\ time) \end{aligned} \quad (30)$$

Flicker reliability noise measurements could be performed. For example, a noise measurement of the material such as a carbon resistor is initially performed. Then the material is stressed at an elevated temperature in an oven over time which creates disorder in the material. Then the material is removed and a final noise measurement at room temperature is performed and compared with the initial noise measurement.

The spectral *1/f* characteristic of the material and its material-stress interaction likely provide a unique spectral characteristic and there may be even a possibility to build a *1/f* library similar to FTIR spectroscopy's method for identifying organic material.

4.1.1 Thin Film Resistors

Thin film resistors are known to age as a power law in aging time t over temperature (for example $\Delta s_R = kt^n$). Since it is known that thin film resistance increases with temperature over time, then flicker noise entropy will increase. However, now the option is available to look at aging rates at lower temperatures to observe the flicker aging law and if needed, transfer it to the time domain and compare it to gross measurements (i.e. higher temperatures and longer macroscopic gross measurements).

4.1.2 Biological Aging Experiment in Living Systems

The human heart is known to have different noise characteristics for Congestive Heart Failure (CHF) compared to a healthy heart. However, now the option may be available to study aging in normal healthy hearts using flicker noise measurement over a person's lifetime. Here we might suggest both long-term tracking of a group of people and also looking at different aging groups. All measurements should be first done with a calibration standard.

4.2 Entropy prognostics of complex systems using flicker noise measurements

A key characteristic of system entropy is its additive property. In a complex system, in terms of entropy, the whole is equal to the sum of *i* parts so that the entropy of the system is cumulative, i.e., $s_{System} = \sum_i s_i$ and according to the second law, the entropy change is $\Delta s_{system} \geq 0$. We can think of damage in terms of a complex partitioned system treated this way. In the mechanical macro view, cumulative entropy damage (i.e., generated entropy) can be treated in a form similar to Miner's fatigue rule [22] for cumulative damage $\left(\sum_i \frac{n_i}{N_i}\right)$. In the entropy view, this is also applicable to 'flicker cumulative damage and should be applicable in prognostics where applicable as needed for sensitive measurements. Similar to Miner's fatigue rule we write [22].

$$Cum\ Damage = \sum_i \frac{s_i}{s_{\max\_i}} = \sum_i \frac{\Delta s_i}{\Delta s_{\max\_i}}$$

$$= \frac{S(f)}{S_{\max}(f)} = \sum_i \frac{\frac{1}{f} s_i(0)}{\frac{1}{f} s_{\max i}(0)} : \sum_i \frac{n_i}{N_i} \quad (31)$$

We conclude that noise; in particular, 1/f flicker noise should have strong applications in the area of prognostics of aging systems. The noise spectra can be read and should provide clues to areas in complex sensitive systems where entropy damage requires maintenance for particular parts, due to their aging. This would be a highly sensitive prognostic tool that likely could detect issues well before other type of measurements.

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