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Research Article

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Kinetic Nonequilibrium Signatures in the Distribution Function of Earth-Escaping Hydrogen Atoms

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Abstract

The lightest atmospheric gas constituents, like H- and He- atoms, are known to escape from planetary gravitational fields to open space. Hereby it counts that not only the uppermost atmospheric layer, the so-called exobase, contributes to this planetary gas escape, but the layers below do contribute as well, especially since from below a nonthermal character of the final particle outflow is induced.We consider the outflow of H - atoms from lower levels of a planetary oxygen-dominated upper atmosphere, stratified by the planet's gravitational field - as given in case of the Earth. The terrestrial H-atom outflow is locally modified by elastic collisions of upwards flying H-atoms with the heavy major atmospheric background constituent. This causes a collision–induced velocity-modulation of the upwards directed H-atom flow and induces nonthermal kinetic signatures of the local H - atom distribution function. An important, hitherto unrespected point hereby is that angle-integrated elastic O - H- collision cross sections are velocity-dependent, falling off with increasing velocity v like (1/v). Consequently this modulation influences low velocity H-atoms stronger than high-velocity ones, which changes the kinetic profile of the escaping H-atoms and causes deviations from the classic Jeans escape. Deeply down in the lower thermosphere the local H-atoms, like as well the O-atoms, indeed are in a thermodynamical equilibrium characterized by Maxwell distributions with a common temperature $T_H = T_O$. Nevertheless, at the upper exobase border of the atmosphere the resulting H-atom escape flow turns out to be a, non-equilibrium flow with non-thermal escape-relevant properties. Here we describe this collisional modification of the H-escape flow and quantify the upcome of kinetic non-equilibrium features like power laws in the wings of the H-distribution function. This collisional modulation via velocity-dependent collision cross-sections acts as a typical process to convert equilibrium distributions into non-equilibrium kappa-like distribution functions. On the basis of this theoretical approach and stabilizing the upper atmosphere by the type of so-called "iso-baric Kappa functions" which all represent the same H - atom- pressure we can calculate the effective escape flux of H- atoms to open space and can quantify its difference with respect to the classical Jeans escape value. While finding again that the classical Jeans formula slightly overestimates the actual escape flux, we now for the first time taking into account the nonthermal influence of collisional modulations can show that the actual escape with respect to the actual Jeans value is even enhanced by factors 2 to 3.

Key words: Nonthermal Distributions, Knetic Theory, Planetary Escape, Hydrogen Loss, Planetary Atmospheres

Thermospheric Deviations from Thermal Equilibrium

Here in this paper we shall start from a physically well-posed problem connected with a planetary atmosphere stratified by the action of the planet´s gravitational field and connected with locally thermal equilibrium distribution functions of the various gas components. We will demonstrate that the collision-modulated escape flow – coming from far down in the thermosphere and arriving at its top level, i.e. the exobase, does show clearly pronounced signatures of a non-thermal kinetic distribution function though at the lower levels of the atmosphere one usually expects to have all atmospheric atoms under thermodynamic equilibrium conditions with local Maxwellians as prevailing distribution functions. That this concept of a local thermodynamic equilibrium is a bit too idealistic and needs to be corrected was already clear very early in the past [1]. The evident reason for perturbations in the thermodynamic equilibrium conditions comes up with the phenomenon of the H -atom escape from exobase levels, since all the upwards moving atoms with over-escape velocities ($v \geq vesc$) are practically lost from the atmosphere and can not immediately be replaced in the downward branch of the distribution function. This means that the exobase must be a region of anisotropic, non-Maxwellian velocity distributions (i..e: Non-LTE condition!, see) [2-7].

With this preamble we are furthermore interested to describe the particle outflow from the top of the atmosphere, taken as one-dimensionally stratified by the earth´s gravity field between coordinates $x = x_0$ (thermosbase) and $x = x_1$ (exobase). Below the top layer x_1 the gas, especially the heavy component, i.e. the mono-atomic O-atom gas, is well enough kept under thermodynamical equilibrium conditions with a common local temperature $T(x)$ as function of x. The particle distribution functions of the atomic gases can be assumed to be isotropic, local Maxwellians $f(v, x) = Max(v, T(x))$. From below the level x_1 particles like H- atoms, distributed according to such a local Maxwellian, are emitted towards the top layer, but before arriving there, they will undergo elastic collisions with other heavier gas particles, like, in case of the earth, especially the locally most abundant O- atoms, in view of their dominant number densities in this region. What comes out under such conditions is a "sheath-modified"- H - atom outflow with a collisionally modified, kinetically transformed, non-Maxwellian distribution function. This latter function finally determines the effective H-escape from the atmosphere to open space, and it is just the collision-modified kinetics of this escape function which we are aiming at in the article that follows.

The H-atom emissivity

Taking a 1d-structured atmosphere with an H/O - gas density distribution given by barometric densities $n_{H,O} = n_{H,O}(x) = n_{H,O}(x_0)\ exp[-(x - x_0)/S_{H,O}]$, with a height-coordinate x, and with the atmospheric scale heights $S_{H,O} = kT(x)/(g \cdot m_{H,O})$ specific for H- and O- atoms, where g denotes the gravitational acceleration within this layer, one obtains a local hydrogen emissivity $\vec{J}_H$ into a direction ϑ of

$$\vec{J}_H(v,\vartheta)dvd\vartheta = n_H(x)[\vec{v} \cdot (\frac{m_H}{kT(x)})^{3/2} \exp[-\frac{m_H v^2}{kT(x)}]v^2 dv \cos\vartheta d\vartheta]$$

Now one must pay attention to the fact that the upward flux $\vec{J}_H(v, \vartheta \simeq 0^\circ)$ of H-atoms around a space angle $\vartheta \simeq 0^o$ originating at a coordinate x, is reduced on its way up before it reaches the top layer at $x = x_1$, due to elastic collisions with the dominant background gas constituent. In case of the Earth's atmosphere, these are the mono-atomic O-atoms. This resulting reduction can be described by a transmission function $T_r(x, v)$, assuming that the colliding, low-mass H-atoms by H - O - collisions are completely redistributed to other directions $\vartheta' \gtrless \vartheta$ i.e. representing in essence a loss for H- atoms in the original flow $\vec{J}_H(v, \vartheta \simeq 0^o)$ which come along the upward direction $\vartheta \simeq 0^\circ$ from below, and hence one obtains an effective transmission given by:

$$Tr(x,v,\vartheta) = \exp[-\sigma_O(v)\int_x^{x_1} n_O(z)\frac{dz}{\cos\vartheta}]$$

Hereby $\sigma_O(v)$ denotes the angle-averaged elastic collision cross section between O-and H- atoms at a relative velocity

This cross section $\sigma_O(v)$ must be described as a type of a polarization cross section with a central interaction potential $V_{H,O}(r) \sim r^{-4}$ (i.e. Maxwell model!) between the collision partners H and O (i.e. polarized atomic shell) , effectively leading to the following velocity-dependence $\sigma_O(v) = \sigma_O(v_0) \cdot (v/v_0)^{-1}$. In case of H - O – atom collisions a reference cross section of $\sigma_O(v_0 = \sqrt{kT_0/m_H})$ $= 3 \cdot 10^{-17} cm^2$ or $\sigma_O = 8 \cdot 10\text{-}17 cm^2$ can be used or, when averaged over the scattering angle, of $\sigma_O = 10^{-15}\text{cm}^2$ [8-11]. Taking these facts together we obtain for velocities $v \geq v_0$ the following hydrogen transmission function:

$$Tr(x,v) = \exp[-\sigma_O \cdot (\frac{v_0}{v})n_O(x)\int_x^{x_1}\exp[-\frac{(z-z_0)}{S_O}]\frac{dz}{\cos\vartheta}] = \exp[-\sigma_O \cdot (\frac{v_0}{v})n_O(x)S_O[\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]$$

One may remind that the general validity with respect of the zenith inclination angle $\vartheta \simeq 0^o$ can only be taken as a valid approximation within a small range of inclination values, say $\vartheta \leq \vartheta_c \simeq 30^\circ$, since the one-dimensional atmospheric approach used here naturally requires limitations in view of the actual sphericity of the real planetary atmosphere. With these precautions, and taking advantage of the very large scale height of hydrogen (i.e. practically constant H-density!), we are lead to the following total H-atom emissivty $J_H(v, \vartheta)$ upwards from the top layer at $x = x_1$, i.e. the exobase:

$$\vec{J}_H(v,\vartheta)dvd\vartheta = \frac{1}{x_1 - x_0}\int_{x_0}^{x_1}\frac{dx}{\cos\vartheta}n_H(x) \cdot [\vec{v}(\frac{m_H}{kT(x)})^{3/2}\exp[-\frac{m_H v^2}{kT(x)}] \cdot \exp[-\sigma_O \cdot (\frac{v_0}{v})n_O(x)S_O[\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]v^2 dv\cos\vartheta d\vartheta$$

The above expression can be simplified introducing

$$v_x^2 = kT(x)/m_H = v_0^2 \cdot (T_0/T(x)),$$

where $v_0 = \sqrt{kT_0/m_H} = 5.8 km/s \simeq 0.5 v_{esc,H}$, $v_{esc,H}$ denoting the exobasic value of the H - escape velocity, and then leads to the following expression:

$$\vec{J}_H(v,\vartheta)dvd\vartheta = \frac{1}{x_1 - x_0}\int_{x_0}^{x_1} dx \cdot n_H(x)[\vec{v} \cdot (\frac{1}{v_0})^3(\frac{T(x)}{T_0})^{3/2} \exp[-\frac{v^2}{v_0^2}\frac{T(x)}{T_0}]\exp[-\sigma_O(\frac{v_0}{v})^1 n_O(x)S_O \cdot \quad \# \quad [\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]v^2 dvd\vartheta$$

which finally with $w = v/v_0$ and $n_H(x) \simeq n_H$ can be presented in the following form:

$$\vec{J}_H(w,\vartheta)dwd\vartheta = v_0\frac{n_H}{x_1 - x_0}\int_{x_0}^{x_1} dx \cdot [w \cdot (\frac{T(x)}{T_0})^{3/2}\exp[-w^2\frac{T(x)}{T_0}]\exp[-\sigma_O\frac{n_O(x)}{w}S_O \cdot [\exp(-\frac{x}{S_O\cos\vartheta}) - \exp(-\frac{x_1}{S_O\cos\vartheta})]]w^2 dwd\vartheta$$

Reference conditions for the terrestrial atmosphere

For the following investigations we take as a standard atmosphere the one at $14.^{00}\,h$ day time for medium solar irradiance conditions, i.e. for a solar radio flux of $F_{10.7} = 150$.

According to CIRA, we then have to use the following input numbers [12]:

$$n_H(x) \simeq n_H = 10^4 cm^{-3}$$

$$n_O(x) = n_{0,o}\exp[-(x - x_0)/S_0] = 10^{10} \cdot \exp[-(x - x_0)/50]cm^{-3}$$

Oxygen scale height: $So = 50km$

The gas temperature between 200 km and 500 km:

$$T(x) = T_0 + \frac{T_1 - T_0}{x_1 - x_0} \cdot (x - x_0) = T_0 + \frac{\Delta T}{\Delta x} \cdot (x - x_0)$$

With

T_0= 700K

and

T_1=1400K

Yielding:

ΔT= 700K

and

Δx=300km

The collision-modulated H-emissivity at the exobase

With these above standard atmospheric input numbers one obtains the following expression for the modulated H-emissivity:

$$\vec{J}_H(w,\vartheta)dwd\vartheta = v_0 n_H \frac{1}{x_1 - x_0}\int_{x_0}^{x_1} dx \cdot [w \cdot (\frac{T_0 + (\Delta T/\Delta x)(x - x_0)}{T_0})^{3/2} \exp[-w^2 \frac{T_0 + (\Delta T/\Delta x)(x - x_0)}{T_0}] \quad \#$$

$$\exp[-\sigma_O \frac{n_O(x)}{w} 5 \exp[-(z-4)][\exp(-\frac{x}{50\cos\vartheta}) - \exp(-\frac{500}{50\cos\vartheta})]]w^2 dwd\vartheta$$

This is then numerically expressed, using the cross section value given by Massey with $\sigma_O = 8 \cdot 10^{-17} cm^2$ and introducing the quantity $\Delta = \Delta T/T_0 \Delta x$, by the following expression [9]:

$$\vec{J}_H(w,\vartheta)dwd\vartheta = v_0 n_H w^3 dwd\vartheta \frac{1}{x_1 - x_0}\int_{x_0}^{x_1} dx \cdot (1 + \Delta(x - x_0))^{3/2} \exp[-w^2(1 + \Delta(x - x_0)) \quad \#$$

$$\exp[-\frac{4}{w}\exp[-(x - x_0)/S_O] \cdot (\exp(-\frac{x}{50\cos\vartheta}) - \exp(-\frac{10}{\cos\vartheta}))]$$

Introducing now the normalized integration variable by $z = x/So$, one finally obtains the following expression:

$$\vec{J}_H(w,\vartheta) = v_0 n_{H0} \frac{S_O}{x_1 - x_0} w^3 \int_4^{10} dz \cdot (1 + \Delta(z-4))^{3/2} \exp[-w^2(1 + \Delta(z-4)) \cdot \quad \#$$

$$\# \quad \exp[-\frac{4}{w}\exp[-(z-4)] \cdot (\exp(-\frac{z}{\cos\vartheta}) - \exp(-\frac{10}{\cos\vartheta}))]$$

Hereby the quantity Δ evaluates to the following expression:

$$\Delta = S_O \Delta T/T_0 \Delta x = \frac{(1400 - 700)}{700 \cdot (300/50)} = \frac{S_O}{x_1 - x_0} = \frac{1}{6}$$

First Results

According to the expression above derived for the upward hydrogen flux $\vec{J}_H(w,\vartheta)$ at the exobase $x = x_1 = 500km$ one obtains the flux values $J_H(x_1, w, \vartheta = 0)$ shown in Figure 1 where we have plotted the logarithms of these fluxes, i.e. $Log[J_H(w, x)]$, because this type of plot more clearly manifests the non thermal characteristics of the function $J_H(x_1, w, \vartheta = 0)$, since w - power law characteristics then show clearly up as straight-line regions, i.e. see the linear dependences on w. As one can see, beyond the velocity $w = 2.5$ this kind of power law characteristic of the velocity spectrum becomes visible in the expression for $Log(J_H(x_1, w, \vartheta = 0))$ which is shown in Figure 1.

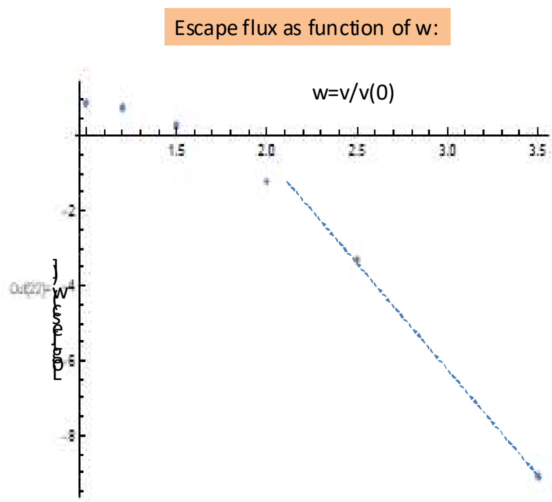

Figure 1: The logarithm of upward hydrogen flux $J_H(w, \vartheta = 0)$ *at* $x = x_1$ (exobase) as function of the normalized velocity $w = v/v_0$ for the standard CIRA atmosphere [12].

One can see that beyond velocities $w = v/v0 = 2.5$ the function $J_H(w, \vartheta)$ turns into a power-law behaviour. The critical occurence point of this turn is dependent on the magnitude of the responsible collision cross section. Since there exist substantial differences in the recommended values of the relevant collision cross sections, expressed in the quantity of the reference cross-section $\sigma_o = \sigma_o(v_o)$ that are given either by the quantity $\sigma_o(v_o) = \sigma^{(1)} = 10^{-17} cm^2$ (Schäfer and Trefftz,1970) or by the quantity $\sigma_o(v_o) = \sigma^{(2)} = 10^{-15} cm^2$. In Figure 2 we show the differences that are solely due to these cross section differences concerning the upcoming results for $Log[J_H(esc, w]$ as function of $w = v/v(o)$.

Log[j(esc,w)] for different cross-sections:

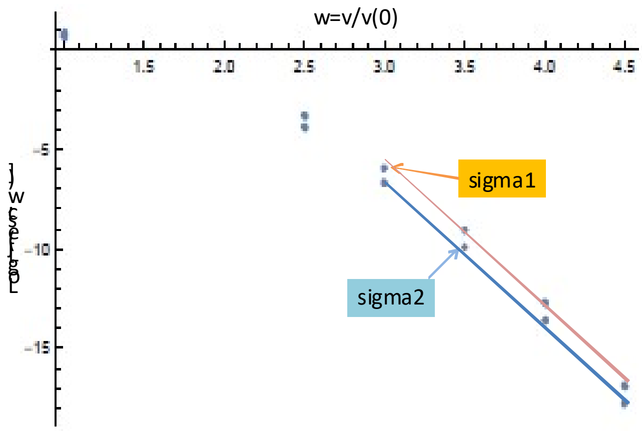

Figure 2: Log [J_H(x₁, w)] is shown as function of w = v/v(o) for two different cross section values: sigma1 =10.^(-17)cm^2 and sigma2=10.^(-15) cm^2.

Modelling of the Non-Equilibrium features

In the aforegoing sections we have shown that hydrogen atoms ascending from far below the exobase, while moving up to the exobase, do undergo collisions with the most numerous gas component, i.e. the mono-atomic oxygen. Treating this velocity-dependned, collisional modulation effect kinetically we have shown in the sections before that due to these collisions the distribution function $f_H(v, x)$ of ascending hydrogen atoms does systematically change its character from an LTE - Maxwellian to a non-LTE function with an increasingly higher statistical weight for the higher velocity H - atoms at increasing heights x.

While this phenomenon has physical relevance and has to be respected, it must, however, in addition be taken care of the atmospheric stability of the hydrogen quasi-isodensity structure with height x, otherwise the H-atmosphere would be substantially disbalanced into a new H-density profile. To guarantee the stable isodensity structure of the H-atmosphere between thermobase and exobase we thus also have to fullfill the condition of "isobaricity" of the H-atmosphere, namely the fact that the region between thermobase and exobase (x_0 up to x_1) should be characterized by about the same hydrogen pressure $P_H \simeq$ const, though we have seen the evident tendency of H - atoms to increase with height x the statistical weight of higher-velocity particles. Thus the outstanding problem now is, how to model this unusual situation?

Here one first could spontaneously think of using non-Maxwellian, kappa-like forms of velocity distribution functions which perhaps would best cover these needs. But to simply think of applying non-LTE functions in the form of general kappa functions $f_H = f\kappa(v)$ given by [13]:

$$f_\kappa(v) = \frac{n}{\pi^{3/2}\kappa^{3/2}\Theta^3}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \bullet [1+\frac{v^2}{\kappa\Theta^2}]^{-(\kappa+1)}$$

where n denotes the density, Γ(ζ) is the Gamma function of the argument ζ, and κ and Θ are two independent kappa-function parameters, is as easily evident not yet the adequate help for our needs here. This is because normal kappa functions typically have two independent parameters Θ and κ, and due to this fact they represent different pressures $P = P(\Theta, \kappa)$ for each set of these parameters. This, however, means they would change the H - pressure with height z, and would hence dissolve the stability of the atmospheric H-density stratification.

The adequate remedy here can, however, be found in so-called "isobaric kappa-functions f_κ^P" recently introduced by Fahr and Fichtner [14]. These functions are found by first producing the kappa-function pressure P_κ as the corresponding velocity moment of the above kappa-function and thus consequently obtaining:

$$P_\kappa = \frac{4\pi m}{3}\int_0^\infty f_\kappa(v)v^4dv = \frac{m}{2}n\Theta^2\frac{\kappa-3/2}{\kappa}.$$

This result is then, however, expressing the interesting fact that those kappa-functions are all "isobaric", i.e. belonging to the same pressure P_κ, which have two coupled parameters of the function $f\kappa\ (v, \kappa, \Theta)$, namely related to each other by the following relation:

$$\Theta^2(\kappa) = 2P_\kappa\frac{\kappa-3/2}{mn\kappa} = \Theta^2_{\kappa,M}\frac{\kappa-3/2}{\kappa}$$

where $\Theta_{\kappa,M}$ is the thermal spread of the associated Maxwellian (i.e. for $\kappa \to \infty$) given by:

$$P_\kappa(\kappa\to\infty) = \frac{mn}{2}\Theta^2_{\kappa,M}$$

Introducing now this upper interrelation of the two parameters κ, and Θ into the upper expression for the general kappa function will then evidently lead to the following family of "isobaric kappa functions $f_\kappa^M(v)$" given by:

$$f_\kappa^M(v) = \frac{n}{\pi^{3/2}(\kappa-3/2)^{3/2}\Theta^3_{\kappa,M}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}[1+\frac{v^2}{\Theta^2_{\kappa,M}(\kappa-3/2)}]^{-(\kappa-1)}$$

In the next Figure 3 we show how such isobaric kappa-functions look as velocity-space distributions, and one can clearly see that with smaller kappa indices the wings of the distribution function are systematically lifted up:

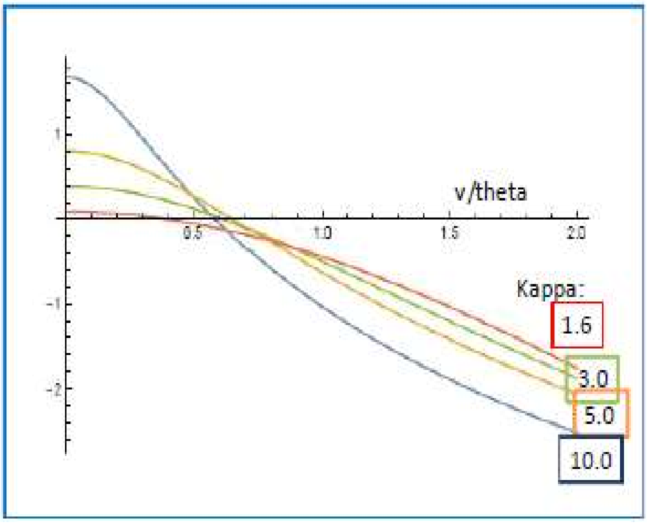

Figure 3: Shown is a set of isobaric kappa-functions $f_\kappa^M(v)$. For comparison purposes the curve for kappa =10 shows practically the set-Maxwellian of this isobaric sample (i.e. asymptotic approach).

In Figure 3 one can see that the statistical weight of differential velocity space particle populations at higher velocities systematically increases with lower and lower kappa parameters down to the value $\kappa = 1.5$, but nevertheless one should have in mind that all these functions represent the same pressure $P = P_\kappa^M$ due to corresponding depletions at lower velocities. Now it is interesting to recognize furtheron that in addition to this eminent property these isobaric kappa's have the additional property of leading to the same value of the Boltzmann-Gibbs entropy $S = S_\kappa^P$ which is given by the following expression:

$$s_\kappa^M = -4\pi k_B \int_0^\infty f_\kappa^M(v) \cdot [\ln f_\kappa^M(v) - 1] v^2 dv$$

This upper expression when further evaluated for isobaric kappa functions $f_\kappa^M(v)$ then leads to the following more explicit expression:

$$s_\kappa^M = -\ln(n) + 3\ln(\Theta_{\kappa,M}) - \ln[\frac{1}{\pi^{3/2}(\kappa - 3/2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}]$$
$$+ \frac{4\pi(\kappa+1)}{\pi^{3/2}(\kappa-3/2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}$$
$$\cdot \int_0^\infty \ln[1 + \frac{x^2}{(\kappa-3/2)}][1 + \frac{x^2}{\kappa-3/2}]^{-(\kappa+1)} x^2 dx$$

which we have visualized by numerical integrations shown in the plots of Figure 4. Hereby the value of the associated Maxwellian temperature is taken as $T^M = P^M/(3nmk/2) = 1400$K.

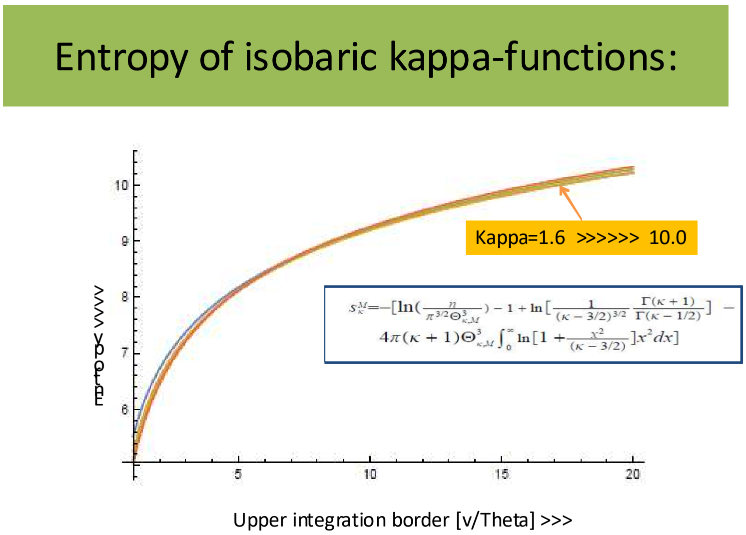

Figure 4: The entropy of isobaric kappa-functions for different kappa-function parameters as function of the upper velocity integration border

As evident in Figure 4 all the expressions for the different entropies $s_\kappa^M(\kappa)$ for values $\kappa = 1.6$ up to $\kappa = 10$ lead to the same entropy value. That expresses the astonishing fact that isobaric kappa-functions $f_\kappa^M(v)$ not only lead to the same density and pressure, they also describe thermodynamical states with identical entropies, i.e. with identical statistical probabilities. This means that such systems can freely communicate with eachother without exchanging informations and changing its states. This is the situation illustrated in Figure 5 below, namely the lower atmosphere being at thermodynamic equilibrium with Maxwellian distributions of the gas particles (hydrogen) and with non-equilibrium, isobaric Kappa-functions in the upper atmosphere. Since no information transport takes place between these two systems, the state of the two systems, i.e. the Upper and lower one!) is stable, even if a free particle exchange between the lower and the upper atmosphere is allowed to take place. To guarantee this persistence of this state the half-side, hemispheric main momentum fluxes (upward, and respectively downward fluxes) of mass, momentum end energy need to cancel eachother.

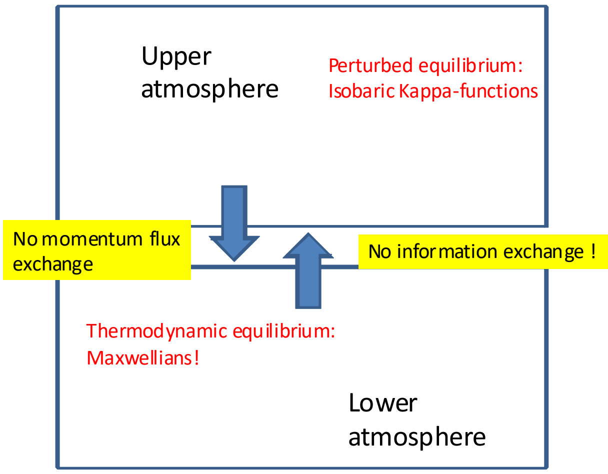

Figure 5: Illustrative view of the upper and lower atmosphere coupled by compensating half-side, hemispheric momentum fluxes of isobaric Kappa-functions from above and of Maxwellians from below.

To guarantee this compensation of the first most essential upward and downward momentum fluxes like A: mass-, B: momentum-, and C: energy fluxes, one needs to have fulfilled the following three relations [14]:

$$A : n_d u_d = n_{up}\Theta \frac{\sqrt{\kappa}\,\Gamma(\kappa)}{(\kappa-1)}\Gamma(\kappa-1)$$

$$B : n_d u_d^2 = n_{up}\Theta^2 \frac{3\kappa}{2\kappa-3}$$

$$C : n_d u_d^3 = n_{up}\Theta^3 \frac{\sqrt{\kappa}\,\Gamma(\kappa)}{(\kappa-1)(\kappa-2)\Gamma(\kappa-\frac{1}{2})}$$

As evident from the fact that for general kappa-functions the left sides contain only two free parameters, n_d and u_d, while the right sides contain three, n_{up}, κ and Θ, no unique, definite solution of the system of equations for general kappa functions is possible, However, for isobaric kappa-functions $f_\kappa^p(v)$ which have their two kappa-parameters related to eachother by $\Theta = \Theta\ (\kappa)$, the upper system of equations has only one definite, unique solution, namely:

$$n_d = n_{up}$$

and:

$$u_d^2 = \Theta_{\kappa,M}^2 \frac{\kappa-3/2}{\kappa}\frac{3\kappa}{2(\kappa-3/2)} = \frac{3}{2}\Theta_{\kappa,M}^2$$

which allows for the unique solution of an appropriate isobaric kappa-function given by:

$$\kappa = 1.6 \text{ and: } \Theta = \frac{\Theta_{\kappa,M}}{4}.$$

and describing the nonthermal escape function of hydrogen at and from the exobase.

The Collision-Modulated Planetary Escape Flux

At the end of this article, the important question must be put how these effects of a collision-induced transformation of the H-atom distribution function into an exobasic, non-equilibrium function do finally influence the hydrogen escape flow from the earth´s exobase to space. This escape flow namely determines the total hydrogen loss of the Earth´s atmosphere per time to space. This escape flow must be quantified now under the new auspices treated in the sections above.

Conventually, for the purpose to determine this escape flux, the classical Jeans approach is used leading to the result that this flux J_{jeans} is given by [15- 17];

$$J_{jeans} = 2\pi \int_{v_{esc}}^{\infty}\int_{\vartheta=0}^{\vartheta=\pi/2} f_{H,MAX}(v)\,v\cos\vartheta\, d(\cos\vartheta)\,v^2 dv$$

with v_{esc} being the escape velocity at the exobase level, and with the thermal exobase hydrogen distribution function given at a central exobase distance $r_{exo} = r_E + r_{ex}$ *with* $r_{ex} = 500km$ by

$$f_{H,MAX}(v) = n_H \cdot \left(\frac{m_H}{2kT_H}\right)^{3/2}\exp\left[-\frac{m_H v^2}{2kT_H}\right]$$

which leads to the well known result [1]:

$$J_{jeans} = \frac{n_H}{2\sqrt{\pi}}\left(\frac{2kT_H}{m_H}\right)^{1/2}(1+\lambda_H)\exp[-\lambda_H]$$

where λ_H is the escape energy parameter given by:

$$\lambda_H = \frac{Gm_H M}{kT_H r_{exo}}$$

where M and $r_{exo} = r_e + 500km$ here denote the mass of the earth and the central exobase radius.

Now, in comparison to that classical result from Jeans, we here in this article have found the following first result for the collision-modified escape flux $J_{H,esc}$ using the modified collision-modulated H-emissivity [15]:

$$J_{H,esc} = v_0 n_{H0}\Delta \int_{w_{esc}}^{\infty} w^3 \int_4^{10} dz \cdot (1+\Delta(z-4))^{3/2}\exp[-w^2(1+\Delta(z-4)) \cdot$$

$$\exp[-\frac{4}{w}\exp[-(z-4)] \cdot (\exp(-\frac{z}{\cos\vartheta}) - \exp(-\frac{10}{\cos\vartheta}))]$$

$$= v_0 n_{H0}\Delta \cdot \frac{1}{4}\int_{w_{esc}}^{\infty} J_H(w,\vartheta=0)dw$$

Let us remember that we have used as normalization of the H-atom velocity:

$kT(x)/m_H = v_0^2 \cdot (T_0/T(x))$, with the normalizing velocity

$v_0 = \sqrt{kT_0/m_H} = 5.8 km/s \simeq 0.5 v_{esc,H}$. This means that the lower border in the upper

w -integral delivering J_{esc} is given by $w_{esc} = v_{esc},H/v0 = 2$. Thus, we obtain the modulated Jeans escape flow by:

$$J_{esc} = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} J_H(w, \vartheta = 0) dw$$

The numerically determined dependence of $J_H (w, \vartheta = 0)$ beyond values of $w = 2$ as can be seen in Figure 1 appears power-law-like, i.e. yielding in the range $w \geq 2$ a straight line when plotting $Log\ [J_H(w)]$ against w with the following algebra:

$$Log[J_H(w)] = A + B \cdot w$$

and when fitting this algebraic expression to the plot shown in Figure 1 one obtains the following result:

$$Log[J_H(w)] = Log[v_0 n_{H0} \Delta] + 12.87 - 6.05 \cdot w$$

Since the escape flux is an integral over the differential flux , and not over its logarithm, we have to use the following expression:

$$J_H(w) = v_0 n_{H0} \Delta \cdot 10.^{(12.87 - 6.05 w)}$$

yielding the total escape flux according to Equ.(??) in the following form:

$$J_{esc} = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} J_H(w, \vartheta = 0) dw = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} 10.^{(12.87 - 6.05 w)} dw$$

The remaining integral evaluates to $\int_2^{\infty} 10.^{(12.87 - 6.05 w)} dw = 0.163$ and hence one finally finds

$$J_{esc} = v_0 n_{H0} \Delta \frac{1}{4} \int_2^{\infty} J_H(w, \vartheta = 0) dw = 2.8 \cdot 10\text{^}5 \cdot 0.163/12 \cdot n_{H0} = 3.81 \cdot 10^3 \cdot n_{H0}$$

Comparing this above result with earlier results obtained by using the classical Jeans expression (Jeans, 1923, see also Fahr and Shizgal (1983, especially their Figure 3 for concretes) we find as a relative surprise that the present value $J_{esc} = 3.8 \cdot 10^3{}_{nH0}$ obtained for a thermally structured atmosphere with a lower temperature $T_0(x_0) = 700K$ and an upper temperature of $T_1(x_1) = 1400K$ not only, as expectable, is larger than the Jeans flux for the lower temperature, i.e. J_{jeans} (T_0 = 700K) =80$nH0$, but, less expectable, is slightly smaller than the Jeans flux for the higher temperature, namely J_{jeans} (T_1=1400K) =7000$_{nH0}$. [1, 15]. Therefore one can say that the classical Jeans formula so far in our investigations does lead to a slight overestimation of the actual hydrogen escape flow from the earth [14].

This result also came already out from several earlier studies following different aspects of the escape problem like those considered by Brinkmann, Fahr (1976), Fahr and Weidner, Lindenfeld and Shizgal, Shizgal and Blackmore or Pierrard [4, 6, 7,18, 19]. In Fahr (1976) it was considered that the H-population at exobase heights in its upward velocity branch contains particles that escape from the earth's gravity field, i.e particles that do not return to the exobase from above, meaning that this part of the population in the downward velocity branch is permanently missing at exobase heights, i.e. it thus does not appear in the downward branch of the distribution function and somehow needs to be replaced via collisions. This loss of escaping particles can be expressed as a permanent loss of thermal energy from the exobasic H -population cooling down the exobasic hydrogen gas by about 80K relative to oxygen (Fahr, 1976) and thereby reducing the Jeans escape rate by about a factor of 0. 5.

A similar reduction of the Jeans flux values is elaborated in a study by Fahr and Weidner determining the influence on the H-escape rate in the sub-exobasic atmospheric layers due to collisions with O-atoms, however, treated in this case as hard-core elastic collisions with velocity-independent cross sections [4]. For the atmospheric exobasic temperature of 1400 K the authors find a similar reduction of the Jeans escape value by a factor of 0. 35.

Putting things so far together, it would turn out that this present study is not the first one demonstrating that classical Jeans escape rates are undermined by present day more realistic results, if collisional effects in the thermosphere of the Earth are taken into account paying attention to velocity-dependences of these collision cross sections.

However, the present study now shows for the first time that the effect of elastic collisions of escaping H-atoms with O-atoms leads to a transformation of the original thermal Maxwell distribution into a non-thermal kappa-like distribution with power-law characteristics at larger velocities of $v \geq 2v_0 = 2\sqrt{kT_0/m_H}$.

As we now have shown in the section ahead this change in the H-distribution function only leads an atmospherically stable stratification, if the conversion of the lower atmospheric Maxwellian towards a Kappa-like nonthermal distribution at greater heights occurs along the line of isobaric, isentropic kappa distributions $f_\kappa^p(v)$ which have the property to always represent the same pressure P which latter fact just can guarantee the H-atmospheric stability. Calculating, however, the Jeans- escape flow now with the help of isobaric kappa-functions $f_\kappa^p(v)$then consequently leads to the following more complicated, but also much better justified expression:

$$J_{\kappa,jeans}^P = 2\pi \int_{v_{esc}}^{\infty} \int_{\vartheta=0}^{\vartheta=\pi/2} f_\kappa^p(v) v \cos\vartheta d(\cos\vartheta) v^2 dv =$$

$$2\pi \int_{v_{esc}}^{\infty} \int_{\vartheta=0}^{\vartheta=\pi/2} [\frac{n_H}{\pi^{3/2}(\kappa - 3/2)^{3/2}\Theta_{\kappa,M}^3} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa - 1/2)} [1 + \frac{v^2}{\Theta_{\kappa,M}^2(\kappa - 3/2)}]^{-(\kappa+1)} \cdot (v\cos\vartheta d(\cos\vartheta) v^2 dv$$

In the figures 6/7 below we thus do show how the kappa-function-induced H-atom emissivity $J_{\kappa,jeans}^P$ at the exobase under such nonthermal conditions does look like, and how, on the basis of this nonthermal emmissivity, then finally this brings up the escape flux values which now are evidently greater than the Jeans escape value by factors of between $j_j = 1.0$ and $j_j = 2.7$. With our favourite value for the exobasic Kappa value of $\kappa = 1.6$ we in fact come to a most probable value of $j_j = 2.7$. One should, however, keep in mind that this factor is dependent on the exobasic

H - temperature TH via the connection $\Theta = \Theta(\kappa=1.6) = \Theta_{\kappa,M}/4$ where $\Theta_{\kappa,M}$ is given by the exobasic H temperature through: $(3/2) m_H\Theta^2_{\kappa,M} = kT_H$. This finally means that the enhancement factor j_j is dependent on the value of the exobase hydrogen temperature which latter is variable according to CIRA with day time and solar activity time and is given by the reference atmosphere CIRA-1965 [12].

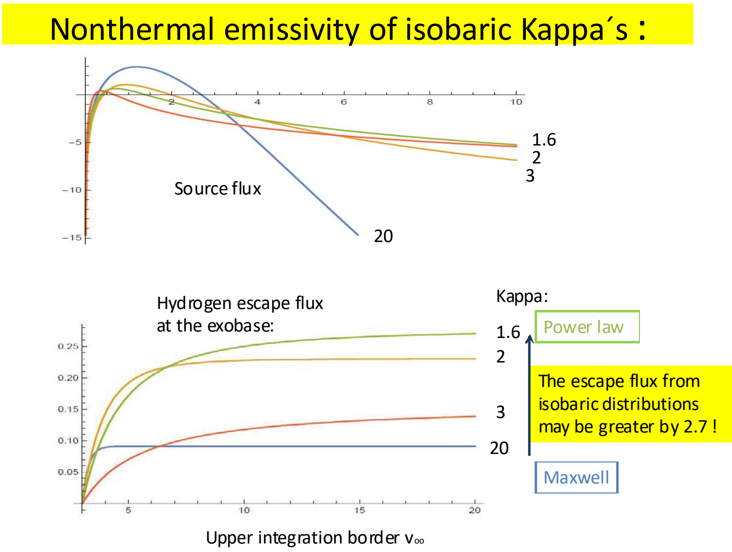

Figure 6: Upper part: The exbasic hydrogen emissivity calculated with isobaric kappa-functions for the parameters kappa = 20, 3, 2, and 1.6 as function of the normalized velocity

Lower part: The hydrogen escape flux from the exobase as function of the upper integration border calculated on the basis of isobaric kappa functions for the parameters kappa = 20, 3, 2, and 1,6.

Conclusions

We have shown in the sections ahead that close to the thermobase (200*km*) indeed the hydrogen atmosphere can be expected to be presented by H - atoms under local equilibrium conditions with local Maxwellian distribution function and a local temperature T_H equal to the local O-temperature T_O. However, those H-atoms ascending from thermobase levels to larger heights thereby are subject to local collisions with ambient O-atoms. The cross sections for such elastic O-H- collisions are velocity-dependent according to $\sim (1/v)$, meaning that hydrogen atoms with higher velocity v when ascending to larger heights are less affected by such elastic collisions compared to slower ones. These velocity-dependent collision-modulations as we do show lead to non-equilibrium features occuring in the hydrogen distribution function which can best be described by so-called Kappa-functions with a central Maxwellian core and power-law wings. Since the H-atmosphere between thermobase and exobase levels is practically an iso-pressure atmosphere, the upcoming H- kappa-functions, in order to keep the atmosphere stable, however, have to be of the type of "so-called isobaric and isentropic kappa functions" [12, 14]. Starting therefore from the assumption that the resulting exobase H-distribution is such a well-fitting, isobaric and isentropic kappa function will then give us a new basis of calculating the final excape flow from exobase levels to space, - and as it turns out from the results presented here (see Figure 6), one can expect to arrive at exobasic H-escape flows that turn out to be greater than the classical Jeans escape limit by factors 2 to 3 [15, 20-22].

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