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A Very Tentative Calculation of Some Coupling Constants and Particle Masses

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Abstract
In this article, we provide tentative first-order calculations of some coupling constants λ and particle mass corrections δm based on the results of prior work. We show that it is possible to derive values which are very roughly within the ballpark of the empirical data, though higher-order corrections are vital for higher accuracy. This article thus may provide a tentative clue on the deeper underlying physics if it exists. The difficulty of interpreting the physicality (if any) of the results are emphasized, and error discussions are made since the inclusion of this is considered vital. Finally, the importance of further work regarding the rigor of the derivations and the accuracy of the physical interpretations are underlined.

Keywords: Particle Mass Corrections, Dimensionless Coupling Constants, Tentative Calculations.

1. Introduction

The derivation from first principles of coupling constants and particle masses should in the author's opinion be required for any aspiring unifying physical formalism. Previous efforts in doing this, in particular those from string theory are known to give precise and rigorous predictions for the - in this case - Higgs mass based on mild assumptions [1]. The latest accurate measurements obviously confirmed this prediction [2]. It is also possible to use quantum field theoretic calculations to give very rough calculations for such quantities through first-order mass corrections, see the appendix (5) for a quick derivation of this.

In this article we will give a very - with an emphasis on very - rough estimate of a few particle masses and coupling constants, and explain our physical interpretation of the following formulae, which interpretations as far as the author of this article understands remains self-consistent with both present and previous work.

The motivation for doing this is to provide a tentative explanation of some coupling constants and particle masses which are very roughly within the ballpark of experimental data [2], where the error is expected to lie in a combination of finding the exact value of the geometrical proportionality constant $A^2 \sim O(-1)$, but may also lie in higher-order coupling constant λ corrections. Using our previously obtained results, we show that it then becomes possible to derive what is interpreted as physical values for the self-coupling constant λ, which measures the strength of the self-interaction of the scalar field ϕ, as well as the particle mass correction δm. Which may or may not, at least up to our first-order $O(\lambda)$ calculation correspond to particle masses of some perhaps neutrally charged elementary particles. It is important to say that it remains very difficult to interpret the physicality (if any) of these results.

2. Results

The physical particle mass m_{phys} is known from quantum field theoretic calculations [3] to first loop order $O(\lambda)$ in the self-coupling constant λ to be given by

$$m_{phys}^2 = m^2 + \delta m^2 = m^2 + \frac{i\lambda}{2}\Delta_F(0) \qquad (1)$$

Where $\Delta_F(0)$ is the Feynman loop propagator in Minkowski space and is thus proportional to the first-order $O(\lambda)$ particle mass correction δm as shown That is the so-called bare particle mass m gets shifted due to the self-interactions of the scalar field of the form $L_I \sim -\lambda\phi^4$, thus λ is dimensionless as desired. Please see section (3) for a discussion of the self-consistency of the entire formalism with regards to turning on self-interactions and the then arising second-order logarithmic amplitude divergence, which does not allow for analytic continuation.

We already know that due to the requirement of conformal invariance that the bare particle mass $m = 0$, thus we are only interested in the first-order particle mass correction $\delta m^2 = \frac{i\lambda}{2}\Delta_F(0)$. Conveniently, we already showed that the divergence of the virtual Feynman loop integral must be treated by analytic continuation, the analytic continuation being physical for all fully virtual diagrams without external lines as far as the author of this article understands as of today, therefore

$$\delta m^2 = \frac{\lambda}{2} i\Delta_F(0) = \frac{\lambda}{2}\Lambda_\eta = \frac{\lambda}{2} A^2\hbar^2 c^2 \tag{2}$$

Where we use the subscript η to denote the specific Lagrange multiplier value in Minkowski space, which may or may not be variable in different spaces. Again we have an implicit lengthscale $L = 1$ m for dimensional consistency and an unknown geometrical proportionality constant $A^2 \sim O(-1)$, since slightly different calculations, all with analytic continuation for divergence treatment give slightly different values for A. Thus

$$\delta m = \sqrt{\frac{\lambda}{2}} A\hbar c \tag{3}$$

Our goal is now to combine equation (3) with one more piece of information as shown below and then solve our system of equations for λ and δm. The other piece of information is that we know that the mass density ρ_m must be of the form

$$\rho_m = \frac{A^2}{8\pi}\frac{\hbar^2 c^4}{G} = \frac{\delta m}{L^3} \tag{4}$$

Where the lengthscale is implicit again for the same reason as before. Thus we arrive at the following equation for the self-coupling constant λ

$$\frac{A^2}{8\pi}\frac{\hbar^2 c^4}{G} = \sqrt{\frac{\lambda}{2}} A\hbar c \tag{5}$$

Which we may solve exactly, given our first-order approximation

$$\lambda = \frac{1}{32\pi^2}(\frac{A\hbar c^3}{G})^2 \approx 5.75A^2 \sim O(-2) \tag{6}$$

Where the exact order of magnitude of λ may vary depending on the exact geometrical value chosen for A. This may help to constrain the exact geometrical value for A since it is required that the dimensionless value for $\lambda << 1$ for our calculation to be valid.

These calculations may also be extended to the Higgs particle (see again the appendix (5)), thus we very roughly expect perhaps neutral particle masses like that of the Higgs H^0, and perhaps some other neutral particle masses to be very roughly of order

$$m_{H^0} \sim \frac{\hbar^2 c^4}{G} \tag{7}$$

In SI-units. Here the error is expected to be explained in the higher-order λ calculations. It is however not clear how to interpret these results, and exactly what (if any) particle masses this corresponds to. Please read the next section for a brief error discussion on this and other aspects of this formalism.

3. Error Discussion

In this section we briefly discuss the potential flaws of this short analysis, and how the formalism maintains consistency at least for all inertial reference frames as far as the author of this article understands today.

Turning on self-interactions of the form $L_I \sim -\lambda\phi^4$ allows in the second-order $O(\lambda^2)$ vertex correction for three logarithmic Mandelstam loop divergences $iA_{u,s,t}$, this logarithmic divergence does not allow for analytic continuation. Previously we modeled the ground state as an ideal fluid, this implies that the fluid is not self-interacting, so $\lambda = 0$ in that case. This will eliminate these logarithmic higher-order divergences in the previous model.

However, in this model where we turn on self-interactions, the higher-order vertex corrections logarithmic loop divergences will be present. As far as is understood today, the application of analytic continuation for divergence treatment in quantum field theories as a way to develop a tentative intuition for how to treat the arising loop divergences in general and as an alternative to the direct use of renormalization and regularization only applies to vacuum diagrams without external lines, thus the analytic continuation is not claimed to be applicable in this case, but the cut-off parameter may very well be identical to the Λ_η-value as used in this article.

In addition, since we are considering a local amplified Higgs fluctuation $H^0(x)$ such that our scalar field ϕ is of the form

$$\phi(x) = \frac{1}{\sqrt{2}}(v + H^0(x)) \tag{8}$$

Where $v = \sqrt{\Lambda_\eta}$ is the standard vacuum expectation value in Minkowski space that we are considering, it is clear that the particle masses originate from the self-interaction of the localized fluctuation (i.e. a standard Higgs mechanism) and is of course not due to the ground state energy directly. Thus, for the sake of accuracy there may or may not be a different geometrical proportionality constant in equation (4). Furthermore, it is clear that when there are no localized fluctuations, that is when $\phi(x) = \phi^0 = v$, (choosing the positive value) as in the previous model, then there will be no self-interactions either.

It is also clear that the author's inability or lack of motivation for making precise calculations is not ideal, but anyone who feels motivated enough to do this based on the formalism as outlined is encouraged to do so. The purpose of this article is simply to provide tentative explanations and calculations, as the possibility for developing some tentative understanding of the potential underlying physics (if any) per unit of work put in is - in the author's opinion - considered the greatest in this way.

Finally, a quick note on the fact that the SI-units calculation yields an extra c^4 factor in equation (1), which cancels out the c^{-4} factor in the interaction Lagrangian density L_I term, so that the same answer, equation (6) for the dimensionless λ in SI-units remains as far as the author understands. This nuance is not included in the calculation for the sake of simplicity. Thus as far as the author understands no factors of c have been overlooked.

4. Conclusions

In this article we provide tentative calculations to first loop order of some dimensionless coupling constants and some particle masses, and show that the latter is very roughly within the ballpark as compared with empirical data [2] where the error is expected to lie in a combination of higher-order λ calculations and in the exact value of the geometrical proportionality constant A We show or argue that the formalism remains self-consistent when you turn on these self-interactions $\lambda \neq 0$, and propose how to treat the second-order vertex correction logarithmic loop divergence for both models.

Further work is needed in order to remedy these errors, either by figuring out the exact value for A or calculate higher-order corrections. It is not clear which (if any) particle masses our results corresponds to, and the difficulty of the interpretation of these results is highly emphasized. In general the tentativeness of our calculations are highly emphasized. The entire formalism remains self-consistent at least for all inertial reference frames as far as the author of this article understands as of today.

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5. Appendix

In this appendix we will derive to first loop order $O(\lambda)$ the immediate implications for the Higgs particle mass in SI-units. This is based on our prior results for λ as outlined above, equation (6) and chapter 13 in the excellent book by Kachelriess [4]. The simplest model with a potential density

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4 \tag{9}$$

that yields a broken symmetry has the following minima

$$v = \pm\sqrt{\frac{\mu^2}{\lambda}} = \pm A\hbar c \tag{10}$$

Now choosing the positive value and expanding as in equation (8) our scalar field ϕ around a localized quantum fluctuation yields the following now positive Higgs mass term

$$m_{H^0} = \sqrt{2}\mu \tag{11}$$

Using our new equation for λ, that is equation (6) we arrive at

$$m_{H^0} = \sqrt{2\lambda}A\hbar c = \frac{1}{4\pi}A^2\frac{\hbar^2 c^4}{G} \tag{12}$$

again with the emphasis on the difficulty of calculating exact geometrical proportionality constants. Therefore, we expect the Higgs mass and perhaps some other neutral particle masses to first-loop order corrections $O(\lambda)$ to be very roughly in the ballpark of

$$m_{H^0} \sim \frac{\hbar^2 c^4}{G} \tag{13}$$

in SI-units. Again if anyone feels motivated to do the higher-order calculations then please do so.

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