

What can be a Classic Syllogism in Fuzzy Logic?

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Abstract

Using the concept of Degree of Fulfillment (DOF) this paper presents 11 different fuzzy logic understanding of a classic syllogism. For each of 11 understandings of the same classic syllogism an example is given. Each example contains computations corresponding to three different fuzzy premises. The computed DOF of the 33 fuzzy syllogisms conclusions reflect the dependence of the “truth value” on fuzzy premises. This kind of results can be useful in evaluation of, premises based, human made decision, particularly within the domains of Autonomy & Unmanned Systems and Data Assisted Decision Making by focusing on how human thinking can be explicitly and computationally integrated into systems of fuzzy reasoning used in technical problems.

1. Introduction

Mathematicians, when arguing a statement, mainly use the following reasoning’s (argumentations): deductive reasoning, mathematical induction, reduction to absurdity or a combination of these.

1.1. Deductive Reasoning

A deductive reasoning is a multi-step argumentation of a statement called a conclusion. This reasoning uses a set of rules and premises. The premises are statements, and if they are true, then the conclusion cannot be false. This argumentation goes from the general to the particular, as the word deductive suggests.

1.2 Syllogism in Classical Logic

The simplest deductive reasoning in classical logic is called a syllogism and contains three parts: Major premise: A general statement; Minor premise: A particular statement, which falls within the general statement; Conclusion: A statement that necessarily follows from the combination of the two premises.

Example 1.1. of a syllogism in mathematics:

Major premise 1: All numbers ending in 0 are divisible by 5.

Minor premise 1: The number 150 ends in 0.

Conclusion 1: The number 150 is divisible by 5.

Example 1.2. of a syllogism in mathematics:

Major premise 2: If two angles are congruent to a third angle, then they are congruent to each other.

Minor premise 2: Angle A and angle B are both congruent to angle C.

Conclusion 2: Angle A is congruent to angle B.

1.3 Necessary and Sufficient Condition for a Valid Syllogism

In order for a set of three categorical propositions taken at random and in random order to form a valid syllogism, they must be able to be brought to a specific logical structure and meet certain strict structural and propositional conditions. If they are not initially arranged in the classical form (Major Premise, Minor Premise, Conclusion), they must meet the following necessary and sufficient conditions:

a. Structural Condition (terms)

The set of three propositions must contain exactly three terms:

Major Term (P): the logical predicate of the conclusion.

Minor Term (S): the logical subject of the conclusion.

Middle Term (M): the connecting term that appears in both premises, but never in the conclusion.

b. Propositional Condition (logic of judgments)

The three propositions must simultaneously satisfy the general laws of syllogism:

Distribution of Terms:

The middle term must be distributed in at least one of the premises. A term cannot be distributed in the conclusion if it has not been previously distributed in the premise.

Law of Quality:

There is no syllogism from two negative premises. If a premise is negative, the conclusion is negative.

An affirmative conclusion necessarily follows from two affirmative premises.

Law of Quantity:

There is no syllogism from two particular premises.

If at least one premise is particular, the conclusion must be particular.

The conclusion follows the weakest part: The conclusion takes the quality of the weaker premise (negative or particular).

c. Reconstituting the correct Order

Once taken at random, the sentences must be sorted logically:

Conclusion: The sentence containing the terms S and P. Contains derivation indicators (e.g., therefore, consequently).

Premises: The other two sentences, each of which must contain the middle term (M) and one of the extreme terms.

Note: To be completely valid, the syllogism must fit into one of the 24 known valid schemes (classified into 4 syllogistic figures, depending on the position of M).

Example 1.3. three mathematical propositions that cannot form a syllogism.

An example of three mathematical sentences taken at random that cannot form a syllogism because they violate the elementary structural rules (number of terms and structure of premises):

Proposition 1: "Every square has four equal sides."

Proposition 2: "Some prime numbers are odd."

Proposition 3: "No triangle has diagonals."

a). Why can't I form a syllogism?

1. There are too many terms (Rule of 3 terms is violated): A syllogism needs exactly 3 terms (Major, Minor, Middle). In this set we have 6 different terms: square, four equal sides, prime numbers, odd numbers, triangle and diagonals.

2. Missing Middle Term (M): There is no common connecting element between the three sentences that would allow a logical deduction. they are completely independent.

3. A conclusion cannot be established: None of the sentences can logically result from the combination of the other two.

Example 1.4. Three mathematical propositions that can form a syllogism.

For three mathematical propositions taken at random to form a syllogism, they must use exactly three terms (Subject, Predicate, Middle) and be able to be rearranged into a valid structure.

Here are three propositions taken in completely random order:

Proposition A: "Some quadrilaterals are squares."

Proposition B: "All squares have right angles."

Proposition C: "Some quadrilaterals have right angles."

Why do they satisfy the necessary and sufficient condition?

If we analyze this set of sentences, we see that they meet all the required conditions:

They have exactly 3 terms:

S (Minor): quadrilaterals

P (Major): figures with right angles

M (Middle): squares

They can be arranged in the standard structure of a valid syllogism:

Major Premise (M-P): All squares have right angles. (Proposition B)

Minor Premise (S-M): Some quadrilaterals are squares. (Proposition A)

Conclusion (S-P): Therefore, some quadrilaterals have right angles. (Proposition C)

This structure corresponds to the valid syllogistic mode. All the laws of distribution and quality/quantity are respected, and the conclusion necessarily follows from the premises.

In classical logic there are infinitely many possible syllogisms, but only 256 logically distinct types and only 24 valid types.

It should be emphasized that the validity of the syllogism does not mean the validity of the conclusion. The latter requires verification of the validity of the premises.

2. Motivation for constructing fuzzy variant for the validity of the conclusion of a valid syllogism

The conclusion of a valid classic syllogism is true if and only if the major premise and the minor premise of the syllogism are true propositions. There are examples of classic syllogisms in mathematics that are valid but their conclusion is not a true proposition.

Example 2.1. A mathematical example of a valid classic syllogism with a false conclusion:

Major Premise (False): All prime numbers are odd

Minor Premise (True): The number 2 is prime.

Conclusion (False) The number 2 is odd.

Why is this logical syllogism valid but the conclusion is false?

Structurally, the syllogism obeys the law of form. However, because the major premise is false (the number 2 is prime but even), the final conclusion is erroneous.

This possibility requires verification of the premises, which can often be subjective. Here we have in mind the fact that, the verification of the validity of premises is done by a group of people who can have different degrees of exigency, as is the case of a medical board in the case of a patient, as is a jury in a figure skating competition or as is the grading of a student by a teacher [1,2]. Also, nowadays it is common to classify scientific journals, universities, researchers, people based on numerical parameters obtained by aggregating other measured parameters. Such are the tests for evaluating the knowledge of students in schools, various psychological tests of general IQ and specialized IQ. Following the classification of a group of people based on the results obtained in general IQ and specialized IQ tests, a single, unequivocal result is obtained, qualifying 'the best'. However, a question arises regarding this result: Does the word 'the best' have the same meaning for all of us? This question is natural because this qualifier 'the best' is not defined. 'The Best' is an ambiguous word!

So is the word 'intelligent' for which in we built a mathematical structure that incorporates a certain degree of ambiguity of the word 'human intelligence' [3]. In the following we will build 11 fuzzy variants for a classical syllogism showing the differences between meaning of "truth value".

3. Fuzzy Variants of a Valid Classic Syllogism

3.1 Syllogism in Classical Logic and Syllogism in Fuzzy Logic [3-5]

The structure of a syllogism in classic logic is

$$IF (a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2) \text{ THEN } C \quad (3.1)$$

The two variables (a_1 is A_1), (a_2 is A_2) are statements, which can be false or true. The conclusion C is a statement which can be true or false.

In classical logic the Boolean calculus [6] assign to any statement two values: 0 in case of the statement when this is false and 1 in case of the statement when this is true. The table of the true values are given in case of the application of different logical operators.

Table 1.

A	B	$\text{not } A$	$A \text{ (AND) } B$	$B \text{ (OR) } A$	$A \text{ (XOR) } B$	$A \text{ (imply) } B$
1	1	0	1	1	0	1
1	0	0	0	1	1	0
0	1	1	0	1	1	1
0	0	1	0	0	0	1

Definition 3.1. In fuzzy logic a fuzzy syllogism consists of a set of two variables in the form of fuzzy sets A_1, A_2 with membership functions f_{A_1}, f_{A_2} and premises a_1, a_2 coupled by fuzzy logical operator AND forming a fuzzy logical expression as

$$(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2) \quad (3.2)$$

and a corresponding conclusion, in the form of a fuzzy set C and with a premise c. The fuzzy sets represent the meaning of some fuzzy linguistic variables. [1]-[5]. The structure of a fuzzy syllogism in fuzzy logic is:

$$IF (a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2) \text{ THEN } (c \text{ is } C) \quad (3.3)$$

In analogy with the Table 1, where the ‘truth value’ for the syllogism in classical logic is given, the ‘Degree of Fulfillment’ (DOF) value for the syllogism in fuzzy context has to be defined.

3.2 Degree of Fulfillment of a Fuzzy Syllogism for given Premises

The ‘truth value’ or ‘truth grade’ of a fuzzy syllogism is a degree to which the syllogism can be applied to a particular case.

Definition.3.2. The ‘truth value’ corresponding to the fulfillment of the conditions of a syllogism for given premises a_1, a_2 is called the ‘Degree of Fulfillment’ (DOF) of that syllogism.

There are several possibilities to define the ‘DOF value’ for a classic syllogism in fuzzy context. Each defined ‘DOF value’ is fuzzy syllogism. [1-5].

What is the ‘product fuzzy syllogism’?

Similarly to Definition 3.4 paper, the ‘product fuzzy logic operator AND’, in case of the fuzzy statements $(x \text{ is } A_1), (y \text{ is } A_2)$, transform these fuzzy statements in a new fuzzy statement ‘ $(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ denoted usually by ‘ $\text{prod}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ [4]. The membership function of the fuzzy subset, which represent the fuzzy statement ‘ $\text{prod}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ is $f_{\text{prod}}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$. This fuzzy subset usually is denoted by $\text{prod}(A_1 \cap A_2)$ and is called also the ‘prod fuzzy intersection’ of fuzzy subsets A_1 and A_2 . The new fuzzy statement ‘ $(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ is a fuzzy counterpart of the classic syllogism. This understanding is acceptable if the membership function of the fuzzy set C, representing the conclusion of the fuzzy syllogism in formula (3.2) is $f_C(x,y) = f_{A_1}(x) \times f_{A_2}(y)$.

Definition.3.3 The DOF of the ‘product(maximum) fuzzy syllogism’ is defined by the formula

$$\text{DOF}[\text{prod}(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2). \quad (3.4)$$

For given a_1, a_2 it is in fact a number attached to the fuzzy logical expression (3.2).

Example.3.1. Let consider, the next and valid classic syllogism, and its fuzzy counterpart defined by (3.4):

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and its fuzzy counterpart (3.2).

In (3.2) the fuzzy set A_1 represent the major premise of classic syllogism. Assume that this might be characterized with a membership function $f_{A_1}(x) = 0$ for $x \leq -20$; $f_{A_1}(x) = \frac{x+20}{10}$ for $-20 < x \leq -10$; $f_{A_1}(x) = 1$ for $-10 < x \leq 0$; $f_{A_1}(x) = \frac{4-x}{4}$ for $0 < x \leq 4$; and $f_{A_1}(x) = 0$ for $4 < x \leq 20$.

This is the trapezoidal fuzzy number $(-20, -10, 0, 4)_{TR}$ represented on the next figure.

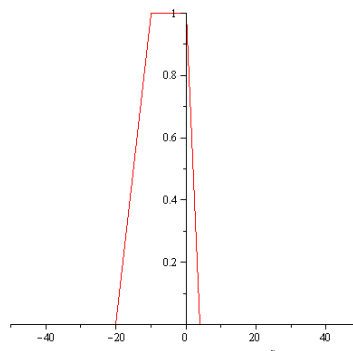


Figure 3.1: The trapezoidal fuzzy number $(-20, -10, 0, 4)_{TR}$

The fuzzy set A_2 represent the minor premise of the classical syllogism. Assume that this might be characterized with a membership function $f_{A_2}(y) = 0$ for $-20 < x \leq -10$; $f_{A_2}(y) = \frac{y+10}{10}$ for $-10 < x \leq 0$; $f_{A_2}(y) = \frac{4-y}{4}$ for $0 < x \leq 4$; and $f_{A_2}(x) = 0$ for $4 < x \leq 20$. This is the triangular fuzzy number $(-10, 0, 4)_T$ represented on the next figure.

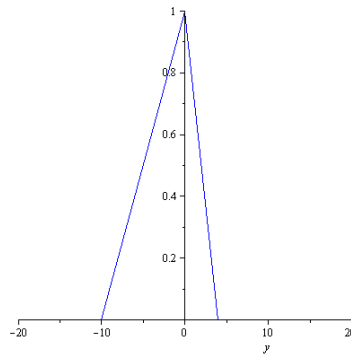


Figure 3.2: The triangular fuzzy number $(-10, 0, 4)_T$

The membership function $f_{\text{prod}}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ of the fuzzy subset, which represent the fuzzy statement 'prod (x is A_1) AND (y is A_2)' is:

$$\begin{aligned}
 f_{\text{prod}}(x, y) &= f_{A_1}(x) \times f_{A_2}(y) = \\
 &\frac{x+20}{10} \cdot 0, x = -20 \dots -10, y = -20 \dots -10 \\
 &\frac{x+20}{10} \cdot \frac{y+10}{10}, x = -20 \dots -10, y = -10 \dots 0 \\
 &\frac{x+20}{10} \cdot \frac{4-y}{4}, x = -20 \dots -10, y = 0 \dots 4 \\
 &\frac{x+20}{10} \cdot 0, x = -20 \dots -10, y = 4 \dots 20 \\
 &1 \cdot 0, x = -10 \dots 0, y = -20 \dots -10 \\
 &1 \cdot \frac{y+10}{10}, x = -10 \dots 0, y = -10 \dots 0 \\
 &1 \cdot \frac{4-y}{4}, x = -10 \dots 0, y = 0 \dots 4 \\
 &1 \cdot 0, x = -10 \dots 0, y = 4 \dots 20, \\
 &\frac{4-x}{4} \cdot 0, x = 0 \dots 4, y = -20 \dots -10 \\
 &\frac{4-x}{4} \cdot \frac{y+10}{10}, x = 0 \dots 4, y = -10 \dots 0
 \end{aligned} \tag{3.5}$$

$$\frac{4-x}{4} \cdot \frac{4-y}{4}, x=0..4, y=0..4$$

$$\frac{4-x}{4} \cdot 0, x=0..4, y=4..20$$

$$0 \cdot 0, x=4..20, y=-20..-10,$$

$$0 \cdot \frac{y+10}{10}, x=4..20, y=-10..0$$

$$0 \cdot \frac{4-y}{10}, x=4..20, y=0..4$$

$$0 \cdot 0, x=4..20, y=4..20$$

The graphic of the membership function $f_{\text{prod}}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$ is given on the next figure.

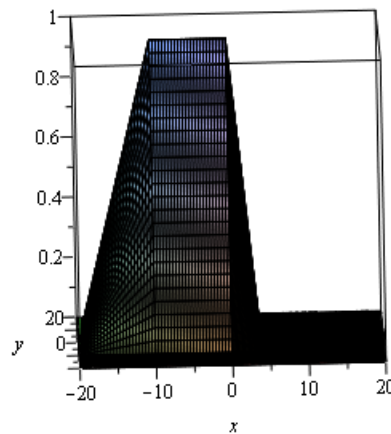


Figure 3.3: The graphic of the membership function $f_{\text{prod}}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$ of the fuzzy subset, which represent the fuzzy statement 'prod (x is A_1) AND (y is A_2)'.

Note that the truth value i.e. the DOF is not a qualitative statement on the accuracy of the fuzzy syllogism. It is a degree to which the fuzzy syllogism can be applied to a particular case.

Remark that:

i). According to (3.3), some computed DOF values for the 'product fuzzy syllogism' corresponding to the classic syllogism given in Example 3.1., for different premise values a_1 and a_2 are:

For $a_1=-10$ and $a_2=0$, $f_{A_1}(-10)=1$ and $f_{A_2}(0)=1$. Therefore $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=1 \times 1=1$

For $a_1=-15$ and $a_2=-5$, $f_{A_1}(-15)=0.5$ and $f_{A_2}(-5)=0.5$. Therefore $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=0.5 \times 0.5=0.25$

For $a_1=-15$ and $a_2=2$, $f_{A_1}(-15)=0.5$ and $f_{A_2}(2)=0.5$. Therefore $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=0.5 \times 0.5=0.25$

ii). In case of premises $a_1=-10$ and $a_2=0$ the DOF value of the product fuzzy syllogism is 4 times of the DOF values of the product fuzzy syllogism in case of premises $a_1=-15$ and $a_2=-5$, $a_1=-15$ and $a_2=2$ respectively.

iii). For $a_1=-10$ and $a_2=0$ and $c=1$ the DOF of fuzzy syllogism conclusion C is $f_C(1)=1$;

For $a_1=-15$ and $a_2=-5$ and $c=1$ the DOF of fuzzy syllogism conclusion C is $f_C(1)=0.25$;

For $a_1=-15$ and $a_2=2$ and $c=1$ the DOF of fuzzy syllogism conclusion C is $f_C(1)=0.25$;

iv. In natural language, these conclusions have the following interpretation.

In the case of premise $a_1=-10$ of argument A_1 (major premise of the syllogism) and premise $a_2=0$ of argument A_2 (minor premise of the syllogism), the truth value of conclusion (c is C) of the fuzzy syllogism for premise $c=1$ is 1.

In the case of premise $a_1 = -15$ of argument A_1 (major premise of the syllogism) and premise $a_2 = -5$ of argument A_2 (minor premise of the syllogism), the truth value of conclusion (c is C) of the fuzzy syllogism for premise $c = 1$ is 0.25.

In the case of premise $a_1 = -15$ of argument A_1 (major premise of the syllogism) and premise $a_2 = 2$ of argument A_2 (minor premise of the syllogism), the truth value of conclusion (c is C) of the fuzzy syllogism for premise $c = 1$ is 0.25.

What is the Minimum Fuzzy Syllogism?

Similarly to Definition 3.3 paper, the 'minimum fuzzy logic operator AND', in case of the fuzzy statements (x is A_1), (y is A_2), transform these fuzzy statements in a new fuzzy statement '(x is A_1) AND (y is A_2)' denoted usually by 'min(x is A_1) AND (y is A_2)' [4]. The membership function of the fuzzy subset, which represent the fuzzy statement 'min (x is A_1) AND (y is A_2)' is $f_{\min}(x,y) = \text{minimum}\{f_{A_1}(x), f_{A_2}(y)\}$. This fuzzy subset usually is denoted by $\min(A_1 \cap A_2)$ and is called also the 'min fuzzy intersection' of fuzzy subsets A_1 and A_2 . The new fuzzy statement 'min (x is A_1) AND (y is A_2)' is a fuzzy counterpart of the classic syllogism. This understanding is acceptable if the membership function of the fuzzy set C, representing the conclusion of the fuzzy syllogism in formula (3.2) is $f_C(x,y) = \text{minimum}\{f_{A_1}(x), f_{A_2}(y)\}$.

Definition.3.4. The DOF of minimum fuzzy syllogism is defined by the next formula

$$\text{DOF}[\min(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} \quad (3.6)$$

This is in fact a number attached to the same fuzzy logical expression (3.2) according to a different formula.

Example.3.2. Let consider again the, valid classic syllogism:

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and its second fuzzy counterpart defined by (3.5) .

The computed DOF values for the same premise values a_1 and a_2 in case of the example 3.1. are:

For $a_1 = -10$ and $a_2 = 0$, $f_{A_1}(-10) = 1$ and $f_{A_2}(0) = 1$. Therefore $\text{DOF}[\min(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \min\{1, 1\} = 1$

For $a_1 = -15$ and $a_2 = -5$, $f_{A_1}(-15) = 0.5$ and $f_{A_2}(-5) = 0.5$. Therefore $\text{DOF}[\min(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \min\{0.5, 0.5\} = 0.5$

For $a_1 = -15$ and $a_2 = 2$, $f_{A_1}(-15) = 0.5$ and $f_{A_2}(2) = 0.5$. Therefore $\text{DOF}[\min(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \min\{0.5, 0.5\} = 0.5$

Note that the truth value i.e. the DOF is not a qualitative statement on the accuracy of the fuzzy syllogism. It is a degree to which the fuzzy syllogism can be applied to a particular case.

The membership function $f_{\min}(x,y) = \text{minimum}\{f_{A_1}(x), f_{A_2}(y)\}$ of the fuzzy subset, which represent the fuzzy statement 'minimum (x is A_1) AND (y is A_2)' is

$$\begin{aligned} f_{\min}(x,y) &= \text{minimum}\{f_{A_1}(x), f_{A_2}(y)\} = \\ 0, & x = -20 \dots -10, y = -20 \dots -10, \\ \frac{x+20}{10}, & x = -20 \dots -10, y = x + 10 \dots 0 \\ \frac{y+10}{10}, & x = -20 \dots -10, y = -10 \dots x + 10, \\ \frac{x+20}{10}, & x = -20 \dots -10, y = 0 \dots -\frac{4}{10} \cdot x - 4, \\ \frac{4-y}{4}, & x = -20 \dots -10, y = -\frac{4}{10} \cdot x - 4 \dots 4 \end{aligned} \quad (3.7)$$

$$\begin{aligned}
&0, x = -20 \dots -10, y = 4 \dots 20 \\
&0, x = -10 \dots 0, y = -20 \dots -10, \\
&\frac{y+10}{10}, x = -10 \dots 0, y = -10 \dots 0 \\
&\frac{4-y}{4}, x = -10 \dots 0, y = 0 \dots 4 \\
&0, x = -10 \dots 0, y = 4 \dots 20 \\
&0, x = 0 \dots 4, y = -20 \dots -10 \\
&\frac{4-x}{4}, x = 0 \dots 4, y = -2.5 \cdot x \dots 0 \\
&\frac{y+10}{10}, x = 0 \dots 4, y = -10 \dots -2.5 \cdot x \\
&\frac{4-x}{4}, x = 0 \dots 4, y = 0 \dots x \\
&\frac{4-y}{10}, x = 0 \dots 4, y = x \dots 4 \\
&0, x = 0 \dots 4, y = 4 \dots 20, \\
&0, x = 4 \dots 20, y = -20 \dots -10, \\
&0, x = 4 \dots 20, y = -10 \dots 0 \\
&0, x = 4 \dots 20, y = 0 \dots 4 \\
&0, x = 4 \dots 20, y = 4 \dots 20,
\end{aligned}$$

The graphic of the membership function $f_{\min}(x,y)=\min\{f_{A_1}(x), f_{A_2}(y)\}$ is given on the next figure.

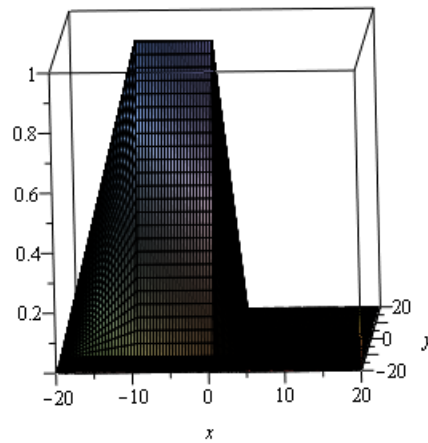


Figure 3.4: The graphic of the membership function $f_{\min}(x,y)=\text{minimum}\{f_{A_1}(x), f_{A_2}(y)\}$ of the fuzzy subset, which represent the fuzzy statement 'minimum (x is A_1) AND (y is A_2)'.

Remark that:

i). According to (3.4), some computed DOF values for the ‘minimum fuzzy syllogism’ corresponding to the classic syllogism given in Example 3.1., for different premise values a_1 and a_2 are:

For $a_1=-10$ and $a_2=0$, $f_{A_1}(-10)=1$ and $f_{A_2}(0)=1$. Therefore, $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=\min\{f_{A_1}(a_1), f_{A_2}(a_2)\}=1$

For $a_1=-15$ and $a_2=-5$, $f_{A_1}(-15)=0.5$ and $f_{A_2}(-5)=0.5$. Therefore, $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=\min\{f_{A_1}(a_1), f_{A_2}(a_2)\}=0.5$

For $a_1=-15$ and $a_2=2$, $f_{A_1}(-15)=0.5$ and $f_{A_2}(2)=0.5$. Therefore, $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=\min\{f_{A_1}(a_1), f_{A_2}(a_2)\}=0.5$

ii). In case of premises $a_1=-10$ and $a_2=0$ the DOF value of the ‘minimum fuzzy syllogism’ is twice of the DOF values of the product fuzzy syllogism in case of premises $a_1=-15$ and $a_2=-5$, $a_1=-15$ and $a_2=2$ respectively.

iii). For $a_1=-10$ and $a_2=0$ and $c=1$ the DOF of fuzzy syllogism conclusion C is $f_C(1)=1$;

For $a_1=-15$ and $a_2=-5$ and $c=1$ the DOF of fuzzy syllogism conclusion C is $f_C(1)=0.5$;

For $a_1=-15$ and $a_2=2$ and $c=1$ the DOF of fuzzy syllogism conclusion C is $f_C(1)=0.5$;

iv. In natural language, these conclusions have the following interpretation.

In the case of premise $a_1=-10$ of argument A_1 (major premise of syllogism) and premise $a_2=0$ of argument A_2 (minor premise of syllogism), the truth value of conclusion (c is C) of the fuzzy syllogism for premise $c=1$ is 1.

In the case of premise $a_1=-15$ of argument A_1 (major premise of syllogism) and premise $a_2=-5$ of argument A_2 (minor premise of syllogism), the truth value of conclusion (c is C) of the fuzzy syllogism for premise $c=1$ is 0.5.

In the case of premise $a_1=-15$ of argument A_1 (major premise of the syllogism) and premise $a_2=2$ of argument A_2 (minor premise of the syllogism), the truth value of conclusion (c is C) of the fuzzy syllogism for premise $c=1$ is 0.5.

What is the fuzzy syllogism ‘both of the classical syllogism premises are true’?

Similarly, to Definition 3.3 paper [4], the ‘fuzzy logic operator AND’, in case of the fuzzy statements (x is A_1), (y is A_2), can transform these fuzzy statements in a new fuzzy statement ‘(x is A_1) AND (y is A_2)’ named by ‘both of the classical syllogism premises are true’ and denoted usually by ‘ $\text{bscpt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’. The membership function of the fuzzy subset, which represent the fuzzy statement ‘ $\text{bscpt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ is $\text{fbscpt}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$. This fuzzy subset usually is denoted by $\min(A_1 \cap A_2)$ and is called also the ‘bscpt fuzzy intersection’ of fuzzy subsets A_1 and A_2 . The new fuzzy statement ‘ $\text{bscpt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ is a new fuzzy counterpart of the classic syllogism. This understanding is acceptable if the membership function of the fuzzy set C, representing the conclusion of the fuzzy syllogism in formula (3.2) is $f_C(x,y)=f_{A_1}(x) \times f_{A_2}(y)$.

Definition.3.4. The DOF of fuzzy syllogism ‘both of the classical syllogism premises are true’ is defined by the formula:

$$\text{DOF}[\text{bscpt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)]=\text{DOF}[(a_1 \text{ is } A_1)] \times \text{DOF}[(a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2) \quad (3.8)$$

It is easy to understand that the fuzzy syllogism ‘both of the classical syllogism premises are true’ sustain in fact the validity of the conclusion of the classical syllogism. It expresses in fuzzy logic the validity of the conclusion of the classical syllogism.

Comparing formulas (3.4) and (3.8) it is easy to see that the ‘product fuzzy syllogism’ and the fuzzy syllogism ‘both of the classical syllogism premises are true’ coincide.

Therefore, computed DOF values of the fuzzy syllogism ‘both of the classical syllogism premises are true’, for different premise values a_1 and a_2 are:

For $a_1=-10$ and $a_2=0$, $f_{A_1}(-10)=1$ and $f_{A_2}(0)=1$. Therefore $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=1 \times 1=1$

For $a_1=-15$ and $a_2=-5$, $f_{A_1}(-15)=0.5$ and $f_{A_2}(-5)=0.5$. Therefore $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=0.5 \times 0.5=0.25$

For $a_1=-15$ and $a_2=2$, $f_{A_1}(-15)=0.5$ and $f_{A_2}(2)=0.5$. Therefore $\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=0.5 \times 0.5=0.25$.

What is the fuzzy syllogism ‘at least one of the classical syllogism premises is true’?

Similarly, to Definition 3.3 paper [4], the ‘fuzzy logic operator AND’, in case of the fuzzy statements (x is A_1), (y is A_2), can transform these fuzzy statements in a new fuzzy statement ‘(x is A_1) AND (y is A_2)’ named ‘at least one of the classical syllogism premises is true’ and is denoted by ‘ $\text{ocspt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’. The membership function of the fuzzy subset, which represent the fuzzy statement ‘ $\text{ocspt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ is $\text{focspt}(x,y)=f_{A_1}(a_1)+f_{A_2}(a_2)-f_{A_1}(a_1) \times f_{A_2}(a_2)$. This fuzzy subset usually is denoted by $\min(A_1 \cap A_2)$ and is called also the ‘ocspt fuzzy intersection’ of fuzzy subsets A_1 and A_2 . The new fuzzy statement ‘ $\text{ocspt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)$ ’ is a new fuzzy counterpart of the classic syllogism. This understanding is acceptable if the membership function of the fuzzy set C, representing

the conclusion of the fuzzy syllogism is $f_c(x,y) = f_{A_1}(a_1) + f_{A_2}(a_2) - f_{A_1}(a_1) \times f_{A_2}(a_2)$.

Definition.3.5. The DOF of ‘at least one of the classical syllogism premises is true’ is defined by the next formula:

$$\text{DOF}[\text{ocspt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)] = \text{DOF}[(a_1 \text{ is } A_1)] + \text{DOF}[(a_2 \text{ is } A_2)] - \text{DOF}[(a_1 \text{ is } A_1)] \times \text{DOF}[(a_2 \text{ is } A_2)] = f_{A_1}(a_1) + f_{A_2}(a_2) - f_{A_1}(a_1) \times f_{A_2}(a_2). \quad (3.9)$$

Example.3.3. Let consider again the, valid classic syllogism:

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and its fuzzy counterpart defined by (3.6).

The computed DOF values of the fuzzy syllogism ‘ocspt (x is A_1) AND (y is A_2)’ for the same premise values a_1 and a_2 as in case of the example 3.1. are:

For $a_1 = -10$ and $a_2 = 0$, $f_{A_1}(-10) = 1$ and $f_{A_2}(0) = 1$. Therefore, $\text{DOF}[\text{‘ocspt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)\text{’}] = 1 + 1 - 1 = 1$

For $a_1 = -15$ and $a_2 = -5$, $f_{A_1}(-15) = 0.5$ and $f_{A_2}(-5) = 0.5$. Therefore, $\text{DOF}[\text{‘ocspt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)\text{’}] = 0.5 + 0.5 - 0.25 = 0.75$

For $a_1 = -15$ and $a_2 = 2$, $f_{A_1}(-15) = 0.5$ and $f_{A_2}(2) = 0.5$. Therefore, $\text{DOF}[\text{‘ocspt}(x \text{ is } A_1) \text{ AND } (y \text{ is } A_2)\text{’}] = 0.5 + 0.5 - 0.25 = 0.75$

3.3 Combination of Two Fuzzy Syllogisms Responses

Fuzzy syllogisms are usually formulated such that several fuzzy syllogisms can be applied to the same situation expressed as a vector of premises. See for example the product and minimum fuzzy syllogism presented above. These fuzzy syllogism’s has not only different DOF-s for the same premises a_1, a_2 , but also different responses. The overall response that can be derived from such fuzzy syllogism’s system has to be a combination of the individual responses which takes into account on the individual DOF-s.

Assume that the procedure for determining the DOF of two fuzzy syllogism’s S_1 and S_2 , $d_1 = \text{DOF}_{S_1}(a_1, a_2)$ and $d_2 = \text{DOF}_{S_2}(a_1, a_2)$ has been selected. Let be C_1 the fuzzy set corresponding to the fuzzy response of product fuzzy syllogism $\text{prod}\{(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)\} = f_{A_1}(a_1) \times f_{A_2}(a_2)$ and C_2 the fuzzy set corresponding to the fuzzy response of minimum fuzzy syllogism $\text{min}\{(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)\} = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\}$. The problem is to define a fuzzy set C on the basis of (d_1, C_1) and (d_2, C_2) .

What is the minimum combination of a set of two fuzzy syllogism’s responses?

The minimum combination of a set of fuzzy syllogism’s responses tries to find a combined fuzzy syllogism conclusion which is at least a certain level in agreement with all applicable syllogism.

Definition 3.5. The minimum combination C of fuzzy responses (d_1, C_1) , (d_2, C_2) is represented by the fuzzy set C which membership function is given by the formula:

$$f_c(x,y) = \min\{d_1 \times f_{C_1}(x,y), d_2 \times f_{C_2}(x,y)\}; \text{for, given}(x,y), \text{and}, d_1 > 0, d_2 > 0 \quad (3.10)$$

Example.3.4. Let consider the ‘product fuzzy syllogism’ and the ‘minimum fuzzy syllogism’ build up in examples 3.1 and 3.2 respectively, and derived from the valid classic syllogism

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and two of its fuzzy counterpart:

What is the minimum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1 = -10$ and $a_2 = 0$ and $y = 0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-10) = 1$ and $f_{A_2}(a_2) = f_{A_2}(0) = 1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0)=f_{A_1}(x) \times f_{A_2}(0)=f_{A_1}(x) \times 1=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x,0) &= \\ f_{C_1}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_1}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_1}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_1}(x,0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.11)$$

The fuzzy response C_2 of the ‘minimum fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function is $f_{C_2}(x,y)=\min\{f_{A_1}(x), f_{A_2}(y)\}$ given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0)=\min\{f_{A_1}(x), f_{A_2}(0)\}=\min\{f_{A_1}(x), 1\}=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x,0) &= \\ f_{C_2}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_2}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_2}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_2}(x,0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.12)$$

For $a_1=-10$ and $a_2=0$ and $y=0$ the ‘minimum combination of responses’ (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x)=f_C(x,0)=\min\{f_{C_1}(x,0), f_{C_2}(x,0)\}=f_{C_1}(x,0)=f_{C_2}(x,0)$ for given x and $d_1=1, d_2=1$.

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned} f_{C_{x,0}}(x) &= \\ \frac{x+20}{10} &&& \text{if } -20 < x \leq -10 ; \\ 1 &&& \text{if } -10 < x \leq 0; \\ \frac{4-x}{4} &; && \text{if } 0 < x \leq 4; \\ 0 &; && \text{if } 4 < x \leq 10; \end{aligned} \quad (3.13)$$

The ‘minimum combination of conclusions’ in this case is represented in the next figure.

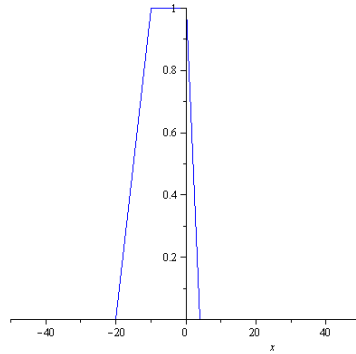


Figure 3.5: The membership function graphic of the ‘minimum combination’ $C_{x,0}$ of responses (d_1, C_1) , (d_2, C_2) , $a_1=-10$ and $a_2=0$ and $y=0$.

Remark that in this case the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$.

What is the minimum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=-5$ and $y=0$?

In this case we have: $f_{A_1}(a_1)=f_{A_1}(-15)=0.5$ and $f_{A_2}(a_2)=f_{A_2}(-5)=0.5$; $d_1=\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=0.5 \times 0.5=0.25 > 0$; $d_2=\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=\text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\}=\text{minimum}\{0.5, 0.5\}=0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0)=f_{A_1}(x) \times f_{A_2}(0)=f_{A_1}(x) \times 1=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x,0) &= \\ f_{C_1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\ f_{C_1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\ f_{C_1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\ f_{C_1}(x,0) &= 0 & \text{if } 4 < x \leq 20; \end{aligned} \quad (3.14)$$

The fuzzy response C_2 of the ‘minimum fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function is $f_{C_2}(x,y)=\min\{f_{A_1}(x), f_{A_2}(y)\}$ given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0)=\min\{f_{A_1}(x), f_{A_2}(0)\}=\min\{f_{A_1}(x), 1\}=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x,0) &= \\ f_{C_2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\ f_{C_2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\ f_{C_2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\ f_{C_2}(x,0) &= 0 & \text{if } 4 < x \leq 20; \end{aligned} \quad (3.15)$$

For $a_1=-15$ and $a_2=-5$ and $y=0$ the minimum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = f_C(x,0) = \text{minimum}\{f_{C_1}(x,0), f_{C_2}(x,0)\} = \text{minimum}\{0.25 \times f_{C_1}(x,0), 0.5 \times f_{C_2}(x,0)\} = \text{minimum}\{0.25 \times f_{C_1}(x,0), 0.5 \times f_{C_1}(x,0)\} = 0.25 \times f_{C_1}(x,0)$, for given x and $d_1=0.25$, $d_2=0.5$.

Therefore, $f_{C_{x,0}}(x)$ is given by

$$\begin{aligned}
 f_{C_{x,0}}(x) = & \\
 & 0.25 \times \frac{x+20}{10} \quad \text{if } -20 < x \leq -10; \\
 & 0.25 \quad \text{if } -10 < x \leq 0; \\
 & 0.25 \times \frac{4-x}{4} \quad \text{if } 0 < x \leq 4; \\
 & 0 \quad \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.16}$$

The minimum combination of conclusions in this case is represented in the next figure.

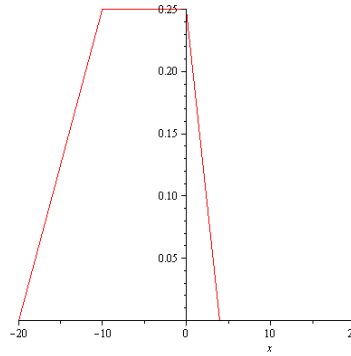


Figure 3.6: The membership function graphic of the minimum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=-5$ and $y=0$.

Remark that in this case the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.25 for any premise c in the range $[-10, 0]$.

What is the minimum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=2$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-15) = 0.5$ and $f_{A_2}(a_2) = f_{A_2}(-5) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_1}(x, 0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
f_{C1}(x,0) &= \\
f_{C1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C1}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.17}$$

The fuzzy response C_2 of the ‘minimum fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function is $f_{C2}(x,y) = \min\{f_{A1}(x), f_{A2}(y)\}$ given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C2}(x,0) = \min\{f_{A1}(x), f_{A2}(0)\} = \min\{f_{A1}(x), 1\} = f_{A1}(x)$ we obtain :

$$\begin{aligned}
f_{C2}(x,0) &= \\
f_{C2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.18}$$

For $a_1=-15$ and $a_2=2$ and $y=0$ the minimum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = f_C(x,0) = \text{minimum}\{f_{C1}(x,0), f_{C2}(x,0)\} = \text{minimum}\{0.25 \times f_{C1}(x,0), 0.5 \times f_{C2}(x,0)\} = \text{minimum}\{0.25 \times f_{C1}(x,0), 0.5 \times f_{C1}(x,0)\} = 0.25 \times f_{C1}(x,0)$ for given x and $d_1=0.25, d_2=0.5$.

Therefore, $f_{C_{x,0}}(x)$ is given by

$$\begin{aligned}
f_{C_{x,0}}(x) &= \\
0.25 \times \frac{x+20}{10} & & \text{if } -20 < x \leq -10; \\
0.25 & & \text{if } -10 < x \leq 0; \\
0.25 \times \frac{4-x}{4} & & \text{if } 0 < x \leq 4; \\
0 & & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.19}$$

The minimum combination of conclusions in this case is represented in the next figure.

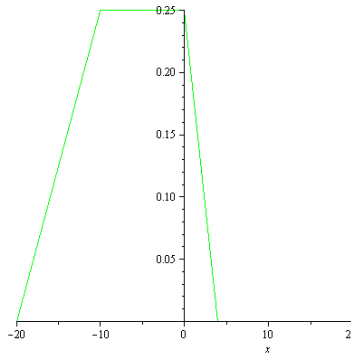


Figure 3.7: The membership function graphic of the minimum combination C of responses(d_1, C_1), (d_2, C_2) $a_1=-15$ and $a_2=2$ and $y=0$

Remark that in this case the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.25 for any premise c in the range $[-10,0]$.

Note that, in case of ‘minimum combination C of responses’, computation reveal that the truth value of the conclusion (c=1 is $C_{y=0}$)=’ Fish is not spoiled’ is highly dependent on the premises a_1 and a_2 . Namely:

If $a_1=-10$ and $a_2=0$ and $y=0$ then the truth value of conclusion ‘Fish is not spoiled’ is 1 for any premise c in the range $[-10,0]$.

If $a_1=-15$ and $a_2=-5$ and $y=0$ then the truth value of conclusion ‘Fish is not spoiled’ is 0.25 for any premise c in the range $[-10,0]$.

If $a_1=-15$ and $a_2=2$ and $y=0$ then the truth value of conclusion ‘Fish is not spoiled’ is 0.25 for any premise c in the range $[-10,0]$.

What is the cresting minimum combination of two fuzzy syllogism’s responses?

Definition 3.6. The cresting minimum combination of responses (d_1, C_1), (d_2, C_2) of two syllogism responses is the fuzzy set C which membership function is defined by the formula:

$$f_C(x,y)=\text{minimum}\{\text{minimum}_{x,y}[(d_1, f_{C1}(x,y)), (d_2, f_{C2}(x,y))]\}, \text{for, given}(x,y) \text{ and } d_1 > 0, d_2 > 0 \quad (3.20)$$

Example 3.5. Let consider again the valid classic syllogism and two of its fuzzy counterpart presented in example 3.2.

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and two of its fuzzy counterpart:

What is the cresting minimum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-10$ and $a_2=0, y=0$?

In this case we have: $f_{A1}(a_1) = f_{A1}(-10)=1$ and $f_{A2}(a_2) = f_{A2}(0)=1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A1}(a_1) \times f_{A2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A1}(a_1), f_{A2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C1}(x,y) = f_{A1}(x) \times f_{A2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C1}(x,0) = f_{A1}(x) \times f_{A2}(0) = f_{A1}(x) \times 1 = f_{A1}(x)$ we obtain :

$$\begin{aligned}
f_{C1}(x,0) &= \\
f_{C1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C1}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.21}$$

The fuzzy response C_2 of the 'minimum fuzzy syllogism' is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function is $f_{C2}(x,y) = \min\{f_{A1}(x), f_{A2}(y)\}$ given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C2}(x,0) = \min\{f_{A1}(x), f_{A2}(0)\} = \min\{f_{A1}(x), 1\} = f_{A1}(x)$ we obtain

$$\begin{aligned}
f_{C2}(x,0) &= \\
f_{C2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.22}$$

Therefore, $\min_{x,0}[(d_1, f_{C1}(x,0)), (d_2, f_{C2}(x,0))] = \min[f_{C1}(x,0), f_{C2}(x,0)] = f_{C1}(x,0) = f_{C2}(x,0)$. It follows that minimum cresting combination of responses in this case is:

$$\begin{aligned}
f_{C_{x,0}}(x) &= \\
\frac{x+20}{10} & \text{if } -20 < x \leq -10 ; \\
1 & \text{if } -10 < x \leq 0; \\
\frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
0 & \text{if } 4 < x \leq 10;
\end{aligned} \tag{3.23}$$

The cresting minimum combination of responses in this case is represented in the next figure.

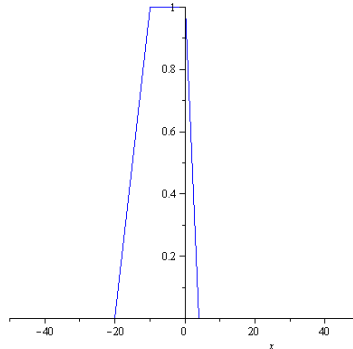


Figure 3.8: The membership function graphic of the cresting minimum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1 = -10$ and $a_2 = 0$ and $y=0$

Remark that in this case the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$.

The cresting minimum combination in this case is the same as the minimum combination Fig.3.3.

What is the cresting minimum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=-5$, $y=0$?

In this case we have: $f_{A_1}(a_1)=f_{A_1}(-15)=0.5$ and $f_{A_2}(a_2)=f_{A_2}(-5)=0.5$; $d_1=\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=0.5 \times 0.5=0.25 > 0$; $d_2=\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=\text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\}=\text{minimum}\{0.5, 0.5\}=0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0)=f_{A_1}(x) \times f_{A_2}(0)=f_{A_1}(x) \times 1=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x,0) &= \\ f_{C_1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\ f_{C_1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\ f_{C_1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\ f_{C_1}(x,0) &= 0 & \text{if } 4 < x \leq 20; \end{aligned} \quad (3.24)$$

The fuzzy response C_2 of the ‘minimum fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function is $f_{C_2}(x,y)=\min\{f_{A_1}(x), f_{A_2}(y)\}$ given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0)=\min\{f_{A_1}(x), f_{A_2}(0)\}=\min\{f_{A_1}(x), 1\}=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x,0) &= \\ f_{C_2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\ f_{C_2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\ f_{C_2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\ f_{C_2}(x,0) &= 0 & \text{if } 4 < x \leq 20; \end{aligned} \quad (3.25)$$

Therefore, $\text{minimum}_x, 0[(d_1, f_{C_1}(x,0)), (d_2, f_{C_2}(x,0))]=\text{minimum}[0.25 \times f_{C_1}(x,0), 0.5 \times f_{C_2}(x,0)]=\text{minimum}[0.25 \times f_{C_1}(x,0), 0.5 \times f_{C_1}(x,0)]=0.25 \times f_{C_1}(x,0)$. It follows that minimum cresting combination of responses in this case is:

$$\begin{aligned} f_{C_x,0}(x) &= \\ 0.25 \times \frac{x+20}{10} & & \text{if } -20 < x \leq -10; \\ 0.25 & & \text{if } -10 < x \leq 0; \\ 0.25 \times \frac{4-x}{4} & & \text{if } 0 < x \leq 4; \\ 0 & & \text{if } 4 < x \leq 20; \end{aligned} \quad (3.26)$$

The cresting minimum combination of responses in this case is represented in the next figure.

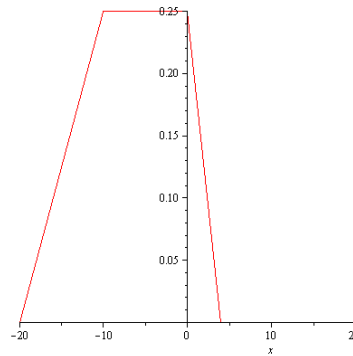


Figure 3.9: The membership function graphic of the cresting minimum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=-5$ and $y=0$

Remark that in this case the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.25 for any premise c in the range $[-10, 0]$.

What is the cresting minimum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=2$, $y=0$?

In this case reasoning similarly as in the above case the cresting minimum combination of responses in case of the product fuzzy syllogism and minimum fuzzy syllogism when $a_1=-15$ and $a_2=-5$ and $y=0$ we obtain the same result.

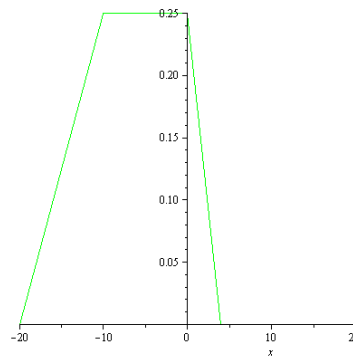


Figure 3.10: The membership function graphic of the cresting minimum combination C of responses (C_1, d_1) , (C_2, d_2) , $a_1=-15$ and $a_2=2$ and $y=0$

Remark that in this case the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.25 for any premise c in the range $[-10, 0]$.

What is the maximum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’?

The maximum combination method tries to find a combined response such that if an outcome has zero membership for all responses for which the DOF is strictly positive then the outcome has a zero membership also for the combined response.

Definition 3.7. The maximum combination C of two fuzzy responses (d_1, C_1) and (d_2, C_2) is defined by the fuzzy set C which membership function is given by the formula:

$$f_C(x, y) = \text{maximum} \{d_1 \times f_{C_1}(x, y), d_2 \times f_{C_2}(x, y)\}; \text{for, given}(x, y), \text{and}, d_1 > 0, d_2 > 0. \quad (3.27)$$

Example.3.6. Let consider again the, valid classic syllogism
Major premise 1: All refrigerated foods do not spoil.
Minor premise 1: Fish is refrigerated.
Conclusion 1: Fish is not spoiled.
and two of its fuzzy counterpart:

What is the maximum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-10$ and $a_2=0$, $y=0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-10)=1$ and $f_{A_2}(a_2) = f_{A_2}(0)=1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x,0) &= \\ f_{C_1}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_1}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_1}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_1}(x,0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.28)$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x,y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x,0) &= \\ f_{C_2}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_2}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_2}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_2}(x,0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.29)$$

For $a_1=-10$ and $a_2=0$, $y=0$ and given x in the range $[-20,20]$ the maximum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = f_c(x,0) = \text{maximum}\{f_{C_1}(x,0), f_{C_2}(x,0)\} = f_{C_1}(x,0) = f_{C_2}(x,0)$
Therefore, $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned} f_{C_{x,0}}(x) &= \\ \frac{x+20}{10} &&& \text{if } -20 < x \leq -10 ; \\ 1 &&& \text{if } -10 < x \leq 0; \\ \frac{4-x}{4} ; &&& \text{if } 0 < x \leq 4; \\ 0 &&& \text{if } 4 < x \leq 10; \end{aligned} \quad (3.30)$$

In this case the maximum combination of conclusions is represented in the next figure.

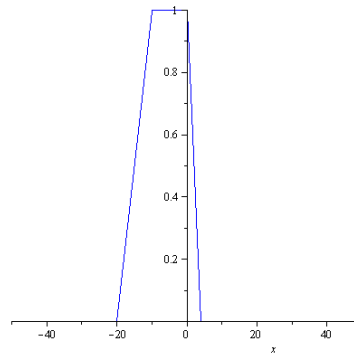


Figure 3.11: The membership function graphic of the maximum combination $C_{x,0}$ of responses (d_1, C_1) , (d_2, C_2) , when $a_1=-10$ and $a_2=0$ and $y=0$

Remark that in this case the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$.

What is the maximum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=-5$, $y=0$?

In this case $f_{A_1}(a_1) = f_{A_1}(-15) = 0.5$ and $f_{A_2}(a_2) = f_{A_2}(-5) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \min\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \min\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x,0) &= \\ f_{C_1}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_1}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_1}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_1}(x,0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.31)$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x,y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x,0) &= \\ f_{C_2}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_2}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_2}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_2}(x,0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.32)$$

The maximum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = f_C(x,0) = \text{maximum} \{0.25 \times f_{C_1}(x,0), 0.5 \times f_{C_2}(x,0)\} = \text{maximum} \{0.25 \times f_{C_1}(x,0), 0.5 \times f_{C_1}(x,0)\} = 0.5 \times f_{C_1}(x,0)$ for $a_1 = -10$ and $a_2 = 0$, $y = 0$ and given x in the range $[-20, 20]$.

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$f_{C_{x,0}}(x) = \begin{cases} 0.5 \times \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\ 0.5 & \text{if } -10 < x \leq 0; \\ 0.5 \times \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\ 0 & \text{if } 4 < x \leq 20; \end{cases} \quad (3.33)$$

The maximum combination of conclusions is represented in the next figure.

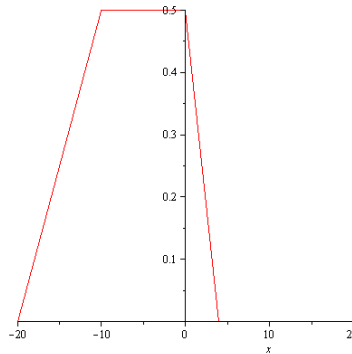


Figure 3.12: The membership function graphic of the maximum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1 = -15$ and $a_2 = -5$ and $y = 0$

Remark that the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.5 for any premise c in the range $[-10, 0]$. This is only the half of the thru value of that in case $a_1 = -10$ and $a_2 = 0$ and $y = 0$

What is the maximum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1 = -15$ and $a_2 = 2$, $y = 0$?

Repeating the above presented reasoning and computation it can be shown that the membership function of maximum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1 = -15$ and $a_2 = 2$, $y = 0$ is given by formula (3.33).

The graphic of maximum combination of conclusions in this case is represented in the next figure.

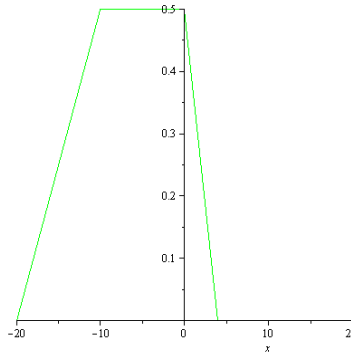


Figure 3.13: The membership function graphic of the maximum combination C of responses $(d_1, C_1), (d_2, C_2)$, $a_1 = -15$ and $a_2 = 2$ and $y = 0$

Remark that the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.5 for any premise c in the range $[-10, 0]$. This is only the half of the thru value of that in case $a_1 = -10$ and $a_2 = 0$ and $y = 0$

What is the cresting maximum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’?

Definition 3.8. The cresting maximum combination of responses $(d_1, C_1), (d_2, C_2)$, is the fuzzy set C which membership function is defined by the formula:

$$f_C(x, y) = \text{maximum} \{ \text{minimum}_x [(d_1, f_{C1}(x, y)), (d_2, f_{C2}(x, y))] ; \text{for, given, } x, y, \text{ and, } d_1 > 0, d_2 > 0 \} \quad (3.34)$$

Example.3.7. Let consider again the valid classic syllogism and two of its fuzzy counterpart presented in example 3.2.

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and two of its fuzzy counterpart:

What is the cresting maximum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ for $a_1 = -10$ and $a_2 = 0$ and $y = 0$?

In this case $f_{A1}(a_1) = f_{A1}(-10) = 1$ and $f_{A2}(a_2) = f_{A2}(0) = 1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A1}(a_1) \times f_{A2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A1}(a_1), f_{A2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$.

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C1}(x, y) = f_{A1}(x) \times f_{A2}(y)$ given by formula (3.5). For $y = 0$ and x given in the range $[-20, 20]$ according to formula $f_{C1}(x, 0) = f_{A1}(x) \times f_{A2}(0) = f_{A1}(x) \times 1 = f_{A1}(x)$ we obtain: $f_{C1}(x, 0) = f_{A1}(x)$,

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x, y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C2}(x, y) = \min\{f_{A1}(x), f_{A2}(y)\}$ is given by formula (3.7). For $y = 0$ and x given in the range $[-20, 20]$ according to formula $f_{C2}(x, 0) = \min\{f_{A1}(x), f_{A2}(0)\} = \min\{f_{A1}(x), 1\} = f_{A1}(x)$ we obtain: $f_{C2}(x, 0) = f_{A1}(x)$

Therefore, $\text{maximum}\{\text{minimum}_x [(1, f_{C1}(x, 0)), (1, f_{C2}(x, 0))]\} = \text{maximum}\{\text{minimum}(1, f_{A1}(x)), (1, f_{A1}(x))\} = f_{A1}(x)$, and $\text{maximum}\{\text{minimum}_x (1, f_{C1}(x, 0)), \text{minimum}_x (1, f_{C2}(x, 0))\} = \text{maximum}\{f_{C1}(x, 0), f_{C2}(x, 0)\} = f_{A1}(x)$. It follows that the cresting maximum combination of responses membership function $f_{C_{x,0}}(x)$ is :

$$\begin{aligned}
 f_{C_x,0}(x) = & \\
 & \frac{x+20}{10} \quad \text{if } -20 < x \leq -10 ; \\
 & 1 \quad \text{if } -10 < x \leq 0; \\
 & \frac{4-x}{4} ; \quad \text{if } 0 < x \leq 4; \\
 & 0 \quad ; \quad \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.35}$$

The cresting maximum combination of responses membership function graphic, in this case, is represented in the next figure.

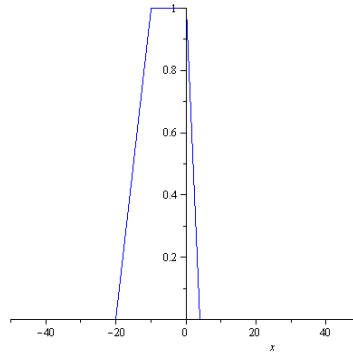


Figure 3.14: The membership function graphic of the cresting maximum combination C of responses (d_1, C_1) , (d_2, C_2) , for $a_1 = -10$ and $a_2 = 0$ and $y = 0$.

Remark that in case $a_1 = -10$ and $a_2 = 0$ and $y = 0$ the cresting maximum combination is the same as the maximum combination. (Fig.3.11.). The thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10, 0]$.

What is the cresting maximum combination in case $a_1 = -15$ and $a_2 = -5$ and $y = 0$?

In this case $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$ and $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$.

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y = 0$ and x given in the range $[-20, 20]$ according to formula $f_{C_1}(x, 0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain: $f_{C_1}(x, 0) = f_{A_1}(x)$.

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x, y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x, y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y = 0$ and x given in the range $[-20, 20]$ according to formula $f_{C_2}(x, 0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain: $f_{C_2}(x, 0) = f_{A_1}(x)$.

Therefore, $\text{maximum}\{\text{minimumx}[(0.25, f_{C_1}(x, 0)), (0.5, f_{C_2}(x, 0))]\} = \text{maximum}\{\text{minimumx}(0.25, f_{A_1}(x)), \text{minimumx}(0.5, f_{A_1}(x))\} =$

$$\begin{aligned}
 & f_{A_1}(x) \quad \text{for } -20 < x < -15 \\
 & 0.5 \quad \text{for } -15 < x < 2 \\
 & f_{A_1}(x) \quad \text{for } 2 < x < 20
 \end{aligned} \tag{3.36}$$

The cresting maximum combination of conclusions is represented in the next figure.

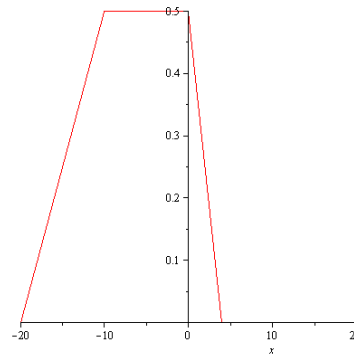


Figure 3.15: The membership function graphic of the cresting maximum combination C of responses $(d_1, C_1), (d_2, C_2)$, $a_1=-15$ and $a_2=-5$ and $y=0$

Remark that in case $a_1=-15$ and $a_2=-5$ and $y=0$ the cresting maximum combination is the same as the maximum combination in the case, $a_1=-15$ and $a_2=-5$ and $y=0$. (Fig.3.12.). The thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.5 for any premise c in the range $[-10,0]$.

What is the cresting maximum combination in case $a_1=-15$ and $a_2=2$ and $y=0$?

In this case $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$ and $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$.

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_1}(x, 0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain: $f_{C_1}(x, 0) = f_{A_1}(x)$.

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x, y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x, y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_2}(x, 0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain : $f_{C_2}(x, 0) = f_{A_1}(x)$.

Therefore, $\text{maximum}\{\text{minimumx}[(0.25, f_{C_1}(x, 0)), (0.5, f_{C_2}(x, 0))]\} = \text{maximum}\{\text{minimumx}(0.25, f_{A_1}(x)), \text{minimumx}(0.5, f_{A_1}(x))\} =$

$$\begin{aligned} & f_{A_1}(x) \quad \text{for} \quad -20 < x < -15 \\ & 0.5 \quad \text{for} \quad -15 < x < 2 \\ & f_{A_1}(x) \quad \text{for} \quad 2 < x < 20 \end{aligned} \tag{3.37}$$

The cresting maximum combination of conclusions is represented in the next figure.

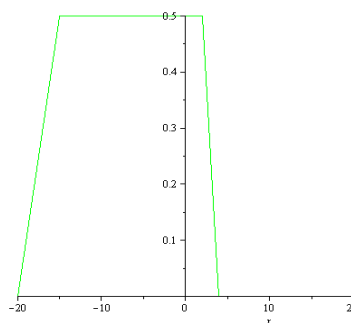


Figure 3.16: The membership function graphic of the cresting maximum combination C of responses $(d_1, C_1), (d_2, C_2)$, $a_1=-15$ and $a_2=2$ and $y=0$

Remark that in case $a_1=-15$ and $a_2=2$ and $y=0$ the cresting maximum combination is the same as the cresting maximum combination in the case $a_1=-15$ and $a_2=-5$. The thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 0.5 for any premise c in the range $[-10,0]$.

What is the weighted sum combination of two responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’?

As a possible compromise between the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ one may select the weighted sum combination.

Definition 3.8. The weighted sum combination of responses (d_1, C_1) (d_2, C_2) is the fuzzy set C which membership function $f_c(x, y)$ is given by:

$$f_c(x, y) = \frac{d_1 \times f_{C_1}(x, y) + d_2 \times f_{C_2}(x, y)}{\max_{x, y} \{d_1 \times f_{C_1}(x, y) + d_2 \times f_{C_2}(x, y)\}} \quad (3.38)$$

Example.3.8. Let consider again, the next valid classic syllogism

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and two of its fuzzy counterpart:

What is the weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-10$ and $a_2=0$, $y=0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-10) = 1$ and $f_{A_2}(a_2) = f_{A_2}(0) = 1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_1}(x, 0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x, 0) &= \\ f_{C_1}(x, 0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_1}(x, 0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_1}(x, 0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_1}(x, 0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.39)$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x, y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x, y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_2}(x, 0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
f_{C_2}(x,0) &= \\
f_{C_2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C_2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C_2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C_2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.40}$$

For $a_1=-10$ and $a_2=0$, $y=0$ and given x in the range $[-20,20]$ the weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{1 \times f_{C_1}(x,0) + 1 \times f_{C_2}(x,0)}{\max\{1 \times f_{C_1}(x,0) + 1 \times f_{C_2}(x,0)\}} = f_{C_1}(x,0) = f_{C_2}(x,0)$ }

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned}
f_{C_{x,0}}(x) &= \\
&\frac{x+20}{10} & \text{if } -20 < x \leq -10 ; \\
&1 & \text{if } -10 < x \leq 0; \\
&\frac{4-x}{4} ; & \text{if } 0 < x \leq 4; \\
&0 ; & \text{if } 4 < x \leq 10;
\end{aligned} \tag{3.41}$$

In this case the weighted sum combination of conclusions is represented in the next figure.

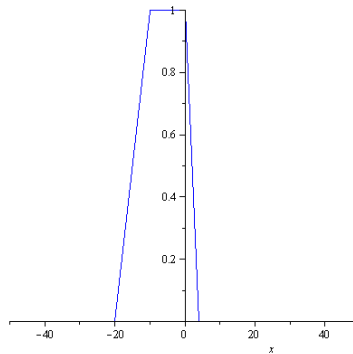


Figure 3.17: The membership function graphic of the weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-10$ and $a_2=0$ and $y=0$

Remark that in case of weighted sum combination, for $a_1=-10$ and $a_2=0$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$.

What is the weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=-5$, $y=0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-15) = 0.5$ and $f_{A_2}(a_2) = f_{A_2}(-5) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to

formula $f_{C_1}(x,0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
 f_{C_1}(x,0) &= \\
 f_{C_1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
 f_{C_1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
 f_{C_1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
 f_{C_1}(x,0) &= 0 & \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.42}$$

The fuzzy response C_2 of the 'minim fuzzy syllogism' is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x,y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
 f_{C_2}(x,0) &= \\
 f_{C_2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
 f_{C_2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
 f_{C_2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
 f_{C_2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.43}$$

For $a_1 = -15$ and $a_2 = -5$, $y=0$ and given x in the range $[-20,20]$ the weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{0.25 \times f_{C_1}(x,0) + 0.5 \times f_{C_2}(x,0)}{\max\{0.25 \times f_{C_1}(x,0) + 0.5 \times f_{C_2}(x,0)\}}$. Because $f_{C_1}(x,0) = f_{C_2}(x,0)$ we obtain that $f_{C_{x,0}}(x) = f_{C_1}(x,0) = f_{C_2}(x,0)$.

Therefore $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned}
 f_{C_{x,0}}(x) &= \\
 \frac{x+20}{10} & \text{if } -20 < x \leq -10 ; \\
 1 & \text{if } -10 < x \leq 0; \\
 \frac{4-x}{4} ; & \text{if } 0 < x \leq 4; \\
 0 & ; \text{if } 4 < x \leq 10;
 \end{aligned} \tag{3.44}$$

In this case the weighted sum combination of conclusions is represented in the next figure.

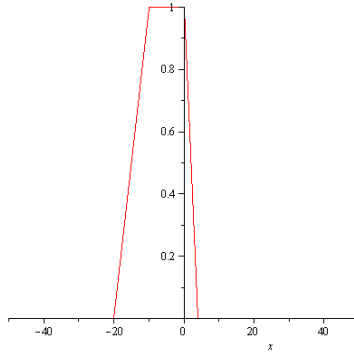


Figure 3.18. The membership function graphic of the weighted sum combination C of responses (d_1, C_1), (d_2, C_2), $a_1=-15$ and $a_2=-5$ and $y=0$

Remark that in case of weighted sum combination, for $a_1=-15$ and $a_2=-5$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$. This is the same as in case $a_1=-10$ and $a_2=0$ and $y=0$.

What is the weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=2$, $y=0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-15) = 0.5$ and $f_{A_2}(a_2) = f_{A_2}(2) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
 f_{C_1}(x,0) &= \\
 f_{C_1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
 f_{C_1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
 f_{C_1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
 f_{C_1}(x,0) &= 0 & \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.45}$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x,y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
 f_{C_2}(x,0) &= \\
 f_{C_2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
 f_{C_2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
 f_{C_2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
 f_{C_2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.46}$$

For $a_1=-15$ and $a_2=-5$, $y=0$ and given x in the range $[-20,20]$ the weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{0.25 \times f_{C_1}(x,0) + 0.5 \times f_{C_2}(x,0)}{\max\{0.25 \times f_{C_1}(x,0) + 0.5 \times f_{C_2}(x,0)\}}$. Because $f_{C_1}(x,0) = f_{C_2}(x,0)$ we obtain that $f_{C_{x,0}}(x) = f_{C_1}(x,0) = f_{C_2}(x,0)$.

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$f_{C_{x,0}}(x) = \begin{cases} \frac{x+20}{10} & \text{if } -20 < x \leq -10 ; \\ 1 & \text{if } -10 < x \leq 0; \\ \frac{4-x}{4} & ; \quad \text{if } 0 < x \leq 4; \\ 0 & ; \quad \text{if } 4 < x \leq 10; \end{cases} \quad (3.47)$$

In this case the weighted sum combination of conclusions is represented in the next figure.

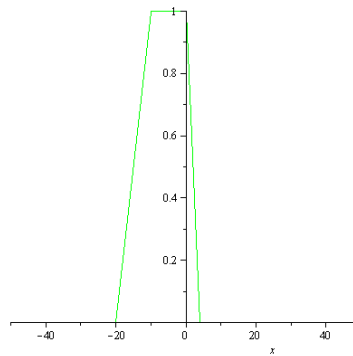


Figure 3.19. The membership function graphic of the weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=2$ and $y=0$

In case of weighted sum combination, for $a_1=-15$ and $a_2=2$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$ and is the same as for $a_1=-10$ and $a_2=0$ and $y=0$ or $a_1=-15$ and $a_2=-5$ and $y=0$.

What is the normed weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’?

As a possible compromise between the product fuzzy syllogism and minimum fuzzy syllogism ’on may select the normed weighted sum combination.

Definition 3.9. The normed weighted sum combination of responses (d_1, C_1) (d_2, C_2) is the fuzzy set C which membership function $f_C(x,y)$ is given by:

$$f_C(x, y) = \frac{d_1 \times f_{C_1}(x,y) \times c_1 + d_2 \times f_{C_2}(x,y) \times c_2}{\max_{x,y}\{d_1 \times f_{C_1}(x,y) \times c_1 + d_2 \times f_{C_2}(x,y) \times c_2\}} \quad (3.48)$$

$$\text{where } c_i = \frac{1}{\int_{-\infty}^{+\infty} f_{C_i}(x,0) dx} \quad \text{for } i=1,2 \quad (3.49)$$

What is the normed weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-10$ and $a_2=0$, $y=0$?

Example.3.9. Let consider again the valid classic syllogism presented in Example 3.1. and two of its fuzzy counterpart defined by formulas (3.4) and (3.5) respectively.

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and two of its fuzzy counterpart:

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-10) = 1$ and $f_{A_2}(a_2) = f_{A_2}(0) = 1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_1}(x, 0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x, 0) &= \\ f_{C_1}(x, 0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_1}(x, 0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_1}(x, 0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_1}(x, 0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.50)$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x, y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x, y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_2}(x, 0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x, 0) &= \\ f_{C_2}(x, 0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_2}(x, 0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_2}(x, 0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_2}(x, 0) &= 0 && \text{if } 4 < x \leq 20; \\ c_1 &= \frac{1}{\int_{-\infty}^{+\infty} f_{C_1}(x, 0) dx} = \frac{1}{\int_{-\infty}^{+\infty} f_{C_2}(x, 0) dx} = c_2 = 0.1428571429 \end{aligned} \quad (3.51)$$

For $a_1=-10$ and $a_2=0$, $y=0$ and given x in the range $[-20, 20]$ the normed weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{1 \times f_{C_1}(x, 0) \times c_1 + 1 \times f_{C_2}(x, 0) \times c_2}{\max\{1 \times f_{C_1}(x, 0) \times c_1 + 1 \times f_{C_2}(x, 0) \times c_2\}} = f_{C_1}(x, 0) = f_{C_2}(x, 0)$ }

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned}
f_{C_x,0}(x) = & \\
& \frac{x+20}{10} & \text{if } -20 < x \leq -10 ; \\
& 1 & \text{if } -10 < x \leq 0; \\
& \frac{4-x}{4} ; & \text{if } 0 < x \leq 4; \\
& 0 ; & \text{if } 4 < x \leq 10;
\end{aligned} \tag{3.52}$$

In this case the normed weighted sum combination of conclusions is represented in the next figure.

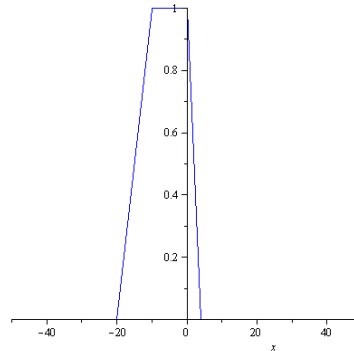


Figure 3.20. The membership function graphic of the normed weighted sum combination C of responses (d_1 , C_1), (d_2 , C_2), $a_1=-10$ and $a_2=0$ and $y=0$

In case of the normed weighted sum combination, for $a_1=-10$ and $a_2=0$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$.

What is the normed weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=-5$, $y=0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-15) = 0.5$ and $f_{A_2}(a_2) = f_{A_2}(-5) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
f_{C_1}(x,0) = & \\
& f_{C_1}(x,0) = \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
& f_{C_1}(x,0) = 1 & \text{if } -10 < x \leq 0; \\
& f_{C_1}(x,0) = \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
& f_{C_1}(x,0) = 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.53}$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x,y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
f_{C2}(x,0) &= \\
f_{C2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C2}(x,0) &= 0 & \text{if } 4 < x \leq 20; \\
c_1 &= \frac{1}{\int_{-\infty}^{+\infty} f_{C1}(x,0)dx} = \frac{1}{\int_{-\infty}^{+\infty} f_{C2}(x,0)dx} = c_2 = 0.1428571429
\end{aligned} \tag{3.54}$$

For $a_1=-15$ and $a_2=-5$, $y=0$ and given x in the range $[-20,20]$ the normed weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{0.25 \times f_{C1}(x,0) \times c_1 + 0.5 \times f_{C2}(x,0) \times c_2}{\max\{0.25 \times f_{C1}(x,0) \times c_1 + 0.5 \times f_{C2}(x,0) \times c_2\}} = f_{C1}(x,0) = f_{C2}(x,0)$ }

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned}
f_{C_{x,0}}(x) &= \\
\frac{x+20}{10} & \quad \text{if } -20 < x \leq -10 ; \\
1 & \quad \text{if } -10 < x \leq 0; \\
\frac{4-x}{4} & ; \quad \text{if } 0 < x \leq 4; \\
0 & ; \quad \text{if } 4 < x \leq 10;
\end{aligned} \tag{3.55}$$

In this case the normed weighted sum combination of conclusions is represented in the next figure.

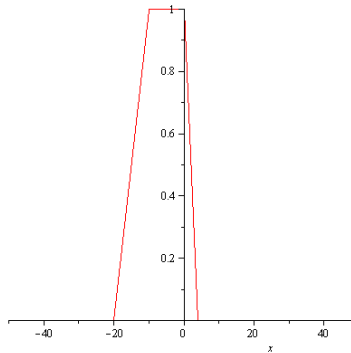


Figure 3.21. The membership function graphic of the normed weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=-5$ and $y=0$

Remark that the normed weighted sum combination, for $a_1=-15$ and $a_2=-5$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$ as for $a_1=-10$ and $a_2=0$ and $y=0$.

What is the normed weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=2$, $y=0$?

In this case we have: $f_{A1}(a_1) = f_{A1}(-15) = 0.5$ and $f_{A2}(a_2) = f_{A2}(2) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A1}(a_1) \times f_{A2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A1}(a_1), f_{A2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0)=f_{A_1}(x) \times f_{A_2}(0)=f_{A_1}(x) \times 1=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x,0) &= \\ f_{C_1}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_1}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_1}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_1}(x,0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned} \quad (3.56)$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x,y)=\min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0)=\min\{f_{A_1}(x), f_{A_2}(0)\}=\min\{f_{A_1}(x), 1\}=f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x,0) &= \\ f_{C_2}(x,0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_2}(x,0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_2}(x,0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ f_{C_2}(x,0) &= 0 && \text{if } 4 < x \leq 20; \\ c_1 &= \frac{1}{\int_{-\infty}^{+\infty} f_{C_1}(x,0)dx} = \frac{1}{\int_{-\infty}^{+\infty} f_{C_2}(x,0)dx} = c_2 = 0.1428571429 \end{aligned} \quad (3.57)$$

For $a_1=-15$ and $a_2=2$, $y=0$ and given x in the range $[-20,20]$ the normed weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{0.25 \times f_{C_1}(x,0) \times c_1 + 0.5 \times f_{C_2}(x,0) \times c_2}{\max\{0.25 \times f_{C_1}(x,0) \times c_1 + 0.5 \times f_{C_2}(x,0) \times c_2\}} = f_{C_1}(x,0) = f_{C_2}(x,0)$ }

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned} f_{C_{x,0}}(x) &= \\ \frac{x+20}{10} &&& \text{if } -20 < x \leq -10 ; \\ 1 &&& \text{if } -10 < x \leq 0; \\ \frac{4-x}{4} &; && \text{if } 0 < x \leq 4; \\ 0 &; && \text{if } 4 < x \leq 10; \end{aligned} \quad (3.58)$$

In this case the normed weighted sum combination of conclusions is represented in the next figure.

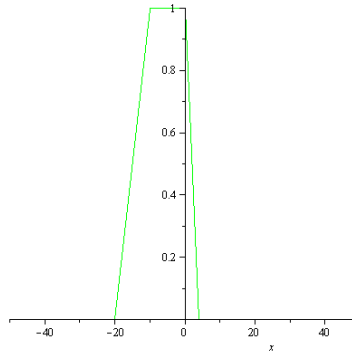


Figure 3.22. The membership function graphic of the normed weighted sum combination C of responses (d_1, C_1), (d_2, C_2), $a_1=-15$ and $a_2=2$ and $y=0$

Remark that in case of normed weighted sum combination, for $a_1=-15$ and $a_2=2$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$ as for $a_1=-10$ and $a_2=0$ and $y=0$ or for $a_1=-15$ and $a_2=-5$ and $y=0$.

What is the cresting weighted sum combination of two responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’?

As a possible compromise between the minimum and maximum combination of responses on may select the cresting weighted sum combination.

Definition 3.10. The cresting weighted sum combination of responses (d_1, C_1), (d_2, C_2), is the fuzzy set C which membership function $f_c(x,y)$ is given by:

$$f_c(x,y) = \frac{\min(d_1, f_{C_1}(x,y)) + \min(d_2, f_{C_2}(x,y))}{\max_{x,y}[\min(d_1, f_{C_1}(x,y)) + \min(d_2, f_{C_2}(x,y))]} \quad (3.59)$$

What is the cresting weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-10$ and $a_2=0, y=0$?

Example.3.10. Let consider again the valid classic syllogism presented in Example 3.1.and two of its fuzzy counterpart defined by formulas (3.4) and (3.5) respectively.

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and two of its fuzzy counterpart:

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-10)=1$ and $f_{A_2}(a_2) = f_{A_2}(0)=1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
f_{C1}(x,0) &= \\
f_{C1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C1}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.60}$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C2}(x,y) = \min\{f_{A1}(x), f_{A2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C2}(x,0) = \min\{f_{A1}(x), f_{A2}(0)\} = \min\{f_{A1}(x), 1\} = f_{A1}(x)$ we obtain :

$$\begin{aligned}
f_{C2}(x,0) &= \\
f_{C2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.61}$$

For $a_1 = -10$ and $a_2 = 0, y=0$ and given x in the range $[-20,20]$ the cresting weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{\min(1, f_{C1}(x,0)) + \min(1, f_{C2}(x,0))}{\max\{\min(1, f_{C1}(x,0)) + \min(1, f_{C2}(x,0))\}} = f_{C1}(x,0) = f_{C2}(x,0) \}$

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$\begin{aligned}
f_{C_{x,0}}(x) &= \\
&\frac{x+20}{10} & \text{if } -20 < x \leq -10 ; \\
&1 & \text{if } -10 < x \leq 0; \\
&\frac{4-x}{4} ; & \text{if } 0 < x \leq 4; \\
&0 ; & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.62}$$

In this case the cresting weighted sum combination is represented in the next figure.

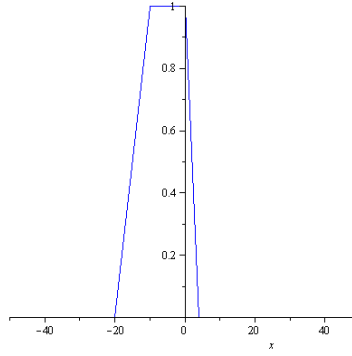


Figure 3.23. The membership function graphic of the cresting weighted sum combination C of responses (d_1, C_1), (d_2, C_2), $a_1=-10$ and $a_2=0$ and $y=0$

Remark that in case the cresting weighted sum combination, for $a_1=-10$ and $a_2=0$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$.

What is the cresting weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=-5$, $y=0$?

In this case we have: $f_{A_1}(a_1)=f_{A_1}(-15)=0.5$ and $f_{A_2}(a_2)=f_{A_2}(-5)=0.5$; $d_1=\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=f_{A_1}(a_1) \times f_{A_2}(a_2)=0.5 \times 0.5=0.25 > 0$; $d_2=\text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)]=\text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\}=\text{minimum}\{0.5, 0.5\}=0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y)=f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0)=f_{A_1}(x) \times f_{A_2}(0)=f_{A_1}(x) \times 1=f_{A_1}(x)$ we obtain :

$$\begin{aligned}
 f_{C_1}(x,0) &= \\
 f_{C_1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
 f_{C_1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
 f_{C_1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
 f_{C_1}(x,0) &= 0 & \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.63}$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x,y)=\min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_2}(x,0)=\min\{f_{A_1}(x), f_{A_2}(0)\}=\min\{f_{A_1}(x), 1\}=f_{A_1}(x)$ we obtain :

$$\begin{aligned}
 f_{C_2}(x,0) &= \\
 f_{C_2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
 f_{C_2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
 f_{C_2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
 f_{C_2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
 \end{aligned} \tag{3.64}$$

For $a_1=-15$ and $a_2=-5$, $y=0$ and given x in the range $[-20,20]$ the cresting weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{\min(0.25, f_{C_1}(x,0)) + \min(0.5, f_{C_2}(x,0))}{\max\{\min(0.25, f_{C_1}(x,0)) + \min(0.5, f_{C_2}(x,0))\}} = \frac{\min(0.25, f_{C_1}(x,0)) + \min(0.5, f_{C_1}(x,0))}{\max\{\min(0.25, f_{C_1}(x,0)) + \min(0.5, f_{C_1}(x,0))\}} =$

$$\begin{aligned} & 2 \times \frac{x+20}{10} \times \frac{1}{0.75} & \text{if } -20 < x \leq -17.5 \\ & (0.25 + \frac{x+20}{10}) \times \frac{1}{0.75} & \text{if } -17.5 < x \leq -15 \end{aligned} \quad (3.65)$$

$$\begin{aligned} & 1 & \text{if } -15 < x \leq 2 \\ & (0.25 + \frac{4-x}{4}) \times \frac{1}{0.75} & \text{if } 2 < x \leq 3 \\ & 2 \times \frac{4-x}{4} \times \frac{1}{0.75} & \text{if } 3 < x \leq 4 \\ & 0, & \text{if } 4 < x \leq 20 \end{aligned}$$

In this case the cresting weighted sum combination is represented in the next figure.

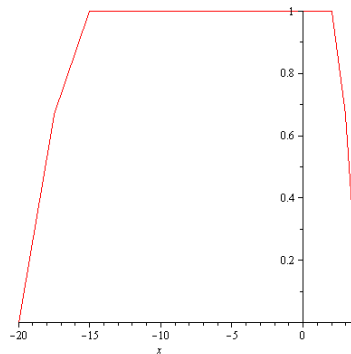


Figure 3.24. The membership function graphic of the cresting weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=-5$ and $y=0$

Remark that in case the cresting weighted sum combination, for $a_1=-15$ and $a_2=-5$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[c \text{ is } C]$ is equal to 1 for any premise c in the range $[-15,2]$

What is the cresting weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=2$, $y=0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-15) = 0.5$ and $f_{A_2}(a_2) = f_{A_2}(2) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \min\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \min\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x,y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x,y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C_1}(x,0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned}
f_{C1}(x,0) &= \\
f_{C1}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C1}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C1}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C1}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.66}$$

The fuzzy response C_2 of the 'minim fuzzy syllogism' is represented by the fuzzy set $C_2(x,y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C2}(x,y) = \min\{f_{A1}(x), f_{A2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20,20]$ according to formula $f_{C2}(x,0) = \min\{f_{A1}(x), f_{A2}(0)\} = \min\{f_{A1}(x), 1\} = f_{A1}(x)$ we obtain :

$$\begin{aligned}
f_{C2}(x,0) &= \\
f_{C2}(x,0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\
f_{C2}(x,0) &= 1 & \text{if } -10 < x \leq 0; \\
f_{C2}(x,0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\
f_{C2}(x,0) &= 0 & \text{if } 4 < x \leq 20;
\end{aligned} \tag{3.67}$$

For $a_1=-15$ and $a_2=2$, $y=0$ and given x in the range $[-20,20]$ the cresting weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{\min(0.25, f_{C1}(x,0)) + \min(0.5, f_{C2}(x,0))}{\max\{\min(0.25, f_{C1}(x,0)) + \min(0.5, f_{C2}(x,0))\}} = \frac{\min(0.25, f_{C1}(x,0)) + \min(0.5, f_{C1}(x,0))}{\max\{\min(0.25, f_{C1}(x,0)) + \min(0.5, f_{C1}(x,0))\}} =$

$$\begin{aligned}
2 \times \frac{x+20}{10} \times \frac{1}{0.75} & & \text{if } -20 < x \leq -17.5 \\
(0.25 + \frac{x+20}{10}) \times \frac{1}{0.75} & & \text{if } -17.5 < x \leq -15 \\
1 & & \text{if } -15 < x \leq 2 \\
(0.25 + \frac{4-x}{4}) \times \frac{1}{0.75} & & \text{if } 2 < x \leq 3 \\
2 \times \frac{4-x}{4} \times \frac{1}{0.75} & & \text{if } 3 < x \leq 4 \\
0 & & \text{if } 4 < x \leq 20
\end{aligned} \tag{3.68}$$

In this case the cresting weighted sum combination is represented in the next figure.

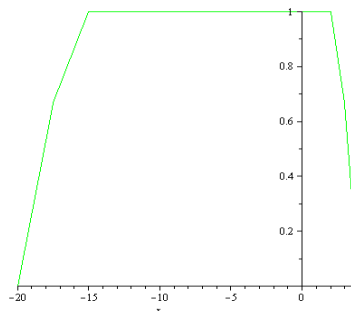


Figure 3.25. The membership function graphic of the cresting weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=2$ and $y=0$

Remark that in case of the cresting weighted sum combination, for $a_1=-15$ and $a_2=2$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-15,2]$ as for $a_1=-15$ and $a_2=-5$ and $y=0$. This computation reveal that when the first premise is $a_1=-15$ the increase of the second premise value from $a_2=-5$ to the value $a_2=2$ does not influence the range $[-15,2]$ of consequence premise c of the overall response C for which the thru value is equal to 1.

What is the cresting normed weighted sum combination of two responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’?

Definition 3.11. The cresting normed weighted sum combination of responses (d_1, C_1) (d_2, C_2) is the fuzzy set C which membership function $f_c(x, y)$ is given by:

$$f_c(x, y) = \frac{\min(d_1, f_{C_1}(x, y)) \times c_1 + \min(d_2, f_{C_2}(x, y)) \times c_2}{\max_{x, y} \{ \min(d_1, f_{C_1}(x, y)) \times c_1 + \min(d_2, f_{C_2}(x, y)) \times c_2 \}} \quad (3.69)$$

$$\text{where } c_i = \frac{1}{\int_{-\infty}^{+\infty} f_{C_i}(x, 0) dx} \quad \text{for } i=1, 2 \quad (3.70)$$

What is the cresting normed weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-10$ and $a_2=0$, $y=0$?

Example.3.11. Let consider again the valid classic syllogism presented in Example 3.1. and two of its fuzzy counterpart defined by formulas (3.4) and (3.5) respectively.

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

and two of its fuzzy counterpart:

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-10) = 1$ and $f_{A_2}(a_2) = f_{A_2}(0) = 1$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 1 \times 1 = 1 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{1, 1\} = 1 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_1}(x, 0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x, 0) &= \\ f_{C_1}(x, 0) &= \frac{x+20}{10} && \text{if } -20 < x \leq -10; \\ f_{C_1}(x, 0) &= 1 && \text{if } -10 < x \leq 0; \\ f_{C_1}(x, 0) &= \frac{4-x}{4} && \text{if } 0 < x \leq 4; \\ (3.71) \\ f_{C_1}(x, 0) &= 0 && \text{if } 4 < x \leq 20; \end{aligned}$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x, y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x, y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_2}(x, 0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$f_{C_2}(x,0)=$$

$$f_{C_2}(x,0)=\frac{x+20}{10} \quad \text{if } -20 < x \leq -10;$$

$$f_{C_2}(x,0)=1 \quad \text{if } -10 < x \leq 0;$$

$$f_{C_2}(x,0)=\frac{4-x}{4} \quad \text{if } 0 < x \leq 4;$$

$$f_{C_2}(x,0)=0 \quad \text{if } 4 < x \leq 20;$$

$$c_1 = \frac{1}{\int_{-\infty}^{+\infty} f_{C_1}(x,0)dx} = \frac{1}{\int_{-\infty}^{+\infty} f_{C_2}(x,0)dx} = c_2 = 0.1428571429$$

For $a_1=-10$ and $a_2=0$, $y=0$ and given x in the range $[-20,20]$ the cresting weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is defined by the formula: $f_{C_{x,0}}(x) = \frac{\min(1, f_{C_1}(x,0)) \times c_1 + \min(1, f_{C_2}(x,0)) \times c_2}{\max\{\min(1, f_{C_1}(x,0)) \times c_1 + \min(1, f_{C_2}(x,0)) \times c_2\}} = \frac{0.1428571429 \times [\min(1, f_{C_1}(x,0)) + \min(1, f_{C_2}(x,0))]}{0.1428571429 \times \max\{\min(1, f_{C_1}(x,0)) + \min(1, f_{C_2}(x,0))\}} = \frac{\min(1, f_{C_1}(x,0)) + \min(1, f_{C_2}(x,0))}{\max\{\min(1, f_{C_1}(x,0)) + \min(1, f_{C_2}(x,0))\}} = f_{C_1}(x,0) = f_{C_2}(x,0)$

Therefore, $f_{C_{x,0}}(x)$ is given by:

$$f_{C_{x,0}}(x) = \begin{cases} \frac{x+20}{10} & \text{if } -20 < x \leq -10 ; \\ 1 & \text{if } -10 < x \leq 0 ; \\ \frac{4-x}{4} & \text{if } 0 < x \leq 4 ; \\ 0 & \text{if } 4 < x \leq 20 ; \end{cases} \quad (3.73)$$

In this case the cresting normed weighted sum combination is represented in the next figure.

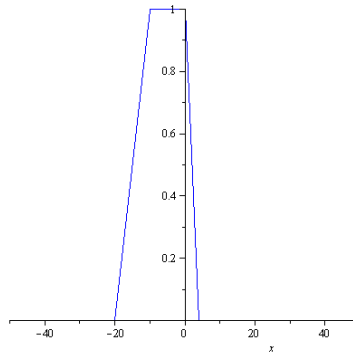


Figure 3.26. The membership function graphic of the cresting normed weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-10$ and $a_2=0$ and $y=0$

Remark that in case of the cresting normed weighted sum combination, for $a_1=-10$ and $a_2=0$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-10,0]$ as in case of the cresting weighted sum combination (Fig.3.23) or in case of normed weighted sum combination (3.20).

What is the cresting normed weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=-5$, $y=0$?

In this case we have: $f_{A_1}(a_1) = f_{A_1}(-15) = 0.5$ and $f_{A_2}(a_2) = f_{A_2}(-5) = 0.5$; $d_1 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = f_{A_1}(a_1) \times f_{A_2}(a_2) = 0.5 \times 0.5 = 0.25 > 0$; $d_2 = \text{DOF}[(a_1 \text{ is } A_1) \text{ AND } (a_2 \text{ is } A_2)] = \text{minimum}\{f_{A_1}(a_1), f_{A_2}(a_2)\} = \text{minimum}\{0.5, 0.5\} = 0.5 > 0$

The fuzzy response C_1 of the ‘product fuzzy syllogism’ is represented by the fuzzy set $C_1(x, y)$ which graphic can be seen in Fig.3.3, and which membership function is $f_{C_1}(x, y) = f_{A_1}(x) \times f_{A_2}(y)$ given by formula (3.5). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_1}(x, 0) = f_{A_1}(x) \times f_{A_2}(0) = f_{A_1}(x) \times 1 = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_1}(x, 0) &= \\ f_{C_1}(x, 0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\ f_{C_1}(x, 0) &= 1 & \text{if } -10 < x \leq 0; \\ f_{C_1}(x, 0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\ f_{C_1}(x, 0) &= 0 & \text{if } 4 < x \leq 20; \end{aligned} \quad (3.74)$$

The fuzzy response C_2 of the ‘minim fuzzy syllogism’ is represented by the fuzzy set $C_2(x, y)$ which graphic can be seen in Fig.3.4, and which membership function $f_{C_2}(x, y) = \min\{f_{A_1}(x), f_{A_2}(y)\}$ is given by formula (3.7). For $y=0$ and x given in the range $[-20, 20]$ according to formula $f_{C_2}(x, 0) = \min\{f_{A_1}(x), f_{A_2}(0)\} = \min\{f_{A_1}(x), 1\} = f_{A_1}(x)$ we obtain :

$$\begin{aligned} f_{C_2}(x, 0) &= \\ f_{C_2}(x, 0) &= \frac{x+20}{10} & \text{if } -20 < x \leq -10; \\ f_{C_2}(x, 0) &= 1 & \text{if } -10 < x \leq 0; \\ f_{C_2}(x, 0) &= \frac{4-x}{4} & \text{if } 0 < x \leq 4; \\ f_{C_2}(x, 0) &= 0 & \text{if } 4 < x \leq 20; \end{aligned} \quad (3.75)$$

For $a_1=-15$ and $a_2=-5$, $y=0$ and given x in the range $[-20, 20]$ the cresting normed weighted sum combination of responses (d_1, C_1) and

(d_2, C_2) is the fuzzy set $C_{x,0}$ which membership function is given by : $f_{C_{x,0}}(x) = \frac{\min(0.25, f_{C_1}(x, 0)) \times c_1 + \min(0.5, f_{C_2}(x, 0)) \times c_2}{\max\{\min(0.25, f_{C_1}(x, 0)) \times c_1 + \min(0.5, f_{C_2}(x, 0)) \times c_2\}} = \frac{0.1428571429 \times [\min(0.25, f_{C_1}(x, 0)) + \min(0.5, f_{C_1}(x, 0))]}{[\min(0.25, f_{C_1}(x, 0)) + \min(0.5, f_{C_1}(x, 0))]} = \frac{0.1428571429 \times \max\{\min(0.25, f_{C_1}(x, 0)) + \min(0.5, f_{C_1}(x, 0))\}}{\max\{\min(0.25, f_{C_1}(x, 0)) + \min(0.5, f_{C_1}(x, 0))\}}$

$$\begin{aligned} 2 \times \frac{x+20}{10} \times \frac{1}{0.75} & & \text{if } -20 < x \leq -17.5 \\ (0.25 + \frac{x+20}{10}) \times \frac{1}{0.75} & & \text{if } -17.5 < x \leq -15 \\ 1 & & \text{if } -15 < x \leq -2 \\ (0.25 + \frac{4-x}{4}) \times \frac{1}{0.75} & & \text{if } -2 < x \leq 3 \\ 2 \times \frac{4-x}{4} \times \frac{1}{0.75} & & \text{if } 3 < x \leq 4 \\ 0, & & \text{if } 4 < x \leq 20 \end{aligned} \quad (3.76)$$

In this case the cresting normed weighted sum combination is represented in the next figure.

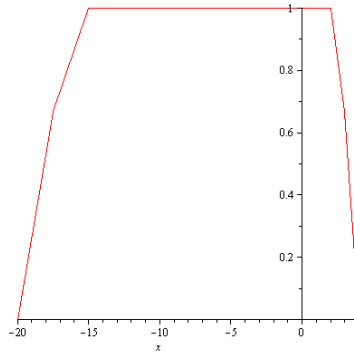


Figure 3.27. The membership function graphic of the cresting normed weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=-5$ and $y=0$

Remark that in case the cresting normed weighted sum combination, for $a_1=-15$ and $a_2=-5$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-15, 2]$ as is in the case of normed weighted sum combination.

What is the cresting normed weighted sum combination of responses in case of the ‘product fuzzy syllogism’ and ‘minimum fuzzy syllogism’ when $a_1=-15$ and $a_2=2$, $y=0$?

In this case following the same reasoning and computation as in the case $a_1=-15$ and $a_2=-5$, $y=0$ we obtained that, for $a_1=-15$ and $a_2=2$, $y=0$ and given x in the range $[-20, 20]$ the cresting normed weighted sum combination of responses (d_1, C_1) and (d_2, C_2) is the fuzzy set C x,0 which membership function is given by : $f_{C_{x,0}}(x)=$

$$\begin{aligned}
 & 2 \times \frac{x+20}{10} \times \frac{1}{0.75} && \text{if } -20 < x \leq -17.5 \\
 & (0.25 + \frac{x+20}{10}) \times \frac{1}{0.75} && \text{if } -17.5 < x \leq -15 \\
 & 1 && \text{if } -15 < x \leq 2 \\
 & (0.25 + \frac{4-x}{4}) \times \frac{1}{0.75} && \text{if } 2 < x \leq 3 \\
 & 2 \times \frac{4-x}{4} \times \frac{1}{0.75} && \text{if } 3 < x \leq 4 \\
 & 0 && \text{if } 4 < x \leq 20
 \end{aligned} \tag{3.77}$$

In this case the cresting normed weighted sum combination is represented in the next figure.

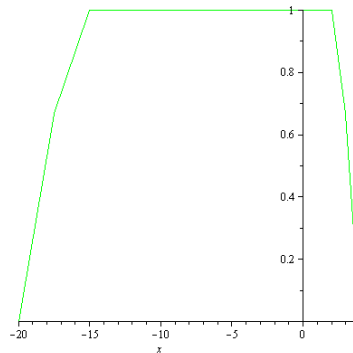


Figure 3.28. The membership function graphic of the cresting normed weighted sum combination C of responses (d_1, C_1) , (d_2, C_2) , $a_1=-15$ and $a_2=2$ and $y=0$

Remark that in case the cresting normed weighted sum combination, for $a_1=-15$ and $a_2=2$ and $y=0$ the thru value of the overall conclusion (c is C) i.e. $\text{DOF}[(c \text{ is } C)]$ is equal to 1 for any premise c in the range $[-15,2]$ as is in the case of normed weighted sum combination.

4. Results

For any valid classical (Aristotelian) syllogism, 11 variants of fuzzy syllogisms have been defined within fuzzy logic. The calculations made for the valid classical syllogism

Major premise 1: All refrigerated foods do not spoil.

Minor premise 1: Fish is refrigerated.

Conclusion 1: Fish is not spoiled.

show the dependence of the truth value of the conclusions of fuzzy syllogisms on both the considered variant and the truth values of the premises.

5. Comments

If we accept that, in the case of ambiguity of the terms used as well as ambiguous formulations, human thinking is fuzzy, then what is shown in this paper can explain many human decisions encountered in life, that are sometimes strange from the point of view of classical (Aristotelian) logic. Our option for fuzzy human thinking in disadvantage of probabilistic (frequency based) thinking is explained by the fact that we refer to decisions in the case of accidental (which cannot be repeated) and not random event.

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