

Viscoelastic and Viscoplastic Glucose Theory (VGT #29): Applying viscoelastic Glucose Theory with CGM Sensor Measured Fasting Plasma Glucose (FPG) Values as the Strain along with the FPG Change Rate Multiplied by a Viscosity Factor as the Stress, and Finally Utilizing the Viscoelastic Perturbation Model to Calculate the Predicted FPG Values to Compare Against the Measured FPG Values Based on GH-Method: Math-Physical Medicine (No. 610)

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Abstract

The author has studied strength of materials and theory of elasticity through his undergraduate courses at the University of Iowa. He also conducted research work to earn a master's degree in Biomechanics under Professor James Andrews. At that time, he used the spring and dashpot models to simulate the behaviors of human joints, bones, muscles, and tendons in order to investigate the human-weapon interactions during the Vietnam war era. Later, he went to MIT to pursue his PhD study under Professor Norman Jones, who taught him theory of plasticity and dynamic plastic behaviors of various structure elements. He also took additional graduate courses in the field of fluid dynamics and thermodynamics.

*Since then, many advancements have been made in the biomechanics branch, especially with human body tissues that possess certain viscoelastic characteristics, such as bones, muscles, cartilages, tendons (connect bone to muscle), ligaments (connect bone to bone), fascia, and skin. For example, the author suffered plantar fasciitis for many years. He understood that the night splint dorsiflexes forefoot, at the back of the foot, increases plantar fascia tension to offer stress-relaxation for the pain. This model where muscles and tendons connect the lower leg and foot is a form of viscoelastic study for medical problem solving. When dealing with human internal organs, it is not easy to conduct live experiments to obtain accurate measurements for the biomedical material properties. Blood itself is a viscous material (time-dependent) and its viscosity factor may fall between water, honey, syrup, or gel. **The author's research focuses on "glucose" where the blood sugar amount is produced by the liver and carried by red blood cells, not the blood itself. Fasting plasma glucose (FPG) has nothing to do with either diet or exercise during sleep hours and is mainly relying on the health condition of both liver and pancreatic beta cells, which produces glucose and releases insulin, to control the glucose level in blood. Therefore, it is nearly impossible to measure the material geometry or material properties to determine the viscosity of "glucose" like in engineering research work. As a result, the best the author could do is to apply the "concept of viscoelasticity and/or viscoplasticity" to construct an analogy model of time-dependent glucose behaviors.***

The author's background includes mathematics, physics, and various engineering disciplines, not including biology and chemistry. He can only investigate the observed biophysical phenomena in the medical field using his ready-learned math-physical tools. He studied both modern physics and quantum mechanics during his school days; therefore, he applied the theory of relativity on interactions among the organs in the human body (inner space) which is similar to the inter-relationships among the planets in the universe (outer space). This analogy on the theory of relativity has proven the inter-connectivity of human internal organs.

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glucose behaviors using elasticity theory and plasticity theory (both static and dynamic), which allowed him to write a few articles on his research findings.

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In this particular viscoelasticity study, he utilized a continuous glucose monitoring (CGM) device to collect FPG values during the 7 hours of sleep as the strain along with the body weight and body temperature measured in the early morning as the input. In order to include the time-dependent characteristics, he uses the FPG change rate multiplied with a **viscosity factor (a combined value from both normalized body weight and normalized body temperature)** in his stress-strain analysis and viscoelastic perturbation analysis.

In his earlier research work, he has identified FPG having a higher degree of correlation (90+%) with either body weight or body temperature measured in the early morning. In this study, he has chosen a combined factor of body weight and body temperature as the viscosity factor which influences FPG level as the output strain. Finally, he adds the strain value with the stress value to obtain a predicted strain i.e., predicted FPG, using his defined "viscoelastic perturbation model".

Nevertheless, the medical field is quite different from the engineering field, where the engineering materials such as steel, copper, concrete, and aluminum are **inorganic** in most cases. These material properties do not change significantly over their expected lifespans. However, in medicine, the body with its organs and cells are organic and go through many distinct stages over their natural lifespans, such as birth, splitting, growth, mutation, development, repair, sickness, and death. As a result, **the biomedical properties are "moving targets" which vary with the individual person, severity of diabetes, and selected different time-windows**. In other words, **they are both time-dependent and specimen-dependent**, because these fundamental characteristics, calculations for the cross-section of the subject, bending moment of resistance, or the shape factors in solid mechanics are not applicable in this biomedical glucose analogy study of elasticity/plasticity or viscoelasticity/viscoplasticity. In the author's opinion, the most important part is to **apply the concept** of elasticity/plasticity theory or viscoelasticity/viscoplasticity theory on understanding or illustrating the observed biomedical phenomena. This is extremely useful to explore deep insights or enable the prediction of glucoses, particularly for **hyperglycemic conditions (leading into various internal organ complications) and hypoglycemic conditions (insulin shock leading to possible sudden death)**. In regard to FPG, the possibility of having insulin shock especially during sleeping hours is dangerous.

To offer a simple explanation to readers who do not have a physics or engineering background, the author includes a brief excerpt from Wikipedia regarding the description of basic concepts for elasticity, plasticity, viscoelasticity, viscoplasticity, and perturbation theories from the disciplines of engineering and physics in the Method section.

In conclusion, the author has defined his stress-strain equations as follows:

strain

= ϵ

= **Sensor FPG value of each month**

Viscosity Factor

= η

= **((body weight / averaged body weight) + (body temperature / averaged body temperature)) / 2 of each month**

However, he has utilized the FPG change rate multiplied with the above described viscosity factor to obtain the stress:

Stress

= $\eta * (d\epsilon/dt)$

= $\eta * (d\text{-strain}/d\text{-time})$

= **((viscosity factor of present month) * (FPG of present month - FPG of previous month) / 30)**

Where 30 indicates the 30-day timespan of his collected monthly data.

Based on the stress-strain analysis in spatial-domain (SD), viscoelasticity or viscoplasticity theory, and perturbation model, the following two distinct observations are evident:

(1) Observing from the stress-strain diagram in SD, the measured FPG using combined weight and temperature as the viscosity factor has shown a viscoelastic behavior (starting FPG and ending FPG are extremely close to each other). It also has both stress-creeping and strain-relaxation phenomena (upper right and right portion).

(2) The predicted FPG values using the viscoelastic perturbation method have achieved extremely high prediction accuracy of 100% and correlation coefficient of 100% in comparison with the measured FPG. This means that the predicted FPG equations using the combined effect from the average of normalized body weight and normalized body temperature can generate highly accurate predicted FPG values.

In summary, from a daily practice viewpoint, using an average normalized body weight and normalized body temperature as the viscosity factor while applying the viscoelastic perturbation model can produce a satisfied prediction of FPG in the early morning.

Introduction

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Methods

Elasticity, Plasticity, Viscoelasticity and Viscoplasticity

The Difference Between Elastic Materials and Viscoelastic Materials

(from “Soborthans, innovating shock and vibration solutions”)

What are Elastic Materials?

Elasticity is the tendency of solid materials to return to their original shape after forces are applied on them. When the forces are removed, the object will return to its initial shape and size if the material is elastic.

What are Viscous Materials?

Viscosity is a measure of a fluid’s resistance to flow. A fluid with large viscosity resists motion. A fluid with low viscosity flows. For example, water flows more easily than syrup because it has a lower viscosity. High viscosity materials might include honey, syrups, or gels – generally things that resist flow. Water is a low viscosity material, as it flows readily. Viscous materials are thick or sticky or adhesive. Since heating reduces viscosity, these materials don’t flow easily. For example, warm syrup flows more easily than cold.

What is Viscoelastic?

Viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. Synthetic polymers, wood, and human tissue, as well as met-

als at high temperature, display significant viscoelastic effects. In some applications, even a small viscoelastic response can be significant.

Elastic Behavior Versus Viscoelastic Behavior

The difference between elastic materials and viscoelastic materials is that viscoelastic materials have a viscosity factor and the elastic ones don’t. Because viscoelastic materials have the viscosity factor, they have a strain rate dependent on time. Purely elastic materials do not dissipate energy (heat) when a load is applied, then removed; however, a viscoelastic substance does.

The following brief introductions are excerpts from Wikipedia:

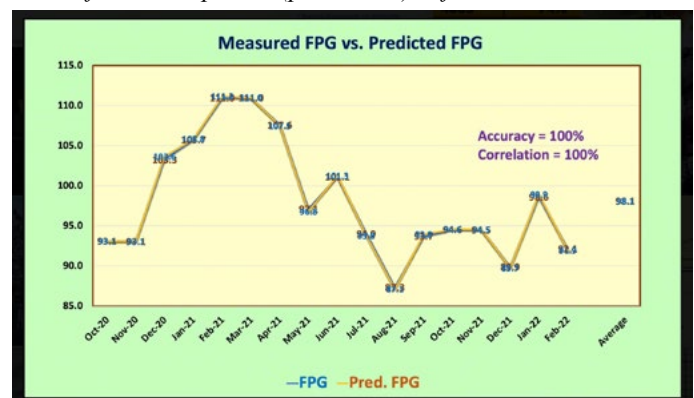
“Elasticity (Physics)

Physical property when materials or objects return to original shape after deformation

In physics and materials science, elasticity is the ability of a body to resist a distorting influence and to return to its original size and shape when that influence or force is removed. Solid objects will deform when adequate loads are applied to them; if the material is elastic, the object will return to its initial shape and size after removal. This is in contrast to plasticity, in which the object fails to do so and instead remains in its deformed state.

The physical reasons for elastic behavior can be quite different for different materials. In metals, the atomic lattice changes size and shape when forces are applied (energy is added to the system). When forces are removed, the lattice goes back to the original lower energy state. For rubbers and other polymers, elasticity is caused by the stretching of polymer chains when forces are applied.

Hooke’s law states that the force required to deform elastic objects should be directly proportional to the distance of deformation, regardless of how large that distance becomes. This is known as perfect elasticity, in which a given object will return to its original shape no matter how strongly it is deformed. This is an ideal concept only; most materials which possess elasticity in practice remain purely elastic only up to very small deformations, after which plastic (permanent) deformation occurs.



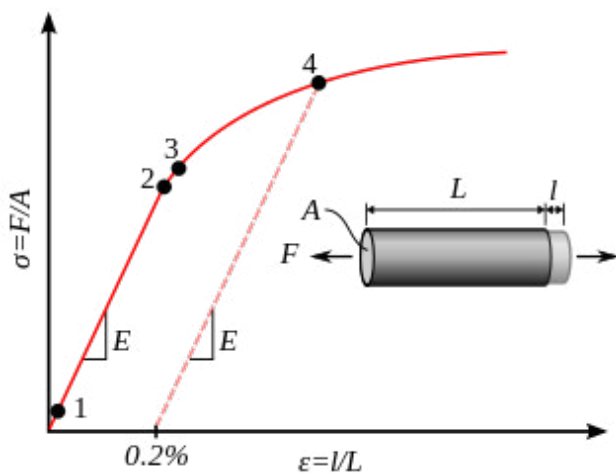
In engineering, the elasticity of a material is quantified by the elastic modulus such as the Young’s modulus, bulk modulus or shear modulus which measure the amount of stress needed to achieve a unit of strain; a higher modulus indicates that the ma-

material is harder to deform. The material's elastic limit or yield strength is the maximum stress that can arise before the onset of plastic deformation.

Plasticity (Physics)

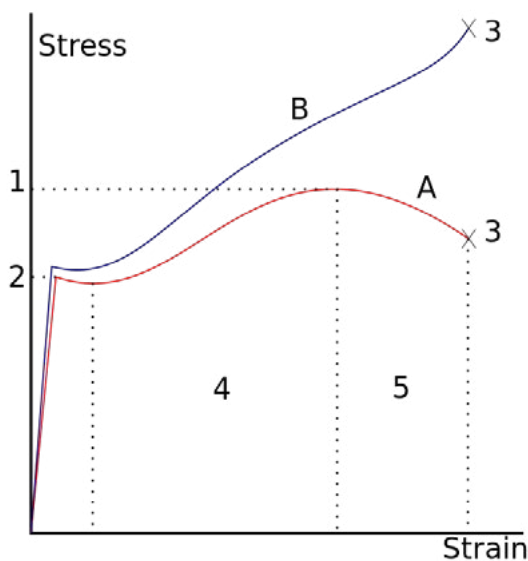
Deformation of a solid material undergoing non-reversible changes of shape in response to applied forces.

In physics and materials science, plasticity, also known as plastic deformation, is the ability of a solid material to undergo permanent deformation, a non-reversible change of shape in response to applied forces. For example, a solid piece of metal being bent or pounded into a new shape displays plasticity as permanent changes occur within the material itself. In engineering, the transition from elastic behavior to plastic behavior is known as yielding.



Stress–strain curve showing typical yield behavior for nonferrous alloys.

1. True elastic limit
2. Proportionality limit
3. Elastic limit
4. Offset yield strength



A stress–strain curve typical of structural steel.

- 1: Ultimate strength
- 2: Yield strength (yield point)
- 3: Rupture
- 4: Strain hardening region
- 5: Necking region
- A: Apparent stress (F/A_0)
- B: Actual stress (F/A)

Plastic deformation is observed in most materials, particularly metals, soils, rocks, concrete, and foams. However, the physical mechanisms that cause plastic deformation can vary widely. At a crystalline scale, plasticity in metals is usually a consequence of dislocations. Such defects are relatively rare in most crystalline materials, but are numerous in some and part of their crystal structure; in such cases, plastic crystallinity can result. In brittle materials such as rock, concrete and bone, plasticity is caused predominantly by slip at microcracks. In cellular materials such as liquid foams or biological tissues, plasticity is mainly a consequence of bubble or cell rearrangements, notably T1 processes.

For many ductile metals, tensile loading applied to a sample will cause it to behave in an elastic manner. Each increment of load is accompanied by a proportional increment in extension. When the load is removed, the piece returns to its original size. However, once the load exceeds a threshold – the yield strength – the extension increases more rapidly than in the elastic region; now when the load is removed, some degree of extension will remain.

Elastic deformation, however, is an approximation and its quality depends on the time frame considered and loading speed. If, as indicated in the graph opposite, the deformation includes elastic deformation, it is also often referred to as "elasto-plastic deformation" or "elastic-plastic deformation".

Perfect plasticity is a property of materials to undergo irreversible deformation without any increase in stresses or loads. Plastic materials that have been hardened by prior deformation, such as cold forming, may need increasingly higher stresses to deform further. Generally, plastic deformation is also dependent on the deformation speed, i.e. higher stresses usually have to be applied to increase the rate of deformation. Such materials are said to deform visco-plastically."

Viscoelasticity

Property of materials with both viscous and elastic characteristics under deformation

In materials science and continuum mechanics, viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. Viscous materials, like water, resist shear flow and strain linearly with time when a stress is applied. Elastic materials strain when stretched and immediately return to their original state once the stress is removed.

Viscoelastic materials have elements of both of these properties and, as such, exhibit time-dependent strain. Whereas elasticity is usually the result of bond stretching along crystallographic planes in an ordered solid, viscosity is the result of the diffusion of atoms or molecules inside an amorphous material.

In the nineteenth century, physicists such as Maxwell, Boltzmann, and Kelvin researched and experimented with creep and

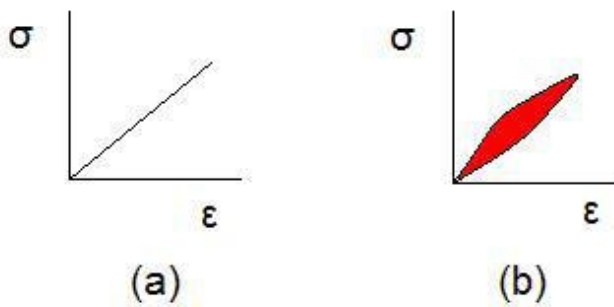
recovery of glasses, metals, and rubbers. Viscoelasticity was further examined in the late twentieth century when synthetic polymers were engineered and used in a variety of applications. Viscoelasticity calculations depend heavily on the viscosity variable, η . The inverse of η is also known as fluidity, ϕ . The value of either can be derived as a function of temperature or as a given value (i.e. for a dashpot).

Depending on the change of strain rate versus stress inside a material, the viscosity can be categorized as having a linear, non-linear, or plastic response. When a material exhibits a linear response it is categorized as a Newtonian material. In this case the stress is linearly proportional to the strain rate. If the material exhibits a non-linear response to the strain rate, it is categorized as Non-Newtonian fluid. There is also an interesting case where the viscosity decreases as the shear/strain rate remains constant. A material which exhibits this type of behavior is known as thixotropic. In addition, when the stress is independent of this strain rate, the material exhibits plastic deformation. Many viscoelastic materials exhibit rubber-like behavior explained by the thermodynamic theory of polymer elasticity.

Cracking occurs when the strain is applied quickly and outside of the elastic limit. Ligaments and tendons are viscoelastic, so the extent of the potential damage to them depends both on the rate of the change of their length as well as on the force applied. **A viscoelastic material has the following properties:**

- hysteresis is seen in the stress–strain curve
- stress relaxation occurs: step constant strain causes decreasing stress
- creep occurs: step constant stress causes increasing strain
- its stiffness depends on the strain rate or the stress rate.

Elastic versus viscoelastic behavior



Stress–strain curves for a purely elastic material (a) and a viscoelastic material (b). The red area is a hysteresis loop and shows the amount of energy lost (as heat) in a loading and unloading cycle. It is equal to

$$\oint \sigma d\epsilon$$

where σ is stress and ϵ is strain.

Unlike purely elastic substances, a viscoelastic substance has an elastic component and a viscous component. **The viscosity of a viscoelastic substance gives the substance a strain rate dependence on time.** Purely elastic materials do not dissipate energy (heat) when a load is applied, then removed. However, a viscoelastic substance dissipates energy when a load is applied, then removed. Hysteresis is observed in the stress–strain curve, with the area of the loop being equal to the energy lost during the loading cycle. Since viscosity is the resistance to thermally activated plastic deformation, a viscous material will lose ener-

gy through a loading cycle. Plastic deformation results in lost energy, which is uncharacteristic of a purely elastic material's reaction to a loading cycle.

Specifically, viscoelasticity is a molecular rearrangement. When a stress is applied to a viscoelastic material such as a polymer, parts of the long polymer chain change positions. This movement or rearrangement is called “creep”. Polymers remain a solid material even when these parts of their chains are rearranging in order to accompany the stress, and as this occurs, it creates a back stress in the material. When the back stress is the same magnitude as the applied stress, the material no longer creeps. When the original stress is taken away, the accumulated back stresses will cause the polymer to return to its original form. **The material creeps, which gives the prefix visco-, and the material fully recovers, which gives the suffix -elasticity.**

Viscoplasticity

Viscoplasticity is a theory in continuum mechanics that describes the rate-dependent inelastic behavior of solids. Rate-dependence in this context means that the deformation of the material depends on the rate at which loads are applied. The inelastic behavior that is the subject of viscoplasticity is plastic deformation which means that the material undergoes unrecoverable deformations when a load level is reached. Rate-dependent plasticity is important for transient plasticity calculations. The main difference between rate-independent plastic and viscoplastic material models is that the latter exhibit not only permanent deformations after the application of loads but continue to undergo a creep flow as a function of time under the influence of the applied load.

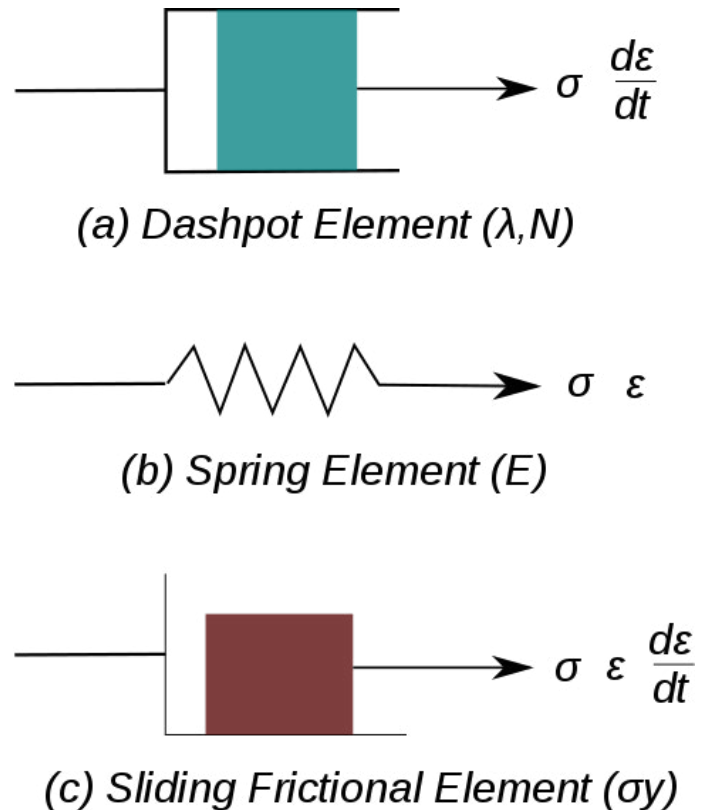


Figure 1. Elements used in one-dimensional models of viscoplastic materials.

The elastic response of viscoplastic materials can be represented

in one-dimension by Hookean spring elements. Rate-dependence can be represented by nonlinear dashpot elements in a manner similar to viscoelasticity. Plasticity can be accounted for by adding sliding frictional elements as shown in Figure 1. In the figure E is the modulus of elasticity, λ is the viscosity parameter and N is a power-law type parameter that represents non-linear dashpot [$\sigma(d\varepsilon/dt) = \sigma = \lambda(d\varepsilon/dt)^{1/N}$]. The sliding element can have a yield stress (σ_y) that is strain rate dependent, or even constant, as shown in Figure 1c.

Viscoplasticity is usually modeled in three-dimensions using overstress models of the Perzyna or Duvaut-Lions types. In these models, the stress is allowed to increase beyond the rate-independent yield surface upon application of a load and then allowed to relax back to the yield surface over time. The yield surface is usually assumed not to be rate-dependent in such models. An alternative approach is to add a strain rate dependence to the yield stress and use the techniques of rate independent plasticity to calculate the response of a material

For metals and alloys, viscoplasticity is the macroscopic behavior caused by a mechanism linked to the movement of dislocations in grains, with superposed effects of inter-crystalline gliding. The mechanism usually becomes dominant at temperatures greater than approximately one third of the absolute melting temperature. However, certain alloys exhibit viscoplasticity at room temperature (300K). For polymers, wood, and bitumen, the theory of viscoplasticity is required to describe behavior beyond the limit of elasticity or viscoelasticity.

In general, viscoplasticity theories are useful in areas such as

- the calculation of permanent deformations,
- the prediction of the plastic collapse of structures,
- the investigation of stability,
- crash simulations,
- systems exposed to high temperatures such as turbines in engines, e.g. a power plant,
- dynamic problems and systems exposed to high strain rates.

Phenomenology

For a qualitative analysis, several characteristic tests are performed to describe the phenomenology of viscoplastic materials. Some examples of these tests are

1. hardening tests at constant stress or strain rate,
2. creep tests at constant force, and
3. stress relaxation at constant elongation.

Strain Hardening Test

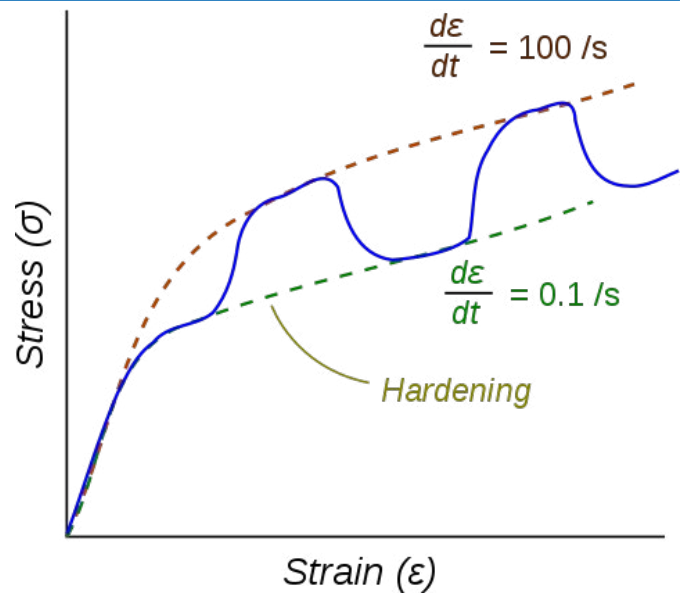


Figure 2. Stress–strain response of a viscoplastic material at different strain rates.

The dotted lines show the response if the strain-rate is held constant. The blue line shows the response when the strain rate is changed suddenly.

One consequence of yielding is that as plastic deformation proceeds, an increase in stress is required to produce additional strain. This phenomenon is known as Strain/Work hardening. For a viscoplastic material the hardening curves are not significantly different from those of rate-independent plastic material. Nevertheless, three essential differences can be observed.

1. At the same strain, the higher the rate of strain the higher the stress
2. A change in the rate of strain during the test results in an immediate change in the stress–strain curve.
3. The concept of a plastic yield limit is no longer strictly applicable.

The hypothesis of partitioning the strains by decoupling the elastic and plastic parts is still applicable where the strains are small, i.e.,

$$\varepsilon = \varepsilon_e + \varepsilon_{vp}$$

where ε_e is the elastic strain and ε_{vp} is the viscoplastic strain.

To obtain the stress–strain behavior shown in blue in the figure, the material is initially loaded at a strain rate of 0.1/s. The strain rate is then instantaneously raised to 100/s and held constant at that value for some time. At the end of that time period the strain rate is dropped instantaneously back to 0.1/s and the cycle is continued for increasing values of strain. There is clearly a lag between the strain-rate change and the stress response. This lag is modeled quite accurately by overstress models (such as the Perzyna model) but not by models of rate-independent plasticity that have a rate-dependent yield stress.”

Perturbation Theory

This article is about perturbation theory as a general mathematical method. In mathematics and applied mathematics, per-

turbation theory comprises methods for finding an approximate solution to a problem, by starting from the exact solution of a related, simpler problem. A critical feature of the technique is a middle step that breaks the problem into "solvable" and "perturbative" parts. In perturbation theory, the solution is expressed as a power series in a small parameter ϵ . The first term is the known solution to the solvable problem. Successive terms in the series at higher powers of ϵ usually become smaller. An approximate 'perturbation solution' is obtained by truncating the series, usually by keeping only the first two terms, the solution to the known problem and the 'first order' perturbation correction.

Perturbation theory is used in a wide range of fields, and reaches its most sophisticated and advanced forms in quantum field theory. Perturbation theory (quantum mechanics) describes the use of this method in quantum mechanics. The field in general remains actively and heavily researched across multiple disciplines.

Description

Perturbation theory develops an expression for the desired solution in terms of a formal power series known as a perturbation series in some "small" parameter, which quantifies the deviation from the exactly solvable problem. The leading term in this power series is the solution of the exactly solvable problem, while further terms describe the deviation in the solution, due to the deviation from the initial problem. Formally, we have for the approximation to the full solution A, a series in the small parameter (here called ϵ), like the following:

$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + \dots$$

In this example, A_0 would be the known solution to the exactly solvable initial problem and A_1, A_2, \dots represent the first-order, second-order and higher-order terms, which may be found iteratively by a mechanistic procedure. For small ϵ these higher-order terms in the series generally (but not always) become successively smaller. An approximate "perturbative solution" is obtained by truncating the series, often by keeping only the first two terms, expressing the final solution as a sum of the initial (exact) solution and the "first-order" perturbative correction $A \approx A_0 + \epsilon A_1(\epsilon \rightarrow 0)$

Some authors use big O notation to indicate the order of the error in the approximate solution:

$$A = A_0 + \epsilon A_1 + O(\epsilon^2).$$

If the power series in ϵ converges with a nonzero radius of convergence, the perturbation problem is called a regular perturbation problem. In regular perturbation problems, the asymptotic solution smoothly approaches the exact solution. However, the perturbation series can also diverge, and the truncated series can still be a good approximation to the true solution if it is truncated at a point at which its elements are minimum. This is called an asymptotic series. If the perturbation series is divergent or not a power series (e.g., the asymptotic expansion has non-integer powers $\epsilon^{1/2}$ or negative powers ϵ^{-2}) then the perturbation problem is called a singular perturbation problem. Many special techniques in perturbation theory have been developed to analyze singular perturbation problems."

Results

Figure 1 displays the data table of the input data and calculation results.

1/21/21	Strain	Stress	Stress	Weight+Temp	Pred. FPG	1/21/22	Stress	Stress	Weight/Avg	Temp/Avg	Stress
Oct-20	93.1	0.0	99.6%	93.1		Oct-20	167.2	97.6	99%	100%	99.6%
Nov-20	93.1	0.0	99.6%	93.1		Nov-20	165.9	97.9	99%	100%	99.6%
Dec-20	105.3	0.3	99.9%	105.6		Dec-20	166.5	97.8	99%	100%	99.5%
Jan-21	105.7	0.1	99.9%	105.8		Jan-21	167.5	98.0	99%	100%	99.9%
Feb-21	111.0	0.2	100.2%	111.2		Feb-21	168.3	98.0	100%	100%	100.1%
Mar-21	111.0	0.8	100.8%	111.0		Mar-21	168.0	97.9	100%	100%	100.0%
Apr-21	107.6	-0.1	100.3%	107.5		Apr-21	169.1	97.8	100%	100%	100.3%
May-21	97.1	-0.3	100.1%	96.8		May-21	168.9	97.6	100%	100%	100.1%
Jun-21	105.1	0.1	100.3%	105.3		Jun-21	169.3	97.7	101%	100%	100.3%
Jul-21	94.0	-0.2	100.3%	93.8		Jul-21	169.3	97.7	101%	100%	100.3%
Aug-21	87.3	-0.2	99.9%	87.1		Aug-21	168.0	97.7	100%	100%	99.9%
Sep-21	93.7	0.2	99.9%	93.9		Sep-21	168.1	97.7	100%	100%	99.9%
Oct-21	94.6	0.0	100.0%	94.6		Oct-21	168.4	97.7	100%	100%	100.0%
Nov-21	94.5	0.0	100.2%	94.5		Nov-21	169.0	97.7	100%	100%	100.2%
Dec-21	89.9	-0.1	100.2%	89.7		Dec-21	169.1	97.6	100%	100%	100.2%
Jan-22	98.6	0.3	100.2%	98.9		Jan-22	169.2	97.6	101%	100%	100.2%
Feb-22	92.1	-0.2	100.2%	91.9		Feb-22	169.3	97.6	101%	100%	100.2%
Average	98.1	0.0	100.0%	98.1		Average	168.3	97.7	100%	100%	100.0%
Correlation			100%			Correlation	100.0%	-4%			

Figure 1: Data table of both input data and calculation results

Figure 2 shows the results from the viscoelastic study. This is the stress-strain diagram that demonstrates the viscoelastic characteristics.

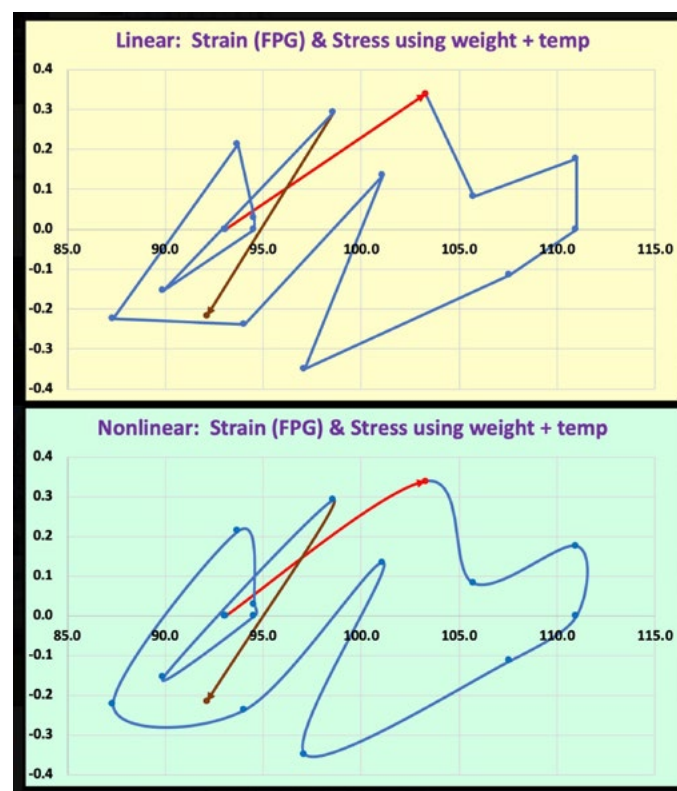


Figure 2: Viscoelastic stress-strain diagram of measured FPG with viscosity factor of averaged value of normalized body weight and normalized body temperature

Figure 3 illustrates the comparison of measured FPG versus predicted FPG using an average slur of normalized body weight and normalized body temperature via the viscoelastic perturbation model. Both prediction accuracy and correlation coefficient between this predicted FPG and the measured FPG are 100%.

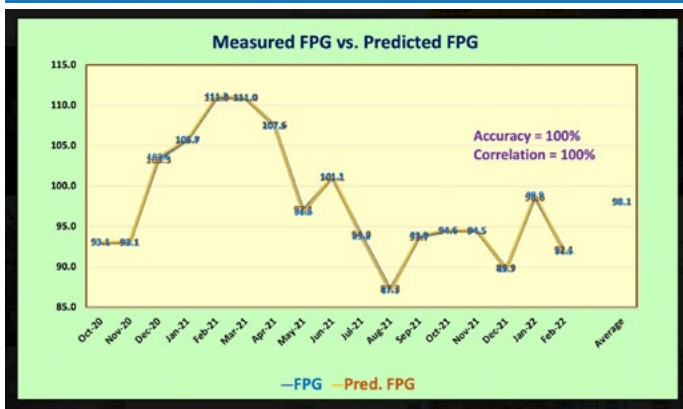


Figure 3: Time-domain comparison between measured FPG and predicted FPG using viscoelastic perturbation model

Conclusion

In conclusion, the author has defined his stress-strain equations as follows:

strain

$$= \epsilon$$

= *Sensor FPG value of each month*

Viscosity Factor

$$= \eta$$

= $((\text{body weight} / \text{averaged body weight}) + (\text{body temperature} / \text{averaged body temperature})) / 2$ of each month

However, he has utilized the FPG change rate multiplied with the above described viscosity factor to obtain the stress:

Stress

$$= \eta * (d\epsilon/dt)$$

$$= \eta * (d\text{-strain}/d\text{-time})$$

$$= ((\text{viscosity factor of present month}) * (\text{FPG of present month} - \text{FPG of previous month}) / 30)$$

Where 30 indicates the 30-day timespan of his collected monthly data.

Based on the stress-strain analysis in spatial-domain (SD), viscoelasticity or viscoplasticity theory, and perturbation model, the following two distinct observations are evident:

(1) Observing from the stress-strain diagram in SD, the measured FPG using combined weight and temperature as the viscosity factor has shown a viscoelastic behavior (starting FPG and ending FPG are extremely close to each other). It also has both stress-creeping and strain-relaxation phenomena (upper right and right portion).

(2) The predicted FPG values using the viscoelastic perturbation method have achieved extremely high prediction accuracy of 100% and correlation coefficient of 100% in comparison with the measured FPG. This means that the predicted FPG equations using the combined effect from the average of normalized body weight and normalized body temperature can generate highly accurate predicted FPG values.

In summary, from a daily practice viewpoint, using **an average normalized body weight and normalized body temperature as the viscosity factor** while applying the **viscoelastic perturbation model** can produce a satisfied prediction of FPG in the early morning.

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Without Professor Norman Jones at MIT as his academic advisor, the author would not be able to conduct his medical research project and also published 500+ research papers. The author has never forgotten his advice to him that he should always enhance his strength on foundations, such as mathematics and physics, in order to make further improvement and advancement. Professor Jones has also provided him with a personal example of doing outstanding teaching and research jobs with an excellent work attitude, extreme dedication, and ultimate commitment to advancing both science and engineering.

References

For editing purposes, majority of the references in this paper, which are self-references, have been removed. Only references from other authors' published sources remain. The bibliography of the author's original self-references can be viewed at www.eclairmd.com.

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