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Vacuons and Quasi-phonons: The Hidden Side of Bogoliubov Collective Excitations

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Abstract
In a series of papers [1, 2, 3], the author exactly diagonalized the truncated Hamiltonian H_c, proposed by Bogoliubov [4, 5], as a low energy approximation for the weakly interacting boson gas. In addition to the well know collective excitations (CEs) resulting from the Bogoliubov Canonic Approximation (BCA) [6, 7], and denoted as quasi-phonons (QPs), the exact eigenstates of H_c exhibit a new kind of CEs (the `hidden side' which the title alludes to), that we denote as vacuons. Those CEs are created/annihilated by adsorp-tion/emission of a quantum of energy twice as large as the activation energy of a QP. Being momentless, they are reminiscent of Cooper pairs of bosons, with opposite moments. The e_ects of the vacuons on the dynamics of the gas are discussed, with their possible exper-imental evidence.

Keywords: Boson systems; Interacting Boson models; Bose-Einstein condensates; Superuidity.

Introduction
The truncated canonic Hamiltonian

$$H_c = \overbrace{\frac{\widehat{u}(0)N^2}{2}}^{E_{in}} + \sum_{\mathbf{k}\neq 0} \overbrace{\left[\mathcal{T}(k) + \widetilde{N}_{in}\,\widehat{u}(k)\right]}^{\widetilde{\epsilon}_1(k)} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} +$$

$$+\frac{1}{2}\sum_{\mathbf{k}\neq 0}\widehat{u}(k)\left[b^\dagger_{\mathbf{k}} b^\dagger_{-\mathbf{k}}(\,b_{\mathbf{0}}\,)^2 + b_{\mathbf{k}} b_{-\mathbf{k}}(\,b^\dagger_{\mathbf{0}}\,)^2\right], \qquad (1)$$

was assumed by Bogoliubov [4, 5] as a low energy approximation for the dynamics of the weakly interacting bosons gas, in the thermodynamic limit (TL) . The operators by $b^\dagger_k$ and b_k create/destroy a spinless boson in the freeparticle state $\langle\mathbf{r}\,|\,\mathbf{k}\,\rangle = e^{i\mathbf{k}\,\mathbf{r}}/\sqrt{V}$ and

$$\widehat{u}(q) = \frac{1}{V}\int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\,\mathbf{r}}\,u(r)\,,$$

is the Fourier transform of the *repulsive* interaction energy $u(r)$ (> 0). The number operator $\widetilde{N}_{in} = b^\dagger_o b_o$ refers to the bosons in the free particle ground state. Accordingly, $\widetilde{N}_{out} = N - \widetilde{N}_{in}$ is the number operator for bosons in the excited states, N being the conserved total number of bosons. Overtilded symbols indicate operators, to avoid confusion with their (non overtilded) eigenvalues.

Bogoliubov's next step is reducing H_c to a bi-linear form, which is realized in the TL, by assuming $|\,N_{in} \pm 2;\ N_{out}\,\rangle \approx |N_{in}, N_{out}$ for the bosonic Fock states [1]. Adams and Bru [6, 7] complete the so called Bogoliubov Canonical Approximation (BCA) by transforming eq.n (1) into a new a canonic (i.e. N-conserving) Hamiltonian H_{BCA}, which is bi-linear in suitably dened creation/annihilation operators. Thanks to bi-linearity, an appropriate Bogoliubov transformation can be applied, which gives H_{BCA} the well known form:

$$H_{BCA} = \sum_{\mathbf{k}}\left[\left(B^\dagger_{\mathbf{k}} B_{\mathbf{k}} + 1/2\right)\epsilon(k) - \frac{\epsilon_1(k)}{2}\right], \qquad (2a)$$

where $B^\dagger_{\mathbf{k}}$ and $B_{\mathbf{k}}$ create/annihilate new 'particles' which is costumary to call 'quasiphonons'(QPs). Actually, the QPs behave like massless bosons, carrying a nite moment $\hbar$k and a quantum of energy $\epsilon(k)$:

$$\epsilon(k) = \sqrt{\epsilon_1^2(k) - N^2\widehat{u}^2(k)}\,, \qquad (2b)$$

with

$$\epsilon_1(k) = \mathcal{T}(k) + N\widehat{u}(k) \qquad (2c)$$

The eigenvalues of H_{BCA} are, obviously:

$$E_{BCA}(\eta,\,k) = \left[\epsilon(k)\left(\eta + \frac{1}{2}\right) - \frac{\epsilon_1(k)}{2}\right] \quad (\eta = 0,\,1,\,\ldots)\,, \qquad (3)$$

resulting from the creation of η QPs.

In a series of papers [1, 3, 2, 8], I performed the exact diagonalization of H_c, which showed some non trivial dierences from the currently accepted theory described above. The rst one is the existence of a new kind of collective excitations (CEs), which I called `pseudobosons' in ref. [3], but could be better denoted as 'vacuons', as we shall see in what follows. They have been found in ref. [1], since now on referred to as (I), by diagonalizing the Hamiltonian (1) exactly, *in* *the special subspace spanned by the states* $|\, j,\ \mathbf{k}\,\rangle_0$ *with the same number j of bosons in* $|\,\mathbf{k}\,\rangle$ *and* $|\,-\mathbf{k}\,\rangle$ *and* $N-2j$ *bosons in* $|\,\mathbf{0}\,\rangle$*:*

$$|\, j,\ \mathbf{k}\,\rangle_0 = \frac{(b_{\mathbf{0}}^\dagger)^{N-2j}}{\sqrt{(N-2j)!}}\frac{(b_{\mathbf{k}}^\dagger)^j(b_{-\mathbf{k}}^\dagger)^j}{j!}|\emptyset\,\rangle \qquad (4)$$

($|\emptyset\,\rangle$ being the real bosons' vacuum). The resulting eigenvalues ES(k; 0) (see, in particular, [2]) turn out to be *twice as large as* the BCA energies eq.n (3), reported in the current literature [10]:

$$E_S(k,0) = 2\underbrace{\left[\epsilon(k)\left(S+\frac{1}{2}\right)-\frac{\epsilon_1(k)}{2}\right]}_{E_{BCA}(S,k)} \quad (S=0,\,1,\,\dots)\,, \qquad (5)$$

The exact eigenstates of H_c calculated in (I) are, in turn, quite dierent from the BCA eigenstates: the latter have total moment $\eta\hbar\mathbf{k}$, corresponding to a number η of QPs, while the formers are superpositions of free bosons states, with opposite moments, carrying a *momentless* energy $E_S(k;0)$. The physical meaning of those new CEs, and their relationship with the *exact* QPs, follow from the complete diagonalization of H_c, including the subspaces containing a dierent number of bosons with opposite moment:

It is worthwhile recalling that exact eigenstates, reminiscent of the pseudobosons, had been found in a dierent context and for dierent aims in ref. [9].

$$|\, j,\ \mathbf{k}\,\rangle_\eta = \frac{(b_{\mathbf{0}}^\dagger)^{N-2j-\eta}}{\sqrt{(N-2j-\eta)!}}\frac{(b_{\mathbf{k}}^\dagger)^{j+\eta}(b_{-\mathbf{k}}^\dagger)^j}{\sqrt{j!(j+\eta)!}}|\emptyset\,\rangle\,, \qquad (6)$$

with j + η bosons in $|\,\mathbf{k}\,\rangle$, j bosons in $|-\mathbf{k}\rangle$ and $N-2j-\eta$ bosons in $|\,0\,\rangle$, so that the total moment is, manifestly, $\eta\hbar\mathbf{k}$. I performed such diagonalization in refs [3, 8], by using the method outlined below.

Complete diagonalization of H_c

Hamiltonian H_c can be written as a sum of independent one-moment Hamiltonians

$$H_c = E_{in} + \sum_{\mathbf{k}\neq 0} h_c(\mathbf{k})\,, \qquad (7a)$$

where:

$$h_c(\mathbf{k}) = \frac{1}{2}\tilde{\epsilon}_1(k)[b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + b_{-\mathbf{k}}^\dagger b_{-\mathbf{k}}] +$$

$$+\frac{1}{2}\hat{u}(k)\left[b_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger(\,b_{\mathbf{0}}\,)^2 + b_{\mathbf{k}}b_{-\mathbf{k}}(\,b_{\mathbf{0}}^\dagger\,)^2\right]. \qquad (7b)$$

Hence the whole problem can be reduced to the study of the exact eigenstates of $h_c(\mathbf{k})$, by solving the eigenvalue equation

$$h_c|S,\ \mathbf{k},\ \eta\,\rangle = \mathcal{E}_S(k,\eta)|S,\ \mathbf{k},\ \eta\,\rangle\,, \qquad (8)$$

with exact QPs expressed as linear combinations of the states eq.n (6). Thanks to a suitable transformation [8], eq.n (8) can be reduced to the same problem already solved *analytically* in (I). This makes it possible to write the eigentates of h_c (eq.n (7b)) as :

$$|S,\ \mathbf{k},\ \eta\,\rangle = \sum_{j=0}^{\infty}\overbrace{x^j(k)\sqrt{\binom{j+\eta}{j}}\sum_{m=0}^{S}C_{S,\eta}(m,k)j^m}^{\phi_{S,\eta}(j,k)}\,|j,\ \mathbf{k}\,\rangle_\eta\,, \qquad (9)$$

with boundary conditions $\lim_{j\to\infty}\phi_{S,\eta}(j,k)=0$ (necessary for normalizability) and $\phi_{S,\eta}(-1;k)=0$ (exclusion of negative populations). It should be noticed that $|\phi_{S,\eta}(j,k)|^2 \propto j^{2S+\eta}\,x^{2j}(k)$ for $j >> 1$, i.e. the preexponential factor in the probability amplitude on the Fock states $|\,j,\ \mathbf{k}\,\rangle\eta$ (eq.n (6)) tends to a polinomial of degree $2S+\eta$ in $j >> 1$. The quantity $x(k)$, the coecients $C_{S,\eta}(m,k)$ and the eigenvalue $\mathcal{E}_S(k,\eta)$ are determined by the following system of S + 2 equations (see ref. [8]):

$$(\underline{\epsilon}_1\eta - 2\underline{\mathcal{E}})\,C(n) + 2\underline{\epsilon}_1 C(n-1) +$$

$$+x\left[(1+\eta)\sum_{m=n}^{S}C(m)\binom{m}{n} + \sum_{m=n-1}^{S}C(m)\binom{m}{n-1}\right] -$$

$$+\frac{1}{x}\sum_{m=n-1}^{S}C(m)\binom{m}{n-1}(-1)^{n-m+1} = 0 \;\; (n=0,\,1,\,\dots,\,S+1)\,, \qquad (10)$$

where $C(S+1)=C(-1)=0$ by denition and the dependence on $\mathbf{k}$, S, η has been provisionally dropped. The energies are expressed in units of $N\hat{u}(k)$:

$$\underline{\epsilon}_1 = \frac{\epsilon_1(k)}{N\hat{u}(k)},\ \underline{\mathcal{E}} = \frac{\mathcal{E}_S(k,\eta)}{N\hat{u}(k)},\ \underline{\epsilon} = \frac{\epsilon(k)}{N\hat{u}(k)}\,.$$

The algebraic structure of the eigenvalue problem (10) is fairly peculiar: the two highest order equations ($n = S+1$ and $n = S$) determine $x(k)$ (i.e. the exponential decay in j) and the eigenvalue $\underline{\mathcal{E}}$, independently from $C(S)$ and $C(S-1)$:

$$x(k) = \underline{\epsilon}(k) - \underline{\epsilon}_1(k) = \frac{\epsilon(k)-\epsilon_1(k)}{N\hat{u}(k)} \qquad (11a)$$

$$\mathcal{E}_S(k,\ \eta) = \frac{\epsilon(k)}{2}(\eta+2S) -$$

$$-\frac{\epsilon_1(k)-\epsilon(k)}{2} \quad (S,\ \eta=0,\,1,\,\dots)\,, \qquad (11b)$$

with all entries restored. Notice that $x(k)$ is negative and smaller than 1 in modulus, which ensure normalizability. The next S equations determine the S unknowns $C(1)$; $C(2)$, . . , $C(S)$ in terms of, say, $C(0)$, that will be determined by normalization. Figure 1 shows the resulting probability amplitudes $\phi_{S,\eta}(j,k)|^2$ for $S = 0, 1, 2$ and η = 0 (1(a)), or = 1 (1(b)).

Since $h_c(\mathbf{k}) = h_c(-\mathbf{k})$ (eq.n (7b)), $\mathcal{E}_S(k,\eta)$ must be counted twice in the sum eq.n (7a). Hence the energy eigenvalues of $H_c - E_{in}$ are:

$$E_S(k,\ \eta) = 2\mathcal{E}_S(k,\ \eta) = \epsilon(k)\,(\eta+2S) - [\epsilon_1(k)-\epsilon(k)] \qquad (12a)$$

$$= \epsilon(k)\left(\eta+\frac{1}{2}\right) + 2\epsilon(k)\left(S+\frac{1}{2}\right) - \underbrace{\left[\epsilon_1(k)+\frac{\epsilon(k)}{2}\right]}_{\mathcal{E}_0(k)}\,. \qquad (12b)$$

Vacuons as bosonic Cooper pairs

The limit of large k >> $\xi^{-1} \equiv \sqrt{2M\widehat{u}(0)N}/\hbar$, that marks the passage from collective to single-particle dynamics, yields $\epsilon_1(k) \to T(k)$ and $\epsilon(k) - \epsilon_1(k) \to 0$. Hence, from eq.n (12), one has:

$$E_S(k,\, \eta) \to (2S+\eta)\mathcal{T}(k) \quad (k >> \xi^{-1})\,. \tag{13}$$

Since the CEs become non interacting *real* bosons when their kinetic energy $T(k) = \hbar^2k^2/(2M)$ largely exceeds the interaction energy, the number $2S + \eta$ corresponds to the total number of real bosons excited in the limit k >> ξ^{-1}. However, while the kinetic energy refers to $2S + \eta$ real bosons, the total moment $\hbar\eta$k corresponds just to η particles. This suggests the physical interpretation of the exact results obtained so far: S numerates a sort of bosonic Cooper pairs, formed by *two* bosons in $|\pm \boldsymbol{k}\rangle$, with opposite moments and identical kinetic energy. In contrast, η numerates the additional bosons in $|\,\mathbf{k}\,\rangle$, carrying the non vanishing moment $\hbar\mathbf{k}$. Hence the states $|S, \mathbf{k}, 0\,\rangle$ do not carry any QP: each of them is nothing but a possible 'vacuum' of QPs. This explains why the transition $S \to S \pm 1$ is conveniently defined as the creation/annihilation of a quantum of vacuum (*vacuon*), on which the QPs can be created in turn.

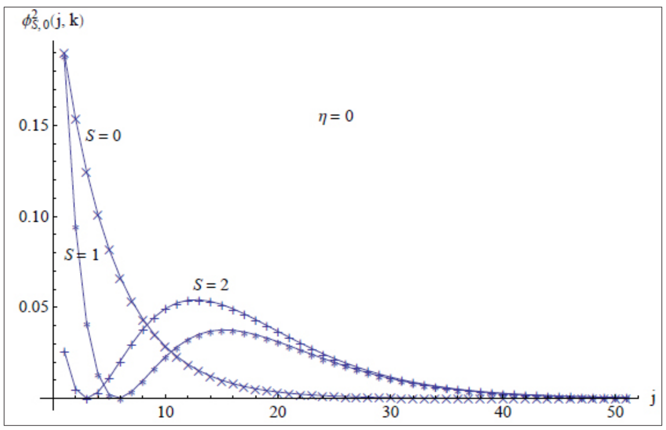

Figure 1 (a)

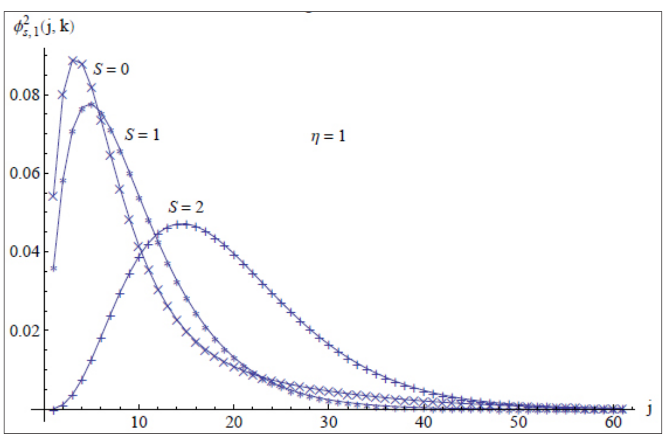

Figure 1(b)

Figure 1: Probability amplitude of exact collective excitations ($x(\boldsymbol{k}) = -0.9$).
(a): CEs corresponding to QP-vacua (η= 0), originated by creation of S = 0; 1; 2 vacuons (bosonic Cooper pairs). **(b)**: CEs corresponding to 1 QP (= 1), created from S = 0; 1; 2-level vacuum.
j is the number of opposite moment pairs ***i***n the subspace spanned by the states $|j,\mathbf{k}\rangle_0$ **(a)** (eq.n (4)) or $|j,\,\mathbf{k}\rangle_1$ **(b)** (eq.n (6)). The collective excitation regime k << ξ^{-1} implies $1 - |x(\mathbf{k})| << 1$.

Comparison with the BCA theory

What precedes marks a relevant dierence with respect to Bogoliubov's theory, where the QPs are obtained by repeated application of the creation operator B to a single vacuum state. Therefore, the BCA eigenvalues of the number operator $B^\dagger_k B_k$ are assumed to be *non degenerate*, in as much as they represent the number of a single species of quasiparticles. As a consequence, the BCA spectrum corresponds to a *single*, *non degenerate* oscillator, with frequency $\omega_Q(k) = \epsilon(k)/\hbar$ (recall $\mathcal{E}_S(k, 0)$, in eq.n (5)). In contrast, a lengthy but straightforward calculation leads to the following exact result:

$$B^\dagger_{\mathbf{k}} B_{\mathbf{k}} |\, S, \mathbf{k},\, \eta\,\rangle = (\eta + S)|\, S, \mathbf{k},\, \eta\,\rangle$$

showing that:

(A) The M-th eigenvalue of $B^\dagger_k B_k$ is M + 1-times *degenerate*, since M = S +η includes both the QPs and the vacuons number. The exact energy spectrum is degenerate too, resulting from the sum of *two* oscillators (eq.n (12b)), one labeled by the non negative integer η, with the same frequency $\omega_Q(k)$ as BCA, the other, labeled by the non negative integer S, with doubled frequency $\omega_P(k) = 2\epsilon(k)/\hbar$ (see Figure 2).

It is worthwhile noticing that $B^\dagger_k B_k$ still preserves its meaning of number operator, counting the *total* number of CEs (QPs + vacuons). However, despite S and η can be varied independently, $B^\dagger_k B_k$ cannot be split into two distinct operators, counting QPs and vacuons separately. This is because, in general, $B^\dagger_k$and B_k are no longer creation/annihilation operators, though their product behaves like a number operator. The only exception is the case of zero vacuons S = 0: by solving the system of equations for the $C_{S\eta}(m, k)$ explicitly [8], in fact, it is possible to show that

$$B^\dagger_{\mathbf{k}}|0,\, \mathbf{k},\, \eta\,\rangle = \sqrt{\eta+1}|0,\, \mathbf{k},\, \eta+1\,\rangle\,. \tag{14}$$

Since in (I) it was shown that the lowest-order vacuum $|0,\, \mathbf{k},\, 0\,\rangle$ coincides with the vacuum of Bogoliubov QPs, equation (14) shows that Bogoliubov QPs coincide with the exact QPs, in the absence of vacuons. In this special case BCA leads to the exact CEs $|\, 0, \mathbf{k}, \eta\rangle$ (eq.n (9)), corresponding to η QPs, each carrying a moment $\hbar\mathbf{k}$, with energy $2\mathcal{E}_0(k, 1) = (k)$ and velocity $v_Q(k) = \nabla_k \omega_Q(k)$ (the 'sound velocity'). In the limit of small k << ξ^{-1}, the sound velocity attains, in modulus, the constant value:

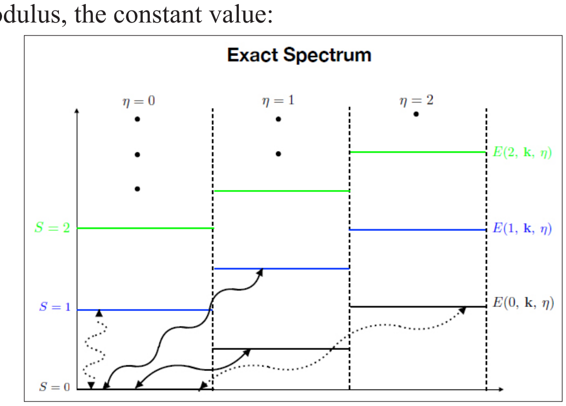

Figure 2: Exact spectrum of H_c.
The lowest energy `staircase'(S = 0), with steps of heigth (k), coincides with the BCA energy eigenvalues. The higher energy staircases, separated by 2(k) (S = 1; 2;), are the additional eigenvalues ignored by the BCA theory. Two rst-order allowed transitions (full wavy lines) and two rst-order forbidden transitions (dotted wavy lines) are also indicated (see the text).

$$v_Q = \sqrt{\frac{N\hat{u}(0)}{M}} \,. \qquad \textbf{(15)}$$

However:

(B) The possible QP-vacua resulting from the exact diagonalization are all the eigenstates |S, **k**, 0 ⟩ (S = 0, 1, . . .). They originate from the activation of S momentless energy quanta ħωₚ (**k**), that we call vacuons, and form an *immobile* 'sea' of opposite moment pairs (Fig. 1(a)). The exact eigenstates |S, k,η ⟩ correspond to η QPs, each with nite moment $\hbar$**k**, created from the S-th vacuum, as unpaired bosons producing an asymmetry in the opposite moment populations.

Hence:

(C) There are innite dierent QPs, originated by the simoultaneous creation of S ≠ 0 vacuons (Fig. 1(b)), whose velocity $v_Q(k) = \nabla_{k}\omega_Q(k)$ is the same as Bogoliubov's velocity, but whose energy is $\epsilon(k)(1+2S)$. Those quasiparticles are totally ignored by Bogoliubov's theory. In practice, SBA exactly reproduces just the part of the whole spectrum corresponding to S = 0.

Allowed and forbidden transitions involving vacuons

Though the vacuons *alone* cannot be responsible for the dissipation processes á *la* Landau [11], since they have zero moment, they can in uence the dynamics of the dissipation processes, by opening new emission/adsorption channels for the QPs themselves. This can be easily seen by refreshing Landau's semiclassical picture of a particle (of mass M_p), moving with initial velocity $\mathbf{v}_i$ in the superfuid at zero temperature, and causing the emission of a single QP, *via* a scattering process. The well known conclusion, based on BCA, is that the particle cannot dissipate its kinetic energy (relative to the super uid), unless $v_i > v_Q$ (eq.n (15)). Taking the super uid in the ground state (S = 0, η= 0) initially, the exact expressions (12) yield:

$$\frac{M_p v_i^2}{2} = \frac{M_p v_f^2}{2} + \epsilon(k)(1+2S) \qquad (16a)$$

$$M_p \mathbf{v}_i = M_p \mathbf{v}_f + \hbar\mathbf{k}\,, \qquad (16b)$$

from energy/moment conservation, with S vacuons and 1 QP in the nal state of the super uid. At small k, equations (16) yield a succession of critical velocities

$$v_c(S) = v_Q(1+2S) \;\; (S = 0,\, 1,\, \ldots)\,, \qquad (17)$$

such that passing from $v_i < v_c(S)$ to $v_i > v_c(S)$ (*or vice versa*) causes the switching on (off) of a new channel of QP emission, corresponding to the simultaneous creation (annihilation) of a vacuon. Note that this is not a standard multiparticle process, usually forbidden at rst order. Actually, a straightforward calculation yields a non vanishing matrix element, in the TL, connecting the initial and nal states of the process (16):

$$\langle\, \mathbf{k}_i | \otimes \langle 0,\, \mathbf{q},\, 0\, | U | S,\, \mathbf{q},\, \eta\,\rangle \otimes |\, \mathbf{k}_f\rangle =$$

$$= W_0\, \delta_{\eta,1} \delta_{\Delta\mathbf{k},\mathbf{q}} \sqrt{N} \sum_{j=0}^{\infty} \phi_{0,0}(j,\mathbf{q}) \phi_{S,1}(j,\mathbf{q}) \sqrt{j+1}\,, \qquad (18a)$$

where Δk = ki – kf and

$$U = \frac{W_0}{V} \sum_{\mathbf{k},\mathbf{k}'} \mathrm{e}^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}} b^\dagger_{\mathbf{k}'} b_{\mathbf{k}} \qquad (18b)$$

represents a repulsive -shaped interaction (W_0 > 0), between bosons in the superfuid and the scattering particle, with initial and nal velocity $v_{i,f} = \hbar\mathbf{k}_{i,f} / M_p$. Expression (18b) adopts the 2nd quantization representation for the super uid, and the co-ordinate (**r**) representation for the scattering particle. Unlike the simultaneous creation/annihilation of a *single* QP and S vacuons, it is easy to see that the creation/annhilation of 2 or more QPs is a multiparticle process, forbidden at rst order. In fact:

$$\langle\, \mathbf{k}_i | \otimes \langle 0,\, \mathbf{q},\, 0\, | U | S,\, \mathbf{q},\, \eta\,\rangle \otimes |\, \mathbf{k}_f\rangle = 0 \quad \text{if } \eta > 1\,. \qquad (19)$$

What precedes leads one to conclude that in any *scattering* process, the rst-order allowed transitions correspond to *odd* multiples $\epsilon(k)$ $(2S+1)$ of the adsorbed/emitted energy, while the *even* order ones are forbidden, as shown in Fig. 2. In particular, the energy $2\epsilon(k)$ can be revealed just as a second-order eect.

Experimental verications and conclusions

As for the experimental verication of the new results obtained, doublephoton Bragg spectroscopy is a current method to observe QPs [12, 13, 14, 15], thanks to which it is possible to estimate the resonant values $\omega_{res}(k)$ of the dierence between the two incident photons' frequencies, in the phonon regime $k < \xi^{-1}$, or in the free particle regime $k > \xi^{-1}$ (see, for example, Fig. 2 of ref. [12]). The knowledge of $\omega_{res}(k)$ leads to the static structure factor $F_0(k)$ [16], from which, however, no dierence can be revealed, between exact results and Bogoliubov theory. In fact, $F_0(k)$ involves just the ground state (S = 0, η = 0), which is the same in both cases, as shown in (I). However, a direct observation of the vacuons could stem from the measured $\omega_{res}(k)$s themselves. On a qualitative feet, in fact, the double oscillator spectrum eq.ns (12) should result in a succession of resonant frequencies $\omega_{res} = \omega(k)\,[1 + 2S]$, each corresponding to a hump in the transferred moment per particle. In particular, the activation of a single QP (η = 1) with zero or one vacuon (S = 0; 1) is expected to produce *two* humps at $\omega_{res} = \omega(k)$; $3\omega(k)$. The possible hump at !res = 2!(k), corresponding to the creation of two QPs and zero vacuons, is expected to be a second-order eect, as discussed above. All the way, the data presently available (to the author's knowledge) do not cover a range of frequencies large enough to include the second (predicted) hump at !res = 3!(k) (see Figure 3). Extending the measurements in ref. [12] up to frequencies values of order 2 Π X10^4Hz could provide a hint, in the interacting bosons regime ($k < \xi^{-1}$). Unfortunately, the data for the non interacting case ($k > \xi^{-1}$), as reported in Fig 3 of ref. [12], stop just below the value $3\omega(k)$, at which the second hump is expected to occur [17].

In principle, the proliferation of critical velocities in eq.n (17) could be observed by angle-resolved spectroscopy of bullet particles, impinging on a thin layered (single-scattering approximation) super uid. Due to the switching on of a new channel of scattering, one would expect a hump in the ux of scattered particles at nite angle, whenever the incident beam's velocity crosses (from below) one of the critical values eq.n (17). The scattering particle velocities involved in experiments with Na [12] or Rb [13] atoms (≈10 mm=s) are unfeasible, in a real experiment, while

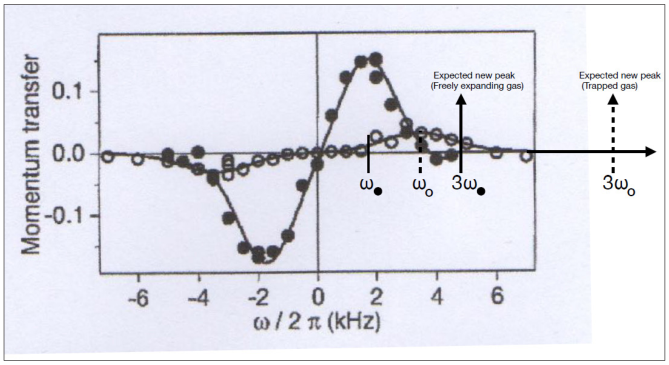

Figure 3: Measured momentum transfer from ref. [12].

Full and empty circles refer to freely expanding and trapped particles regime, respectively. The resonant frequencies $\omega_\bullet$ and ω_o and the positions of the expected peaks at $3\omega_\bullet$ and $3\omega_o$ are indicated accordingly. The antisymmetric part of negative frequencies is ignored.

those involved in exciton-polariton condensates ($\approx 10^7$cm/s for electrons, 10^5cm=s for neutrons), as roughly estimated from data in ref. [14], look more accessible.

Thermal depletion could provide an indirect evidence of the cohexistence of QPs and vacuons. From the exact energy (12a), one denes

$$S_T = \frac{1}{e^{2\beta\epsilon(k)} - 1} \quad , \quad \eta_T = \frac{1}{e^{\beta\epsilon(k)} - 1} \tag{20}$$

as the number of thermally activated vacuons and QPs. Then the exact thermal depletion follows from the calculated number of excited bosons in $| \pm \mathrm{k} \rangle$:

$$\langle\, n_{-\mathbf{k}} \,\rangle_T = \sum^{\infty} j\, \phi^2_{S_T,\eta_T}(j\,, k) \ ,$$

$$\langle\, n_{\mathbf{k}} \,\rangle_T = \sum_{j=0}^{\infty} j\, \phi^2_{S_T,\eta_T}(j\,, k) + \eta_T, \tag{21}$$

to be compared with measured values, as those reported, for instance, in ref. [15]. However, in the experimental procedure and in the theoretical calculations of ref. [15] there are a number of details that must be carefully accounted for, in view of a reliable comparison with eq.ns (21). This is a program for the future.

In conclusion, the exact diagonalization of the truncated Hamiltonian Hc (eq.n (1)) reveals a hidden side of Bogoliubov collective excitations, i.e. the existence of an equispaced ladder of zero-point energies, each corresponding to an *immobile* 'sea' of boson pairs with opposite moment, on which QPs can be created/annihilated, as unpaired bosons producing an asymmetry in the opposite moment populations. Bogoliubov theory accounts just for the QPs created from the lowest-level vacuum, but ignores all the higher-level CEs. Passing from one vacuum to another corresponds to the creation/annihilation of what we call *vacuons*. Those CEs, reminiscent of bosonic Cooper pairs, are expected to produce observable eects, whose experimental verication represents a new challenging item for future investigations.

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