ISSN: 2993-8643

Research Article

Engineering: Open Access

Utilising General Relativity to Predict the Percentage of Dark Energy and To Refine Einstein's Space-Time Equations

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Submitted: 2025, June 01; **Accepted:** 2025, June 26; **Published:** 2025, July 01

Citation: Babb, E. (2025). Utilising General Relativity to Predict the Percentage of Dark Energy and To Refine Einstein's Space-Time Equations. *Eng OA, 3*(7), 01-11.

Abstract

Dark energy is believed to drive the accelerated expansion of the universe, comprising approximately 71.35% of the total energy density. This work proposes that gravitational potential energy and dark energy are equivalent, and derives expressions predicting both the percentage of dark energy and spatial variation in the speed of light. Results reproduce known light bending near massive bodies and resolve issues such as the black hole time-stop paradox. The approach is based solely on classical mechanics and relativity, without introducing new physical laws.

Keywords: Dark Energy Theory, Gravity, Quantum Gravity, Quark, Black Hole, General Relativity

1. Introduction

1.1 Problem

Dark energy is hypothesized to drive the observed accelerated expansion of the universe, despite the attractive nature of the gravity force Newton [1]. Based on the Wilkinson Microwave Anisotropy Probe, the estimate of dark energy is 71.35%. Dark energy theories are of different types, as described by Bennett, Amendola, Misner, and Padmanabhan (e.g., quintessence, modified gravity, and cosmic acceleration) [2-5]. The Lambda Cold Dark Matter (Lambda-CDM) model uses only experimental data in the dark energy theory, yielding the value for the cosmological constant Λ in Einstein's field equation. Farnes used a negative mass to explain the dark energy expansion [6].

Einstein's field equations are an excellent description of the interaction between matter and radiation in low-gravity fields. The theory of general relativity (GR) (Cheng [7] and Katti [8]) was initially verified by Einstein [9,10], who calculated the bending angle of light by the Sun to be 1.7 secs of arc (p. 105). GR does not explain the dark energy or dark matter. It does not define the detailed curvature of space between two masses. At the event horizon of a black hole, it predicts that the speed of light drops to zero, referred to as the time-stop problem. It does not relate to quantum theory.

1.2 Objectives

The approach involves logically deriving equations based on classical gravity and relativity. No new laws are assumed. Equations are derived for the percentage of dark energy and the speed of light (curvature of space) at any location. These equations are validated by experiment where possible.

1.3 Summary of Methods

The methods are given in section 2. This paper proposes two important new equations. The first equation is for the percentage of dark energy Babb [21]. The second equation is used to calculate the speed of light at any location, enabling the prediction of light bending near the Sun. Additional expressions are presented for the expansion rate and 3D energy density of dark energy.

1.4 Summary of Results

The results are given in section 3. The percentage of dark energy is calculated by a dark energy equation. The equation gives 71.5% compared to the experimental value of 71.35% for dark energy. By contrast, the other dark energy models (quintessence, modified gravity, etc.) give no prediction of the percentage of dark energy.

The speed of light equation uses the density of dark energy. The equation is validated by predicting the same bending of light by the sun as Einstein see [9,10].

The equation predicts a localized variation ('kink') in dark energy density above each mass, such as between the Earth and Moon (Figure 1). Near a black hole, the speed of light equation changes the optical behavior quite considerably. In particular, the speed of light now never drops to zero, thus solving the time-stop problem.

The expansion of dark energy is explained by the radiation from the stars creating kinetic energy and so creating potential and therefore dark energy.

1.5 Assumptions or Axioms

The main assumptions are listed in subsection 2.1. The dark energy equation calculates the level of potential energy released when all quarks collapse together. The result was given in Babb [21]. It was so close to the experimental value; it was assumed that potential energy and dark energy are the same. The other main assumptions is that Newtons inverse square law is true. Finally, it is assumed the speed-of-light ratio is inversely proportional to the relative potential energy density D" in space. For example, double the density of dark energy halves the speed of light.

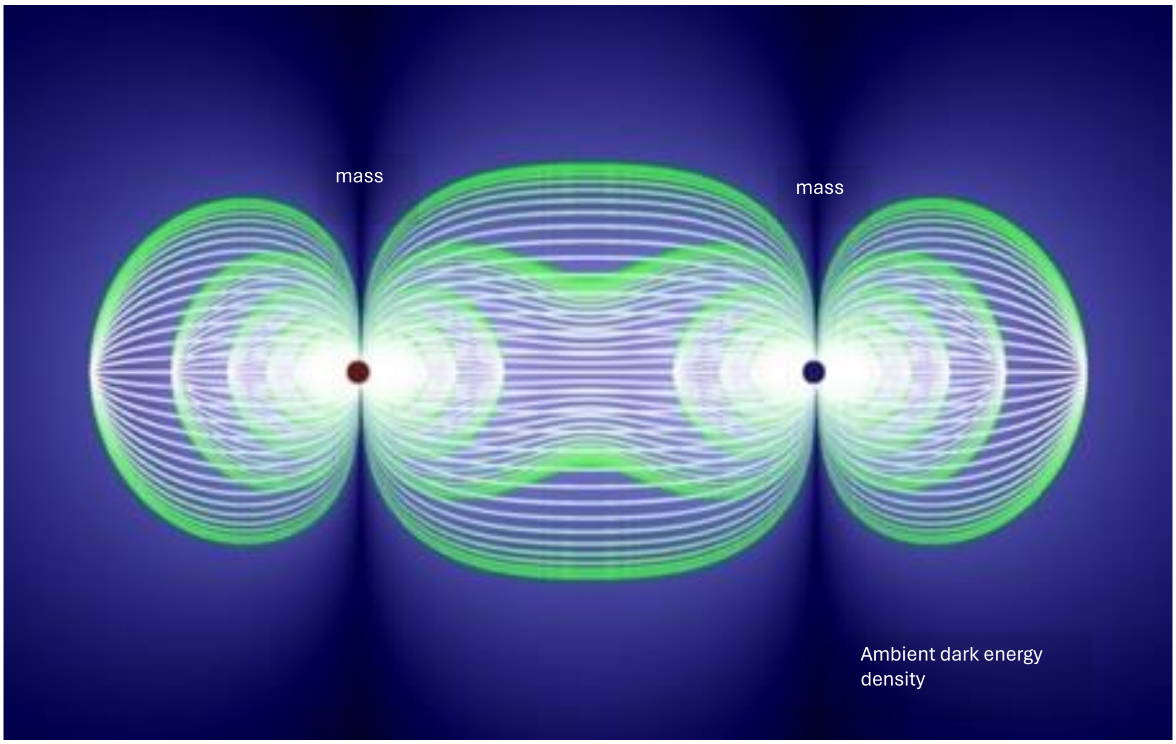

Figure 1: 3D Contours of Dark Energy Density Between Two Masses

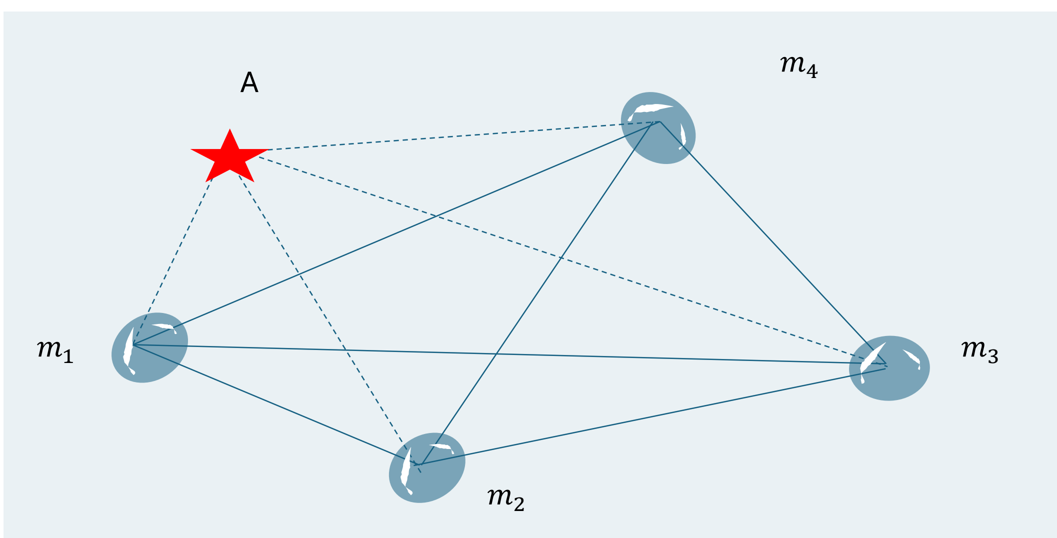

Figure 2: Dark Energy Density Between n Masses

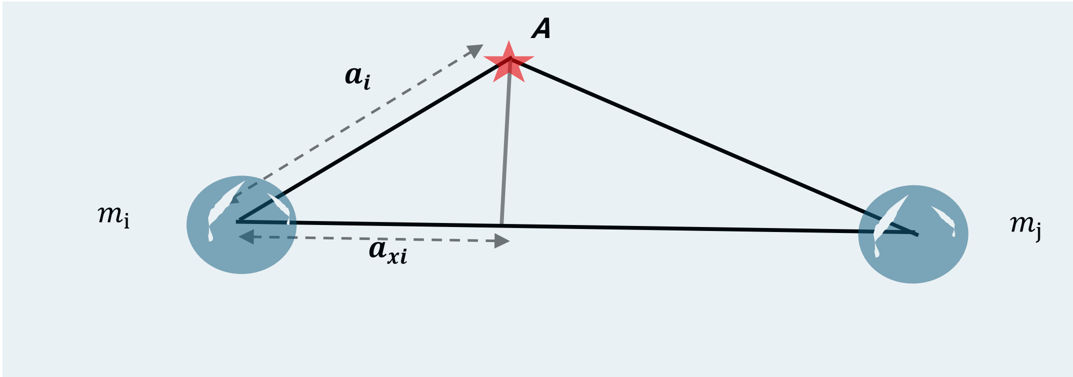

Figure 3: The Location of a Relative to m_i and m_j

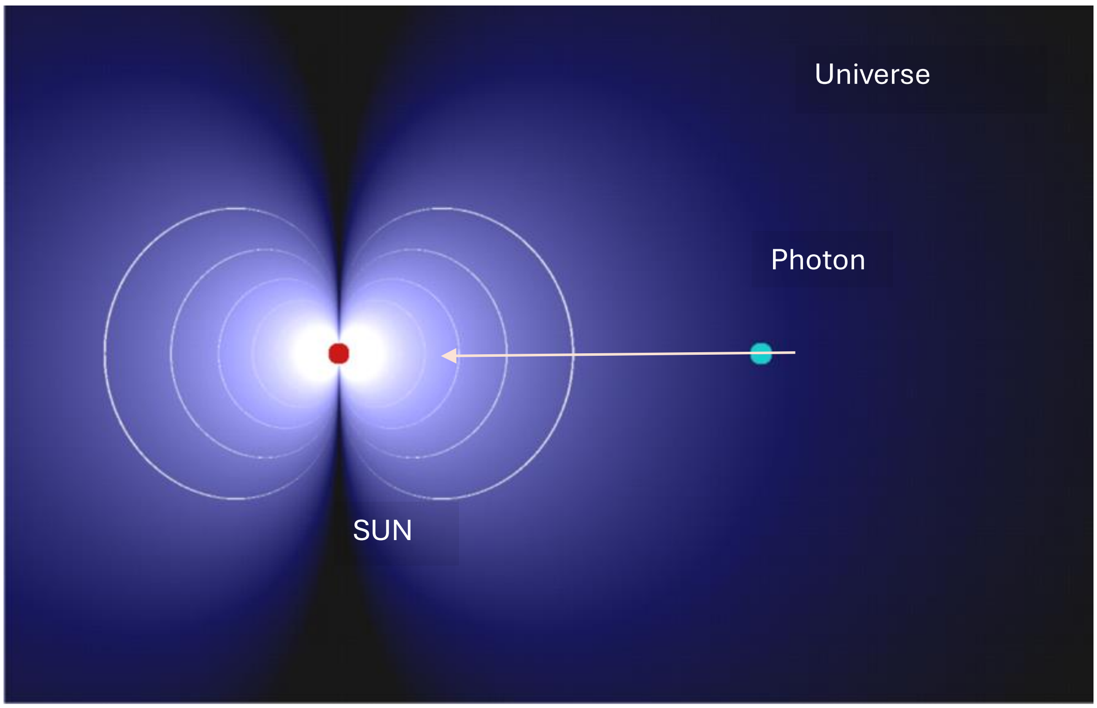

Figure 4: Marginal Dark Energy Density from The Sun Affecting A Photon

2. Method

2.1 The Equation for The Percentage of Dark Energy

When spherical particles of equal mass $\boldsymbol{m_p}$ and radius $\boldsymbol{r_p}$ fall together and surround particle P, they generate kinetic energy $\boldsymbol{E_{sp}}$ and remove $\boldsymbol{E_{sp}}$ of dark energy. The particles fall together into a lattice where the maximum distance between quarks is $\boldsymbol{n_{max}}$

$$E_{Sp} = \frac{G\, m_p{}^2}{24\, r_p{}^3} \sum\nolimits_{i,j,k} \frac{1}{d_{i,j,k}{}^3}\; if\; (0 > d_{i,j,k} < n_{max}) \quad (2.1)$$

This equation, from Babb [21, 16], assumes that all the masses in the universe collapse together so that they touch, releasing all the potential or dark energy stored in the elasticity of space. The particle surrounded by the most dark energy is the quark. In this model, all quarks are collapsed into contact, forming a theoretical minimum-energy configuration. This equation describes the collapse and provides the energy, specifically the kinetic energy released when they all touch. It is assumed that this kinetic energy is from potential energy stored in space.

2.2 The Equation for The Speed of Light at Any Location

The speed of light in any location is illustrated in Figure 2 is given by Equation (10.1) and proved in section 10:

$$c'' = \frac{1}{1 + \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{a_i}} \quad (2.2)$$

where the lengths $\boldsymbol{a_x}$ and $\boldsymbol{a}$ are calculated from the location vectors for $\boldsymbol{A}$, $\boldsymbol{m_i}$, $\boldsymbol{m_j}$ (Figure 3).

The equation above uses Newtonian gravity plus relativity as axioms. The three central axioms are as follows:

1. Newton's inverse square gravity law yields a force or rate of change of kinetic energy:

$$\textbf{Force} = \frac{dE}{dx} = \frac{Gm_i m_j}{x^2} \tag{2.3}$$

where m_i , m_j represents the two masses, and x is the distance between them.

2. Potential and dark energy are the same [21]. Therefore:

$$\Delta E_{dark\ energy} = \Delta E_{potential\ energy} = -\Delta E_{kinetic} \tag{2.4}$$

3. The speed-of-light ratio [21] is inversely proportional to the relative potential energy density $D'' = \frac{D}{D_A}$ in space:

$$c'' = \frac{D}{D_A}\ \text{, where } c'' = {c'}/{c} \tag{2.5}$$

At location A, the 1D dark energy density is D_A, and the speed of light is c'.

At the observer location, the 1D dark energy density is D, and the speed of light is always c. Einstein used these axioms, excluding axioms 2 and 3, to derive his equations for the curvature of space [9,10]. The new speed-of-light equation above is confirmed in Babb [21] using these axioms.

Bending of light by the Sun using the speed-of-light equation
As depicted in Figure 4, the significant mass, m_{sun} is the Sun. Therefore, m_i is m_{sun}. The vector location of A is just distance a from the centre of the Sun to a photon passing by the Sun. The location of A is the location of the photon, so $a_x = a$. This yields the speed of light at the photon's location as:

$$c'' = \frac{1}{1 + \frac{Gm_{\text{sun}}}{\text{a}\, c^2}} \tag{2.6}$$

If the G term is small, then the above can be approximated by:

$$c'' = 1 - \frac{Gm_{\text{sun}}}{\text{a}\, c^2} \tag{2.7}$$

This speed-of-light ratio c'' is that obtained by Einstein [9,10]. He used his equivalence principle to get his equation. The rate of change in c'' with respect to a causes photon bending. After differentiating with respect to a, the equation is as follows:

$$\therefore \quad dc''/da = \frac{G\,m}{a^2\,\text{c}} \tag{2-8}$$

Again, this is the result used by Einstein [9, 10] see section 12. The term dc"/da indicates the force on a stationary particle with the mass of a photon. It also bends a moving particle, such as a photon, using Huygens' principle.

2.3 Expansion Rate of Dark Energy

Radiation from stars creates dark energy because the photons lose kinetic energy, which is converted into potential or dark energy. The loss of kinetic energy changes the wavelength of the photons. The estimated rate of expansion was given by Babb [17].

2.4 The Density of Dark Energy at Any Location

The 3D energy density dE/dV at location A is given by Equation (8.18) as follows:

$$\frac{dE}{dV} = \sum_i \sum_j \frac{Gm_i m_j}{4\pi\, x_{ij}{}^2} \frac{a_{xi}}{a_i{}^3} \tag{2.9}$$

The lengths a_{xij} , $a_{i,j}$, x_{ij} (Figure 5) are calculated from the location vectors for A, m_i, m_j. The range of i, j covers all the masses in the universe. Ideally, these masses would be elementary particles.

For just two masses m_1,m_2, the 3D energy density becomes:

$$\frac{dE}{dV} = \frac{Gm_1 m_2}{4\pi\, x_{1,2}{}^2} \frac{a_{x1}}{a_1{}^3} + \frac{Gm_2 m_1}{4\pi\, x_{1,2}{}^2} \frac{a_{x2}}{a_2{}^3} \tag{2.10}$$

$$\frac{dE}{dV} = \frac{Gm_1 m_2}{4\pi\, x_{1,2}{}^2} \left(\frac{a_{x1}}{a_1{}^3} + \frac{a_{x2}}{a_2{}^3}\right) \tag{2.11}$$

This equation produces the dark energy density above ambient and is plotted in Figure 1, which illustrates the kink above each mass.

3. Results

The results are equations that yield the percentage of dark energy, the speed of light at any location, and its expansion rate, a solution to the time-stop problem.

3.1 Percentage of Dark Energy Results

Equation (2.1) given in methods cannot be computed directly. This relationship is resolved using a polar-coordinate equation, which also yields the ratio of dark energy to total energy. It is derived by Babb [21, 16] as follows:

$$E_s = \frac{G'\,\pi\sqrt{2}\, m_q{}^2}{6\, r_Q{}^3} \left(\ln\left(\sqrt[3]{\frac{3\, m_U}{m_q\, 4\pi\sqrt{2}}}\right)\right),\ \text{G}' = \frac{G\, m_q{}^2}{24\, r_Q{}^3} \tag{3.1}$$

and

$$m_Q = 4/7 m_{up} + 3/7 m_{down} \tag{3.2}$$

where the r_Q is the radius of the quark, m_q is average mass of up quark, m_U is the mass of the universe.

Applying this equation to the experimental data presented below resulted in a calculation of 71.50% dark energy in relation to the total energy. This value is close to the experimental value from NASA, which estimates dark energy at 71.35% (70.39 to 72.25). Because it closely predicted the experimental ratio of dark energy, it was concluded that potential and dark energy are the same.

The experimental data used in this equation is listed below:

Radius of quark, Zeus [13] r_Q	is	4.3E-19 metres,
Mass of up quark [14] m_{up}	is	3.82E-30 Kgms.
Mass of down quark [14] m_{down}	is	8.36E-30 Kgms.
Mass of universe [15] m_U	is	1E+53 Kgms.

3.2 The Speed of Light at any Location

The speed of light at any location A is given by Equation (10.15) as follows:

$$c'' = \frac{1}{1 + \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{a_i}} \tag{3.3}$$

where the lengths a_x and a are calculated from the location vectors for A, m_i, m_j (Figure 3).

The relationship of location A to adjacent masses is illustrated in Figure 2. This equation suggests a kink in the speed of light when the variable $\boldsymbol{a_{xi}}$ is zero. In the black area above the masses in Figure 1, the 3D dark energy density is much lower than along the centre line of the two masses. For example, between the Earth and the Moon, the speed of light would be measured at this location on the Earth or the Moon and might be detected by measuring the slight reduction in the bending of a photon.

3.3 The Bending of Light by the Sun

Einstein [9, 10] assumed a spherically symmetric gravitational field around each mass. He did not include the Newtonian force between pairs of masses. In terms of the above equation, it means $\boldsymbol{a_{xi}}$ is assumed to be the same as $\boldsymbol{a_i}$. Therefore, the term $\frac{a_{xi}}{a_i}$ can be removed. Einstein calculated the bending of light by the Sun at an angle of 4.23766E-06. The result given in section 12 for both methods is the same angle of 4.23766E-06. In practice, his spherical assumption makes little difference to this angle prediction.

3.4 The Rate of Expansion of Dark Energy

The theory by Babb [17] provides the expansion rate of dark energy, the Hubble constant, at an almost exact 71 km/sec/megaparsec.

3.5 The Density of Dark Energy at any Location

Equation (2.11) is used for just two masses, repeated as follows:

$$\frac{dE}{dV} = \frac{Gm_1, m_2,}{4\pi\, x_{1,2}{}^2}\left(\frac{a_{x1}}{a_1{}^3} + \frac{a_{x2}}{a_2{}^3}\right) \tag{3.4}$$

This equation predicts an increase in dark energy density relative to the ambient background, as visualized in Figure 1 through localized distortions near each mass. Between the Earth $\boldsymbol{m_1}$ and the moon $\boldsymbol{m_2}$ this kink might produce a measurable reduction in the speed of light at right angles to the axis between them. The kink is caused by the $\boldsymbol{a_{x1}}$ droping to zero.

3.6 Solution to Time-Stop Problem with Black Holes

Einstein [9, 10] calculates the speed-of-light ratio c" as

$$c'' = 1 - \frac{G\,m}{x\,c^2},\; c'' = c'/c \tag{3.5}$$

It is given in Section 12 as Equation (12.4). Section 12 repeats Einstein's calculation of light bent by the Sun. For a black hole, this equation incorrectly becomes negative inside the event horizon, where m is large, and x is small.

Based on the new theory, the speed-of-light ratio near the Sun mass m (which represents any mass, instead of the more specialised $\boldsymbol{m_{sun}}$) is as follows:

$$c'' = \frac{1}{1 + \frac{G\mathrm{m}}{a\,c^2}} \tag{3.6}$$

It approximates to Equation (3.5) when $\frac{G\mathrm{m}}{a\,c^2}$ is small. However, for very large $\frac{G\mathrm{m}}{a\,c^2}$ this becomes smaller but not negative.

4. Discussion

4.1 Dark Energy Wave Equation

The 3D energy density $\boldsymbol{dE / dV}$ at location A in Figure 5 is given by Equation (8.18) as follows:

$$\frac{dE}{dV} = \sum_i \sum_j \frac{Gm_i, m_j}{4\pi\, x_{ij}{}^2}\frac{a_{xi}}{a_i{}^3} \tag{4.1}$$

where

$$E' = \frac{dE}{dV} \tag{4.2}$$

Because space has elasticity and mass, it can transmit waves. The new theory leads to a 4D wave equation where the dark energy in the force of space $\boldsymbol{dE'}$/dx is exchanged for the inertial force 1/c^2 dE'/dt. This is the same as Newton's wave equation for a gas [20], except that elasticity and inertia are both derived from 3D dark energy density E' where the speed of light c replaces gas velocity v:

$$\textit{Pressure} = \textit{Space Elasticity} = \frac{dE'}{dx} + \frac{dE'}{dy} + \frac{dE'}{dz} \tag{4.3}$$

$$\textit{Pressure} = \textit{Space Inertia} = \frac{1}{c^2}\frac{dE'}{dt} \tag{4.4}$$

Equating (4.3) and (4.4) yields the 4D dark energy wave equation:

$$\therefore \quad \frac{dE'}{dx} + \frac{dE'}{dy} + \frac{dE'}{dz} = \frac{1}{c^2}\frac{dE'}{dt} \tag{4.5}$$

$$\therefore \quad \nabla E' = \frac{1}{c^2}\frac{dE'}{dt} \tag{4.6}$$

4.2 Dark Matter and Black Holes

Travelling in this elastic dark energy field of space are dark energy waves with a magnitude bounded by the density of space. These waves may be dark matter.

High concentrations of matter form black holes. Inside the black hole, the speed of light drops to a fraction of its normal value. Most of the radiation is trapped by internal reflection. The black hole core may be modelled as a region with high internal reflectivity, analogous to a glass sphere. Some radiation can escape Babb [18]. It is hypothesized that photons could be trapped within dark energy wave structures, potentially resembling micro black holes in behaviour. Particles such as quarks may be micro black holes containing trapped photons.

The general equation for the speed of light at any location and dark energy density is given by Equation (11.1) as follows:

$$c'' = \frac{1}{1 + \frac{G}{c^2} \sum_i \frac{a_{xi}}{a_i} \left(\frac{m_i}{a_{i,j}} c'' + \frac{m_i}{3\, a_{i,j}{}^3} (1 - c'') \right)} \tag{4.7}$$

Note the speed of light ratio is also on RHS. Its solved iteratively by assuming c" =1 on RHS to give a new c" on LHS and so on.

If the sum of m_i is the mass of the black hole or a particle Babb [18] then this mass drops to zero at the centre and so removes the singularity. It has a significant impact on the theory of black holes, which currently relies on Einstein's equation which doesn't use dark energy.

5. Conclusion

GR plus relativity plus the assumption that potential energy and dark energy are the same yields two novel equations.

Equation 1 is the percentage of dark energy, utilising an important theorem that generalises Newton's law to include the speed-of-light ratio. When the whole universe shrinks together, the inverse square law becomes an inverse fourth power law. The results of the new theory were compared with the experimental results. The theory used the mass and radius of a quark to estimate the percentage of dark energy at 71.50%. The experimental result from the Wilkinson-Microwave-Anisotropy-Probe NASA Bennet [2] is 70.39% to 72.25%. It was concluded that this was sufficiently close to establishing a new equation as correct and that potential and dark energy are the same.

Equation 2 is the speed of light, or curvature of space, at any location. The study calculated the bending of light by the Sun and produced the same result as Einstein. For very large masses such as a black hole, the time-stop and singularity problems are removed, suggesting a revision to the theory of black holes, with the possibility of conventional radiation.

Few existing models directly predict the proportion of dark energy using classical or relativistic mechanics. The approach using dark energy improves the speed-of-light equation. The new equations suggest two forms of energy. The potential energy stored in space is referred to as dark energy. Wave energy, which travels in this space, is either trapped in particles or loose in space.

This work presents an alternative to General Relativity-based models in specific contexts, providing measurements of the percentage of dark energy and the speed of light at any location.

6. Statements and Declarations

The author declares no conflicts of interest. This work was not supported by any external funding. E. Babb was the sole author of this study. The author has read and approved the final manuscript. The author would like to thank Sandra S, Peter B., Dr. Richard B., Peter K, Dr. Andrew B., Prof. Volodymyr K, Dr. Hao C, Dr. Tahir, and staff at Imperial College and Oxford University.

7. Appendices

The following theorems and algorithms are logical consequences of Newton's inverse square law, relativity, and the potential energy being equivalent to dark energy.

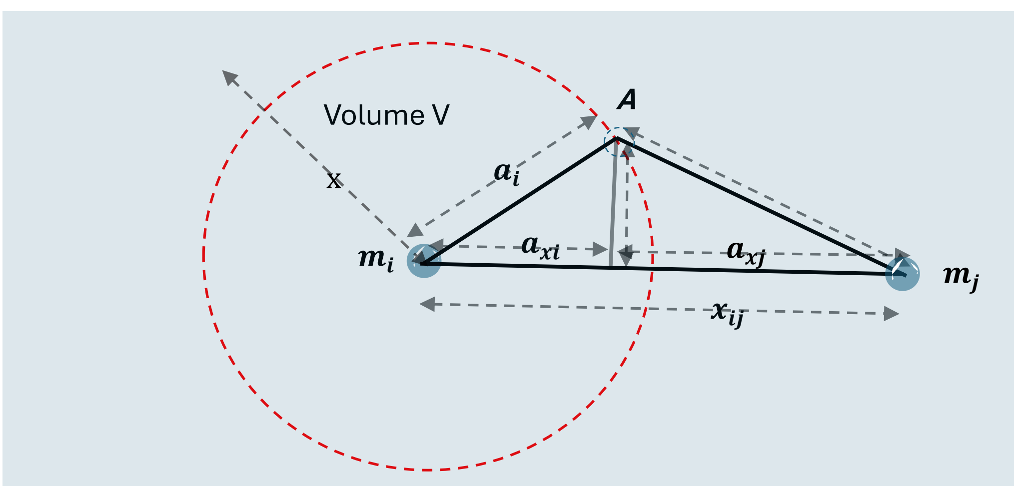

Figure 5: Energy Density at location A

8. Theorem 3D Marginal Dark Energy Density anywhere

8.1 Theorem

The 3D dark energy density ***dE / dV*** at location A is given by Equation (8.18) as follows:

$$\frac{dE}{dV} = \sum_i \sum_j \frac{Gm_i m_j}{4\pi\, x_{ij}^2} \frac{a_{xi}}{a_i^3} \tag{8.1}$$

The lengths a_{xij}, $a_{i,j}$, x_{ij}, depicted in Figure 5, are calculated from the location vectors for ***A***, m_i, m_j. The range of i, j covers all the masses in the universe. Ideally, these masses would be elementary particles.

8.2 Proof

The force between two masses m_i, m_j is given by Newton's inverse square law:

$$\text{Force} = \frac{Gm_i m_j}{x_{ij}^2} \tag{8.2}$$

Because energy is force times distance, force is the change in energy ***E*** divided by the change in distance x_{ij}:

$$\text{Force} = \frac{dE}{dx_{ij}} \tag{8.3}$$

$$\therefore \quad \frac{dE}{dx_{ij}} = \frac{GmM}{x_{ij}^2} \tag{8.4}$$

The two masses m_i, m_j are x apart. When moved toward each other by an increment ***dx***, the masses gain energy ***dE*** by removing an element space ***dV*** from around m_i, m_j. The change in volume at location ***A*** propagates as the surface of a sphere moves toward m_i, m_j. The change in the volume of this sphere is given by the formula for the volume of a sphere radius of a_i:

$$V = \frac{4}{3}\pi a_i^3 \tag{8.5}$$

The change in volume for a change in length a_i is given by:

$$\frac{dV}{da_i} = 4\pi a_i^2 \tag{8.6}$$

The shape of the triangle with sides (a, a_x,c) remained constant. Therefore, a change in a_i relative to the change in a_{xi} is given by the ratio of a_i to a_{xi}:

$$\frac{dV}{da_i} = \frac{a_i}{a_{xi}} \tag{8.7}$$

The change in volume relative to the change in a_{xi} is given by:

$$\frac{dV}{da_{xi}} = \frac{dV}{da_i} \frac{da_i}{da_{xi}} \tag{8.8}$$

Substituting $\frac{dV}{da_i}$ from Equation (8.6) and $\frac{dV}{da_i}$ from Equation (8.7) yields

$$\frac{dV}{da_{xi}} = 4\pi a_i^2 \frac{a_i}{a_{xi}} \tag{8.9}$$

Taking the reciprocal of both sides of the equation yields:

$$\therefore \quad \frac{da_{xi}}{dV} = \frac{a_{xi}}{4\pi a_i^3} \tag{8.10}$$

Furthermore, $x_{ij} = a_{xi} + a_{xj}$., yielding:

$$\therefore \quad dx_{ij} = da_{xi} + da_{xj} \tag{8.11}$$

However, a_{xj} does not change, so da_{xj} is zero and yields:

$$\therefore \quad dx_{ij} = da_{xi} \tag{8.12}$$

Dividing both sides by ***dV*** yields:

$$\frac{dx_{ij}}{dV} = \frac{da_{xi}}{dV}, \tag{8.13}$$

Substituting Equation (8.10) yields:

$$\frac{dx_{ij}}{dV} = \frac{a_{xi}}{4\pi a_i^3} \tag{8.14}$$

The density of energy, or rate of change of energy E with volume V is given by:

$$\frac{dE}{dV} = \frac{dE}{dx_{ij}} \frac{dx_{ij}}{dV} \tag{8.15}$$

Substituting from equations (8.4), (8.12) and (8.14) yields the 3D density of dark energy:

$$\frac{dE_{ij}}{dV} = \frac{Gm_i m_j}{4\pi\, x^2} \frac{a_{xi}}{a_i^3} \tag{8.16}$$

This is the dark energy density for two masses i and j. The sum of the effect of all mass pairs yields the total energy density between all combinations of masses. For example, if there were four masses, then mass 1 would interact with masses 2, 3, and 4. The same applies to mass 2 interacting with 1, 3, and 4. The sum of all these combinations produces the total energy density:

$$\frac{dE}{dV} = \sum_i \sum_j \frac{dE_{ij}}{dV} \tag{8.17}$$

Substituting from Equation (8.18) yields:

$$\frac{dE}{dV} = \sum_i \sum_j \frac{Gm_i m_j}{4\pi\, x_{ij}^2} \frac{a_{xi}}{a_i^3} \tag{8.18}$$

The lengths a_{x,i,j_i}, $a_{i,j}$, x_{ij}, depicted in Figure 5, are calculated from the location vectors for ***A***, m_i, m_j. The range of i and j is to cover all the masses in the universe. Ideally, these masses would be elementary particles.

QED

9. Theorem 1D Marginal Dark Energy density anywhere

9.1 Theorem

The 1D dark energy density is as follows:

$$\frac{dE_{ij}}{da_i} = G\, m_i, m_j\, \frac{a_{xi}}{a_i{}^3} \quad (9.1)$$

The lengths a_{x,i,j_i} , $a_{i,j}$, x_{ij} , depicted in Figure 5, are calculated from the location vectors for A, m_i, m_j. The range of i,j is to cover all the masses in the universe. Ideally, these masses would be elementary particles.

9.2 Proof

The 1D dark energy density $\frac{dE}{da_i}$ is derived as follows:

The volume V of a sphere radius a_i is given by:

$$V = \frac{4}{3}\pi a_i{}^3 \quad (9.2)$$

The rate of change of volume with x is given by:

$$\frac{dV}{da_i} = 4\pi a_i{}^2 \quad (9.3)$$

The 1D dark energy density is given by:

$$\frac{dE}{da_i} = \frac{dE}{dV}\frac{dV}{da_i} \quad (9.4)$$

Substituting from Equations (9.3) and (8.18) yields:

$$\frac{dE}{da_i} = 4\pi\, a_i{}^2\, \frac{Gm_i, m_j}{4\pi\, a_i{}^2}\, \frac{a_{xi}}{a_i{}^3} \quad (9.5)$$

$$\therefore \quad \frac{dE_{ij}}{da_i} = G\, m_i, m_j\, \frac{a_{xi}}{a_i{}^3} \quad (9.6)$$

QED

10. The Speed of Light at any Location.

10.1 Theorem.

The speed of light at any location A, as illustrated in

Figure 2, and is given by Equation (10.15)

$$c'' = \frac{1}{1 + \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{a_i}} \quad (10.1)$$

where the lengths a_x and a are calculated from the location vectors for A, m_i, m_j.

10.2 Proof.

Doubling the 1D dark energy density [21] reduces the speed of light to half because there is now double the amount of space to travel through.

The ratio of total 1D dark energy yields the speed-of-light ratio c" at location A of Figure 5:

$$c'' = \frac{\Lambda E_U}{\Lambda E_U + \Lambda E_A} \quad (10.2)$$

$$\therefore \quad c'' = \frac{1}{1 + E_A/E_U} \quad (10.3)$$

The dark energy relating to all masses m_j is determined by assuming that its energy is converted entirely into dark energy when each mass m_j is separately converted into radiation, which goes to infinity.

$$E_U = \sum_j^{pmax} m_j\; c^2 \quad (10.4)$$

The energy E_A is derived as follows. The 1D energy density is given by Equation (9.6) as follows:

$$\frac{dE}{da_i} = G\, m_i, m_j\, \frac{a_{xi}}{a_i{}^3} \quad (10.5)$$

This is the rate of change of dark energy at location A with respect to da_i . The total energy at location A is found by integrating this expression from a_i to infinity:

$$\therefore \quad E_{Aij} = -\int_{a_i}^{\infty} \frac{dE}{da_i} da_i \quad (10.6)$$

Substituting from Equation (10.5) yields

$$\therefore \quad E_{Aij} = -\int_{a_i}^{\infty} \frac{G\, m_i, m_j}{a_i{}^2}\, \frac{a_{xi}}{a_i} da_i \quad (10.7)$$

The ratio $\frac{a_{xi}}{a_i}$ remains constant because doubling one doubles the other. Therefore:

$$\therefore \quad E_{Aij} = \frac{G\, m_i\, m_j}{a_i}\, \frac{a_{xi}}{a_i} \quad (10.8)$$

The total energy at location A is given by summing the dark energy from of all combinations of masses i and j:

$$\therefore \quad E_{Aij} = \sum_i \sum_j \frac{G\, m_i\, m_j}{a_i}\, \frac{a_{xi}}{a_i} \quad (10.9)$$

$$\therefore \quad E_A = \sum_i \frac{G\, m_i}{a_i}\, \frac{a_{xi}}{a_i} \sum_j m_j \quad (10.10)$$

The speed-of-light ratio is given by Equation (10.3) as follows:

$$\therefore \quad c'' = \frac{1}{1 + E_A/E_U} \quad (10.11)$$

Substituting Equation (10.10) into the ratio E_A / E_U yields:

$$E_A/E_U = \frac{\sum_i \frac{G\, m_i}{a_i}\frac{a_{xi}}{a_i}\sum_j m_j}{\sum_j m_j\; c^2} \quad (10.12)$$

Repeating terms are cancelled:

$$E_A/E_U = \frac{\sum_i \frac{G\,m_i}{a_i}\frac{a_{xi}}{a_i}}{c^2} \tag{10.13}$$

which yields:

$$E_A/E_U = \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{a_i} \tag{10.14}$$

The speed-of-light ratio is given by:

$$c" = \frac{1}{1 + \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{a_i}} \tag{10.15}$$

QED

11. Theorem Speed of Light for High Dark Energy Density.

When the speed of dark energy density is only slightly higher than the ambient density, Equation (10.15) is sufficient. However, if there is a region where the speed of light generally falls, then this equation is only an approximation.

11.1 Theorem.

The speed of light for a location with low or high dark energy density is given by equation (11.16):

$$c" = \frac{1}{1 + \frac{G}{c^2}\sum_i \frac{a_{xi}}{a_i}\left(\frac{m_i}{a_{i,j}}c" + \frac{m_i}{3\,a_{i,j}{}^3}(1 - c")\right)} \tag{11.1}$$

11.2 Proof

The previous theorem assumed that the energy density was low above ambient and thus c" was close to 1. However, near a black hole, Babb [18]., the speed-of-light ratio c" inside the event horizon is very low the Newtonian law in Equation (8.4) repeated as follows:

$$\frac{dE}{dx_{ij}} = \frac{GmM}{x_{ij}{}^2} \tag{11.2}$$

and becomes an inverse fourth power law:

$$\frac{dE}{dx_{ij}} = \frac{GmM}{x_{ij}{}^4} \tag{11.3}$$

This inverse fourth power [21] was used to calculate the percentage of dark energy in the universe.

$$E_{ij} = \frac{GmM}{3\,x_{ij}{}^3} \tag{11.4}$$

The equations (10.6) to (10.10) now change using this fourth power law.

$$\therefore \quad E_{Aij} = -\int_{a_i}^{\infty}\frac{dE}{da_i}da_i \tag{11.5}$$

Substituting form Equation (10.5) yields:

$$\therefore \quad E_{4Aij} = -\int_{a_i}^{\infty}\frac{G\,m_i, m_j}{a_i{}^4}\frac{a_{xi}}{a_i}da_i \tag{11.6}$$

The ratio $\frac{a_{xi}}{a_i}$ remains constant because doubling one doubles the other. Therefore:

$$\therefore \quad E_{4Aij} = \frac{G\,m_i\,m_j}{3a_i{}^3}\frac{a_{xi}}{a_i} \tag{11.7}$$

The total energy at location A is given by summing the effect of all combinations of masses i and j:

$$\therefore \quad E_{4Aij} = \sum_i\sum_j \frac{G\,m_i\,m_j}{3a_i{}^3}\frac{a_{xi}}{a_i} \tag{11.8}$$

$$\therefore \quad E_{4Aij} = \sum_i \frac{G\,m_i}{3a_i{}^3}\frac{a_{xi}}{a_i}\sum_j m_j \tag{11.9}$$

The speed-of-light ratio is given by Equation (10.3) as follows:

$$\therefore \quad c" = \frac{1}{1 + E_{4Aij}/E_U} \tag{11.10}$$

Substituting Equation (11.9) into the ratio E_A / E_U yields:

$$E_{4Aij}/E_U = \frac{\sum_i \frac{G\,m_i}{3a_i{}^3}\frac{a_{xi}}{a_i}\sum_j m_j}{\sum_j m_j\;c^2} \tag{11.11}$$

Repeating terms are cancelled:

$$E_{4Aij}/E_U = \frac{\sum_i \frac{G\,m_i}{3a_i{}^3}\frac{a_{xi}}{a_i}}{c^2} \tag{11.12}$$

which yields:

$$E_{4Aij}/E_U = \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{3a_i{}^3} \tag{11.13}$$

The equation for high dark energy density is:

$$c" = \frac{1}{1 + \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{3a_i{}^3}} \tag{11.14}$$

The equation (10.15) given earlier gives the speed of light for ambient dark energy density when the speed of light ration c" is close one.

$$c'' = \frac{1}{1 + \frac{G}{c^2}\frac{a_{xi}}{a_i}\sum_i \frac{m_i}{a_i}} \quad (11.15)$$

They can be combined into a single equation by multiplying ambient dark energy term by c". The non-ambient term is then multiplied by (1-c"):

$$c'' = \frac{1}{1 + \frac{G}{c^2}\sum_i \frac{a_{xi}}{a_i}\left(\frac{m_i}{a_{i,j}}c'' + \frac{m_i}{3\,a_{i,j}{}^3}(1 - c'')\right)} \quad (11.16)$$

So, c" close to the value of 1 gives equation (11.14) and close to zero gives equation (11.15). A value for c" of 1/2 uses 50% of each energy term to calculate c". This equation (11.16) can be solved through iteration, using c" to start until the result does not change.

QED

12. Einstein's Approach to Bending Light by The Sun

Einstein [8,10] uses the gravitational potential φ to calculate the change in the speed of light near the Sun. The gravitational potential is given by:

$$\varphi = \frac{G\,m_{sun}}{x} \quad (12.1)$$

$$c' = c\,(1 - \varphi/c^2) \quad (12.2)$$

If c close c'

$$c' = c + \varphi/c \quad (12.3)$$

or

$$c'' = 1 + \frac{\varphi}{c^2},\ c'' = c'/c, \quad (12.4)$$

Therefore

$$\frac{dc'}{dx} = \frac{1}{c}\frac{d\varphi}{dx} \quad (12.5)$$

where

$$d\varphi/dx = \frac{Gm}{x^2} \quad (12.6)$$

The angle of deflection α per x for photon travelling at speed c is given by the rate of change of the speed of light c':

$$d\alpha/dx = \frac{1}{c}\frac{dc'}{dx} \quad (12.7)$$

Based on Equation (12.5)

$$d\alpha/dx = \frac{1}{c}\frac{1}{c}\frac{d\varphi}{dx} \quad (12.8)$$

Based on Equation (12.6)

$$d\alpha/dx = \frac{1}{c^2}\frac{Gm}{x^2} \quad (12.9)$$

Einstein then used this rate of change α from the Sun to calculate the bending of light as it passes toward the Sun. It moves from an angle of $\theta = -\frac{\pi}{2}$ to $\theta = \frac{\pi}{2}$.

$$\alpha = \int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}} d\alpha/dx \cos\theta\ ds \quad (12.10)$$

Based on Equation (12.9):

$$\alpha = -\frac{1}{c^2}\int_{\theta=-\frac{\pi}{2}}^{\theta=\frac{\pi}{2}} \frac{Gm}{x^2}\cos\theta\ ds \quad (12.11)$$

$$\alpha = 2\left[\frac{Gm}{c^2\,x}\sin\theta\right],\ \theta = -\frac{\pi}{2}\ to\ \frac{\pi}{2} \quad (12.12)$$

$$\alpha = \frac{Gm}{c^2\,x}(1+1) \quad (12.13)$$

$$\alpha = 2\frac{Gm}{c^2\,x_{sun}} \quad (12.14)$$

$$\alpha = 2\frac{6.67E{-}11 * 1.989E{+}30}{3E{+}08\,\hat{}2 * 6.957E{+}08} \quad (12.15)$$

$$\alpha = 4.23766E{-}06 \quad (12.16)$$

This value was obtained by Einstein et al. The actual bending is double because, in addition to the bending of space, a direct gravitational force is exerted on the photon.

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