

# Urban Congestion as a Volatility Pricing Problem: A Call–Put Options Framework with Black–Scholes Travel Time Valuation

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## Abstract

This paper develops a unified financial-economics framework for urban congestion in which travel time is treated as a stochastic asset with density-dependent volatility. Automobile travel is modeled as a call option on low-congestion travel time, while transit and avoidance behavior act as put-like hedges against congestion risk. We embed a Black–Scholes-type valuation framework to price travel-time uncertainty and derive a congestion pricing rule equivalent to an option premium. Marginal external costs are shown to scale convexly with congestion-induced volatility, providing a structural basis for congestion pricing in urban systems.

## 1. Introduction

Urban congestion is traditionally modeled using marginal cost pricing:

$$MB = MC + MEC$$

However, this formulation omits a key empirical feature: travel time is stochastic and volatility increases with density. This paper reframes congestion as a financial options market over uncertain travel-time outcomes, where congestion externalities arise from unpriced volatility risk.

## 2. Travel Time as a Stochastic Process

$$T = T_0 + \varepsilon$$

$$\varepsilon \sim N(0, \sigma^2(q))$$

$$\frac{d\sigma^2}{dq} > 0$$

Thus, congestion increases both:

- Mean delay (first moment)
- Uncertainty (second moment)

## 3. Utility Function

$$U = V - c - \alpha T$$

Substituting:

$$U = V - c - \alpha T_0 - \alpha \varepsilon$$

Expected utility:

$$E[U] = V - c - \alpha T_0 - \frac{1}{2} \alpha^2 \sigma^2(q)$$

## 4. Call Option Interpretation (Automobile Travel)

Driving is modeled as a call option:

$$C = \max(V - c - \alpha T, 0)$$

Expected value:

$$E[U_{car}] = V - c - \alpha T_0 - \frac{1}{2} \alpha^2 \sigma^2(q)$$

Interpretation:

- Upside in low congestion
- Downside in congestion spikes
- Convex exposure to volatility

## 5. Put Option Interpretation (Transit / Avoidance)

Transit acts as downside protection:

$$U_{transit} = V - c_t - \alpha(1.69T_0)$$

Interpretation:

- Higher mean travel time
- Low variance exposure
- Behaves like a put-on congestion risk

## 6. Marginal External Cost (Volatility Externality)

$$MEC = \frac{1}{2}\alpha^2\sigma^2(q)$$

Interpretation:

- Each trip increases system volatility
- This volatility is not priced
- MEC is equivalent to unpriced short volatility exposure

## 7. Black–Scholes Framework for Travel Time Valuation

### 7.1. Define the “Travel Asset”

Let:

Variable	Meaning
$S$	realized travel speed
$K$	free-flow speed
$\sigma$	congestion uncertainty
$C$	value of driving option

**Table 1**

### 7.4. Key Insight

From Black–Scholes:

$$\frac{\partial C}{\partial \sigma} > 0$$

Thus:

congestion volatility increases the value of flexibility (driving option) but also increases system inefficiency.

## 8. Congestion Pricing from Black–Scholes Sensitivity

The marginal congestion toll is derived from volatility sensitivity:

$$\tau(q) \approx \frac{\partial C}{\partial \sigma} \cdot \frac{d\sigma}{dq}$$

Which reduces to:

$$\tau(q) = \alpha\sigma(q) \frac{d\sigma(q)}{dq}$$

or equivalently:

$$\tau(q) = \frac{1}{2}\alpha^2\sigma^2(q)$$

- $S$  = realized travel efficiency (inverse travel time)
- $K$  = free-flow benchmark efficiency
- $\sigma$  = congestion volatility
- $t$  = travel horizon
- $r$  = social discount rate

## 7.2. Black–Scholes Call Option (Driving Value)

$$C = SN(d_1) - Ke^{-rt}N(d_2)$$

where:

$$d_1 = \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

## 7.3. Interpretation in Transportation Terms

## 9. CMA Empirical Structure

For each CMA  $i$ :

- $D_i$ : density
- $T_i^{car}$ : car travel time
- $T_i^{transit}$ : transit travel time
- $\sigma_i^2$ : congestion volatility

### • Volatility Equation

$$\sigma_i = 0.05 + 0.00015D_i$$

$$\sigma_i^2 = a + bD_i + cD_i^2$$

### • External Cost Equation

$$MEC_i = \frac{1}{2}\alpha^2\sigma_i^2$$

### • Transit Gap

$$T^{transit} = 1.69T^{car}$$

$$Gap = 0.69T^{car}$$

## 10. Structural Results

### • Proposition 1

Congestion volatility increases convexly with density:

$$\frac{\partial^2 \sigma^2}{\partial D^2} > 0$$

### • Proposition 2

External costs scale linearly with volatility:

$$MEC \propto \sigma^2$$

### • Proposition 3

Transit disadvantage increases with congestion risk.

## 11. Conclusion

This paper shows that urban congestion can be modeled as a Black–Scholes-style options market over stochastic travel time. Automobile travel behaves as a call option on free-flow mobility, while congestion externalities correspond to unpriced volatility exposure. Optimal congestion pricing is equivalent to an option premium that internalizes travel-time risk.

### 11.1. Final Core Result

Urban congestion is a Black–Scholes-style mispriced volatility market in which travel behaves as a call option on stochastic mobility, and marginal external costs arise from unpriced sensitivity to travel-time variance.

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