ISSN: 2639-0108

Research Article

Advances in Theoretical & Computational Physics

Topologically Stabilized Torsion in Weak-Field Gravity: A Ricci–Flow Framework

Elisa Varani*

Università Cattolica del Sacro Cuore Milano, Italy

*Corresponding Author
Elisa Varani, Università Cattolica del Sacro Cuore Milano, *Italy*.

Submitted: 2026, Jul 30; **Accepted:** 2026; Aug 27; **Published:** 2026, Sep 16

Citation: Varani, E. (2026). Topologically Stabilized Torsion in Weak-Field Gravity: A Ricci–Flow Framework. *Adv Theo Comp Phy, 9*(3), 01-09.

Abstract
We investigate stationary torsional configurations supported by chiral Majorana neutrino currents in linearized gravity. A Ricci–flow–inspired geometric relaxation (with no physical time interpretation) is introduced to drive the tetrad perturbation toward fixed points sustained by chiral sources while keeping curvature invariants negligible. We show that divergence-free chiral currents can support globally non-trivial torsional holonomy, classified by π_1 (S^1), and that three-dimensional torsional domains admit an independent winding classification associated with π_3 (S^3). Toroidal skyrmionic domains emerge when one chirality dominates, whereas a chiral-flip interference sector supports Möbius-type bridges between opposite-chirality regions, characterized by a $\mathbb{Z}_2$ spinorial holonomy associated with π_1 (SO(3)). In the static limit, a Green-function formulation provides a finite-range Yukawatype response governed by the neutrino coherence length. These results identify a purely torsional mechanism independent of local curvature through which coherent chiral currents may influence effective gravitational behavior in neutrino-rich environments.

Keywords: Torsion and Gravity, Chiral Spinor Fields, Majorana Neutrinos, Ricci Flow, Stationary Solutions, Topological Structures, Linearized Gravity

1. Introduction

Spinor fields in gravitational theories with torsion couple to the antisymmetric part of the affine connection [1,2]. Majorana neutrinos, with intrinsic chirality, are natural sources for torsional effects and may form coherent domains. This short communication focuses on stationary torsional solutions supported by chiral currents, which are topologically non-trivial while the curvature remains negligible. Coherent neutrino populations are expected to arise in several high-density astrophysical environments from supernova interiors to the early Universe neutrino background making torsion-induced effects of potential relevance beyond laboratory scales. We also compare the resulting torsional structures with the loop-like configurations that arise in the post-Newtonian regime once strict spherical symmetry is broken, in order to make explicit that the two mechanisms, while similar in morphology, are supported by different sources and belong to different sectors of the connection. A broader development, including extended classification and topological aspects, appears in a companion study [3]. Related Yukawa–torsion effects are discussed in [4].

1.1. Context and Related Developments

Gravitational theories with torsion have been investigated in several complementary directions over the past decades, particularly in connection with spinor matter and extended geometric structures. Within the Einstein–Cartan and gauge–gravity frameworks, torsion naturally emerges as the antisymmetric part of the affine connection and couples to spin and chiral currents, providing a consistent geometric extension of purely metric formulations [1,2,5]. A rigorous derivation of spin–torsion coupling and its physical implications in metric–affine gravity is provided in Ref. [6]. In this context, fermionic fields with intrinsic chirality including Majorana neutrinos represent natural geometric sources of torsion, especially in regimes where coherent or collective spinor configurations may arise. More recent developments have emphasized the role of global geometric constraints and topological structures in the classification of physically admissible configurations, shifting the focus from local field equations to the structure of the space of solutions itself. In parallel, geometric and topological methods ranging from moduli-based classifications to the use of geometric flows as organizing principles have been employed to analyze stationary or selfconsistent gravitational configurations without invoking dynamical evolution

[7,8]. Within this broader context, torsion-supported solutions may be interpreted as elements of distinct geometric sectors characterized by nontrivial topology, motivating the search for analytically controlled stationary configurations in which curvature effects remain negligible while global geometric features persist.

1.2. Chiral Currents and Weak–Field Setting

We consider the weak–field expansion of the tetrad,

$$e_\mu{}^a = \delta_\mu{}^a + \tfrac{1}{2} h_\mu{}^a, \qquad |h_{\mu\nu}| \ll 1, \tag{1}$$

where $h_{\mu\nu} = h_{(\mu\nu)} + h_{[\mu\nu]}$ decomposes into a symmetric part, identified with the ordinary metric perturbation through $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and an antisymmetric part $h_{[\mu\nu]}$. In purely metric (Riemannian) gravity the antisymmetric part can be removed by a local Lorentz transformation and carries no physical content. In the present Einstein–Cartan setting, however, we adopt the working assumption that $h_{[\mu\nu]}$ is not a gauge degree of freedom but an effective dynamical variable of the torsional sector, whose dynamics is governed by the chiral spinor currents introduced below. This assumption is consistent with the fact that in Einstein–Cartan theory torsion is algebraically sourced by the spin density of matter, so that in the presence of coherent spinor currents the antisymmetric sector acquires a physical, current-driven character. All curvature invariants built from the symmetric sector remain negligible in the regimes considered. We focus on Majorana spinor fields, whose left- and right-handed components are related by charge conjugation. Their intrinsic chirality makes them natural sources for torsional degrees of freedom. The left- and right-handed fermionic currents are defined as

$$J_L^\mu = \bar{\psi}\gamma^\mu P_L \psi, \qquad J_R^\mu = \bar{\psi}\gamma^\mu P_R \psi, \qquad P_{L,R} = \frac{1 \mp \gamma_5}{2}, \tag{2}$$

and encode the local transport of chiral charge. In spatial components,

$$J_{L,R}^k = \bar{\psi}_{L,R}\, \sigma^k\, \psi_{L,R}, \tag{3}$$

with σ^k the Pauli matrices.

Torsion is sourced by the chiral imbalance between the left- and right-handed sectors, schematically (J_L-J_R), and by an interference (flip) sector built from mixed bilinear of the form $\bar{\psi}_L \psi_R$. The latter enables the formation of globally twisted torsional structures with a non-trivial internal spinorial sector, despite locally vanishing curvature.

1.3. Ricci Flow Evolution

To model the geometric response of the torsional sector to chiral currents, we use a flow parameter τ that evolves the tetrad perturbation $h_{\mu\nu}$ toward stationary configurations without representing physical time. In the weak–field regime, the evolution equation takes the form

$$\frac{\partial h_{\mu\nu}}{\partial \tau} = -2\, R_{\mu\nu}(h), \tag{4}$$

inspired by the classical Ricci flow framework introduced by Hamilton and extended to include entropy functionals by Perelman [7,8]. Here $R_{\mu\nu}(h)$ denotes the linearized Ricci tensor of the torsionful connection, whose antisymmetric part is non-vanishing in the Einstein–Cartan framework and encodes the torsional contribution generated by chiral currents [3]. Using the Levi–Civita structure ϵ_{abc}, the most relevant components can be expressed schematically as

$$\frac{\partial h_{[0c]}}{\partial \tau} \propto \int dx^b\, \epsilon_{bck} \left(\bar{\psi}_L \sigma^k \psi_L - \bar{\psi}_R \sigma^k \psi_R \right), \tag{5}$$

$$\frac{\partial h_{[ac]}}{\partial \tau} \propto \int dx^0\, \epsilon_{ack} \left(\bar{\psi}_L \sigma^k \psi_L - \bar{\psi}_R \sigma^k \psi_R \right), \tag{6}$$

$$\left. \frac{\partial h_{[ac]}}{\partial \tau} \right|_{\text{flip}} \propto \int dx^b\, \epsilon_{bca} \left(\bar{\psi}_L \psi_R - \bar{\psi}_R \psi_L \right), \tag{7}$$

demonstrating that left- and right-handed currents induce torsion with opposite sign, while the flip bilinear couples the two chiral components and enables the formation of globally coherent, twisted configurations with non-trivial spinorial holonomy. The aim of the flow evolution is to identify geometry–current combinations that satisfy $\partial_\tau h_{\mu\nu} = 0$, i.e. stationary torsional states, which are analyzed in the next section.

1.4. Left/Right and Chiral-Flip Contributions

The Levi–Civita tensor ϵ_{abc} in Equations (5) - (7) enforces an opposite orientation of torsion generated by left- and right-handed currents. Domains dominated by J_L and J_R therefore induce counterrotating local frames, realizing geometrically distinct chiral phases with vanishing curvature. The flip sector, driven by mixed bilinears of the form $\bar{\psi}_L \psi_R$, couples the two chiral components and allows for a continuous interpolation between them. This mechanism acts as a torsional "stitching" that preserves stationarity while allowing a non-trivial global twist of the internal chiral sector.

Figure 1: Schematic Möbius-like torsional bridge generated by chiral-flip stitching between domains of opposite chirality. Local curvature remains negligible while the internal spinorial sector may carry a non-trivial $\mathbb{Z}_2$ holonomy.

Coherent domains connected through the flip sector can thus produce globally non-trivial torsional structures such as toroidal skyrmionic regions or Möbius-like bridges where all curvature invariants remain arbitrarily small. These configurations will be classified topologically in the next section.

2. Topological Characterization

Stationary torsional configurations fall into distinct topological classes depending on whether torsion is confined along a closed line or distributed over a tubular domain. These properties are captured by global invariants associated with the holonomy of the torsional connection and its mapping into an internal chiral space. In this sense, topological properties are not introduced as external constraints, but emerge as intrinsic features of the space of admissible gravitational configurations.

2.1. Holonomy and Torsional Vortices

The torsional sector can be represented by a gravitational vector potential A_μ^T [3], such that parallel transport acquires a phase depending solely on the global structure of the connection. The torsional circulation along a closed loop Γ is quantified by the holonomy:

$$\Phi_T = \oint_\Gamma A_\mu^T \, dx^\mu = 2\pi n, \qquad n \in \mathbb{Z}. \tag{8}$$

A non-zero integer n signals the presence of a line-like torsional vortex, classified by the first homotopy group $\pi_1(S^1)$. The spacetime remains locally flat but exhibits a globally non-trivial phase a torsional analogue of the Aharonov–Bohm effect in gauge theory. Thus, Φ_T detects the existence of a quantized torsional defect.

The interpretation of Φ_T as a contour integral encircling a defect admits a natural counterpart in a complementary geometric framework. In the spinor–twistor description of Ref. [9], field sources are represented as poles of suitable twistor functions, and the corresponding contour integrals of Cauchy type evaluate cohomological functionals whose residues are interpreted as evidence of field torsion. The two descriptions share a common structural feature, in that the global information is carried by a circulation around a defect rather than by local curvature invariants, which remain negligible in the regimes considered here.

2.2. Skyrmionic Torsion and Toroidal Domains

Beyond line-like defects, torsion can also form volumetric domains where the orientation of the chiral frame winds non-trivially throughout a 3D region. The local chiral frame is related to a reference frame by an $SU(2)$ rotation $U(x) = n^0(x)\mathbb{1} + i\, n^k(x)\sigma^k$, so that the configuration is described by a unit field $n^a(x)$, $a = 0,1,2,3$, with $n^a n_a = 1$, taking values in $SU(2) \simeq S^3$. Assuming $U(x)$ approaches a constant value at spatial infinity, n^a defines a map $S^3 \to S^3$, whose global wrapping defines the topological charge:

$$Q_S = \frac{1}{12\pi^2} \int d^3x \, \epsilon^{ijk} \epsilon_{abcd} \, n^a \, \partial_i n^b \, \partial_j n^c \, \partial_k n^d \in \mathbb{Z}. \tag{9}$$

A non-zero Q_S corresponds to a $\pi_3(S^3)$ classification, ensuring that the torsional domain cannot be continuously unwound without destroying or severing the region that supports it. In this sense, $Q_S \neq 0$ stabilizes winding-protected torsional domains as stationary solutions, independently of the holonomy associated with Φ_T.

Concretely, n^a is obtained from the coherent chiral amplitudes themselves. Since ψ_L and ψ_R are related by charge conjugation, the normalized chiral components may be written as

$$\hat{\psi}_L = \frac{\psi_L}{\sqrt{\psi_L^\dagger \psi_L}}, \qquad \hat{\psi}_R = -i\sigma^2 \hat{\psi}_L^*, \tag{10}$$

up to the conventional Majorana phase. The pair $(\hat{\psi}_L, \hat{\psi}_R)$ then defines the columns of an $SU(2)$ element,

$$U = \left(\hat{\psi}_L, \, \hat{\psi}_R\right), \qquad U^\dagger U = \not{1}, \qquad \det U = 1, \tag{11}$$

whose expansion $U = n^0 \not{1} + i \, n^k \, \sigma^k$ identifies the four real fields n^a entering Eq. (9).
The spatial current introduced in Eq. (3) is recovered as

$$J_L^k = |\psi_L|^2 \, \hat{\psi}_L^\dagger \sigma^k \hat{\psi}_L, \tag{12}$$

so that the normalized current direction is obtained from the full spinorial variable U through the standard projection onto the local chiral direction. The current alone therefore does not retain the complete information contained in the $SU(2)$ frame, the phase degree of freedom being lost in this projection; retaining the full pair $\bar{\psi}_L \psi_R$ is what allows the $S^3 \to S^3$ winding classification entering Q_S.

The construction is well defined wherever $\psi_L^\dagger \, \psi_L \neq 0$. At zeros of the coherent spinor amplitude the normalized chiral frame ceases to be defined, providing the natural location of possible defect cores. In summary, Φ_T characterizes the holonomy of line-like torsional defects, whereas Q_S characterizes the topological winding of three-dimensional torsional domains. The two invariants therefore describe complementary aspects of the global torsional structure. A third, $\mathbb{Z}_2$ invariant associated with the chiral-flip sector is introduced in Sec. 3.2.

3. Examples of Stationary Torsional Domains

Stationary solutions supported by chiral currents naturally organize into coherent domains whose topology is fixed by the structure of the currents. Here we present two representative geometries: a toroidal domain carrying a non-zero $\pi_3(S^3)$ winding charge Q_S, and a Möbius-like torsional bridge in which the flip sector carries the $\mathbb{Z}_2$ spinorial holonomy ν_{spin} introduced below. The torsional holonomy Φ_T of Sec. 2.1 may be present in either case, since it characterizes line-like defects independently of the two mechanisms considered here.

3.1.Toroidal Stationary Domain (Skyrmion-Like)

A domain in which left-handed currents circulate within a closed tubular region may support a non-trivial S^3 winding of the internal chiral frame. When $Q_S \neq 0$ the resulting configuration belongs to a non-trivial $\pi_3(S^3)$ sector and is protected against continuous unwinding. The torsion remains localized within the torus while curvature stays negligible everywhere; the geometry is sketched in Figure 2.

3.2. Chiral-Flip Stitching: Möbius-Like Bridge

When two regions dominated by opposite chirality are connected through the flip sector, the resulting torsional configuration acquires a non-trivial twist while remaining flat locally. A loop encircling the bridge may carry a torsional holonomy Φ_T of the type discussed in Sec. 2.1, as in Aharonov–Bohm-type defects; the Möbius character of the configuration, however, is a distinct property, characterized as follows.

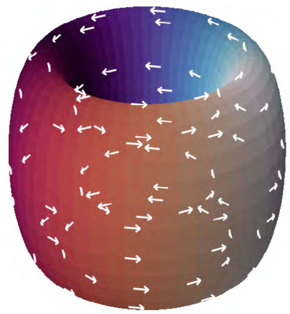

Figure 2: Schematic toroidal torsional domain supported by circulating chiral currents. When the winding charge Q_S is non-zero the configuration is protected against continuous unwinding

The Möbius-type character of such a configuration can be made precise as a statement about the internal spinorial sector rather than about spacetime itself. Since the connection takes values in the proper Lorentz algebra, its holonomy preserves spacetime orientation. Consider, however, a closed loop crossing the flip region twice, so that the internal chiral frame undergoes two successive π rotations. The associated $SO(3)$ rotation returns to the identity, whereas its lift to Spin(3) $\simeq SU(2)$ changes sign, $U \to -U$. The corresponding loop therefore represents the non-trivial element of

$$\pi_1(SO(3)) \simeq \mathbb{Z}_2. \tag{13}$$

More generally, for an even number n of flip crossings the lifted spinorial transformation acquires the sign$(-1)^{n/2}$, defining the invariant

$$\nu_{\text{spin}} = \frac{n}{2} \pmod 2. \tag{14}$$

Thus, the chiral-flip sector may carry a non-trivial $\mathbb{Z}_2$ spinorial holonomy. This sign reversal is structurally analogous to the transition function of a Möbius real line bundle over S^1, and it is in this precise sense that the resulting configuration is referred to here as Möbius-type. No non-orientability of the spacetime manifold itself is implied. These structures demonstrate that topologically non-trivial torsion can be sustained solely by chiral imbalance, providing explicit realizations of the stationary solutions described above.

3.3. Relation to Post-Newtonian Gravitomagnetic Structures

It is useful to distinguish the torsional toroidal configurations considered here from loop-like structures that may arise in the conventional post-Newtonian gravitational field. When strict spherical symmetry is broken, for instance by rotation, the weak-field metric develops a vector sector associated with mass currents. In the standard gravitomagnetic description, this sector may be represented by a vector potential A_i^{GM}, with associated field

$$\mathbf{B}_{\text{GM}} = \nabla \times \mathbf{A}_{\text{GM}}. \tag{15}$$

For rotating sources, this sector is responsible for frame-dragging effects and can generate closed, circulation-like field structures.

The mechanism considered in the present work is complementary to this one. Here the relevant circulation belongs to the torsional sector and is sourced by coherent chiral spinor

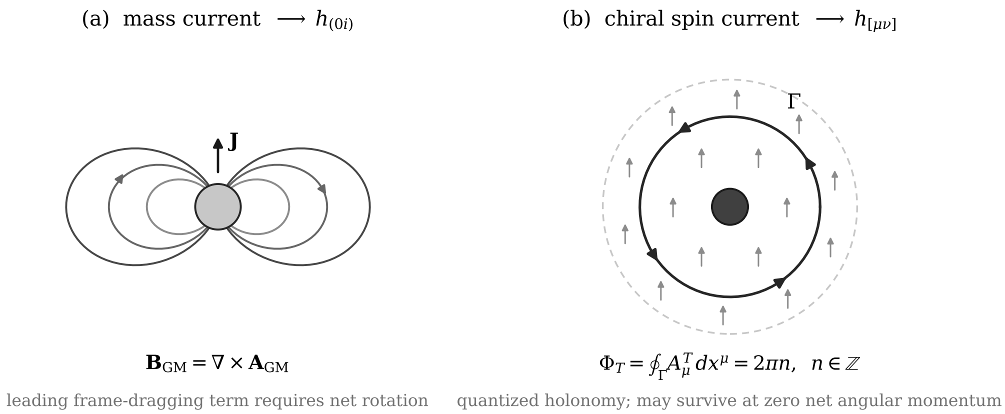

Figure 3: Comparison between the two mechanisms. (a) A rotating mass current sources the gravitomagnetic potential A_{GM}, a component of the symmetric perturbation $h_{(0i)}$; the resulting field lines close about the rotation axis, and the leading frame-dragging contribution varies continuously with the net angular momentum **J** and vanishes for **J** = 0. (b) A coherent chiral spin current (grey arrows) supports a torsional defect belonging to the antisymmetric sector $h_{[\mu\nu]}$; the global content is measured by the holonomy Φ_T along a loop Γ enclosing the core, is quantized, and may persist in a region of vanishing total angular momentum where all curvature invariants built from the symmetric sector remain negligible.

currents rather than by ordinary rotational mass currents. The corresponding global information is encoded in the torsional holonomy (8), where $A_\mu{}^T$ denotes the effective torsional potential introduced in Sec. 2.1. This quantity can remain non-trivial even in a regime in which the curvature associated with the metric sector is negligible. Thus, although gravitomagnetic and torsional configurations may both exhibit loop-like geometries, they originate from different sources and belong to different sectors of the connection. Since in the Einstein–Cartan framework torsion is sourced locally by the spin density, a torsional configuration may persist even when the total angular momentum of the region vanishes. In such a case the leading gravitomagnetic contribution associated with net rotation is absent, while higher-order mass-current multipoles, if present, need not vanish. The two cases are compared in Figure 3.

Schematically, the two mechanisms may be summarized as

mass current → gravitomagnetic vector sector →frame dragging, (16)

whereas in the present construction

chiral spin current→torsional connection→non-trivial torsional holonomy (17)

This comparison suggests that loop-like or toroidal structures need not be associated exclusively with rotational metric effects. In a torsionful geometry, analogous global structures may also be supported by the spinorial sector, including configurations for which local metric curvature remains negligible. A connection between rotating sources and the generation of torsion has likewise been conjectured, in a twistor-theoretic setting, in Ref. [9]. A multipolar analysis of the chiral sources, developed in the following section, provides a more systematic way to characterize the relation between the two mechanisms.

4. Static Limit and Green-Function Formulation

The stationary condition in the flow parameter τ does not necessarily imply the absence of residual spatial variations. It is thus useful to complement the Ricci–flow–based description with a static limit, where $\partial_0 h_{\mu\nu} = 0$ and time derivatives vanish in physical coordinates. In this regime, the linearized field equations reduce to an elliptic system

$$\nabla^2 h_{\mu\nu}(\mathbf{x}) = -2\, T^{\mathrm{eff}}_{\mu\nu}(\mathbf{x}), \tag{18}$$

where $T^{eff}_{\mu\nu}$ is built directly from localized chiral sources (including the flip interference). This formulation highlights the finite-range response of the torsional sector to the underlying spinor currents.

A screened Green function,

$$G(\mathbf{x}, \mathbf{x}') = \frac{e^{-\mu|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|}, \tag{19}$$

accounts for a characteristic coherence length μ^{-1} of the torsion-inducing neutrino currents. The tetrad perturbation can then be written in compact Yukawa-type form:

$$h_{[0c]}(\mathbf{x}) = \frac{1}{3}\int d^3x'\, G(\mathbf{x},\mathbf{x}')\,\epsilon_{bck}\left(\bar{\psi}_L\sigma^k\psi_L - \bar{\psi}_R\sigma^k\psi_R\right)(\mathbf{x}'), \tag{20}$$

$$h^{(L/R)}_{[ac]}(\mathbf{x}) = -\frac{1}{3}\int d^3x'\, G(\mathbf{x},\mathbf{x}')\,\epsilon_{ack}\left(\bar{\psi}_L\sigma^k\psi_L - \bar{\psi}_R\sigma^k\psi_R\right)(\mathbf{x}'), \tag{21}$$

$$h^{(\mathrm{flip})}_{[ac]}(\mathbf{x}) = -\frac{1}{3}\int d^3x'\, G(\mathbf{x},\mathbf{x}')\,\epsilon_{bca}\left(\bar{\psi}_L\psi_R - \bar{\psi}_R\psi_L\right)(\mathbf{x}'). \tag{22}$$

These expressions show that left- and right-handed contributions retain opposite sign, while the flip sector couples' chiral domains, thus enabling globally twisted stationary configurations with non-trivial internal spinorial holonomy, such as the one depicted in Figure 1. Complementary Yukawa–torsion analysis is developed by the author in [4].

4.1. Multipolar Structure of the Screened Torsional Response

The Green-function representation also provides a natural way to investigate the angular structure of the torsional response. For localized chiral sources, the screened kernel entering Equations (20) - (22) can be expanded in spherical harmonics as

$$G(\mathbf{x},\mathbf{x}') = \mu\sum_{\ell m} i_\ell(\mu r_<)\,k_\ell(\mu r_>)\,Y_{\ell m}(\hat{\mathbf{x}})\,Y^*_{\ell m}(\hat{\mathbf{x}}'), \tag{23}$$

where $r_< = min(r,r')$, $r_> = max(r,r')$, and i_1, k_1 are modified spherical Bessel functions normalized so that $i_0(z) = \sin h\, z/z$ and $k_0(z) = e^{-z/z}$ The Yukawa screening therefore modifies the radial dependence of the individual multipoles while preserving their angular decomposition.

Of particular interest for the configurations of Sec. 3 is the occurrence of a toroidal current moment. For a localized current distribution $\mathbf{J}(\mathbf{x})$ the toroidal dipole moment is defined, up to convention-dependent normalization, by

$$\mathbf{T} = \frac{1}{10}\int d^3x\left[(\mathbf{x}\cdot\mathbf{J})\,\mathbf{x} - 2r^2\mathbf{J}\right], \tag{24}$$

and is associated with closed current distributions possessing poloidal circulation [10].

As an explicit realization compatible with the domains of Sec. 3.1, consider the effective chiral source $\boldsymbol{J}_\chi = \boldsymbol{J}_L - \boldsymbol{J}_R$ confined to the surface of a torus of major radius R and minor radius a, with purely poloidal circulation (Figure 4)

$$\mathbf{K}(\theta,\phi) = \frac{K_0}{R + a\cos\theta}\,\mathbf{e}_\theta, \tag{25}$$

which is divergence-free on the torus. For this distribution the ordinary magnetic dipole moment vanishes identically,

$$\mathbf{m} = \tfrac{1}{2}\int \mathbf{x}\times\mathbf{J}_\chi\, d^3x = 0, \tag{26}$$

since$(\boldsymbol{x}\times\boldsymbol{e}_\theta)_z$ vanishes pointwise and the transverse components integrate to zero, whereas the toroidal moment is non-vanishing and aligned with the symmetry axis,

$$\mathbf{T} = -\pi^2 K_0\, R\, a^2\,\hat{\mathbf{z}}, \tag{27}$$

for the orientation and normalization adopted here. This configuration provides an idealized realization of a toroidal current moment with vanishing ordinary magnetic dipole moment, and shows that a conserved chiral current with poloidal geometry may possess a toroidal sector with no competing magnetic dipole.

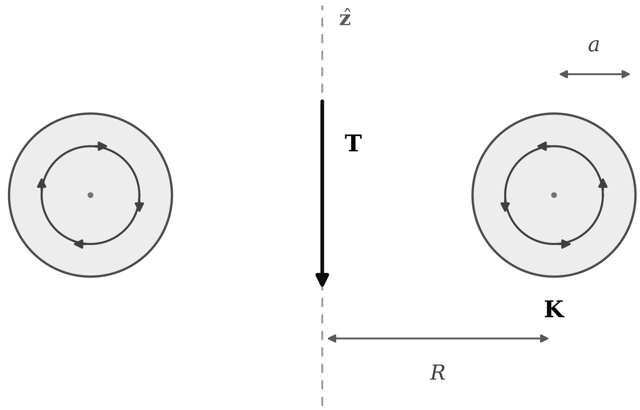

Figure 4: Meridional cross-section of the poloidal chiral-current distribution of Equation (25), used in the multipolar analysis of this section. The two disks are the two intersections of the same tube with a plane through the symmetry axis; R and a denote the major and minor radii. The effective current **K** circulates along the poloidal direction e_θ, wrapping around the minor section of the torus rather than around the hole, so that it flows upward on the outer side of the tube and downward through the hole on both sides. For this axially symmetric configuration the ordinary magnetic dipole moment vanishes, **m** = 0, whereas the toroidal moment is non-zero and directed along the symmetry axis, with the orientation shown corresponding to the sign in Equation (27) for $K_0 > 0$.

We stress that this conclusion follows from the poloidal geometry and axial symmetry of the source, and is therefore a statement about the admissible multipolar structure of the domains introduced in Sec. 3.1 rather than a consequence of chirality as such. Two points remain open. First, the sources entering Equations. (20) - (22) involve a Levi-Civita contraction of the chiral current rather than the current itself, so the multipolar content of the torsional response is not directly given by the moments of $\mathbf{J}_\chi$ and requires a dedicated analysis. Second, the relation between a non-vanishing **T** and the quantized holonomy (8) introduced in Sec. 2.1 is not established here: the two quantities characterize, respectively, the local structure of the source and a global topological invariant, and their possible correspondence is left for further investigation. The flip contribution in Equation (22) requires a separate treatment as well, being generated by mixed chiral bilinears rather than by the conserved current difference alone.

5. Conclusion

We have shown that chiral Majorana neutrino currents can sustain stationary torsional structures in the weak-field regime through a Ricci–flow–inspired evolution of the tetrad perturbation. These configurations include toroidal domains, characterized by a winding charge associated with π_3 (S^3), and Möbius-like bridges in which the chiral-flip sector carries a $\mathbb{Z}_2$ spinorial holonomy while local curvature remains negligible and the spacetime manifold itself remains orientable. The resulting states are characterized by three distinct topological invariants, $\Phi_T \in 2\pi\mathbb{Z}$, $Q_S \in \mathbb{Z}$ and $\nu_{spin} \in \mathbb{Z}_2$ and a finite-range response encoded by a Yukawa-type Green function. The screened Green–function used here is derived and discussed in [4].

Two further results follow from the analysis. First, the toroidal structures obtained here are complementary to the loop-like configurations of the post-Newtonian gravitomagnetic sector: the latter are sourced by rotational mass currents and belong to the ordinary metric perturbation, whereas the former are sourced by chiral spin currents and survive in a regime where curvature invariants are negligible. Toroidal morphology alone therefore does not identify the mechanism that produces it. Second, the multipolar expansion of the screened response shows that a conserved chiral current with poloidal geometry may carry a toroidal moment while its ordinary magnetic dipole vanishes, so that the toroidal domains of Sec. 3.1 are compatible with a well-defined multipolar structure of the source. The relation between this moment and the quantized holonomy Φ_T remains to be established.

Such torsional stationary domains could, in principle, emerge on scales where coherent neutrino currents are present, potentially influencing the effective gravitational behavior of regions that do not contain ordinary matter. Left-handed configurations may contribute to localized attractive geometric effects, while right-handed sectors provide an opposite response. A network of such stationary torsional structures may therefore play a role in large-scale geometry, possibly contributing to gravitational effects that are not directly associated with visible matter. A detailed investigation of these broader implications, including their potential relevance in astrophysical and cosmological contexts, is developed in the extended companion study [3].

Finally, the present work also suggests a broader mathematical perspective. While our analysis has been developed entirely within the framework of weak-field Einstein–Cartan gravity and Ricci-flow dynamics, the stationary torsional configurations obtained here possess a well-defined geometric and topological structure. This naturally raises the question of whether such configurations admit a deeper mathematical interpretation in terms of differential cohomology or Hodge theory. In particular, it would be interesting to investigate whether the stationary torsional configurations correspond to distinguished representatives of suitable cohomology classes or, more generally, to eigenforms of a suitably defined Laplace-type operator associated with the torsional sector. Such a formulation could establish a bridge between the local Clifford-algebraic origin of spin-induced torsion and its global topological characterization. Recent mathematical approaches describing field configurations through hypercohomology groups and Hodge decompositions provide a possible framework for exploring this direction [11]. At present, however, this interpretation remains speculative and lies beyond the scope of the present work. Establishing whether such a correspondence exists constitutes an interesting open problem for future investigations.

Acknowledgements
The author thanks Francisco Bulnes for stimulating discussions and helpful suggestions.

References

1. Hehl, F. W., Von der Heyde, P., Kerlick, G. D., & Nester, J. M. (1976). General relativity with spin and torsion: Foundations and prospects. *Reviews of Modern Physics, 48*(3), 393.
2. Shapiro, I. L. (2002). Physical aspects of the space–time torsion. *Physics Reports, 357*(2), 113-213.
3. Varani, E. (2025). Geometric evolution of torsional curvature from Majorana neutrinos: A Ricci flow approach. *Transnational Journal of Mathematical Analysis 13* (1), 53–104.
4. Varani, E. (2025). Torsional Effects in the Coupling between Gravity and Spinors--Yukawa Gravity. *arXiv preprint arXiv:2505.17158*.
5. Hehl, F. W., & Obukhov, Y. N. (2001). How does the electromagnetic field couple to gravity, in particular to metric, nonmetricity, torsion, and curvature?. In *Gyros, Clocks, Interferometers...: Testing Relativistic Graviy in Space* (pp. 479-504). Berlin, Heidelberg: Springer Berlin Heidelberg.
6. Puetzfeld, D., & Obukhov, Y. N. (2013). Covariant equations of motion for test bodies in gravitational theories with general nonminimal coupling. *Physical Review D—Particles, Fields, Gravitation, and Cosmology, 87*(4), 044045.
7. Hamilton, R. S. (1982). Three-manifolds with positive Ricci curvature. *Journal of Differential geometry, 17*(2), 255-306.
8. Perelman, G. (2002). The entropy formula for the Ricci flow and its geometric applications. arXiv preprint math/0211159.
9. Bulnes, F. (2023). SINGULARITIES, TORSION, CAUCHY INTEGRALS AND THEIR SPECTRA ON SPACE-TIME. *Transylvanian Journal of Mathematics & Mechanics, 15*(1/2), 29.
10. Dubovik, V. M., & Tugushev, V. V. (1990). Toroid moments in electrodynamics and solid-state physics. *Physics reports, 187*(4), 145-202.
11. Bulnes, F. (2026). Correspondence between hypercohomology groups to solutions as H-states with Hodge decomposition of a connection. *Int. J. Adv. Appl. Math. Mech. 14* (1), 64–71.