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Short Communication

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Theoretical Justification for the Effectiveness of the New Estimate for the Doppler Spectrum when Operating Spaceborne SAR over the Ocean

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Abstract
A complete theoretical and partial experimental justification for the proposed method [1] for processing the Doppler spectrum is given.

In the work [1] a new spectral estimate was proposed for the Doppler spectrum when operating spaceborne SAR over the ocean:

$$G_{new}(\omega) \propto \left\{\int d\omega'\, G_s(\omega') \exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega')^2\right]\right\}^2 \quad (1)$$

Here: $G_s(\omega')$ is the simple spectral estimate obtained through the FFT of the only realization of the single look complex signal, Δ_{SAR} is the SAR nominal azimuthal resolution, $V \cong 8\, km/s$ is the speed of SAR carrier.

Even at first glance at the formula (1), it is natural to expect that the proposed transformation of the strongly fluctuating estimate $G_s(\omega)$ will significantly smooth out the fluctuations. Strictly speaking, to confirm this expectation, a fairly large amount of experimental data is needed, which the author does not have. Nevertheless, the results of the theoretical analysis (see below) and the processing of, unfortunately, only two experimental Doppler spectra, fully correspond to the expectation expressed above.

The basis of (1) is the convolution of $G_s(\omega)$ with the shape factor $\tilde{G}(\omega)$ of the average Doppler spectrum [2]:

$$\tilde{G}(\omega) = \exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega_0)^2\right] \quad (2)$$

where $\omega_0 = 2k\bar{v}_{rad}$ and $\bar{v}_{rad}$ is the regular part of the velocity radial component on the ocean surface. Accordingly, the shape factor of the initial estimate is represented as

$$\tilde{G}_s(\omega) = [1+\xi(\omega)]\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega_0)^2\right] \quad (3)$$

where $\xi(\omega)$ is the random variable with zero mean and variance σ_ξ^2. Note that according to [2], in the case of spaceborne SAR, the shape factor of the average Doppler spectrum depends weakly on the surface state, which cannot be said about the fluctuation part.

Without specifying the amplitude factor, which is unimportant in this case, we will omit it and write

$$\langle G_{new}(\omega)\rangle \propto I_1^2 + \langle I_2^2\rangle \qquad (4)$$

Here, I_1 and I_2 and denote the integrals

$$I_1 = \int d\,\omega' \exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega')^2\right]\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega'-\omega_0)^2\right] \qquad (5)$$

$$I_2 = \int d\omega'\,\xi(\omega')\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega')^2\right]\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega'-\omega_0)^2\right] \qquad (6)$$

The angle brackets indicate averaging over random realizations of the spectral estimate.

The first integral can be taken:

$$I_1 = \frac{\pi\sqrt{\pi}}{2}\frac{V}{\Delta_{SAR}}\exp\left[-\frac{\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega_0)^2\right] \qquad (7)$$

As for the second term of the sum (4), it is written as

$$\langle I_2^2\rangle = \iint d\omega'\,d\omega''\,\langle \xi(\omega')\xi(\omega'')\rangle \exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega'-\omega_0)^2\right]\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega')^2\right]\cdot$$

$$\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega''-\omega_0)^2\right]\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega-\omega'')^2\right] \qquad (8)$$

We introduce a correlation function $R_\xi = \langle \xi(\omega')\xi(\omega'')\rangle = \sigma_\xi^2 r_\xi(\omega'-\omega'')$; the function $r_\xi(\omega'-\omega'')$ is shown in Fig.1. It is quite obvious that the scale Δ_ξ is extremely small compared to the width of the original spectrum, i.e.

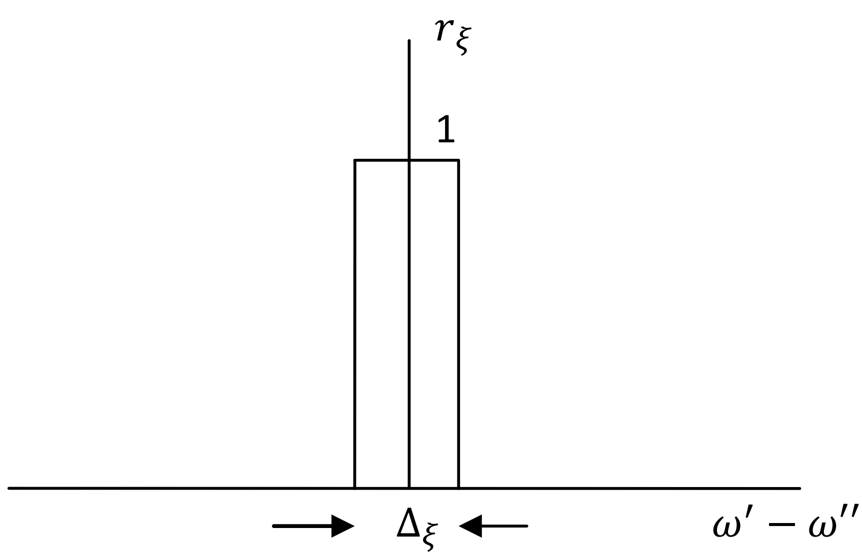

Figure 1

$$\Delta_\xi \ll \frac{V}{\Delta_{SAR}} \quad (9)$$

Taking this circumstance into account, the double integral (8) is taken, and we obtain

$$\langle I_2^2 \rangle = \sigma_\xi^2 \Delta_\xi \frac{\pi\sqrt{\pi}}{2\sqrt{2}} \frac{V}{\Delta_{SAR}} \exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega - \omega_0)^2\right] \quad (10)$$

Next, after performing elementary calculations, we arrive at the formula

$$\langle G_{new}(\omega) \rangle \propto \left[\pi \left(\frac{\pi}{2}\right)^{1/2} \frac{1}{\Delta_\xi} \frac{V}{\Delta_{SAR}} + \sigma_\xi^2\right] \exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega - \omega_0)^2\right] \quad (11)$$

It is clear that the shape factors of the average spectrum and its average estimate coincide.

The degree of smoothing of the original spectrum can be estimated as follows.

$$G_{new}(\omega) \propto (I_1 + I_2)^2 = I_1^2 + 2I_1 I_2 + I_2^2 \quad (12)$$

$$G_{new}^2 \propto I_1^4 + 4I_1^2 I_2^2 + I_2^4 + 4I_1^3 I_2 + 2I_1^2 I_2^2 + 4I_1 I_2^3 \quad (13)$$

Let us perform averaging (12) and (13), considering the odd moments of the random variable ξ equal to zero:

$$\langle G_{new}^2(\omega) \rangle \propto I_1^4 + 6I_1^2 \langle I_2^2 \rangle + \langle I_2^4 \rangle \quad (14)$$

$$\langle G_{new}(\omega) \rangle^2 \propto I_1^4 + 2I_1^2 \langle I_2^2 \rangle + \langle I_2^2 \rangle^2 \quad (15)$$

$$\langle G_{new}^2(\omega) \rangle - \langle G_{new}(\omega) \rangle^2 \propto 4I_1^2 \langle I_2^2 \rangle + \langle I_2^4 \rangle - \langle I_2^2 \rangle^2 \quad (16)$$

On the right-hand side of (16) we neglect the terms of order σ_ξ^4:

$$\frac{\langle G_{new}^2(\omega) \rangle - \langle G_{new}(\omega) \rangle^2}{\langle G_{new}(\omega) \rangle^2} \cong \frac{4I_1^2 \langle I_2^2 \rangle}{I_1^4} = \frac{4\langle I_2^2 \rangle}{I_1^2} \quad (17)$$

Taking in account (7) and (10) we obtain:

$$\frac{\langle G_{new}^2(\omega) \rangle - \langle G_{new}(\omega) \rangle^2}{\langle G_{new}(\omega) \rangle^2} = \frac{4\sqrt{2}}{\pi\sqrt{\pi}} \Delta_\xi \frac{\Delta_{SAR}}{V} \sigma_\xi^2 \cong \Delta_\xi \frac{\Delta_{SAR}}{V} \sigma_\xi^2 \quad (18)$$

At the same time for the initial spectrum (3) we have:

$$\frac{\langle G_s^2(\omega) \rangle - \langle G_s(\omega) \rangle^2}{\langle G_s(\omega) \rangle^2} = \sigma_\xi^2 \quad (19)$$

The coefficient in front of σ_ξ^2 on the right-hand side of (18) is the ratio of the characteristic scale of fluctuations to the width of the initial spectrum, which according to (9) we consider extremely small. From this we conclude that as a result of processing using the proposed method, fluctuations practically disappear.

Note that the proposed method for processing a Gaussian Doppler spectrum is applicable to any system; it is only necessary to specify the width of the average spectrum. In particular, Figs.2 and 3 show the results of processing the Doppler spectrum of the hydroacoustic system [3]. The original spectra are shown in black, and the processed ones in red. Present in the pictures, the vertical shift of the red curves relative to the black ones is explained by the fact that both of them are placed on the same graph without preliminary normalization. But the main result of the processing is obvious – the red curves turned out absolutely smooth.

We believe that everything presented above provides a compelling justification for conducting a full test of the proposed Doppler spectrum processing method.

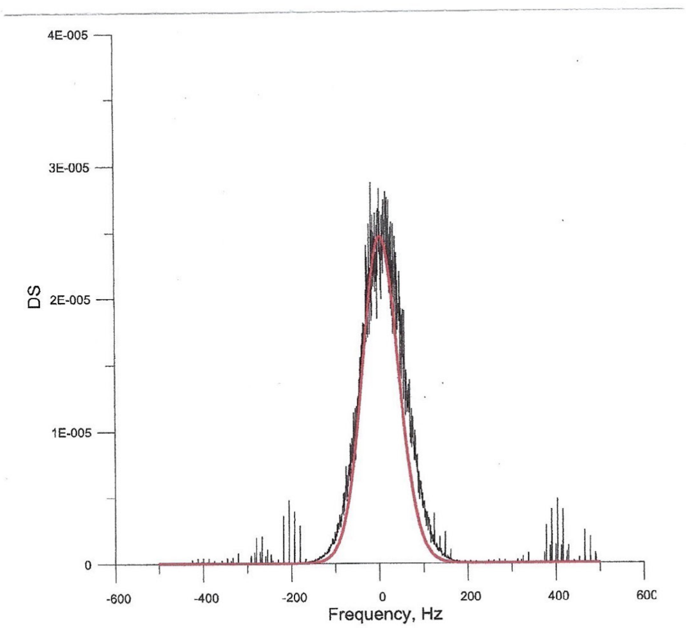

Figure 2

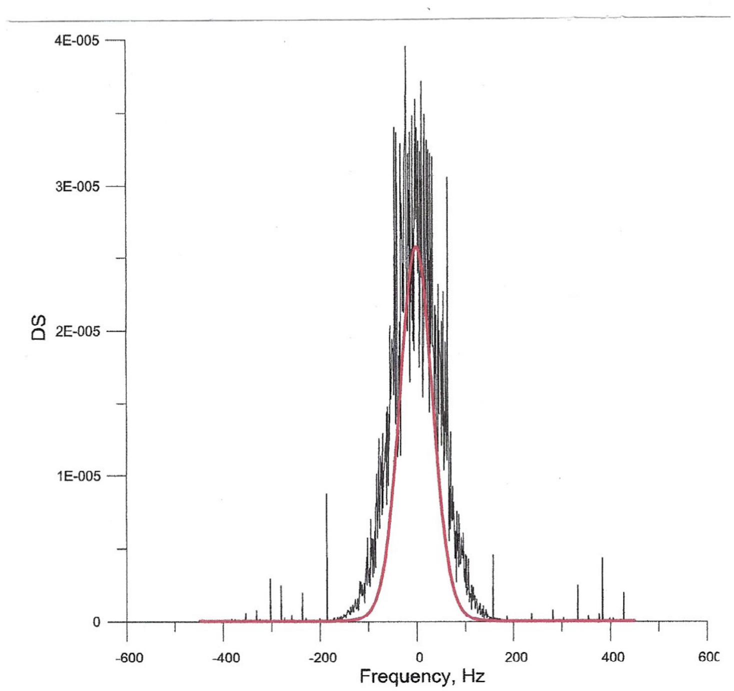

Figure 3

References

1. Kanevsky, M. B. (2025). A new estimate for Doppler spectrum when operating space-based SAR over the ocean. *J Mari Sci Res Ocean, 8*(2), 01-02.
2. Kanevsky, M.B. (2024). On the Doppler spectrum for the signal of a spaceborne SAR operated over the ocean. *J Mari Sci Res Ocean, 7*(2), 01-05.
3. Private communication.