Review Article

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# The Use of Fractional Order in Solving Avian Influenza Epidemic Model with Quarantine and Vaccination

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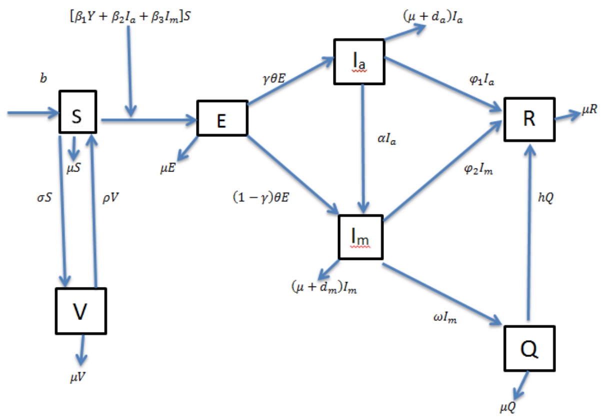
## Abstract
*In this paper, a fractional order avian influenza epidemic model with quarantine and vaccination class is formulated and numerically solved using the Laplace Pade Differential Transform Method (LPDTM).*

## Introduction
Avian influenza A viruses are very contagious among birds and some of these viruses can sicken and even kill certain domesticated bird's species including chickens, duck and turkeys (FAO, [10]; Causey and Edwards [5]. Avian influenza (AI) is an important disease of zoonotic origin that has caused morbidity and mortality in domestic animals, wildlife and humans [25, 28, 11]. It is known to be caused by type A viruses of the family Orthomyxoviridae, and they are classified by their hemagglutinin (HA) and neuraminidase (NA) surface glycoproteins. Highly Pathogenic Avian Influenza (HPAI) has caused respiratory disease and deaths in poultry and poultry handlers that were inappropriately exposed to aerosols generated from handling chickens [27]. In recent times, several studies carried out in the area of mathematical modelling of physical phenomena involving nonlinear dynamics has continue to gain popularity success by applying fractional calculus in modelling which include reduction of errors from neglected parameters [30], avoid large computational work [3].

## Literature Review
Many mathematical models have been proposed or developed on Avian influenza (Bird Flu), with the aim of getting efficient inventive curative and the best strategies to control or curtal bird flu.[26] analyzed a model to examine the role of hospital and commonly control measures, artificial drugs are vaccination in combating a potential flu pandemic in a population. [12] incorporated the dynamics of both wild and domestic birds and the isolated of individuals with symptoms of both the avian and mutant straps.[16] presented a dynamic behavior of the avian-human. influenza epidemic model by using efficient computational algorithm, namely the Multistage Differential Transform method (MSDTM)Also [17] applied Homotopy Analysis method (HAM) and expanded it to Hybrid Numeric Analysis Method known as Multislage HAM (MSHAM) in solving avian-human influenza epidemic model. [22] Proposed a Mathematical model on avian influenza with quarantine and vaccination. [19] constructed two avian influenza birds-to- human transmission models with different growth laws of the avian population, one with Logistive growth and the other with avian effect, and analyzed their dynamicbehaviors.

## Model Formulation
The fractional- order Avian Influenza epidemic model is obtained by extending the work of [17] and [22]. The parameters α,ρ and h used by [22] were cooperated. The basic assumptions and flow diagram are shown respectively below:

### Basic Assumption for Model
1. Individuals are only recruited into susceptible sub-population
2. The number of susceptible for the bird population is increased by newrecruitment but reduced through natural death and infection.
3. The avian influenza virus is not contagious from an infective human toa susceptible human. It is only contagious from an infective avian to a susceptible human
4. An infected avian keeps in the state of disease and cannot recover butan infected human can recover the recovered human has permanent immunity.
5. Susceptible individuals are vaccinated.



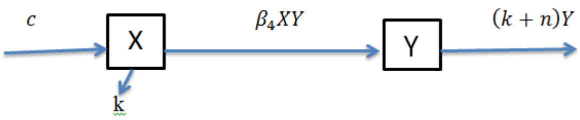



**Figure 3.2:** Flow Diagram of Bird population

$$
\begin{aligned}
\frac{dS}{dt} &= b + \rho V(t) - [\beta_1 Y(t) + \beta_2 I_a(t) + \beta_3 I_m(t)]S(t) + (\mu+\sigma)S(t)] \\
\frac{dE}{dt} &= [\beta_1 Y(t) + \beta_2 I_a(t) + \beta_3 I_m(t)]S(t) - (\mu+\theta)E(t) \\
\frac{dI_a}{dt} &= \gamma\theta E(t) - (\alpha + \mu + d_a + \varphi_1)I_a(t) \\
\frac{dI_m}{dt} &= (1-\gamma)\theta E(t) + \alpha I_a(t) - (\alpha + \mu + d_m + \varphi_2 + \omega)I_m(t) \\
\frac{dR}{dt} &= \varphi_1 I_a(t) + \varphi_2 I_m(t) + hQ(t) - \mu R(t) \\
\frac{dQ}{dt} &= \omega I_m(t) - (h+\mu)Q(t) \\
\frac{dV}{dt} &= \sigma S(t) - (\rho+\mu)V(t) \\
\frac{dX}{dt} &= c - \beta_4 X(t)Y(t) - kX(t) \\
\frac{dY}{dt} &= \beta_4 X(t)Y(t) - (k+n)Y(t)
\end{aligned} \tag{3.1}
$$

**Table 3.1: Variable Description**

| VARIABLE | DESCRIPTION |
|---|---|
| $S$ | Susceptible Individuals |
| $E$ | Exposed Individuals |
| $I_a$ | Infected Individuals with avian strain |
| $I_m$ | Infected Individuals with mutant strain |
| $R$ | Recovered Individuals |
| $V$ | Vaccinated Individuals |
| $Q$ | Quarantine Individuals |
| $X$ | Susceptible Birds |
| $Y$ | Infected Birds |

Initial conditions is S(0) = 21, E(0) = 12, Ia(0) = 5, Im(0) = 2, R(0) = 10, V (0) = 15, X(0) = 990, Y (0) = 10, Q(0) = 4

**Table 3.2: Parameters Description**

| PARAMETER | DESCRIPTION | VALUE | REFERENCE |
|---|---|---|---|
| $b$ | Recruitment rate for human | 30 | Jabbari et al. [17] |
| $c$ | Bird inflow | 1000 | Assumed |
| $\beta_1$ | Rate at which bird to human avian influenza is contacted | 0.2 | Jabbari et al. [17] |
| $\beta_2$ | Rate at which human-human avian influenza is contracted | 0.4 | Jabbari et al. [17] |
| $\beta_3$ | Rate at which human-human mutant influenza is contracted | 0.0015 | Jabbari et al. [17] |
| $\beta_4$ | Rate at which bird contract avian influenza | 0.001 | Jabbari et al. [17] |
| $\mu$ | Natural death rate for human | $3.91 \times 10^{-5}$ | Liu et al. [19] |
| $\alpha$ | Mutation rate | 0.01 | Gumel [12] |
| $d_a$ | Deduced death rate due to avian strain in human | 1 | Iwami et al. [13] |
| $d_m$ | Induced death rate due to mutant strain in human | 0.06 | Jabari et al. [18] |
| $k$ | Natural death rate for birds | $3.4246 \times 10^{-4}$ | Liu et al. [19] |
| $n$ | Induced death rate due to avian strain in birds | 5 | Iwami et al. [13] |
| PARAMETER | DESCRIPTION | VALUE | REFERENCE |
| $\phi_1$ | Recovery rate of human with avian strain | 0.2669 | Lucche et al. [20] |
| $\phi_2$ | Recovery rate of human with mutant strain | 0.05 | Gumel [12] |
| $\theta$ | Disease progression rate of human from exposed to infectious | 0.8 | Assumed |
| $\gamma$ | Proportion of human that are infected with avian strain | 0.3 | Assumed |
| $\sigma$ | Rate of transmission from susceptible to vaccinated humans | 0.1 | Mishra et al. [22] |
| $\omega$ | Rate of transmission from infected individual with mutant strain strain to quarantined individual | 0.08 | Mishra et al. [22] |
| $h$ | Rate of transmission from quarantined to recovered human | 0.6 | Mishra et al. [22] |
| $\rho$ | Rate of transmission from vaccinated to susceptible humans | 0.02 | Mishra et al. [22] |

Now equation (3.1) is written in terms of time dependent integrations as follows

$$\begin{aligned}
\frac{dS}{dt} &= \int_0^t f(t-g)[b+\rho V(g)-[\beta_1 Y(g)+\beta_2 I_a(g)+\beta_3 I_m(g)]S(g)+(\mu+\sigma)S(g)\\
\frac{dE}{dt} &= \int_0^t f(t-g)[[\beta_1 Y(g)+\beta_2 I_a(g)+\beta_3 I_m(g)]S(g)-(\mu+\theta)E(g)]dg\\
\frac{dI_a}{dt} &= \int_0^t f(t-g)[\gamma\theta E(g)-(\alpha+\mu+d_a+\varphi_1)I_a(g)]dg\\
\frac{dI_m}{dt} &= \int_0^t f(t-g)[(1-\gamma)\theta E(g)+\alpha I_a(g)-(\alpha+\mu+d_m+\varphi_2+\omega)I_m(g)]dg\\
\frac{dR}{dt} &= \int_0^t f(t-g)[\varphi_1 I_a(g)+\varphi_2 I_m(g)+hQ(g)-\mu R(g)]dg\\
\frac{dQ}{dt} &= \int_0^t f(t-g)[\omega I_m(g)-(h+\mu)Q(g)]dg\\
\frac{dV}{dt} &= \int_0^t f(t-g)[\sigma S(g)-(\rho+\mu)V(g)]dg\\
\frac{dX}{dt} &= \int_0^t f(t-g)[c-\beta_4 X(g)Y(g)-kX(g)]dg\\
\frac{dY}{dt} &= \int_0^t f(t-g)[\beta_4 X(g)Y(g)-(k+n)Y(g)]dg
\end{aligned} \tag{3.2}$$

Where $f(t-g)$ is a kernel with respect to time. The power-law kernel is define by

$$f(t-g)=\frac{1}{\Gamma(p-1)}(t-g)^{p-2} \tag{3.3}$$

Substituting (3.3) into (3.2) to have

$$\begin{aligned}
\frac{dS}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[b+\rho V(g)-[\beta_1 Y(g)+\beta_2 I_a(g)+\beta_3 I_m(g)]S(g)-(\mu+\sigma)S(g)]dg\\
\frac{dE}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[[\beta_1 Y(g)+\beta_2 I_a(g)+\beta_3 I_m(g)]S(g)-(\mu+\theta)E(g)]dg\\
\frac{dI_a}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[\gamma\theta E(g)-(\alpha+\mu+d_a+\varphi_1)I_a(g)]dg\\
\frac{dI_m}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[(1-\gamma)\theta E(g)+\alpha I_a(g)-(\alpha+\mu+d_m+\varphi_2+\omega)I_m(g)]dg\\
\frac{dR}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[\varphi_1 I_a(g)+\varphi_2 I_m(g)+hQ(g)-\mu R(g)]dg\\
\frac{dQ}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[\omega I_m(g)-(h+\mu)Q(g)]dg\\
\frac{dV}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[\sigma S(g)-(\rho+\mu)V(g)]dg\\
\frac{dX}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[c-\beta_4 X(g)Y(g)-kX(g)]dg\\
\frac{dY}{dt} &= \frac{1}{\Gamma(p-1)}\int_0^t (t-g)^{p-2}[\beta_4 X(g)Y(g)-(k+n)Y(g)]dg
\end{aligned} \tag{3.4}$$

By definition of fractional integral, system (3.4) becomes

$$\frac{dS}{dt} = J^{p-1}[b + \rho V(g) - [\beta_1 Y(g) + \beta_2 I_a(g) + \beta_3 I_m(g)]S(g) - (\mu + \sigma)S(g)]$$
$$\frac{dE}{dt} = J^{p-1}[[\beta_1 Y(g) + \beta_2 I_a(g) + \beta_3 I_m(g)]S(g) - (\mu + \theta)E(g)]$$
$$\frac{dI_a}{dt} = J^{p-1}[\gamma\theta E(g) - (\alpha + \mu + d_a + \varphi_1)I_a(g)]$$
$$\frac{dI_m}{dt} = J^{p-1}[(1-\gamma)\theta E(g) + \alpha I_a(g) - (\alpha + \mu + d_m + \varphi_2 + \omega)I_m(g)]$$
$$\frac{dR}{dt} = J^{p-1}[\varphi_1 I_a(g) + \varphi_2 I_m(g) + hQ(g) - \mu R(g)]$$
$$\frac{dQ}{dt} = J^{p-1}[\omega I_m(g) - (h + \mu)Q(g)] \tag{3.5}$$
$$\frac{dV}{dt} = J^{p-1}[\sigma S(g) - (\rho + \mu)V(g)]$$
$$\frac{dX}{dt} = J^{p-1}[c - \beta_4 X(g)Y(g) - kX(g)]$$
$$\frac{dY}{dt} = J^{p-1}[\beta_4 X(g)Y(g) - (k + n)Y(g)]$$

By applying fractional derivative of order $p - 1$ $(J^1 - p)$ on both sides of preceding system, we have

$$D^p_* S(t) = [b + \rho V(t) - [\beta_1 Y(t) + \beta_2 I_a(t) + \beta_3 I_m(t)]S(t) - (\mu + \sigma)S(t)]$$
$$D^p_* E(t) = [[\beta_1 Y(t) + \beta_2 I_a(t) + \beta_3 I_m(t)]S(t) - (\mu + \theta)E(t)]$$
$$D^p_* I_a(t) = [\gamma\theta E(t) - (\alpha + \mu + d_a + \varphi_1)I_a(t)]$$
$$D^p_* I_m(t) = [(1-\gamma)\theta E(t) + \alpha I_a(t) - (\alpha + \mu + d_m + \varphi_2 + \omega)I_m(t)]$$
$$D^p_* R(t) = [\varphi_1 I_a(t) + \varphi_2 I_m(t) + hQ(t) - \mu R(t)]$$
$$D^p_* Q(t) = [\omega I_m(t) - (h + \mu)Q(t)] \tag{3.6}$$
$$D^p_* V(t) = [\sigma S(t) - (\rho + \mu)V(t)]$$
$$D^p_* X(t) = [c - \beta_4 X(t)Y(t) - kX(t)]$$
$$D^p_* Y(t) = [\beta_4 X(t)Y(t) - (k + n)Y(t)]$$

Thus, a fractional order mathematical model that describes the avian influenza with the vaccination and quarantine class is proposed by system (3.6).

## The Concept and Application of Laplace Differential Transform Method (LPDTM)

Several approximate methods provide power series solutions (polynomials). Nevertheless, sometimes, this type of solutions lack large domains of convergence. Therefore, Laplace-Pad`er-essumation method [32, 31], [24] is used in literature to enlarge the domain of convergence of solutions or inclusive to find the exact solutions. Hence, Pad`eapproximants are extensively used to overcome these shortcomings

The Pad`eapproximation of a function *f(t)* of order [m/n] is defined by [23].

$$[m/n] = \frac{a_0 + a_1 t + \cdots + a_m t^m}{1 + b_1 t + \cdots + b_n t^n} \tag{4.1}$$

When we consider $b_0 = 1$, the numerator and denominator have no common factors. It is important to note that the Pad`eapproximant can be obtained through the in-built utility of symbolic computational software such as maple, matlap etc. Thus, the procedure of the Laplace Differential Transformation method to the given system of functional order differential equation is summarized as follows:

1. Apply the differential transformation method to the given system offunctional order differential equation
2. Perform several desirable numbers of iterations (i.e. of K

times) and get the solution in power series form. e.g. For susceptible class we have

$$s_1(G)=\sum_{z=0}^{K}S(z)G^z$$

3. Take the Laplace transform of the power series.
4. Next, s is substituted with $1/G$ in the resulting equation
5. After that, the transformed series is converted into mormorphic function by forming its Pad`eapproximant of order *[m/n]*. *N* and *M* are arbitrary chosen but they should be smaller than order of the power series.

In this step, the Pad`eapproximant extends the domain of the truncated series solution to obtain better accuracy and convergence
6. *G* is substituted by 1/s
7. The inverse Laplace transformation is obtained in term of *G*.
8. Finally, *G* is substituted with $t^p$ to obtain the exact or approximate solution in terms of *t* Using the definition of differential transformation method, equation (3.1) can be re-written thus

$$\begin{aligned}
S(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}\Big[\beta\delta(K,0)-\sum_{i=0}^{K}S(l)\Big[\beta_1Y(K-l)+\beta_2I_a(K-l)\\
&\quad+\beta_3I_m(K-l)\Big]-(\mu+\sigma)S(K)+\rho V(K)\Big]\\
E(K+1))&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}\Big[\sum_{i=0}^{K}S(l)\Big(\beta_1Y(K-l)+\beta_2I_a(K-l)+\beta_3I_m(K-l)\Big)-k_1E(K)\Big]\\
I_a(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}[\gamma\theta E(K)-k_2I_a(K)]\\
I_m(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}[(1-\gamma)\theta E(K)+\alpha I_a(K)-k_3I_m(K)]\\
R(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}\Big[\phi_1I_a(K)+\phi_2I_m(K)+hQ(K)-\mu R(K)\Big]\\
Q(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}[wI_m(K)-k_4Q(K)]\\
V(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}[\sigma S(K)-(\rho+\mu)V(K)]\\
X(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}\Big[c\delta(K,0)-\beta_4\sum_{i=0}^{K}\Big(X(l)Y(K-l)\Big)-kX(K)\Big]\\
Y(K+1)&=\frac{\Gamma(pK+1)}{\Gamma(p(K+1)+1)}\Big[\beta_4\sum_{i=0}^{K}\Big(X(l)Y(K-l)\Big)-k_5Y(K)\Big]
\end{aligned}\tag{4.2}$$

where $k_1=\mu+\theta$, $k_2=\alpha+\mu+d_a+\varphi_1$, $k_3=\varphi_2+w+\mu+d_m$, $k_4=h+\mu$, $k_5=k+n$
using the initial conditions $S(0)=21$, $E(0)=12$, $I_a(0)=5$, $I_m(0)=2$, $R(0)=10$, $V(0)=15$, $Q(0)=4$, $X(0)=990$, $Y(0)=10$ and the parameter table 2.2 with the help of Maple 18, we obtain the series solution below for each compartment.

$$\begin{aligned}
s_1(G)&\cong\sum_{z=0}^{15}S(z)G^z=21-55.8638211000000\,G+213.459041751002\,G^2\\
&\quad-618.248683844327\,G^3+\cdots-6541456.46965407\,G^{15}\\
&\quad+\cdots-6.9534471768499\times10^6t^{15}\\
e(G)&\cong\sum_{z=0}^{81}E(z)G^z=12+74.4625308000000\,G-240.433232485777\,G^2\\
&\quad+675.230521209997\,G^3-\cdots
\end{aligned}$$

$$i_a(G) \cong \sum_{z=0}^{81} I_a(z)G^z = 5 - 3.50469550000000\,G + 11.1731450547720\,G^2 - 23.9904671956655\,G^3 + \cdots$$

$$i_m(G) \cong \sum_{z=0}^{81} I_m(z)G^z = 2 + 6.44992180000000\,G + 20.3158653065288\,G^2 - 45.9274038469550\,G^3 + \cdots$$

$$q(G) \cong \sum_{z=0}^{81} Q(z)G^z = 4 - 2.30015640000000\,G + 0.851339933057620\,G^2 + 0.168318672700162\,G^3 - 0.599341994306988\,G^4 + \cdots$$

$$r(G) \cong \sum_{z=0}^{81} R(z)G^z = 10 + 3.83410900000000\,G - 0.996575446305950\,G^2 + 1.50291620212654\,G^3 - \cdots$$

$$x(G) \cong \sum_{z=0}^{15} X(z)G^z = 990 + 989.760964600000\,G + 14.7329135840316\,G^2 - 14.9896593428025\,G^3 + 10.5693573550055\,G^4 - \cdots$$

$$y(G) \cong \sum_{z=0}^{81} Y(z)G^z = 10 - 40.1034246000000\,G + 85.3630380553945\,G^2 - 127.293497036055\,G^3 + 148.559695512497\,G^4 - \cdots \quad (4.3)$$

$$v(G) \cong \sum_{z=0}^{15} V(z)G^z = 15 + 1.79941350000000\,G - 2.81122036853392\,G^2 + 7.13407950039577\,G^3 - 15.4919572292373\,G^4 - \cdots$$

Applying step 3 to step 8 of the procedure of the Laplace Differential Transformation method to equation (4.3) and when $p = 1$ gives the approximate solution to the fractional order A.I. model presented in (3.6).

$s1(t) = 0.0601435920624958e{-}19.4172245865354t + 1.39956351085885e{-}11.4494665963008t + 8.03001432484090e{-}5.24160405850728t + \cdots$

$e(t) = -0.000450408960911004e{-}30.1796295359648t - 0.0831823106732497e{-}18.7817878007770t - 1.76602629646116e{-}10.9490321566713t - \cdots$

$ia(t) = 0.000148671467499998e{-}23.2346530924505t + 0.0175963022338194e{-}13.1717923124120t + 0.236215399401915e{-}6.65060451745231t + \cdots$

$im(t) = 0.000441167846211876e{-}22.6997518392598t + 0.0516231239419094e{-}12.5558106157352t + 1.00931690441547e{-}5.48260357632302t + \cdots$

$r(t) = -0.00000179851876388121e{-}23.8309721649748t - 0.000515583955933068e{-}13.1932638529236t - 0.0264768480657769e{-}5.91257334781731t \cdots$

$x(t) = 3.11428189434324 - 0.0888939449779140e^{-6.19844613454376t} + 1.76910626064991e{-}3.72065472589920t + \cdots$

$y(t) = 0.0000218973510397729e{-}11.1597063532236t + 1.13489272836264e{-}5.87648270221130t + 5.38912836978742e{-}4.37160992601893t + \cdots$

$v(t) = -0.0000379845977890386e{-}28.3677493227504t - 0.000886216212473279e{-}17.3221558465245t - 0.0303647659826797e{-}9.51067763148441t - \cdots$

$q(t) = -0.000000178972575797361e{-}25.7041970056611t - 0.0000473250827509065e{-}15.2573101765748t - 0.00172192178447173e{-}8.29117429793647t - \cdots$

The same is repeated for $p = 0.2, 0.4, 0.6$ and $0.8$ respectively

## Numerical Simulations

In this section, maple 18 is used to carry out simulations for the fractional order Avian influenza epidemic model with quarantine and vaccine using the following initial conditions $S(0) = 21$, $E(0) = 12$, $I_a(0) = 5$, $I_m(0) = 2$, $R(0) = 10$, $V(0) = 15$, $Q(0) = 4$, $X(0) = 990$, $Y(0) = 10$

Figure 5.1-5.9 display that the results obtained by LPDTM are in excellent agreement with those of Runge-Kutta method and provides correctly the dynamics of the formulated A. I. model

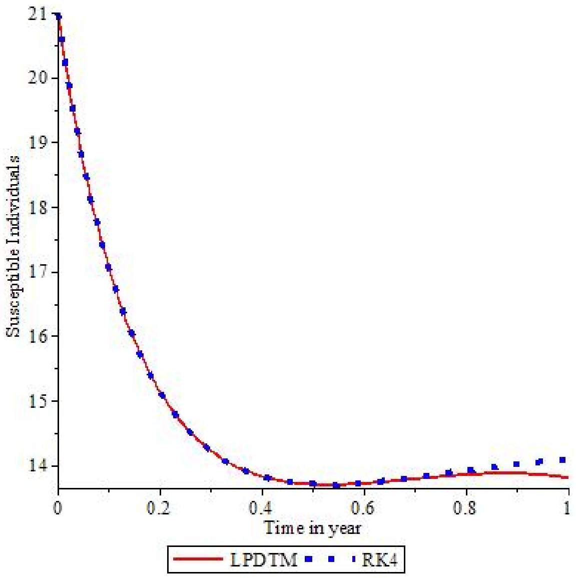



**Figure 5.1:** Graphical Comparison of *S(t)*

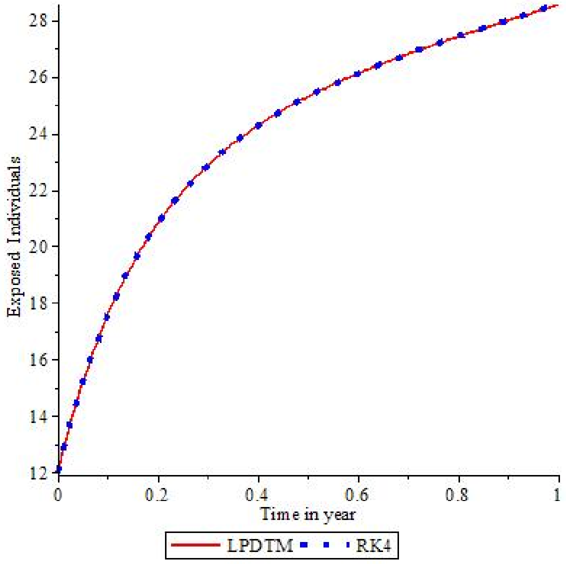



**Figure 5.2:** Graphical Comparison of *E(t)*

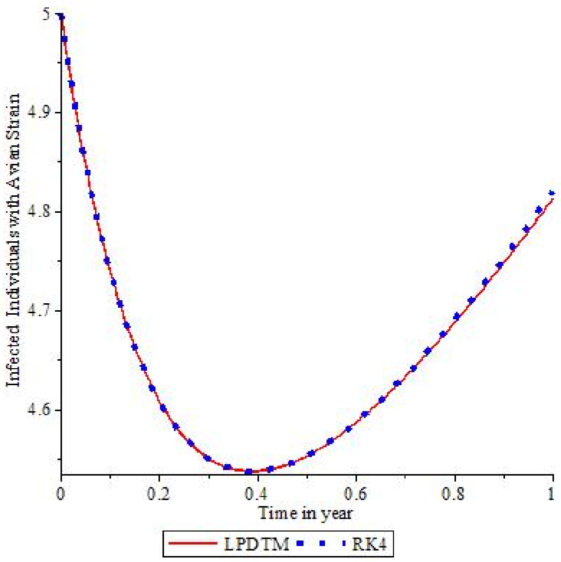



**Figure 5.3:** Graphical Comparison of $I_a(t)$

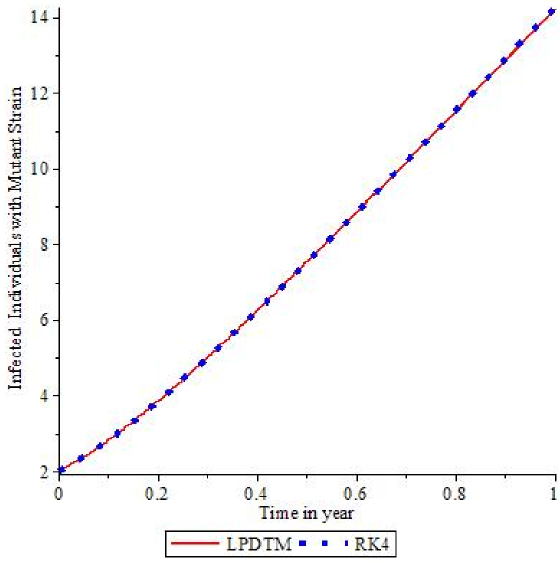



**Figure 5.4:** Graphical Comparison of $I_m(t)$

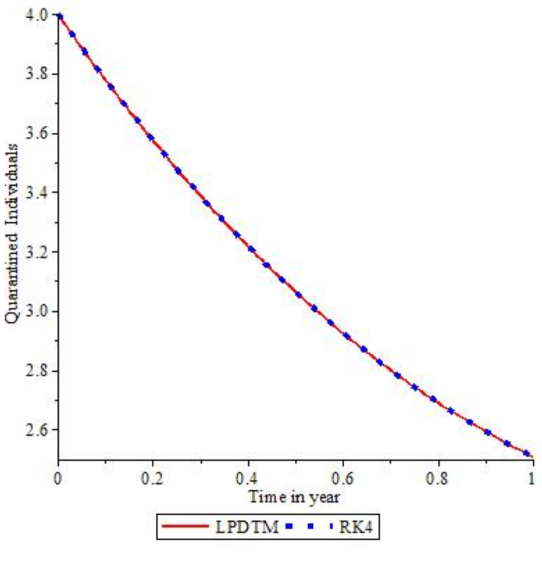



**Figure 5.5:** Graphical Comparison of *Q(t)*

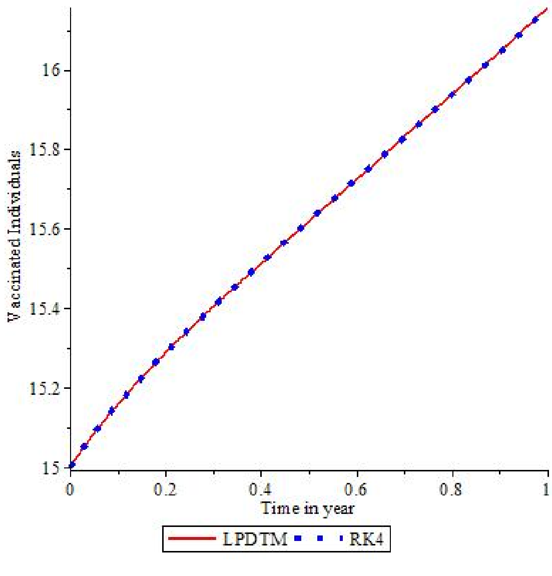



**Figure 5.6:** Graphical Comparison of *V (t)*

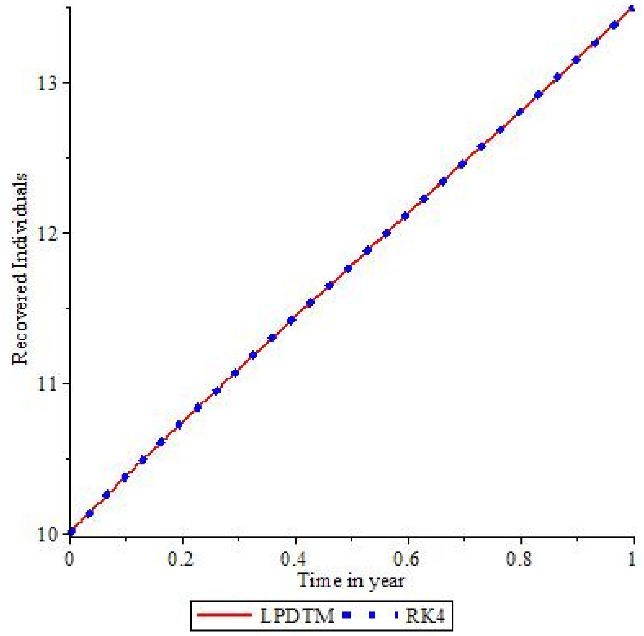



**Figure 5.7:** Graphical Comparison of *R(t)*

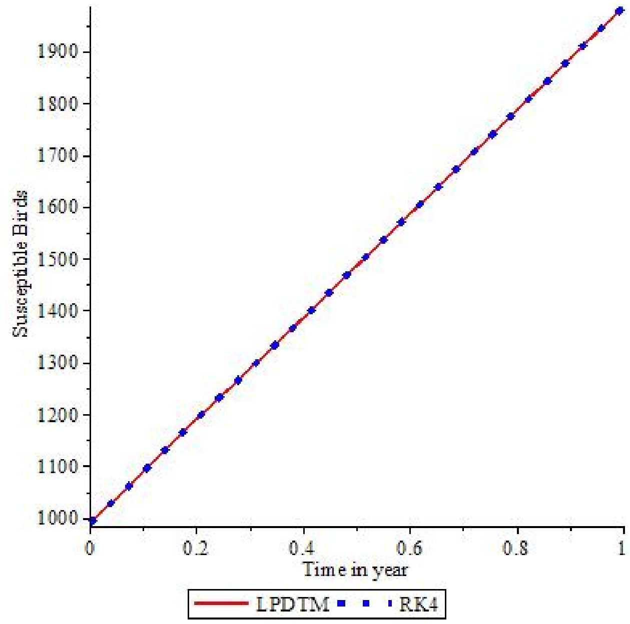



**Figure 5.8:** Graphical Comparison of *X(t)*

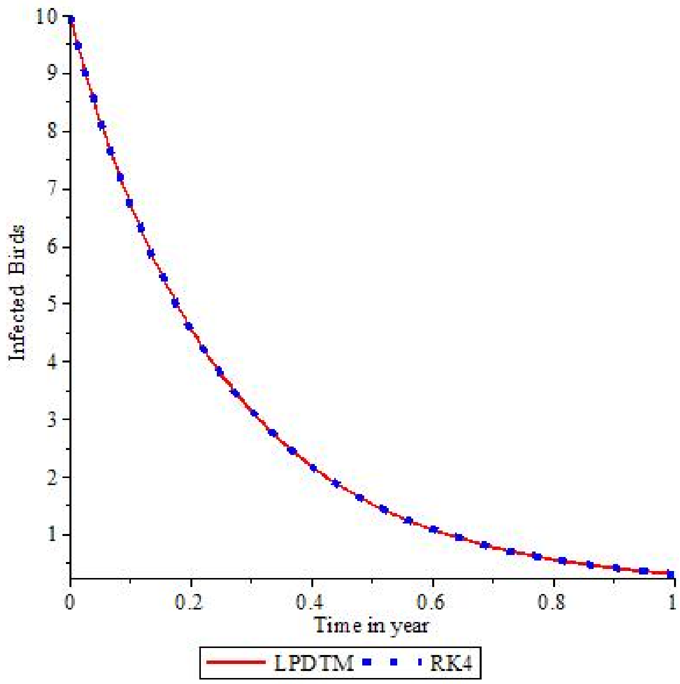



**Figure 5.9:** Graphical Comparison of *Y (t)*

Figure 5.10-5.18 display the result obtained by LPDTM for different values of p. This shows that the dynamics of A.I depends on the value of p. Since small changes in the value of p, greatly influences the population curve of each compartment.

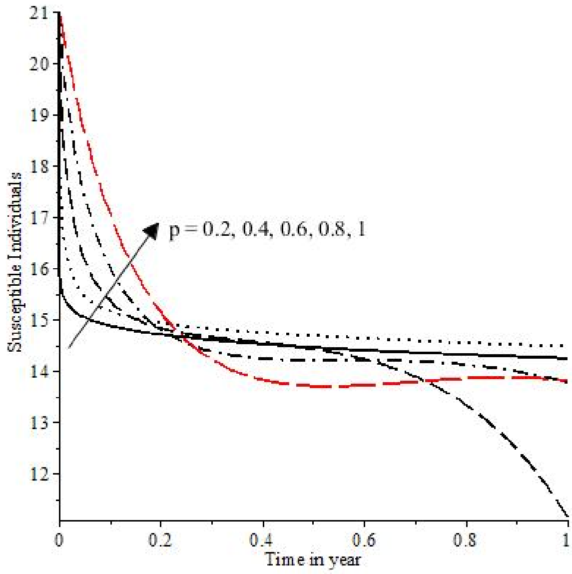



**Figure 5.10:** Graphical Comparison of different values of $p$ for $S(t)$

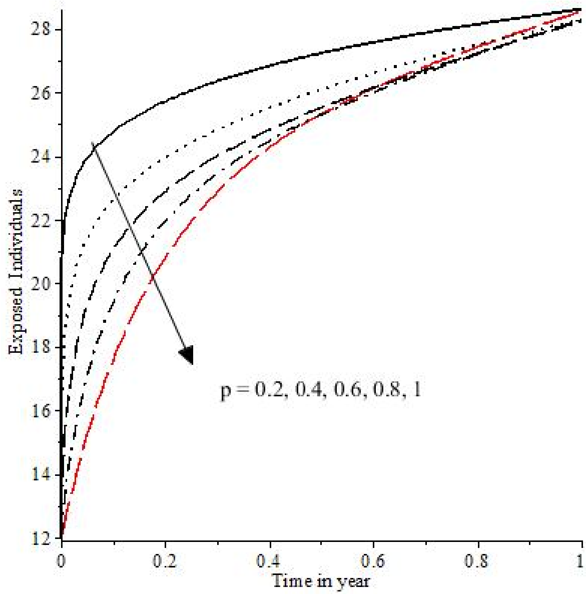



**Figure 5.11:** Graphical Comparison of different values of $p$ for $E(t)$

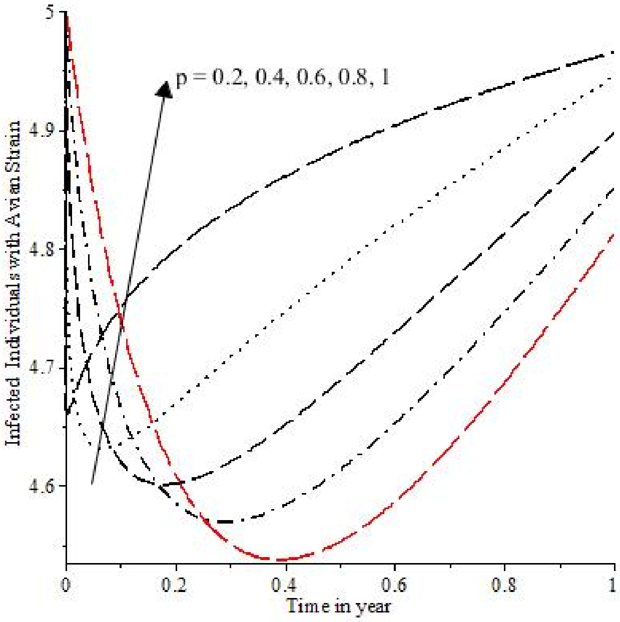



**Figure 5.12:** Graphical Comparison of different values of $p$ for $I_a(t)$

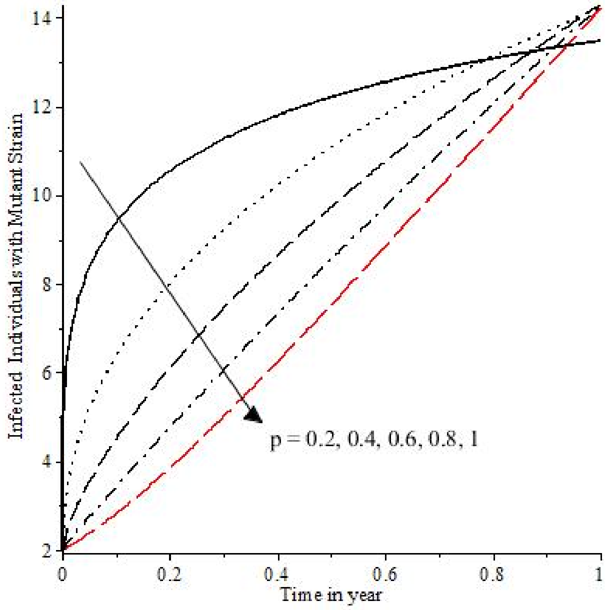



**Figure 5.13:** Graphical Comparison of different values of $p$ for $I_m(t)$

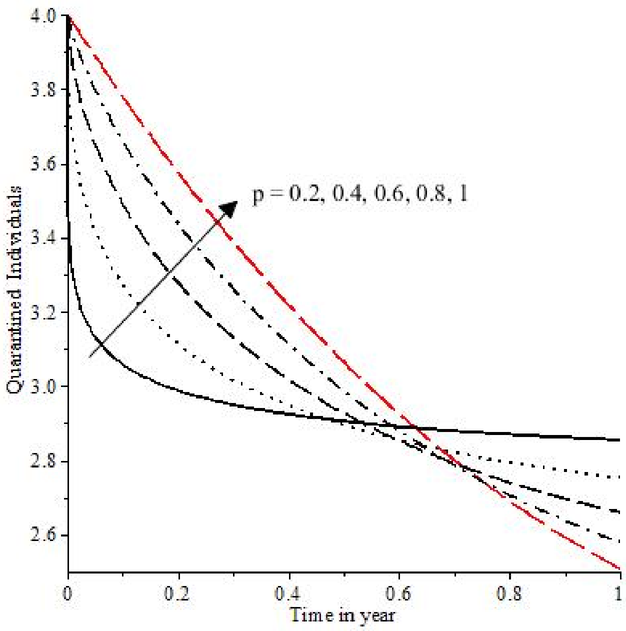



**Figure 5.14:** Graphical Comparison of different values of *p* for *Q(t)*

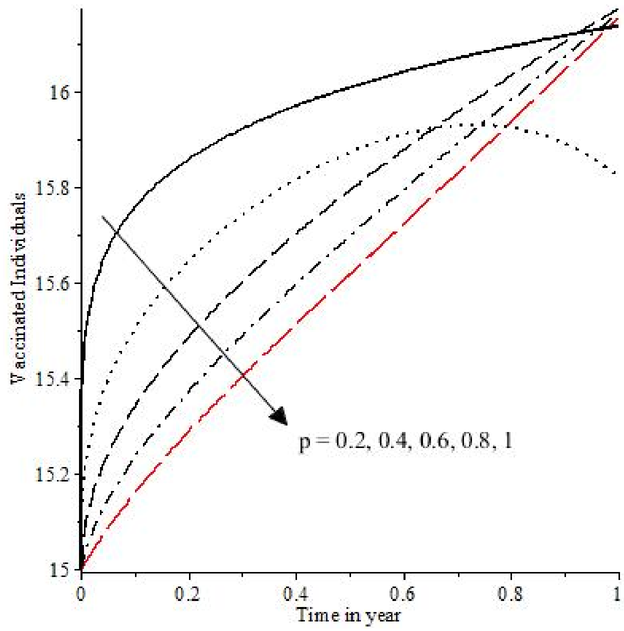



**Figure 5.15:** Graphical Comparison of different values of *p* for *V (t)*

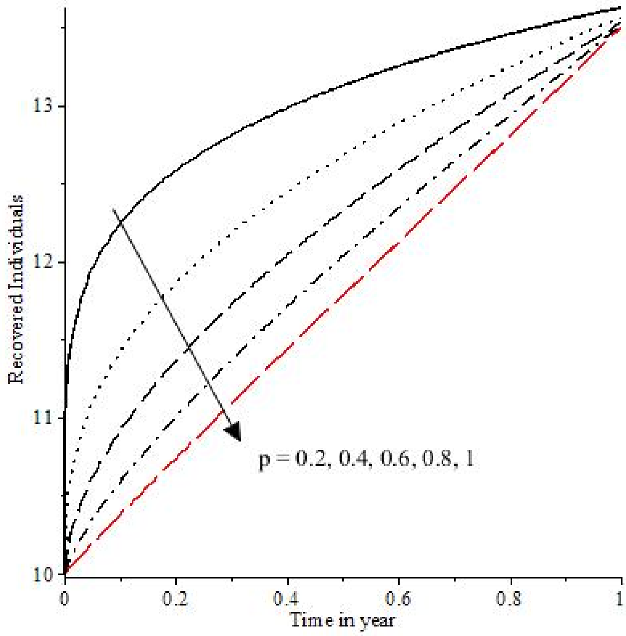



**Figure 5.16:** Graphical Comparison of different values of *p* for *R(t)*

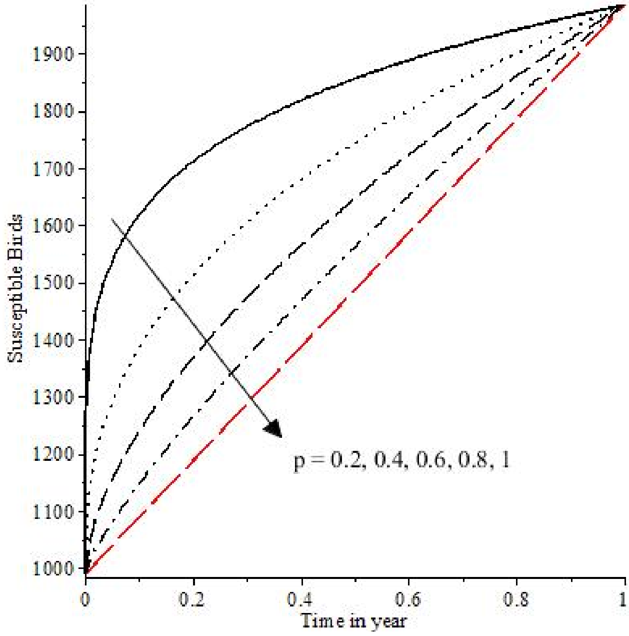



**Figure 5.17:** Graphical Comparison of different values of *p* for *X(t)*

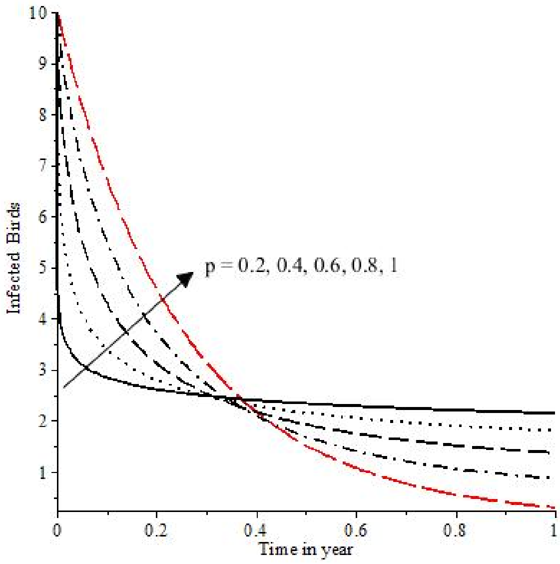



**Figure 5.18:** Graphical Comparison of different values of $p$ for $Y(t)$

## Conclusion

A numerical technique called Laplace Differential Transformation method (LPDTM) is employed to solve a system of fractional order nonlinear differential equation. This system of equations describe the use of vaccination and quarantine as a preventive measures to Avian Influenza. The proposed method can be used to obtain approximate solution and requires no perturbation, linearization or dicretization thus stressing the point that LPDTM should be applied for various nonlinear models.

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