ISSN: 2834-4928

Research Article

Journal of Electrical Electronics Engineering

The Twin Prime Conjecture: An Analytical Approach

Patrick C. DiCarlo*

*Corresponding Author
Patrick C. DiCarlo

Submitted: 2023, Nov 14; **Accepted:** 2023, Dec 20; **Published:** 2024, Jan 08

Citation: DiCarlo, P. C. (2024). The Twin Prime Conjecture: An Analytical Approach. *J Electrical Electron Eng, 3*(1), 01-22.

Abstract

This paper examines the twin prime conjecture. The basic strategy is to first establish that there is no highest prime number by calculating the rates at which the multiples of each successive prime preclude higher numbers from being prime, and then proving that this rate (in the aggregate) can never reach 100%. The same basic methodology is then used to show that there can also be no highest twin prime.

I. No Highest Prime Number

A. Calculating the New Elimination Rate of each prime number

By the definition of a prime number, we know that if a number is not prime, then it is divisible by a smaller number, other than 1. Of course, this is the same as saying that if a number is not prime, then it is a multiple of a smaller number, other than 1. All numbers are either prime or the product of primes.

Therefore, we know that if a number is not prime, it must be the multiple of a smaller prime. We can, thus, say that the multiples of prime numbers define the set of non-prime numbers. This, of course, is simply the sieve method for identifying prime numbers. For example, 2 eliminates all of its multiples from being prime because they are, by definition, divisible by 2, and the same is true for the multiples of each prime number. The prime numbers are then defined as the set of numbers that are not multiples of smaller primes.

Knowing the rate at which P's multiples occur, thus, tells us the rate at which P's multiples preclude numbers between P and infinity from being prime. However, the reciprocal of each prime (> 2) does not reveal its rate of new eliminations because of duplication between the multiples of two primes. For example, although every 3rd number is a multiple of 3, half of 3's multiples are numbers that were already eliminated from consideration as potential prime numbers by multiples of 2. To keep an accurate count of the rate at which the multiples of P eliminate higher numbers from being prime, it is necessary to reduce P's reciprocal (1/P) by duplicate multiples between P and each prime < P. For purposes of this paper, the term "New Elimination Rate" means the rate at which a given prime (P) precludes higher numbers from being prime for the first time (1/P minus duplicate eliminations with primes < P), and the term "Nominal Rate" means a given prime's reciprocal (1/P).

To calculate the New Elimination Rate of P, its Nominal Rate must be reduced to adjust for duplicate multiples of P and all smaller primes. The rate at which duplicate multiples between any two primes occur is the product of their reciprocals (1/P1 x P2). So, for example, 2 and 3 share a duplicate multiple every 6 numbers (2 x 3 = 6), and the rate at which duplicate multiples of 2 and 3 occur is the reciprocal of their product (1/6). Therefore, to calculate the rate of new duplicate multiples between 3 and a larger prime, it is necessary to use 3's New Elimination Rate of 1/6 to avoid double counting.

Similarly, every third multiple of 5 and every 5th multiple of 3 are duplicate multiples of 3 and 5. However, half of 3's multiples are also duplicate multiples of 2. So to calculate 5's New Elimination Rate, the first step is to reduce 5's Nominal Rate by 1/2, to adjust for all duplicate multiples between 5 and 2. To adjust for duplicate multiples with 3, however, it is necessary to use 3's New Elimination Rate, rather than its Nominal Rate, to avoid reducing 5's Nominal Rate by duplicate multiples of 2 and 3 that were already accounted for in step one.

The following examples demonstrate the calculation of the New Elimination Rates of 5 and 7:

P = 5

- Two duplicates 1/2 of 5's multiples. Therefore, 5's elimination potential is cut in half by 2 (1/5 x 1/2 = 1/10).

• Three duplicates 1/3 of 5's multiples. However, half of 3's multiples duplicate multiples of 2. So 3 newly reduces only 1/6 (1/3 x 1/2 = 1/6) of 5's eliminations Three, thus, further reduces the 1/10 established in step one by 1/30 (1/5 x 1/6 = 1/30).
• 5's total New Elimination Rate is 1/15 (1/10 – 1/30 = 1/15). In other words, every 15th number is a multiple of 5 that is not also a multiple of 2 or 3.

P = 7

As a further example (using decimals), consider the aggregate rate at which the multiples of all primes ≥ 7 preclude the set of all numbers > 7 from being prime candidates.

• Multiples of two duplicate 50% of 7's multiples. Therefore, 7's nominal elimination potential (1/7 or .1428) is cut in half by 2 (.1428 x .5 = .0714).
• Three duplicates 33.3% of 7's multiples. However, half of 3's multiples duplicate multiples of 2. So 3 newly reduces only .1666 (.3333 x .5 = .1666) of 7's potential eliminations. Multiples of three, thus, further reduces the .0714 established in step one by .1666.
• Five duplicates 1/5 of 7's multiples. However, half of 5s multiples duplicate multiples of 2, and 1/3 of 5's multiples are duplicate multiples of 3.
• 7's total New Elimination Rate is 1/15 (1/10 – 1/30 = 1/15). In other words, every 15th number is a multiple of 5 that is not also a multiple of 2 or 3.

Similarly, 7's New Elimination Rate can be calculated as follows:

(1/7 x 1/2) – (1/7 x 1/6) – (1/7 x 1/15) = 4/105.

Or in decimals: (.1428 x .5) – (.1428 x .1667) – (.1428 x .0667) = .0381

For any given prime, the New Elimination Rates of that prime and all smaller primes can be aggregated to show the total rate at which the multiples of those primes eliminate all future numbers from being prime (the "Aggregate New Elimination Rate"). The "Remaining Balance" is 100% minus the Aggregate New Elimination Rate at any given prime. In other words, if we only looked at 2 and 3, we could say that 66.67% of all numbers > 3 are multiples of 2 and/or 3 and, hence, not prime, and 33.33% of numbers > 3 may still be prime, at this point in the analysis, depending on whether they are multiples of primes > 3. The following table illustrates these concepts.

Prime #	New Elimination Rate	Aggregate New Elimination Rate	Remaining Balance
2	50.00%	50%	50%
3	16.67%	66.67%	33.33%
5	6.67%	73.34%	26.66%
7	3.80%	77.14%	22.86%
11	2.08%	79.22%	20.78%
13	1.60%	80.82%	19.18%
17	1.19%	82.01%	17.99%
19	.95%	82.96%	17.04%
23	.74%	83.70%	16.3%
29	.56%	84.26%	15.74%
31	.51%	84.77%	15.23%
37	.41%	85.18%	14.82%
41	.36%	85.54%	14.46%

B. The Aggregate New Elimination Rate is the rate by which the next prime's Nominal Elimination Rate is Reduced By All Smaller Primes

The Aggregate New Elimination Rate, at any point, always equals the rate by which all smaller primes reduce the elimination potential of the next prime. For example, to calculate 7's New Elimination Rate, its Nominal Elimination Rate (1/7) is reduced by 1/2 for 2 (50%), 1/6 by 3 (16.6667%), and 1/15 for 5 (6.6667%). The sum of these reduction factors is 73.3334% (50 + 16.6667 + 6.6667 = 73.3334), which is the same as the Aggregate New Elimination Rate at 5. In other words, the sum of the New Elimination Rates of all primes < 7 equals 73.39%, which is the same rate at which 7's new elimination power is reduced by those smaller primes.

This is necessarily true for any prime number because the inputs are the same for both calculations. The New Elimination Rates of each smaller prime that individually reduce the Nominal Rate of a given prime are the same rates that sum to the Aggregate New Elimination Rate for the immediately preceding prime. To illustrate, consider the following two methods of calculating the New Elimination Rate of 11.

• <u>Individual Method</u>

(1/11 x 1/2) – (1/11 x 1/6) – (1/11 x 1/15) – (1/11 x 4/105) = 8/385 (2.08%)

Or in decimals: (.0909 x.5) – (.0909 x .1666) – (.0909 x .0666) – (.0909 x.038) = .0208

• Aggregate Method

A simpler calculation is 1/11 – (1/11 x 27/35) = 8/385.

Or in decimals: .0909 – (.0909 x .7712) = .0208

This method can be expressed as the formula 1/P – (1/P x A), where P = a given prime number, and A equals the Aggregate New Elimination Rate at the prime immediately preceding P. If P = 11 then A = 27/35 (1/2 + 1/6 + 1/15 + 4/105 = 27/35) or 77.142857%. We can aggregate the New Elimination Rates of each smaller prime, reduce 11's Nominal Elimination Rate by this aggregate, and thus determine 11's New Elimination Rate.

1/11 – (1/11 x 27/35) =
1/11 – 27/385 =
8/385
Or in decimals:
9.090909 – (9.090909 x .77142857) =
9.090909 – 7.01298693 =
2.077922

Thus, together 2, 3, 5 and 7 reduce 11's Nominal Rate by 27/35 (1/2 + 1/6 + 1/15 + 4/105 = 27/35) or 77.142857% from 1/11 (9.090909%) to 24/1,155 (2.077922%). The individual method reduces 11's Nominal Rate by each smaller prime's New Elimination Rate one at a time and the aggregate method simply sums the New Elimination Rates of each smaller prime and then reduces 11's Nominal Rate by the aggregate amount.

C. The Aggregate New Elimination Rate Can Never Reach 100%

To prove that the Aggregate New Elimination Rate can never reach 100%, consider a hypothetical in which the Aggregate New Elimination Rate finally reaches 100%. At the immediately prior prime, call it Px, the Aggregate New Elimination Rate, call it A, would be, by definition, some number less than 100%, which would then reduce the Nominal Elimination Rate of the next prime number, call it Py, by some amount less than 100%. This concept is illustrated on the following chart:

Prime #	New Elimination Rate	Aggregate New Elimination Rate	Remaining Balance
Px	Unknown %	A%	B%
Py	1/Py - (1/Py x A)	100%	0%

The question then becomes: if the Aggregate New Elimination Rate at Px (A) is < 100%, can the resulting New Elimination Rate of Py equal the Remaining Balance (B) at Px? In other words, can 1/Py – (1/Py x A) be greater than or equal to B, where B = 100 minus A, and A is < 100?.

First, we can deduce that 1/Py is < 1 (or 100%) because, obviously, 1 over any number > 1 is a fraction that is < 1. If 1/Py is < 1 and A is < 100 and B = 100 – A, then 1/Py – (1/Py x A) must be < B. In other words, if A is < 100, and B = 100 – A, then B must be < 100. This is much like the proverbial flea jumping halfway to a wall; it never gets there because each jump is less than the remaining distance.

Here are a few illustrative examples:

- Let 1/Py = .9 and A = .8 (80%); therefore B must = .2 (100% - 80% = 20%)
  .9 – (.9 x .8) = .18 (18%), which less than B (20%)
- Let 1/Py = .9 and A = .85 (85%); therefore B must = .15 (100% - 85% = 15%)
  .9 – (.9 x .85) = .135 (13.5%), which is less than B (15%)
- Let 1/Py = .9 and A = .99 (99%); therefore B must = .01 (100% - 99% = 1%)
  .9 – (.9 x .99) = .009 (.9%), which is less than B (1%)

Therefore, if 1/Py < 1 and A is < 100, then 1/Py – (1/Py x A) will always be less than B, where B = 100 – A, which means there will always be some Remaining Balance, thus precluding the Aggregate New Elimination Rate from ever reaching 100%. Because the Aggregate New Elimination Rate can never reach 100%, there can be no point at which all higher numbers have been eliminated as prime candidates, and thus no highest prime number.

II. No Highest Twin Prime

A. Thinking of twin primes as a three number set

Twin primes can be thought of as a set of 3 numbers, the middle of which is always an even multiple of 3. For the numbers on both ends of a three number set to be prime, the middle number must be an even multiple of 3 because:

• In any series of 3 numbers, one of them will be a multiple of 3.
• If the multiple of 3 is the first or third number in the set, that number is not prime because it is a multiple of 3.
• If the multiple of 3 in the middle of a 3 number set is odd, the numbers on either side will be even, and thus (for numbers > 2) not prime.

Further, the multiple of 3 in the middle cannot appear in the 4 or 6 column of a chart such as the one below (i.e., end in a 4 or 6), because if it did, there would be a number ending in 5 on one side or the other, and a number ending in 5 (a multiple of 5) cannot be prime for numbers > 5.
A "Twin Prime Middle" (or "TPM") can, thus, be defined as an even multiple of 3 that ends in a 2, 8 or 0. A "Twin Prime Candidate" (or "TPC") can be defined as the numbers on either side of a Twin Prime Middle. A "Candidate Set" can be defined as a series of 3 consecutive numbers, the middle number of which is a Twin Prime Middle. A "Twin Prime Pair" is an actual set of prime numbers that differ from each other by 2.

On the chart below, TPMs are designated in green and TPCs are designated in yellow.
The pattern of distribution of TPMs and TPCs repeats every 30 numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90

(*See also* Attachment 3.)

Each Candidate Set will be a Twin Prime Pair unless one of the two (or both) Twin Prime Candidates in the Set are eliminated from being prime by the multiples of a smaller prime. The methodology discussed above can be used to determine the rate at which each prime number eliminates Twin Prime Candidates, and thus eliminates Candidate Sets from being Twin Prime Pairs. As discussed below, the rate at which TPCs are eliminated is calculated first, and then an adjustment is made to reflect the rate at which Candidate Sets are eliminated from being Twin Prime Pairs. Finally, the same method used to prove that the Aggregate Rate of Elimination of prime numbers cannot be 100% is then used to prove that the rate at which Candidate Sets are eliminated from being Twin Prime Pairs can also never reach 100%.

B. Calculating the TPC New Elimination Rate

Calculating the rate at which each prime eliminates Twin Prime Candidates for the first time (the "TPC New Elimination Rate") is a two-step process. The first step is to identify a consistent pattern to the distribution of TPCs. The second step is to identify a consistent pattern to the rates at which a given prime eliminates TPCs.

1. Calculating the Distribution of TPCs

Because Twin Prime Candidates are the odd numbers on either side of an even multiple of 3, the distribution of Twin Prime Candidates is controlled by the distribution of multiples of 3. As discussed in the following section, there is a general rule governing the distribution of the multiples all primes (other than 2 and 5), which is necessary to understand the distribution of multiples of 3 and important for other purposes later in the analysis.

(a) The Distribution of Any Prime's Multiples Repeats Every 10 Multiples

Consider how the multiples of a prime would be distributed on a table of consecutive numbers broken out into rows of 10 numbers each like the TPM/TPC table above in § II.A. On this type of table, of course, the last digit of a given number controls which column it goes in.

The final digit of the multiples of any prime (other than 2 or 5) necessarily repeats after each set of 10 multiples. Therefore, there will be one multiple of any given prime in each of the 10 columns for each set of 10 consecutive multiples, and then the pattern starts over.

If the number is in the 2, 4, 6, 8 or 10 column, it is even and (for numbers > 2) not prime.
If the number is > 5 and appears in the 5 column, it is divisible by 5, and hence not prime. Therefore, for the given number to be prime (if it is not 2 or 5) it must appear in the 1, 3, 7 or 9 columns. The multiples of primes that end in 1, 3, 7 or 9 proceed through a specific sequence every 10 multiples. If a prime number ends in 1, then from the row on which one multiple appears to the row on which the next multiple appears, the next multiple will be exactly 1 column over to the right. Consider, for example, the distribution of multiples of 11. Eleven is the first multiple and appears in the 1column, next 22 appears in the 2 column, then 33 in the 3 column, 44 in the 4 column, 55 in the 5 column, 66 in the 6 column, 77 in the 7 column, 88 in the 8 column, 99 in the 9 column, 110 in the 10 column, and then the pattern starts over with 121 in the 1 column. The multiples of 31, as a further example, proceed through the same sequence of columns, although the next multiple is always 3 rows down and 1 column over to the right from the previous multiple. [Reference chart.]

If a prime number ends in 3, its multiples will appear in the following column sequence:
3, 6, 9, 2, 5, 8, 1, 4, 7, 10, and then the process starts over in the 3 column (as in 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 and 33). If a prime number ends in 7, its column sequence will be 7, 4, 1, 8, 5, 2, 9, 6, 3, 10 and then back to 7 (7, 14, 21, 28, 35, 42, 49, 56, 63, 70 and 77). If a prime number ends in 9, its column sequence will be (the inverse of one) 9, 8, 7, 6, 5, 4, 3, 2, 1, 10 and then back to 9 (9, 18, 27, 36, 45, 54, 63, 72, 81, 90 and 99). [Examples.]

Because all primes (other than 2 and 5) must end in 1, 3, 7 or 9, and the multiples of numbers appearing in these columns land in each of the 10 columns every 10 multiples, the distribution pattern of multiples of any prime (other than 2 or 5) repeats after each set of 10 multiples.

(b) The Distribution of Multiples of Three

Like all primes, other than 2 and 5, the distribution of 3's multiples repeats every 10 multiples of 3, or every 30 numbers. (Attachment 2.) In other words, there will be one multiple of 3 in each of the 10 columns, and then the process starts over again after the 10th multiple. This same pattern repeats infinitely because every 30 numbers the exact same counting process repeats.

(c) The Twin Prime Candidate Ratio

Once it has been established that the pattern of 3's multiples repeats every 10 multiples to infinity, it is possible to calculate the ratio of Twin Prime Candidates in each set of 10 multiples of 3 (every 30 numbers). There are 5 even multiples of 3 in each set of 10 multiples. Two of the 5 (those ending in 4 or 6) cannot be Twin Prime Middles because they are within one number of a number ending in 5 on either side, and numbers > 5 that end in 5 cannot be prime. This leaves 3 TPMs with 1 TPC on each side for a total of 6 Twin Prime Candidates every 30 numbers. (See Attachment 3.) Six TPCs every 30 numbers = 6/30 = 1/5 (20%). Twin Prime Candidates are, therefore, 20% of all numbers. Further, because there are two TPCs in each Candidate Set, there are three Candidate Sets every 30 numbers (3/30 = 1/10 = 10%).

2. Calculating the Rates of Twin Prime Eliminations

(a) Primes Less Than or Equal to Five

In calculating each prime's TPC New Elimination Rate, the number two is obviously not a factor because the TPCs are all odd. Three is also irrelevant because the TPMs are all, by definition, multiples of 3, and so the numbers immediately adjacent cannot also be. Similarly, 5 is irrelevant because TPCs never appear in the 5 column (i.e., no number ending in 5 is a Twin Prime Candidate). The first prime that can eliminate TPCs, therefore, is 7.

(b) Primes Greater Than Five

To calculate the rate at which 7 eliminates TPCs, it is necessary to identify a consistent pattern between the multiples of 7 and 3, because 3 controls the distribution of TPCs. The pattern of multiples of 3 and 7 (viewed together) starts over every 210 numbers (3 x 7 x 10 = 210). (Attachment 4.) There are 42 TPCs in each group of 210 numbers. (Attachment 3.)

Therefore, 20% of the numbers in each group of 210 numbers is a TPC (42/210 = 1/5 (20%)).
(Id.) There are 30 multiples of 7 in each group of 210 numbers, and 6 of those eliminate Twin Prime Candidates (6/30 = 1/5 (20%)). (Attachment 5.)

Thus, the ratio of TPCs in the set (42) to the total numbers in the set (210) is 1/5 (42/210 = 1/5), and the ratio of the multiples of 7 that hit TPCs in the set (6) to the total number of multiples of 7 in the set (30) is also 1/5 (6/30 = 1/5). This is true because both 30 and 210 are being reduced by the 20% TPC ratio.

- 210 x .2 = 42 (42/210 = 1/5 (20%))
- 30 x .2 = 6 (6/30 = 1/5 (20%))

The ratio of the multiples of 7 that eliminate TPCs to the total number of TPCs in the set is, thus,

6 eliminations by 7's multiples per 42 TPCs for an elimination rate of 1/7 (6/42 = 1/7), which is
7's Nominal Rate. (Id.)

For the same reason, the Nominal Rate at which every prime eliminates Twin Prime Candidates is the same as the Nominal Rate at which that prime eliminates all numbers. In other words, because 80% of numbers are not TPCs, the same percentage of the multiples of a given prime are irrelevant in considering the rate at which a given prime eliminates TPCs. Therefore, every prime's Nominal Rate is also its nominal rate of TPC elimination. The same process described above can be used to adjust for duplicate eliminations.

For example, 7 is the first prime that can eliminate TPCs, and, therefore, its Nominal Rate is also its TPC New Elimination Rate (1/7 or 14.28571%). However, 11's Nominal Rate (1/ 11 or 9.09091%), is reduced by duplicate multiples between 7 and 11 to 7.79221% (9.09091 – (9.09091 x .1428571) = 7.79221). Thirteen's Nominal Rate (1/13 or 7.69231%) is reduced by duplicate multiples between 7 and 13 to 6.59341% (7.69231 – (7.69231 x .1428571) = 6.59341).

Thirteen's Nominal Rate is also reduced by duplicate multiples between 11 and 13 by .5994001 (7.69231 x .077922 = .5994001) to 5.99401% (6.59341 - .5994001 = 5.99401).

The shortcut method takes 13's Nominal Rate (7.69231%) and reduces it by 22.0791% (14.28571 + 7.7922 = 22.07791), which is the Aggregate New Elimination Rate at 11 (7.69231 – (7.69231 x .2207791) = 5.99401).

The following chart illustrates the rate at which each prime number newly eliminates
Twin Prime Candidates, thus eliminating an entire Candidate Set.

Prime #	Twin Prime Candidate New Elimination Rate	Aggregate TPC New Elimination Rate	Remaining Balance
7	14.28571%	14.28571%	85.71429%
11	7.7922%%	22.07791%	77.92209%
13	5.99401%	28.07192%	71.92808%
17	4.23107%	32.30299%	67.69701%
19	3.563%	35.86599%	64.13401%
23	2.78913%	38.65512%	61.34488%
29	2.1153371%	40.770457%	59.22955%
31	1.91063%	42.681087%	57.31892%
37	1.54916%	44.230247%	55.76976%
41	1.0787844%	45.309031%	54.69097%

C. A Prime's Candidate Set New Elimination Rate Is Always < Its TPC New Elimination Rate

Each prime's TPC New Elimination Rate would equal its "Candidate Set New Elimination Rate" but for the fact that each TPC in a Candidate Set can be eliminated by a different prime. The first TPC eliminated by a prime precludes its entire Candidate Set from being a Twin Prime Pair. Therefore, some of the TPC eliminations by a given prime are rendered moot because the other TPC in a given Candidate Set has already been eliminated. As discussed in more detail below, this reduces the rate at which a given prime actually eliminates Candidate Sets.

D. The Aggregate Candidate Set New Elimination Rate can never reach 100%

The Aggregate Candidate Set New Elimination Rate can never reach 100% because: (1) the TPC New Elimination Rate at a given prime (Py) will never be sufficient to completely bridge the distance between the Aggregate TPC New Elimination Rate at the preceding prime (A) and 100%; and (2) a prime's Candidate Set New Elimination Rate is always less than its TPC New Elimination Rate.

Consider a situation in which the Aggregate TPC New Elimination Rate finally reaches 100%. In the immediately prior step, the Aggregate New Elimination Rate (A) would necessarily be some number less than 100%, which would then reduce the elimination power of the following prime number (Py) by some amount less than 100%. The following chart illustrates this hypothetical:

Prime #	TPC New Elimination Rate	Aggregate TPC New Elimination Rate	Remaining Balance
Px	Unknown %	A%	B%
Py	1/Py - (1/Py x A)	100%	0%

As was the case with proving that there can be no highest prime number, the question here is: can 1/Py – (1/Py x A) be greater than or equal to B, where A is < 100, and B = 100 minus A? As discussed above in § I.C., the difference between an A value of less than 100 minus a fixed value will always be less than exactly 100 minus the same fixed value. So the problem works only if 1/Py = 1/1 (a 100% New Elimination Rate).

No prime can have a 100% TPC New Elimination Rate because the TPC New Elimination Rates start at 14.28571%% (for 7) and get smaller as the prime in the denominator gets larger. Because no prime can have a 100% TPC New Elimination Rate, 1/Py must be < 1, and, if A is < 100, then 1/Py – (1/Py x A) must be < B. If 1/Py – (1/Py x A) is always less than B, then there will always be some Remaining Balance, thus precluding the Aggregate TPC New Elimination Rate from ever reaching 100%. Because the Aggregate TPC New Elimination Rate can never reach 100%, there can be no point at which all TPCs have been eliminated from being prime.

Further, a given prime's Candidate Set New Elimination Rate is always less than its TPC New Elimination Rate because the Candidate Set New Elimination Rate is reduced not only by exact duplicates, but also reduced by Duplicate Candidate Set Eliminations. Therefore, 1/Py is not only reduced by an Aggregate Rate (A) less than 100%, but also by Duplicate Candidate Set Eliminations, which reduce its Candidate Set New Elimination Rate even further.

Accordingly, there will always be some Remaining Balance, thus precluding the Aggregate Candidate Set New Elimination Rate from ever reaching 100%. There can, thus, be no point at which all Candidate Sets have been eliminated from being Twin Prime Pairs, which means there can be no highest twin prime.

Attachment 1
Glossary of Terms

"Aggregate Candidate Set New Elimination Rate" – the sum of the Candidate Set New Elimination Rates of a prime and all smaller primes.

"Aggregate New Elimination Rate" - the sum of the New Elimination Rates of a given prime and all smaller primes.

"Aggregate TPC New Elimination Rate" - the sum of the TPC New Elimination Rates of a prime and all smaller primes.

"Candidate Set" - a series of 3 consecutive numbers, the middle number of which is a Twin Prime Middle.

"Candidate Set Duplication Rate" – the rate of Duplicate Candidate Set Eliminations between a prime and an individual smaller prime (6/(P1 x P2 x 30)).

"Candidate Set New Elimination Rate" - the rate at which a prime number eliminates Candidate Sets from being Twin Prime pairs after adjustment for exact duplicate multiples with smaller primes and Duplicate Candidate Set Eliminations.

"Duplicate Candidate Set Eliminations" - TPC eliminations that are moot for purposes of calculating Candidate Set New Elimination Rates because the eliminated TPC is part of a Candidate Set in which the other TPC has already been eliminated.
"Multiples Within Two" - a set of the multiples of two primes that come within exactly two numbers of each other.

The "Maximum Multiple Difference" is the farthest apart any multiple of a smaller prime can get from the closest multiple of a larger prime between duplicate multiples of the two. "New Elimination Rate" - the rate at which a given prime (P) eliminates new numbers from being prime (1/P minus duplicate eliminations).

"Nominal Rate" - a given prime's reciprocal (1/P).
"Remaining Balance" - 100% minus the Aggregate New Elimination Rate (for all numbers, TPCs or Candidate Sets) at a given prime.

"Total Candidate Set Duplication Rate" – the total of the Candidate Set Duplication Rates between a prime and all smaller primes.

"Twin Prime Middle" (or "TPM") - an even multiple of 3 that ends in a 2, 8 or 0.

"Twin Prime Candidate" (or "TPC") - one of the numbers on either side of a Twin Prime Middle.
"Twin Prime Pair" - a set of prime numbers that differ from each other by 2.

"TPC New Elimination Rate" - the rate at which each prime number eliminates Twin Prime Candidates for the first time.

Attachment 2
The Distribution of Multiples of Three

Multiples of 3 are indicated in yellow. The pattern repeats every 30 numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
292	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350
351	352	353	354	355	356	357	358	359	360

Attachment 3
The Distribution of Candidate Sets

Twin Prime Middles (TPMs) are indicated in green. Twin Prime Candidates (TPCs) are indicated in yellow. The pattern repeats every 30 numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
292	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350
351	352	353	354	355	356	357	358	359	360
361	362	363	364	365	366	367	368	369	370
371	372	373	374	375	376	377	378	379	380
381	382	383	384	385	386	387	388	389	390
391	392	393	394	395	396	397	398	399	400
401	402	403	404	405	406	407	408	409	410
411	412	413	414	415	416	417	418	419	420

Attachment 4
The Distribution of Multiples of 3 and 7

Multiples of 3 are indicated yellow. Multiples of 7 are indicated in blue. Duplicate multiples of 3 and 7 are indicated in green. The pattern repeats every 210 numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
292	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350
351	352	353	354	355	356	357	358	359	360
361	362	363	364	365	366	367	368	369	370
371	372	373	374	375	376	377	378	379	380
381	382	383	384	385	386	387	388	389	390
391	392	393	394	395	396	397	398	399	400
401	402	403	404	405	406	407	408	409	410
411	412	413	414	415	416	417	418	419	420

Attachment 5

Twin Prime Candidate Eliminations by Multiples of 7

TPCs are indicated yellow, and TPMs are green. Multiples of 7 that eliminate TPCs are orange. The pattern repeats every 210 numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
292	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350
351	352	353	354	355	356	357	358	359	360
361	362	363	364	365	366	367	368	369	370
371	372	373	374	375	376	377	378	379	380
381	382	383	384	385	386	387	388	389	390
391	392	393	394	395	396	397	398	399	400
401	402	403	404	405	406	407	408	409	410
411	412	413	414	415	416	417	418	419	420

Attachment 6
The Distribution of the Multiples of Two Primes Relative To Each Other Between Duplicate Multiples

The following table shows the distribution of multiples of 7 and 13 between 91 (7 x 13 = 91) and 182 (7 x 13 x 2 = 182). The distance between multiples of 7 and the closest multiple of 11 is indicated in parenthesis. Multiples of either 7 or 13 within 2 of the closest multiple of the other are in bold and underlined.

Multiples of 7 From 91 - 182	Multiples of 13 From 91 - 182
91	91
98 (6)	104
105 (1)	117
112 (5)	130
119 (2)	143
126 (4)	156
133 (3)	169
140 (3)	182
147 (4)	
154 (2)	
161 (5)	
168 (1)	
175 (6)	
182	

The following table shows the distribution of multiples of 7 and 17 between 119 (7 x 17 = 119) and 238 (7 x 17 x 2 = 238).

Multiples of 7 From 119 - 238	Multiples of 17 From 119 - 238
119	119
126 (7)	136
133 (3)	153
140 (4)	170
147 (6)	187
154 (1)	204
161 (8)	221
168 (2)	238
175 (5)	
182 (5)	
189 (2)	
196 (8)	
203 (1)	
210 (6)	
217 (4)	
224 (3)	
231 (7)	
238	

This table shows the distribution of multiples of 11 and 13 between 143 (11 x 13 = 143) and 286 (11 x 13 x 2 = 286).

Multiples of 11 From 143 - 286	Multiples of 13 From 143 - 286
143	143
154 (2)	156
165 (4)	169
176 (6)	182
187 (5)	195
198 (3)	208
209 (1)	221

220 (1)	234
231 (3)	247
242 (5)	260
253 (6)	273
264 (4)	286
275 (2)	
286	

Attachment 7
Duplicate Multiples of 7 and 11 In Relation To
The Distribution of Candidate Sets

TPCs are indicated yellow. TPMs are indicated in green. Duplicate multiples of 7 and 11 that do not eliminate TPCs are indicated in red. Duplicate multiples of 7 and 11 that eliminate TPCs are indicated in orange. Duplicate multiples of 7 and 11 that eliminate TPMs are indicated in black. The pattern repeats every 2,310 numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
292	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350
351	352	353	354	355	356	357	358	359	360

361	362	363	364	365	366	367	368	369	370
371	372	373	374	375	376	377	378	379	380
381	382	383	384	385	386	387	388	389	390
391	392	393	394	395	396	397	398	399	400
401	402	403	404	405	406	407	408	409	410
411	412	413	414	415	416	417	418	419	420
421	422	423	424	425	426	427	428	429	430
431	432	433	434	435	436	437	438	439	440
441	442	443	444	445	446	447	448	449	450
451	452	453	454	455	456	457	458	459	460
461	462	463	464	465	466	467	468	469	470
471	472	473	474	475	476	477	478	479	480
481	482	483	484	485	486	487	488	489	490
491	492	493	494	495	496	497	498	499	500
501	502	503	504	505	506	507	508	509	510
511	512	513	514	515	516	517	518	519	520
521	522	523	524	525	526	527	528	529	530
531	532	533	534	535	536	537	538	539	540
541	542	543	544	545	546	547	548	549	550
551	552	553	554	555	556	557	558	559	560
561	562	563	564	565	566	567	568	569	570
571	572	573	574	575	576	577	578	579	580
581	582	583	584	585	586	587	588	589	590
591	592	593	594	595	596	597	598	599	600
601	602	603	604	605	606	607	608	609	610
611	612	613	614	615	616	617	618	619	620
621	622	623	624	625	626	627	628	629	630
631	632	633	634	635	636	637	638	639	640
641	624	643	644	645	646	647	648	649	650
651	652	653	654	655	656	657	658	659	660
661	662	663	664	665	666	667	668	669	670
671	672	673	674	675	676	677	678	679	680
681	682	683	684	685	686	687	688	689	690
691	692	693	694	695	696	697	698	699	700
701	702	703	704	705	706	707	708	709	710
711	712	713	714	715	716	717	718	719	720
721	722	723	724	725	726	727	728	729	730
731	732	733	734	735	736	737	738	739	740
741	742	743	744	745	746	747	748	749	750
751	752	753	754	755	756	757	758	759	760
761	762	763	764	765	766	767	768	769	770
771	772	773	774	775	776	777	778	779	780
781	782	783	784	785	786	787	788	789	790
791	792	793	794	795	796	797	798	799	800
801	802	803	804	805	806	807	808	809	810
811	812	813	814	815	816	817	818	819	820
821	822	823	824	825	826	827	828	829	830
831	832	833	834	835	836	837	838	839	840
841	842	843	844	845	846	847	848	849	850
851	852	853	854	855	856	857	858	859	860
861	862	863	864	865	866	867	868	869	870
871	872	873	874	875	876	877	878	879	880
881	882	883	884	885	886	887	888	889	890
891	892	893	894	895	896	897	898	899	900
901	902	903	904	905	906	907	908	909	910

911	912	913	914	915	916	917	918	919	920
921	922	923	924	925	926	927	928	929	930
931	932	933	934	935	936	937	938	939	940
941	942	943	944	945	946	947	948	949	950
951	952	953	954	955	956	957	958	959	960
961	962	963	964	965	966	967	968	969	970
971	972	973	974	975	976	977	978	979	980
981	982	983	984	985	986	987	988	989	990
991	992	993	994	995	996	997	998	999	1000
1001	1002	1003	1004	1005	1006	1007	1008	1009	1010
1011	1012	1013	1014	1015	1016	1017	1018	1019	1020
1021	1022	1023	1024	1025	1026	1027	1028	1029	1030
1031	1032	1033	1034	1035	1036	1037	1038	1039	1040
1041	1042	1043	1044	1045	1046	1047	1048	1049	1050
1051	1052	1053	1054	1055	1056	1057	1058	1059	1060
1061	1062	1063	1064	1065	1066	1067	1068	1069	1070
1071	1072	1073	1074	1075	1076	1077	1078	1079	1080
1081	1082	1083	1084	1085	1086	1087	1088	1089	1090
1091	1092	1093	1094	1095	1096	1097	1098	1099	1100
1101	1102	1103	1104	1105	1106	1107	1108	1109	1110
1111	1112	1113	1114	1115	1116	1117	1118	1119	1120
1121	1122	1123	1124	1125	1126	1127	1128	1129	1130
1131	1132	1133	1134	1135	1136	1137	1138	1139	1140
1141	1142	1143	1144	1145	1146	1147	1148	1149	1150
1151	1152	1153	1154	1155	1156	1157	1158	1159	1160
1161	1162	1163	1164	1165	1166	1167	1168	1169	1170
1171	1172	1173	1174	1175	1176	1177	1178	1179	1180
1181	1182	1183	1184	1185	1186	1187	1188	1189	1190
1191	1192	1193	1194	1195	1196	1197	1198	1199	1200
1201	1202	1203	1204	1205	1206	1207	1208	1209	1210
1211	1212	1213	1214	1215	1216	1217	1218	1219	1220
1221	1222	1223	1224	1225	1226	1227	1228	1229	1230
1231	1232	1233	1234	1235	1236	1237	1238	1239	1240
1241	1242	1243	1244	1245	1246	1247	1248	1249	1250
1251	1252	1253	1254	1255	1256	1257	1258	1259	1260
1261	1262	1263	1264	1265	1266	1267	1268	1269	1270
1271	1272	1273	1274	1275	1276	1277	1278	1279	1280
1281	1282	1283	1284	1285	1286	1287	1288	1289	1290
1291	1292	1293	1294	1295	1296	1297	1298	1299	1300
1301	1302	1303	1304	1305	1306	1307	1308	1309	1310
1311	1312	1313	1314	1315	1316	1317	1318	1319	1320
1321	1322	1323	1324	1325	1326	1327	1328	1329	1330
1331	1332	1333	1334	1335	1336	1337	1338	1339	1340
1341	1342	1343	1344	1345	1346	1347	1348	1349	1350
1351	1352	1353	1354	1355	1356	1357	1358	1359	1360
1361	1362	1363	1364	1365	1366	1367	1368	1369	1370
1371	1372	1373	1374	1375	1376	1377	1378	1379	1380
1381	1382	1383	1384	1385	1386	1387	1388	1389	1390
1391	1392	1393	1394	1395	1396	1397	1398	1399	1400
1401	1402	1403	1404	1405	1406	1407	1408	1409	1410
1411	1412	1413	1414	1415	1416	1417	1418	1419	1420
1421	1422	1423	1424	1425	1426	1427	1428	1429	1430
1431	1432	1433	1434	1435	1436	1437	1438	1439	1440
1441	1442	1443	1444	1445	1446	1447	1448	1449	1450
1451	1452	1453	1454	1455	1456	1457	1458	1459	1460

1461	1462	1463	1464	1465	1466	1467	1468	1469	1470
1471	1472	1473	1474	1475	1476	1477	1478	1479	1480
1481	1482	1483	1484	1485	1486	1487	1488	1489	1490
1491	1492	1493	1494	1495	1496	1497	1498	1499	1500
1501	1502	1503	1504	1505	1506	1507	1508	1509	1510
1511	1512	1513	1514	1515	1516	1517	1518	1519	1520
1521	1522	1523	1524	1525	1526	1527	1528	1529	1530
1531	1532	1533	1534	1535	1536	1537	1538	1539	1540
1541	1542	1543	1544	1545	1546	1547	1548	1549	1550
1551	1552	1553	1554	1555	1556	1557	1558	1559	1560
1561	1562	1563	1564	1565	1566	1567	1568	1569	1570
1571	1572	1573	1574	1575	1576	1577	1578	1579	1580
1581	1582	1583	1584	1585	1586	1587	1588	1589	1590
1591	1592	1593	1594	1595	1596	1597	1598	1599	1600
1601	1602	1603	1604	1605	1606	1607	1608	1609	1610
1611	1612	1613	1614	1615	1616	1617	1618	1619	1620
1621	1622	1623	1624	1625	1626	1627	1628	1629	1630
1631	1632	1633	1634	1635	1636	1637	1638	1639	1640
1641	1642	1643	1644	1645	1646	1647	1648	1649	1650
1651	1652	1653	1654	1655	1656	1657	1658	1659	1660
1661	1662	1663	1664	1665	1666	1667	1668	1669	1670
1671	1672	1673	1674	1675	1676	1677	1678	1679	1680
1681	1682	1683	1684	1685	1686	1687	1688	1689	1690
1691	1692	1693	1694	1695	1696	1697	1698	1699	1700
1701	1702	1703	1704	1705	1706	1707	1708	1709	1710
1711	1712	1713	1714	1715	1716	1717	1718	1719	1720
1721	1722	1723	1724	1725	1726	1727	1728	1729	1730
1731	1732	1733	1734	1735	1736	1737	1738	1739	1740
1741	1742	1743	1744	1745	1746	1747	1748	1749	1750
1751	1752	1753	1754	1755	1756	1757	1758	1759	1760
1761	1762	1763	1764	1765	1766	1767	1768	1769	1770
1771	1772	1773	1774	1775	1776	1777	1778	1779	1780
1781	1782	1783	1784	1785	1786	1787	1788	1789	1790
1791	1792	1793	1794	1795	1796	1797	1798	1799	1800
1801	1802	1803	1804	1805	1806	1807	1808	1809	1810
1811	1812	1813	1814	1815	1816	1817	1818	1819	1820
1821	1822	1823	1824	1825	1826	1827	1828	1829	1830
1831	1832	1833	1834	1835	1836	1837	1838	1839	1840
1841	1842	1843	1844	1845	1846	1847	1848	1849	1850
1851	1852	1853	1854	1855	1856	1857	1858	1859	1860
1861	1862	1863	1864	1865	1866	1867	1868	1869	1870
1871	1872	1873	1874	1875	1876	1877	1878	1879	1880
1881	1882	1883	1884	1885	1886	1887	1888	1889	1890
1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
1901	1902	1903	1904	1905	1906	1907	1908	1909	1910
1911	1912	1913	1914	1915	1916	1917	1918	1919	1920
1921	1922	1923	1924	1925	1926	1927	1928	1929	1930
1931	1932	1933	1934	1935	1936	1937	1938	1939	1940
1941	1942	1943	1944	1945	1946	1947	1948	1949	1950
1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
2011	2012	2013	2014	2015	2016	2017	2018	2019	2020

2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
2031	2032	2033	2034	2035	2036	2037	2038	2039	2040
2041	2042	2043	2044	2045	2046	2047	2048	2049	2050
2051	2052	2053	2054	2055	2056	2057	2058	2059	2060
2061	2062	2063	2064	2065	2066	2067	2068	2069	2070
2071	2072	2073	2074	2075	2076	2077	2078	2079	2080
2081	2082	2083	2084	2085	2086	2087	2088	2089	2090
2091	2092	2093	2094	2095	2096	2097	2098	2099	2100
2101	2102	2103	2104	2105	2106	2107	2108	2109	2110
2111	2112	2113	2114	2115	2116	2117	2118	2119	2120
2121	2122	2123	2124	2125	2126	2127	2128	2129	2130
2131	2132	2133	2134	2135	2136	2137	2138	2139	2140
2141	2142	1243	2144	2145	2146	2147	2148	2149	2150
2151	2152	2153	2154	2155	2156	2157	2158	2159	2160
2161	2162	2163	2164	2165	2166	2167	2168	2169	2170
2171	2172	2173	2174	2175	2176	2177	2178	2179	2180
2181	2182	2183	2184	2185	2186	2187	2188	2189	2190
2191	2192	2193	2194	2195	2196	2197	2198	2199	2200
2201	2202	2203	2204	2205	2206	2207	2208	2209	2210
2211	2212	2213	2214	2215	2216	2217	2218	2219	2220
2221	2222	2223	2224	2225	2226	2227	2228	2229	2230
2231	2232	2233	2234	2235	2236	2237	2238	2239	2240
2241	2242	2243	2244	2245	2246	2247	2248	2249	2250
2251	2252	2253	2254	2255	2256	2257	2258	2259	2260
2261	2262	2263	2264	2265	2266	2267	2268	2269	2270
2271	2272	2273	2274	2275	2276	2277	2278	2279	2280
2281	2282	2283	2284	2285	2286	2287	2288	2289	2290
2291	2292	2293	2294	2295	2296	2297	2298	2299	2300
2301	2302	2303	2034	2305	2306	2307	2308	2309	2310
2311	2312	2313	2314	2315	2316	2317	2318	2319	2320
2321	2322	2323	2324	2325	2326	2327	2328	2329	2330
2331	2332	2333	2334	2335	2336	2337	2338	2339	2340
2341	2342	2343	2344	2345	2346	2347	2348	2349	2350
2351	2352	2353	2354	2355	2356	2357	2358	2359	2360
2361	2362	2363	2364	2365	2366	2367	2368	2369	2370
2371	2372	2373	2374	2375	2376	2377	2378	2379	2380
2381	2382	2383	2384	2385	2386	2387	2388	2389	2390
2391	2392	2393	2394	2395	2396	2397	2398	2399	2400
2401	2402	2403	2404	2405	2406	2407	2408	2409	2410
2411	2412	2413	2414	2415	2416	2417	2418	2419	2420
2421	2422	2423	2424	2425	2426	2427	2428	2429	2430
2431	2432	2433	2434	2435	2436	2437	2438	2439	2440
2441	2442	2443	2444	2445	2446	2447	2448	2449	2450
2451	2452	2453	2454	2455	2456	2457	2458	2459	2460
2461	2462	2463	2464	2465	2466	2467	2468	2469	2470

Attachment 8

Duplicate Candidate Set Eliminations by Multiples of 7 and 11

TPCs are indicated yellow. TPMs are indicated in green. Multiples of 7 that do not eliminate TPCs or TPMs are indicated in red. Multiples of 11 that do not eliminate TPCs or TPMs are indicated in dark blue. Multiples of 7 that eliminate TPCs are indicated in orange. Multiples of 11 that eliminate TPCs are indicated in light blue. Multiples of 7 that eliminate TPMs are indicated in dark green. Multiples of 11 that eliminate TPMs are indicated in purple. The pattern repeats every 2,310 numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
292	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
341	342	343	344	345	346	347	348	349	350
351	352	353	354	355	356	357	358	359	360
361	362	363	364	365	366	367	368	369	370
371	372	373	374	375	376	377	378	379	380
381	382	383	384	385	386	387	388	389	390
391	392	393	394	395	396	397	398	399	400
401	402	403	404	405	406	407	408	409	410
411	412	413	414	415	416	417	418	419	420
421	422	423	424	425	426	427	428	429	430
431	432	433	434	435	436	437	438	439	440
441	442	443	444	445	446	447	448	449	450
451	452	453	454	455	456	457	458	459	460
461	462	463	464	465	466	467	468	469	470
471	472	473	474	475	476	477	478	479	480
481	482	483	484	485	486	487	488	489	490
491	492	493	494	495	496	497	498	499	500
501	502	503	504	505	506	507	508	509	510
511	512	513	514	515	516	517	518	519	520
521	522	523	524	525	526	527	528	529	530
531	532	533	534	535	536	537	538	539	540
541	542	543	544	545	546	547	548	549	550

551	552	553	554	555	556	557	558	559	560
561	562	563	564	565	566	567	568	569	570
571	572	573	574	575	576	577	578	579	580
581	582	583	584	585	586	587	588	589	590
591	592	593	594	595	596	597	598	599	600
601	602	603	604	605	606	607	608	609	610
611	612	613	614	615	616	617	618	619	620
621	622	623	624	625	626	627	628	629	630
631	632	633	634	635	636	637	638	639	640
641	624	643	644	645	646	647	648	649	650
651	652	653	654	655	656	657	658	659	660
661	662	663	664	665	666	667	668	669	670
671	672	673	674	675	676	677	678	679	680
681	682	683	684	685	686	687	688	689	690
691	692	693	694	695	696	697	698	699	700
701	702	703	704	705	706	707	708	709	710
711	712	713	714	715	716	717	718	719	720
721	722	723	724	725	726	727	728	729	730
731	732	733	734	735	736	737	738	739	740
741	742	743	744	745	746	747	748	749	750
751	752	753	754	755	756	757	758	759	760
761	762	763	764	765	766	767	768	769	770
771	772	773	774	775	776	777	778	779	780
781	782	783	784	785	786	787	788	789	790
791	792	793	794	795	796	797	798	799	800
801	802	803	804	805	806	807	808	809	810
811	812	813	814	815	816	817	818	819	820
821	822	823	824	825	826	827	828	829	830
831	832	833	834	835	836	837	838	839	840
841	842	843	844	845	846	847	848	849	850
851	852	853	854	855	856	857	858	859	860
861	862	863	864	865	866	867	868	869	870
871	872	873	874	875	876	877	878	879	880
881	882	883	884	885	886	887	888	889	890
891	892	893	894	895	896	897	898	899	900
901	902	903	904	905	906	907	908	909	910
911	912	913	914	915	916	917	918	919	920
921	922	923	924	925	926	927	928	929	930
931	932	933	934	935	936	937	938	939	940
941	942	943	944	945	946	947	948	949	950
951	952	953	954	955	956	957	958	959	960
961	962	963	964	965	966	967	968	969	970
971	972	973	974	975	976	977	978	979	980
981	982	983	984	985	986	987	988	989	990
991	992	993	994	995	996	997	998	999	1000
1001	1002	1003	1004	1005	1006	1007	1008	1009	1010
1011	1012	1013	1014	1015	1016	1017	1018	1019	1020
1021	1022	1023	1024	1025	1026	1027	1028	1029	1030
1031	1032	1033	1034	1035	1036	1037	1038	1039	1040
1041	1042	1043	1044	1045	1046	1047	1048	1049	1050
1051	1052	1053	1054	1055	1056	1057	1058	1059	1060
1061	1062	1063	1064	1065	1066	1067	1068	1069	1070
1071	1072	1073	1074	1075	1076	1077	1078	1079	1080
1081	1082	1083	1084	1085	1086	1087	1088	1089	1090
1091	1092	1093	1094	1095	1096	1097	1098	1099	1100
1101	1102	1103	1104	1105	1106	1107	1108	1109	1110

1111	1112	1113	1114	1115	1116	1117	1118	1119	1120
1121	1122	1123	1124	1125	1126	1127	1128	1129	1130
1131	1132	1133	1134	1135	1136	1137	1138	1139	1140
1141	1142	1143	1144	1145	1146	1147	1148	1149	1150
1151	1152	1153	1154	1155	1156	1157	1158	1159	1160
1161	1162	1163	1164	1165	1166	1167	1168	1169	1170
1171	1172	1173	1174	1175	1176	1177	1178	1179	1180
1181	1182	1183	1184	1185	1186	1187	1188	1189	1190
1191	1192	1193	1194	1195	1196	1197	1198	1199	1200
1201	1202	1203	1204	1205	1206	1207	1208	1209	1210
1211	1212	1213	1214	1215	1216	1217	1218	1219	1220
1221	1222	1223	1224	1225	1226	1227	1228	1229	1230
1231	1232	1233	1234	1235	1236	1237	1238	1239	1240
1241	1242	1243	1244	1245	1246	1247	1248	1249	1250
1251	1252	1253	1254	1255	1256	1257	1258	1259	1260
1261	1262	1263	1264	1265	1266	1267	1268	1269	1270
1271	1272	1273	1274	1275	1276	1277	1278	1279	1280
1281	1282	1283	1284	1285	1286	1287	1288	1289	1290
1291	1292	1293	1294	1295	1296	1297	1298	1299	1300
1301	1302	1303	1304	1305	1306	1307	1308	1309	1310
1311	1312	1313	1314	1315	1316	1317	1318	1319	1320
1321	1322	1323	1324	1325	1326	1327	1328	1329	1330
1331	1332	1333	1334	1335	1336	1337	1338	1339	1340
1341	1342	1343	1344	1345	1346	1347	1348	1349	1350
1351	1352	1353	1354	1355	1356	1357	1358	1359	1360
1361	1362	1363	1364	1365	1366	1367	1368	1369	1370
1371	1372	1373	1374	1375	1376	1377	1378	1379	1380
1381	1382	1383	1384	1385	1386	1387	1388	1389	1390
1391	1392	1393	1394	1395	1396	1397	1398	1399	1400
1401	1402	1403	1404	1405	1406	1407	1408	1409	1410
1411	1412	1413	1414	1415	1416	1417	1418	1419	1420
1421	1422	1423	1424	1425	1426	1427	1428	1429	1430
1431	1432	1433	1434	1435	1436	1437	1438	1439	1440
1441	1442	1443	1444	1445	1446	1447	1448	1449	1450
1451	1452	1453	1454	1455	1456	1457	1458	1459	1460
1461	1462	1463	1464	1465	1466	1467	1468	1469	1470
1471	1472	1473	1474	1475	1476	1477	1478	1479	1480
1481	1482	1483	1484	1485	1486	1487	1488	1489	1490
1491	1492	1493	1494	1495	1496	1497	1498	1499	1500
1501	1502	1503	1504	1505	1506	1507	1508	1509	1510
1511	1512	1513	1514	1515	1516	1517	1518	1519	1520
1521	1522	1523	1524	1525	1526	1527	1528	1529	1530
1531	1532	1533	1534	1535	1536	1537	1538	1539	1540
1541	1542	1543	1544	1545	1546	1547	1548	1549	1550
1551	1552	1553	1554	1555	1556	1557	1558	1559	1560
1561	1562	1563	1564	1565	1566	1567	1568	1569	1570
1571	1572	1573	1574	1575	1576	1577	1578	1579	1580
1581	1582	1583	1584	1585	1586	1587	1588	1589	1590
1591	1592	1593	1594	1595	1596	1597	1598	1599	1600
1601	1602	1603	1604	1605	1606	1607	1608	1609	1610
1611	1612	1613	1614	1615	1616	1617	1618	1619	1620
1621	1622	1623	1624	1625	1626	1627	1628	1629	1630
1631	1632	1633	1634	1635	1636	1637	1638	1639	1640
1641	1642	1643	1644	1645	1646	1647	1648	1649	1650
1651	1652	1653	1654	1655	1656	1657	1658	1659	1660
1661	1662	1663	1664	1665	1666	1667	1668	1669	1670

1671	1672	1673	1674	1675	1676	1677	1678	1679	1680
1681	1682	1683	1684	1685	1686	1687	1688	1689	1690
1691	1692	1693	1694	1695	1696	1697	1698	1699	1700
1701	1702	1703	1704	1705	1706	1707	1708	1709	1710
1711	1712	1713	1714	1715	1716	1717	1718	1719	1720
1721	1722	1723	1724	1725	1726	1727	1728	1729	1730
1731	1732	1733	1734	1735	1736	1737	1738	1739	1740
1741	1742	1743	1744	1745	1746	1747	1748	1749	1750
1751	1752	1753	1754	1755	1756	1757	1758	1759	1760
1761	1762	1763	1764	1765	1766	1767	1768	1769	1770
1771	1772	1773	1774	1775	1776	1777	1778	1779	1780
1781	1782	1783	1784	1785	1786	1787	1788	1789	1790
1791	1792	1793	1794	1795	1796	1797	1798	1799	1800
1801	1802	1803	1804	1805	1806	1807	1808	1809	1810
1811	1812	1813	1814	1815	1816	1817	1818	1819	1820
1821	1822	1823	1824	1825	1826	1827	1828	1829	1830
1831	1832	1833	1834	1835	1836	1837	1838	1839	1840
1841	1842	1843	1844	1845	1846	1847	1848	1849	1850
1851	1852	1853	1854	1855	1856	1857	1858	1859	1860
1861	1862	1863	1864	1865	1866	1867	1868	1869	1870
1871	1872	1873	1874	1875	1876	1877	1878	1879	1880
1881	1882	1883	1884	1885	1886	1887	1888	1889	1890
1891	1892	1893	1894	1895	1896	1897	1898	1899	1900
1901	1902	1903	1904	1905	1906	1907	1908	1909	1910
1911	1912	1913	1914	1915	1916	1917	1918	1919	1920
1921	1922	1923	1924	1925	1926	1927	1928	1929	1930
1931	1932	1933	1934	1935	1936	1937	1938	1939	1940
1941	1942	1943	1944	1945	1946	1947	1948	1949	1950
1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
2031	2032	2033	2034	2035	2036	2037	2038	2039	2040
2041	2042	2043	2044	2045	2046	2047	2048	2049	2050
2051	2052	2053	2054	2055	2056	2057	2058	2059	2060
2061	2062	2063	2064	2065	2066	2067	2068	2069	2070
2071	2072	2073	2074	2075	2076	2077	2078	2079	2080
2081	2082	2083	2084	2085	2086	2087	2088	2089	2090
2091	2092	2093	2094	2095	2096	2097	2098	2099	2100
2101	2102	2103	2104	2105	2106	2107	2108	2109	2110
2111	2112	2113	2114	2115	2116	2117	2118	2119	2120
2121	2122	2123	2124	2125	2126	2127	2128	2129	2130
2131	2132	2133	2134	2135	2136	2137	2138	2139	2140
2141	2142	1243	2144	2145	2146	2147	2148	2149	2150
2151	2152	2153	2154	2155	2156	2157	2158	2159	2160
2161	2162	2163	2164	2165	2166	2167	2168	2169	2170
2171	2172	2173	2174	2175	2176	2177	2178	2179	2180
2181	2182	2183	2184	2185	2186	2187	2188	2189	2190
2191	2192	2193	2194	2195	2196	2197	2198	2199	2200
2201	2202	2203	2204	2205	2206	2207	2208	2209	2210
2211	2212	2213	2214	2215	2216	2217	2218	2219	2220
2221	2222	2223	2224	2225	2226	2227	2228	2229	2230

2231	2232	2233	2234	2235	2236	2237	2238	2239	2240
2241	2242	2243	2244	2245	2246	2247	2248	2249	2250
2251	2252	2253	2254	2255	2256	2257	2258	2259	2260
2261	2262	2263	2264	2265	2266	2267	2268	2269	2270
2271	2272	2273	2274	2275	2276	2277	2278	2279	2280
2281	2282	2283	2284	2285	2286	2287	2288	2289	2290
2291	2292	2293	2294	2295	2296	2297	2298	2299	2300
2301	2302	2303	2034	2305	2306	2307	2308	2309	2310
2311	2312	2313	2314	2315	2316	2317	2318	2319	2320
2321	2322	2323	2324	2325	2326	2327	2328	2329	2330
2331	2332	2333	2334	2335	2336	2337	2338	2339	2340
2341	2342	2343	2344	2345	2346	2347	2348	2349	2350
2351	2352	2353	2354	2355	2356	2357	2358	2359	2360
2361	2362	2363	2364	2365	2366	2367	2368	2369	2370
2371	2372	2373	2374	2375	2376	2377	2378	2379	2380
2381	2382	2383	2384	2385	2386	2387	2388	2389	2390
2391	2392	2393	2394	2395	2396	2397	2398	2399	2400