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The Past, Present and the Futuristic Earth-Moon Orbital-Globe Dynamics - and its Habitability

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Abstract

Using Advanced Kinematic Model (AKM), the unfolding of Earth's eco-system under the influence of the evolving tidally interacting Earth-Moon System (E-M system) is studied. E-M system is born 4.467Gy ago at inner geo-synchronous orbit (aG1 = 15,000Km) which is a Total Energy Maxima Hill Top (+4.18x10^30J). E-M system tidally evolves to its final interlocking orbit in 97Gy from NOW at outer geo-synchronous orbit (aG2 = 534,590.6Km) which is Total Energy Minima Valley (- 2.64x10^28J). Tidal evolution is accompanied with synchronization and circularization of Moon's orbit but co-planarization with Ecliptic plane remains unachievable due to weak tidal coupling between Earth and Moon. E-M system falls in Cassini State 1 in 214My from NOW. In 992.56My from NOW Moon's 'orbit' gets completely circularized and synchronized. In 1.1Gy Sun's brightness increases 10% and Earth becomes moist greenhouse planet. In 1.3458Gy from NOW Earth Obliquity angle reaches an eco-hostile value of 30.42^0 with complete destruction of Habitat trinity. In 2Gy from NOW due to Moon's resonance passage there is large oscillations of obliquity and severe climatic alterations. In 3.5Gy Sun's brightness increases 40% and Earth becomes a runaway greenhouse planet reducing to a molten remnant. If E-M system survives Red Giant Branch stage, Thermal Pulse-Asymtotic Giant Branch stage, the Planetary Nebular phase and if it survives the present phase of Big-Bang of our Bouncing Universe then in 97Gy from NOW Moon will get interlocked with Earth in a triple synchrony orbital period of 44.7473 d at aG2 = 534,590.6Km.

Key Points

Key Point 1. Advanced Kinematic Model (AKM) is developed which includes Astor-metric parameters of Earth-Moon system (E-M system).
Key Point 2. Based on AKM, E-M system evolved from inner geo-synchronous orbit to the present orbit in Fits and Bound.
Key Point 3. E-M system will evolve from the present orbit to outer geo-synchronous orbit, synchronizing and circularizing in transit.

Plain Language Summary

In 4.467 Gy Earth-Moon System has evolved from semi-major axis 18,000Km to the present orbit 384,400Km in Fits and Bound and Moon is receding from Earth at 3.7cm/y as measured by Lunar Laser Ranging Experiments. In 214My from NOW Moon will fall in Casini State I. In 992.56My from NOW Moon's eccentricity gets nullified and Moon's orbit is fully circularized and synchronized. In 1.1Gy from NOW, luminosity of Sun will increase by 10% leading to moist green-house Earth. Earth's obliquity deteriorates to eco-hostile value of 30.4° in 1.3458Gy which triggers the destruction of Habitat Trinity. In 1.3458Gy to 2.5Gy due to resonance passage of Moon, chaotic Earth's obliquity is caused leading to extreme climate conditions. In 3.5Gy from NOW, luminosity of Sun will increase by 40% leading to runaway Green House Planet, and Earth will become a molten remnant. If Earth-Moon system was to survive the Red-Giant Phase, strong wind from Sun in post-AGB phase and if Earth was to survive Planetary Nebula Phase then in 97Gy from NOW Earth-Moon will get inter-locked in triple synchrony state. It will remain thus until solar perturbations deflect Moon onto a death spiral into a glancing angle collision with Earth.

1. The Beginning of Time for Earth-Moon System.

Giant Impact on Earth

Right at the end of the formation of Earth 4.467Gya, Mars-size impactor glancing angle collision with a high obliquity, high angular momentum Earth created a circum-terrestrial debris disk. This debris disk was predominantly made of impactor material. In post-impact state Earth's mantle, atmosphere and disk were not dynamically isolated from one another. As a consequence, they were well mixed and equilibrated. This ensures the identical nature in the isotopic signature of newly accreted Moon and impacted, re-solidified Earth. In the debris disk Moon is accreted beyond Roche's limit which in E-M system is at 18,000Km whereas the inner geo-synchronous orbit is 15,000Km. Hence Moon tumbles into super-synchronous orbit as a consequence full-sized Moon is catapulted, through gravitational sling shot (see S4.1 of SOM), on an expanding spiral path from its orbit of accretion which in this case is 18,000Km. Moon is69 spiralling out to outer geo-synchronous orbit at ~ 534590.6 Km. This scenario of tidal evolution of the Moon explains the near identity between the isotopic signature of Earth and Moon and also gives a pathway to reach Earth's climatically favourable low obliquity of 23.44º from 3RE to 45RE, moon experienced stalled tidal evolution with unpredictable configuration hence it tidally evolved in fits (Cuk et.al. 2016). Only after Moon settled in Cassini State II in a synchronous orbit that it has rapidly evolved and receded to the current orbit of a = 384,400 Km at an anomalously high rate of tidal recession which at present is clocking at 3.8±0.07cm/y as measured by Lunar Laser Measurement [1]. Hence Moon has moved in Fits and presently it is Bounding and therefore Author proposes "fits and bound" model of Earth-Moon system.

The Physics of Tidally interacting Earth=Moon system.

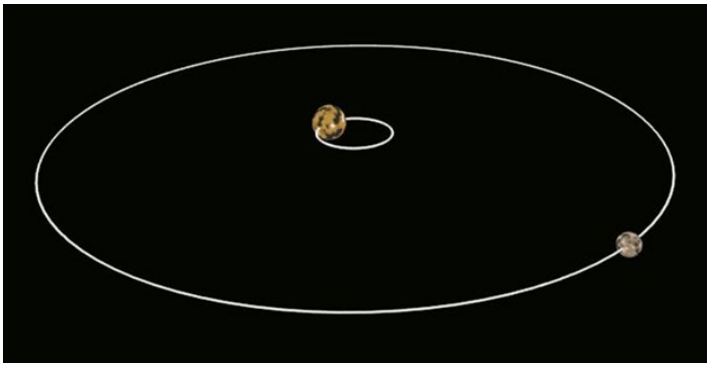
What does theory of tidally interacting binary predict?

The physics of tidally interacting binary pairs of which E-M system is just one example has been Very succinctly placed by Zahn "Eventually the Binary may settle in its state of minimum kinetic energy, in which the orbit is circular, rotation of both stars is synchronized with the orbital motion and the spin axis are perpendicular to the orbital plane [2-5]. Whether the system actually reaches this state is determined by the strength of tidal interaction, thus by the separation of the two components, equivalently the orbital period. But it also depends on the efficiency of the physical process which are responsible for the dissipation of the kinetic energy." At the terminal point of outer geo-synchronous orbit the scalar sum of J0 (orbital angular momentum of E-M system) =3.37088x1034 Kg-m2/s , J1 (Moon's spin angular momentum) =1.44124x1029 Kg-m2/s and J2 (Earth's spin angular momentum)= 1.30317x1032 Kg-m2/s is JTotal*(total scalar sum of J0 + J1 + J2) =3.383926x1034 Kg-m2/s but the sum should be conserved at Jt=3.37492× 10^34 Kg--m2/sec (6). What Zahn implies is that tidal evolution of E-M system should be characterized with synchronization, circularization and co-planarization. In the modern times E-M system has a large Earth's obliquity angle (ϕ = 23.44.º) and a residual inclination (α = 5.14º) of lunar orbital plane with respect to Ecliptic plane. The total resultant angular momentum has been calculated as JT = 3.37492x1034 Kg-m2/s which is vector sum of current era J2 (Earth's spin angular momentum with Earth's Obliquity ϕ =23.44º), J1 (Moon's spin angular momentum with Moon's Obliquity β =1.54º) and J0 (orbital angular momentum with lunar plane inclination α =5.14º) and JT is near perpendicular to the Ecliptic Plane [6]. By conservation of momentum total vector sum of angular momentum will be invariant (in magnitude and direction) for all times to come until it finally locks in at outer geo-synchronous orbit. Since the scalar sum exceeds JT = 3.37492x1034 Kg-m2/s hence tilt in Moon's orbital plane and tilt in Earth's spin axis becomes imperative to satisfy the conservation of angular momentum.

The Kinematic Model (See S4. SOM)

KM unravels the tidally evolving E-M system in Figure 1, Figure 2 and Figure 3. As seen in Figure 1 and Figure 2, Earth-Moon system is born at inner-geo-synchronous orbit. The inner geo-synchronous aG1 is energy maxima corresponding to a total energy level + 4.18x1030Joules which is unstable equilibrium hence Moon has the option to tumble long or tumble short of aG1. If Moon had fallen short of aG1 then our Moon would have been trapped in a death spiral – a gravitationally Runaway collapsing spiral path as Phobos is trapped heading for a glancing angle collision with Mars in next 10My. But our Moon fell long of aG1 because of a happy coincidence. An accretion in impact generated debris disk can occur only beyond Roche's Limit which in our case happens to be 18,000Km whereas the aG1 happens to be 1.46177x107m (See S5.1,SOM, Figure S5.1, Figure S5.2) hence Moon's accretion took place in an orbit which was larger than aG1 (the inner geo-synchronous orbit) hence by necessity our Moon was in a super-synchronous orbit which is an expanding spiral orbit.

As can be seen in Figure 1 and 2, aG1 corresponds to Energy Maxima Hill Top (+4.18x1030Joules) where as aG2 as seen in Figure 3 corresponds to energy Minima (-2.64x1028Joules). Hence tidal evolution is relentlessly driving our Moon to lowest energy state with its accompanying characterization of circularization, synchronization, co-planarization of the orbital planes of Earth and Moon and ultimately through inter-locking of the tidal bulges of Earth-Moon along the elongated axis as it has happened with Pluto- Charon pair as seen in Animation 1. The present study has been unable to achieve co-planarization of Earth's orbit plane and Moon's orbit plane for reasons explained in The Physics of Tidally interacting Binary Pairs.



Animation 1. See Pluto-Charon lock-in Animation_S1.1. Perfect LOCK-IN of Pluto & Charon. The orbital locii of Pluto and Charon are perfect circles concentric on the Barycenter. Both Pluto and Charon are orbiting in perfect circles around the barycenter in equal period of 6.375 Solar Day. The spin periods of both the bodies is 6.375 days. This is triple synchrony state. Charon is locked-in at Outer Clarke's orbit. In perfect tidal lock-in position, the long axis of the tidal bulge of primary and secondary components are exactly aligned and both the components orbit the barycenter as one single body as is the case with Pluto-Charon. This lock-in pair was video photographed by New Horizon. in NH Lorri OPNAV Campaign [7-9]. On 14th July 2015, Pluto-Charon were at the point of closest approach to New Horizon space probe and Pluto Charon were seen making circular paths around the barycenter which lay outside the two globes. The tidal stretching and squeezing completely stops and hence tidal dissipation are zero. This perfect lock-in occurs when the two components are synchronized, the orbit of each component around the barycenter are circularized and the orbital planes of the two components are co-planer

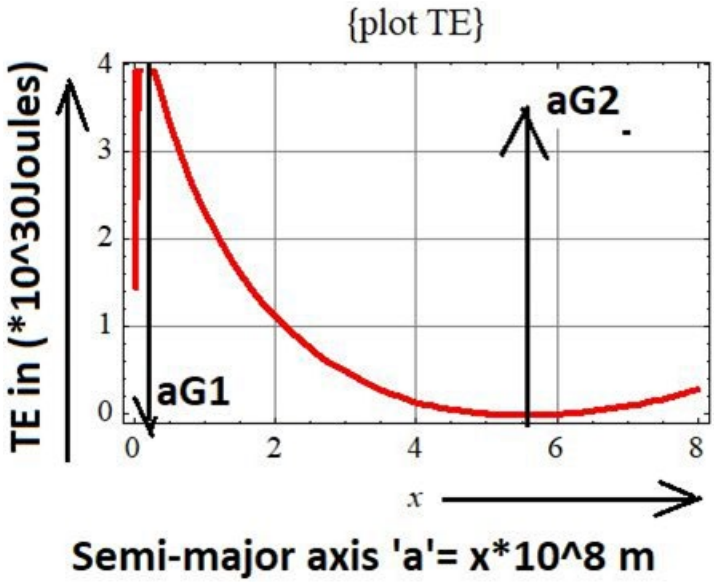


Figure 1. Plot of total energy in the range 0.5×107m and 8×108m along the long tidal history of Moon from its inception to its lock-in at outer geo-synchronous orbit. [Credit: Author]

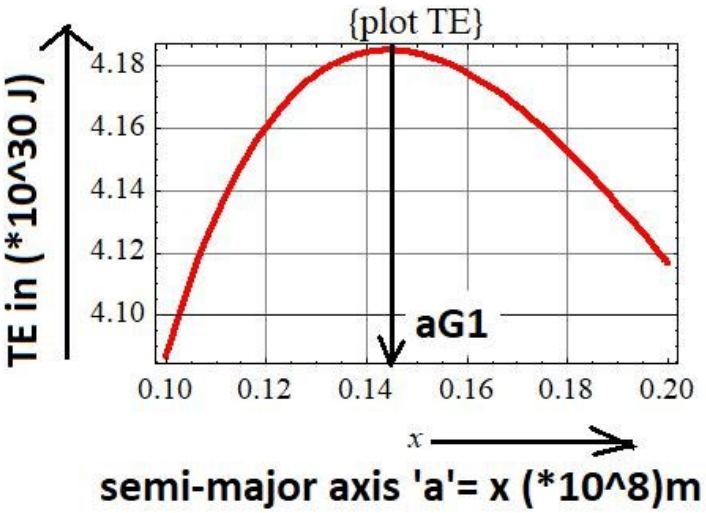


Figure 2. Plot of total energy in the range 1.4×107m and 1.5×107m around the inner geo-synchronous orbit. Earth-Moon system is born at inner geo-synchronous orbit of a = 1.46×107m. [Credit: Author]

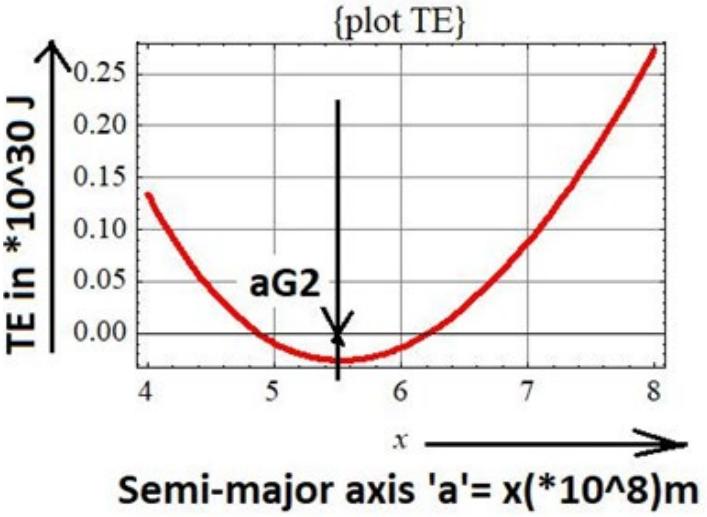


Figure 3. Plot of total energy in the range 5.4×108m and 5.6×108m around the outer geo-synchronous orbit of a = 5.527×108m (in KM). [Credit: Author]

In course of tidal evolution and orbital spiral expansion Moon has reached 384,400Km elliptical orbit and our Moon is gradually coasting along the expanding spiral path up the potential well (potential well is created due to Earth) to its final tidal lock-in orbit at 534,590.6Km. The energy, required for climbing the potential well, was imparted to Moon during gravitational sling shot phase. As seen in Figure 1.5 SOM, Figure S4.2 and Figure S4.3, Moon has an accelerating phase during impulsive torque from aG1 to a2 (2:1 Mean Motion Resonance orbit) with peak of radial acceleration occurring at a1. After this point, recession velocity continuously decelerates until it gets nullified at aG2. Because of this asymptotic approach to outer geo-synchronous orbit, Moon will

take 97Gy from NOW unfathomable future to reach this final point of tidal inter-locking with Earth at aG2. From aG1 to a2 Earth-Moon being in conservative phase, Moon acquired large amount of rotational kinetic energy from the spin-down of Earth due to tidal braking and in effect Moon experienced a massive impulsive torque.

This impulsive torque occurred at a1 and by the time Moon reached a2 (2:1 Mean Motion Resonance orbit) the impulsive torque completely decayed due to the differential between ω (Earth's angular spin rate) and Ω (Earth-Moon angular orbital rate).The rotational Kinetic Energy of Earth released due to secular spin-down of Earth or due to secular lengthening of Length of Day(LOD) was 80% transferred to Moon as orbital acceleration but after a2 rotational energy is dissipated tidally heating, Earth is no more being transferred to Moon but is being dissipated within Earth's body as tidal heating and plate-tectonic movements resulting in Volcanic Eruptions and Earth-quakes. This part is well illustrated in S1.5.1. SOM and This is precisely why real time LOD can become a very important space asset in Early Warning and Forecasting Methods (EWFM) for terrestrial, natural calamities. The unfolding of the evolutionary path of tidally interacting Binary pair, while rolling down from Energy Maxima Hill Top (4.18x1030J) at aG1 to Energy Minima Valley (- 2.64x1028J) at aG2 is characterized with synchronization, circularization and co-planarization of Earth's orbital plane with Moon's orbital plane and the energy released (4.15x1030J) goes to power the tidal heating of Earth's interior and triggering seismic activity which may result in Earth-quakes and Volcanic Eruptions.

This means in future as the E-M system evolves Lunar Orbital plane inclination will move towards null value (it minimizes but remains constant at 5º), eccentricity of the Lunar orbit will move towards null point, synchronization will be maintained as it is being maintained today but Earth's obliquity will deteriorate and progressively lead to eco-hostile configuration and extreme climates. This happens because of the constraint of conservation of total angular momentum of E-M system as already explained in The Physics of Tidally interacting Binary Pairs.

We have studied the Orbital-Globe dynamics of tidally receding Moon till the current lunar orbit (apresent = 384,400Km) for the last 1.2Gy in the prequel Paper [6]. The Orbital-Globe dynamics of E-M system for the remaining part of its expanding spiral path from apresent =384,400Km to aG2 = 534,590.6Km in unfathomable future 97.30Gy from NOW in its non-Keplarian journey to its minimum energy state (--2.64x1028J) will be covered in the present paper assuming E-M system will survive Red-Giant Branch of our Sun post-AGB wind and Planetary Nebula formation with White Dwarf compact stage at the core of Planetary Nebula and the present phase of Big-Bang in the bouncing Universe Model [10-12] . The mathematics for generating the three figures (Figure 1, 2 and 3) is given in S1.5. SOM. Table 1 gives the summary of the Work done by Darwin, Krasinsky and by the Author.

Table 1. Comparative study of the geo-synchronous results obtained by Krasinsky (2002), Sharma [personal communication: arXiv:0805.0100 (2008)]; Sharma (2019), the present paper and Darwin (1879,1880) [13-15].

	aG1/R_{Earth}	aG2/R_{Earth}	Orbital period at aG1	Orbital period at aG2
Krasinsky analysis	2.15	83.8	4.42h	44.8days
BKS analysis (KM)	2.29	86.65	4.8596h	47.0739days
BKS analysis (AKM)	2.333	83.91	5h	44.75days
George Howardating Darwinating	Not available	90.4	Not available	47days

The Research on Futuristic Earth-Moon Till Date.

Till date there is a large amount of research on the futuristic evolution of Earth Moon system, was the first to analyze that Moon is in expanding spiral orbit and it is destined to be tidally interlocked with Earth at 5.345906×108 m = 83.91RE with a triple synchrony at 44.75days [15]. Here the tidal interlocking implies that the tidal bulge of Earth and Moon are perfectly aligned and this alignment rotates around the barycenter of the two bodies at 44.75 days period where Earth's spin period = Moon's spin period = Orbital period of E-M system=44.75d.

Conway (1983) included lunar orbital eccentricity, lunar tidal dissipation, terms beyond the first term in Earth's tidal potential and the solar tidal torque [16]. In his model Moon experiences a spiral expansion up to 74.6RE After reaching the end point Moon falls in collapsing spiral orbit. During spiral expansion phase it asymptotically approaches 74.6RE. In this model Earth's obliquity(ϕ) increases from 23.44° to 80° at the endpoint. Lunar orbital plane inclination (α) remains unchanged at 5.14°. The paper is silent on the evolution of eccentricity of the lunar orbit.

Mignard (1979, 1980, 1981) have revisited the evolution of lunar orbit [17-19]. In Mignard_Part III, Sun's perturbation is included for (a/RE) > 15 and Sun-Earth-Moon is analyzed as three-body system. In the present text, the methodology in Mignard_Part III, is utilized for (a/RE) > 15 and the methodology developed in Part II is utilized for (a/RE) < 15.

Tumalski (2004) has analyzed that the forces which created Moon 4.5Gy ago are responsible for the deformation of Earth and for the continental drift on its surface [20].

Touma and Wisdom (1994), examine the dynamics of Earth-Moon system under much less severe approximation and they conclude that Earth's obliquity is presently increasing and it will increase from 23.44° to 34° [21].

Laskar, Joutal and Robutal (1993) have concluded that Moon stabilizes the obliquity of Earth which in turn helps our Planet to avoid extreme climatic changes [22-24]. This stabilizing factor is missing in case of Mars hence Mars has been experiencing constant wobbliness leading to extreme climatic changes which in turn has caused complete vaporization of water and escape of atmosphere.

Atobe, Ida and Ita (2004) have shown that in case of Earth-Moon system the stability is achieved because the Moon increases the precession rate of Earth's spin axis around the ecliptic normal thereby avoiding resonances between the precession of Earth's spin axis and precession of Earth's orbital plane normal [25].

Hanslmeier (2009) has shown that high obliquity would probably result in extreme changes in the climate and may destroy the planet's habitability [26].

Trurnit (1995) has shown that tectonic based events will continue to occur well into future and Earth's surface will be steadily reshaped by tectonic uplifts, extrusions and erosion [27]. Mount Vesuvius can be expected to erupt 40 times in next 1000years.

Gordon, J.F. MacDonald, (1964) have shown that if Earth and Moon are distance 'r' apart then tides are proportional to r-3 and the tidal torque which arises by the gravitational pull varies as r-6 [28]. Hence changes in orbit elements take place rapidly when 'r' is small but these changes become negligible when 'r' is large. It is also found that increasing obliquity results in a more uniform distribution of insolation over the globe. At ϕ =35°, all latitudes of Earth would receive approximately the same amount of solar energy. Provided insolation is the controlling factor this would result in SUBTROPICAL climate over much of the world [Maddox 1994]. Krasinsky (2002) along with the Author et.al., were the first scientists to point out that E-M system has two-geosynchronous orbits and that Moon is launched just beyond the inner geo-synchronous orbit (aG1) on an expanding spiral orbit [13]. This expanding spiral orbit will terminate at outer geo-synchronous orbit (aG2) in a very distant future about 20Gy from now. Krasinsky and the Author also pointed out that inner geo-synchronous orbit (aG1) is unstable since it is an energy maxima and outer geo-synchronous orbit (aG2) is a stable orbit since it is energy minima. [Maddox 1994].

Schroeder and Smith (2008) have shown that mass loss by the Sun at RGB (Red Giant Branch) stage (0.322M_SUN ,7.59Gy from now) initially gives a significant orbital expansion to Earth, inversely proportional to the remaining solar mass but finally with the present initial condition planet Earth undergoes engulfment by Red Giant Sun because at the tip of the RGB our planet contacts solar cool giant photosphere [11]. To avoid engulfment our planet must be presently at 1.15AU.
Maddox makes similar assessments about Sun-Earth system [10].

Futuristic Evolution of E-M System Using Advanced Kinematic Model.

The purpose of this study is to examine how semi-major axis (a), the terrestrial obliquity(ϕ), lunar obliquity (β), lunar orbit eccentricity (e) and Lunar Plane inclination (α) tidally evolve in the future and how these orbit elements change effect the habitability of humans on Earth. In paper Sharma (2019) study spatial functions in S6, SOM, using the algorithms in Appendix A1 of Sharma 2019, the Author has studied five orbital dynamics parameters namely Moon's orbital plane inclination (α), Moon's obliquity (β), Moon's orbit eccentricity (e), LOM/LOD (Length of Lunar Sidereal Month/Length of Earth Day) and Earth's obliquity (ϕ). The five spatial functions corresponding to these five parameters have been derived in Appendix S1 of Sharma (2019) which gave the evolutionary history of Moon's orbital plane inclination (α), Moon's obliquity (β), Moon's orbit eccentricity (e), LOM/LOD (Length of Lunar Sidereal Month/Length of Eartth Day) and Earth's obliquity (ϕ) over the last 1.2Gy. These five spatial functions are

$$\text{Inclination angle } \alpha$$

$$= \frac{1.18751 \times 10^{25}}{a^3} - \frac{7.1812 \times 10^{16}}{a^2} + \frac{1.44103 \times 10^{8}}{a} - 8.250567342 \times 10^{-3} \quad (1)$$

$$\text{Moon's Obliquity angle } \beta = 3.36402 - 1.37638 \times 10^{-8} a + 1.32216 \times 10^{-17} a^2 \quad (2)$$

$$e = 0.210252 + 8.38285 \times 10^{-10} a - 3.23212 \times 10^{-18} a^2 \quad (3)$$

$$\frac{LOM}{LOD} = \frac{\omega}{\Omega} = -12.0501 + 2.6677 \times 10^{-7} \times a - 4.27538 \times 10^{-16} \times a^2 \quad (4)$$

$$\text{Earth's obliquity angle } \varphi = -0.732299 + 2.97166 \times 10^{-9} \times a \quad (5)$$

All angles are in radians. These five spatial functions were studied in the past up to 1.2Gy
[6]. In the present paper we study these functions in futuristic orbits during the
course of tidal evolution of lunar orbit in the coming eons to come. Ten futuristic orbits are
considered namely: 4×108 m, 4.25×108 m, 4.5×108 m, 4.75×108 m, 5×108 m, 5.25×108 m,
5.3×108 m, 5.345×108 m, 5.3459×108 m, and aG2 = 5.345906×108 m.

2. Materials and Methods

Angular momentum vector diagram gets simplified as shown in Figure 6 and Figure 7 below:

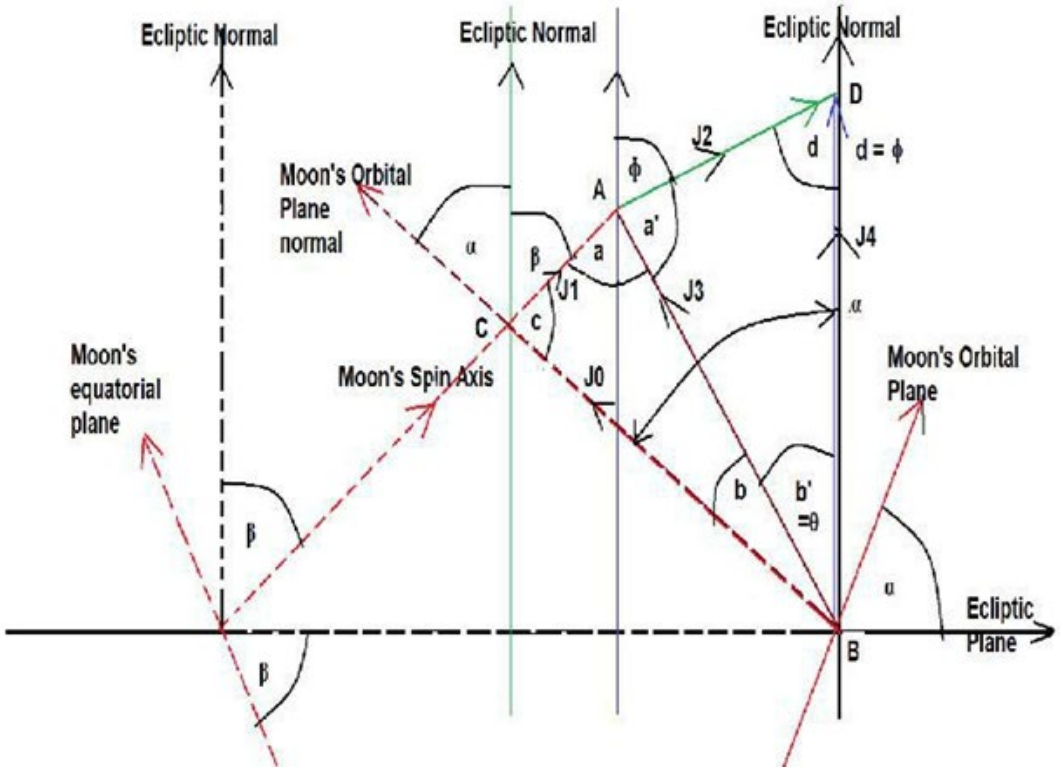


Figure 4. Two triangles of J vectors: Triangle ABC sides J0(orb. J), J1 (Moon's spin J) and J3 where J3 is the vector sum of J0 and J1. Triangle ABD sides J2 (Earth's spin J), J3 (the vector sum of J0 and J1) and J4 (total resultant J of E-M system) {Credit: Author]

Once Moon's obliquity becomes zero as seen in Figure 5:

$$J_3^2 = J_0^2 + J_1^2 - 2J_0J_1\cos[\pi - \alpha]$$

$$\approx (J_0 + J_1)^2 \text{ since in future orbits } \alpha \text{ is always nearly } 5^0 \qquad 6$$

Hence

$$J_3 = J_0 + J_1 \qquad 7$$

Triangle ABC will be considered a straight line and only Triangle ABD remains to be considered. as seen in Figure 5 and in future analysis Eq. (7) will be applicable.

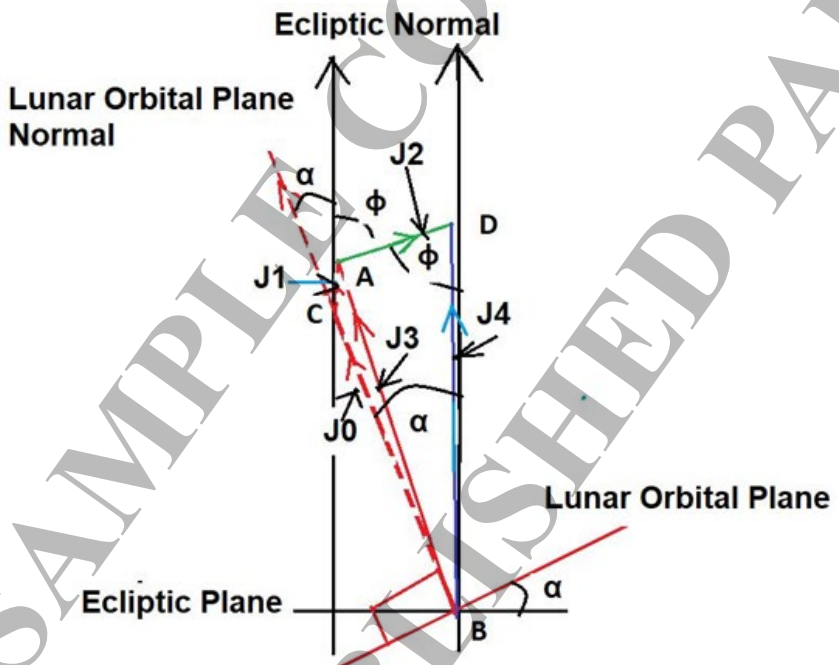


Figure 5. Simplified vector diagram of angular momentum J0, J1, J2 J3 and J4 applicable to futuristic orbits beyond 'a'= 3.92x108m = 61.53RE.

Angular momentum is defined as in Figure 5. The angle α is the inclination angle of lunar orbital plane. Since Lunar obliquity β = 0 therefore lunar orbital angular momentum J0 and Moon's spin angular momentum J1 are nearly aligned and represented by J3. Earth spin angular momentum is J2 tilted with respect to ecliptic normal by terrestrial obliquity angle ϕ. Total angular momentum of Earth-Moon system is J4 = vector sum of J3 and J2. J4 is inclined with respect to ecliptic normal by a very small angle γ = 0.385445°.

From triangle ADB, we obtain:

$$J_4^2 = J_2^2 + J_3^2 - 2J_2J_3Cos\{\pi - \alpha - \varphi\} \qquad (8)$$

On expanding the trigonometric function:

$$J_4^2 = J_2^2 + J_3^2 - 2J_2J_3\{-Cos\alpha Cos\varphi + Sin\alpha Sin\varphi\} \qquad (9)$$

Define:

$$Z = -Cos\alpha Cos\varphi + Sin\alpha Sin\varphi \qquad (10)$$

Eq. (9) is written as:

$$Z = -Cos\alpha Cos\varphi + Sin\alpha Sin\varphi \qquad (11)$$

J4 is the total angular momentum of Earth Moon system which has remained constant since Cassini State transition and which will remain constant in future till the final lock-in at second geo-synchro-

nous orbit. J4 is considered normal to the Ecliptic for all practical purposes.

Eq. (11) L.H.S. remains constant and R.H.S. is written in analytical form:

$$J_4^2 = (C\omega)^2 + (m^*a^2 + I)^2\Omega^2 - 2(C\omega)(m^*a^2 + I)\Omega \times Z \quad (12)$$

In Eq. (12), C= principal spin axis moment of inertia = 8.02×1037Kg. m2; I =spin axis moment of inertia of Moon = 8.73669×1034 Kg.m2; m* = reduced mass of Moon=7.258980539×1022Kg; = orbital angular velocity of Moon = spin angular velocity of Moon; And ω = spin angular velocity of Earth.
This equation is simplified to obtain a quadratic equation of LOM/LOD = X= ω/Ω

$$(\frac{J_4}{C\Omega})^2 = (\frac{\omega}{\Omega})^2 + a^4(\frac{m^* + \frac{I}{a^2}}{C})^2 - 2 \times \frac{\omega}{\Omega} \times \left(\frac{m^* + \frac{I}{a^2}}{C}\right) a^2 \times Z \quad (13)$$

I/a2 is several orders of magnitude smaller than m* hence I/a2 can be neglected.
From Kepler's Third Law:

$$(\frac{1}{\Omega})^2 = \frac{a^3}{B^2} \quad where\ B = \sqrt{G(M+m)} = 2.008433303 \times 10^7 \frac{m^{3/2}}{s}$$

$$Let\ N = \frac{J_4}{B \times C} = 2.09707715634 \times 10^{-11} \frac{1}{m^{\frac{3}{2}}}; X = \frac{\omega}{\Omega}; F = \frac{m^*}{C} = 9.04936 \times 10^{-16} \frac{1}{m^2};$$

Using the new substitution and new definitions Eq. (13) simplifies to:

$$N^2 \times a^3 = X^2 + a^4 \times F^2 - 2 \times a^2 \times F \times X \times Z \quad (14)$$

In the ideal case where, terrestrial obliquity angle = lunar orbital plane inclination = lunar obliquity= is taken as ZERO we get Z = -1 and Eq. (14) reduces to:

$$N \times a^{3/2} = X + F \times a^2 \quad or\ X = N \times a^{3/2} - F \times a^2 \quad (15)$$

Eq. (15) is the equation the Author has been using in classical Kinematic Model.
Eq. (14) will be the cardinal equation to be used for studying the futuristic orbital-globe
dynamics of E-M system.
Determination of Length of (Earth) Day (LOD) in the future.
From definition: X =LOM/LOD
Therefore

$$LOD = \frac{LOM}{X} \quad (16)$$

From Kepler's Third Law:

$$LOM = \frac{2\pi}{B} \times a^{3/2} \quad (17)$$

Substituting (17) in (16):

$$LOD = \frac{2\pi}{B} \times a^{3/2} \times \frac{1}{X} \quad (18)$$

Determination of Transit Time from futuristic lunar orbit3.92v108m to the required
futuristic orbit.

$$V(a) = \frac{2K}{m^*B} \times \frac{\sqrt{a}}{a^Q}[X-1] \times 31.5569088 \times \frac{10^6 m}{y} \quad (19)$$

The value of the constants in Eq. (19) are as follows:

$$K(structure\ constant) = 8.33269 \times 10^{42} N - m^Q, Exponent\ Q = 3.22684,$$

$$m^* = reduced\ mass\ of\ Moon = 7.256742697 \times 10^{22} Kg.$$

$$B = \sqrt{G(M+m)} = 2.008774813 \times 10^7 \frac{m^{3/2}}{s}, a = lunar\ semi - major\ axis.$$

The given value of K (structure constant) and Q (exponent of the structure factor) ensure the modern-day recession velocity of Moon as 3.82±0.07cm/y at the modern-day lunar orbit as measured in the ongoing Lunar Laser Ranging Experiments [1]. Eq. (14) is a quadratic equation in X = LOM/LOD.

Eq. (14) is solved and two roots are obtained. One is negative and the other is positive. The positive root is retained. The value of inclination angle (α in radians) given in Eq. (1) and Eq. (5) for terrestrial obliquity angle (ϕ in radians) is substituted in the expression of Z. This form of X is substituted in (19). (19) is used for calculating the transit time from 3.92×108m to the futuristic orbits. The transit time from 3.844×108m to 3.92×108m is derived using the older algorithm given in Sharma 2019. It comes out to be 214My. The transit time is given by the following time integral:

$$Transit\ time\ from\ a_1\ to\ a_2 = \int_{a_1}^{a_2} \frac{1}{V(a)} da; \quad (20)$$

3. Data in the Text Obtained from Futuristic Analysis

Length of day, Velocity of recession and the transit time into futuristic orbits.
The algorithm given in Appendix A1 of Sharma, 2019, are used for futuristic orbits up to 3.92×108 m in this analysis. 3.92×108 m is a special orbit where Moon's obliquity angle becomes zero and from there onward Moon remains stay-put in Cassini State I and the vector diagram of angular momentum simplifies as seen in Materials and Method. Transit Time of Moon from 3.92×108 m to any intermediate orbit before the outer geo-synchronous orbit (aG2 =5.345906×108 m) is determined using the algorithms given in the Methods and Materials of this paper. The algorithm given in Materials and Methods is used to determine the transit time from 3.92×108 m orbit to the ten futuristic orbits up to aG2 = 5.345906×108 m. The lengthening of LOD due to tidal interaction and the velocity of recession of Moon are derived in these 10 futuristic orbits. These data are tabulated in Table 2

Table 2. Length of Day, Velocity of recession and transit time (TT) from the present to the eleven futuristic orbits.

Semi-majoraxis 'a'(m)	'a' (xRE)	X	LOD (d)	Velocity of recess(cm/y)	TT* to the given orbit(y)(x500My)
3.844×10^8	60.336	27.32	0.998687	3.7cm/y	Present
3.92×10^8	61.5288	26.711	1.0519	3.4cm/y	214M(0.428)
4×10^8	62.7845	26.225	1.10436	3.18cm/y	456.32M(0.9126)
4.25×10^8	66.7085	24.2362	1.38	2.48cm/y	1.3458G(2.6916)
4.5×10^8	70.6326	21.4004	1.61485	1.866cm/y	2.5058G(5.0176)
4.75×10^8	74.5566	17.4738	2.14981	1.3cm/y	4.1658G(8.3316)
5×10^8	78.4806	12.0822	3.35	0.76cm/y	6.652G(13.304)
5.25×10^8	82.4046	4.61707	9.432	0.21cm/y	12.392G(24.784)
5.3×10^8	83.189	2.79316	15.786	0.105	15.627G(31.254)
5.345×10^8	83.8958	1.03656	43.1581	2x10-3	32.668G(65.336)
5.3459×10^8	83.909904	1.00024	44.7365	$1,37x10^{-5}$	72,468G(142.936)
5.345906×10^8	83.909998	1.	44.7473	0 cm/y	97.304G(194.608)
5.346×10^8	83.91	0.9962034	Non-	$- 2.17x10^{-4}$	Moon gets deflected at
*Transit Time (TT) is measured in the future with respect to the present					

The results in Table 2 are consistent with tidally interacting binaries physics as well as with Lunar Laser Ranging experiments [1, 9].

i. E-M system is unravelling to its minima energy orbit through synchronization, circularization, and co-planarization;

ii. LOD is lengthening due to tidal deceleration of spinning Earth;

iii. velocity of Moon's recession progressively approaches a Null value at outer geo-synchronous orbit aG2 = 5.345906×108 m S4.1 of SOM And to S4.2.3 of SOM. Outer geo-synchronous orbit acts as a fire-wall which the orbiting Moon cannot cross. It will get deflected and launched on a death spiral on a gravitationally runaway collapsing spiral path. Hence aG2 defines the sphere of influence or it defines the Hill Sphere of Earth with respect to Sun approximately.

iv. George Howard Darwin had predicted one and half century back that LOD will lengthen up to 47 days at the outer geo-synchronous orbit. Author's analysis in his personal communication (http://arXiv.org/abs/0805.0100) had also predicted that as Moon recedes its velocity of recession will decrease and it will asymptotically approach Null value at outer geo-synchronous orbit where Earth spin period = Moon's spin period = E-M orbital period = 47d. But in this study the Author obtains 44.75d

v. The Table 2 shows that the Moon's tidal recessionary journey is slowing down until Moon interlocks with Earth at outer geo-synchronous orbit aG2 = 5.345906×108 m with orbital period of 44.75 solar days where it will be in triple synchrony state.

vi. This result has been corroborated by Conway (1981), Darwin (1880), Mignard (1981), Gordon (1964) and Krasinsky (2002) [13, 15, 16, 19]. As discussed in Sharma (2019) as well as discussed by Conway (1981), Darwin (1880), Mignard(1981), Gordon (1964) and Krasinsky (2002) [13, 15, 16, 19] .

vii. Moon starts its journey in equatorial plane of Earth and transfers to the ecliptic plane at 'a' = 17RE. This is known as Laplace Plane transition and 'a' = 17RE is Laplace Plane transition orbit.

viii. The lunar orbital expansion is shown in Figure 6/ and lunar recession velocity profile is shown in Figure 7.

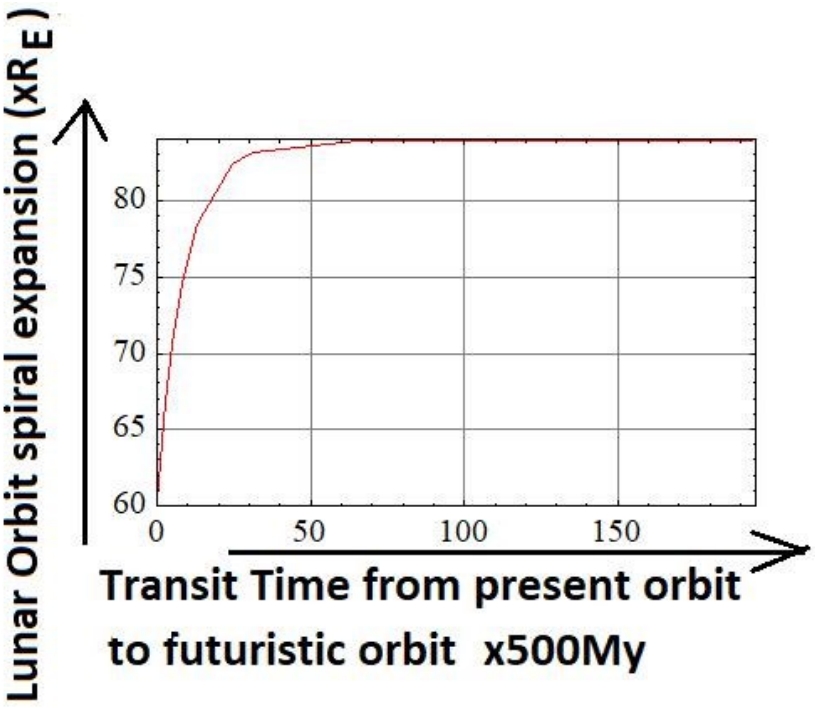


Figure 6. Lunar Orbit spirally expands from 3.884x108m(60.336RE) to the outer geo-synchronous orbit at 5.345906x108m(83.91RE) in 97.304Gy(194.608x500My). Since Moon approaches outer geo-synchronous orbit asymptotically hence there is inordinately long Transit Time longer than the age of Universe (13.8Gy).[Credit: Author]

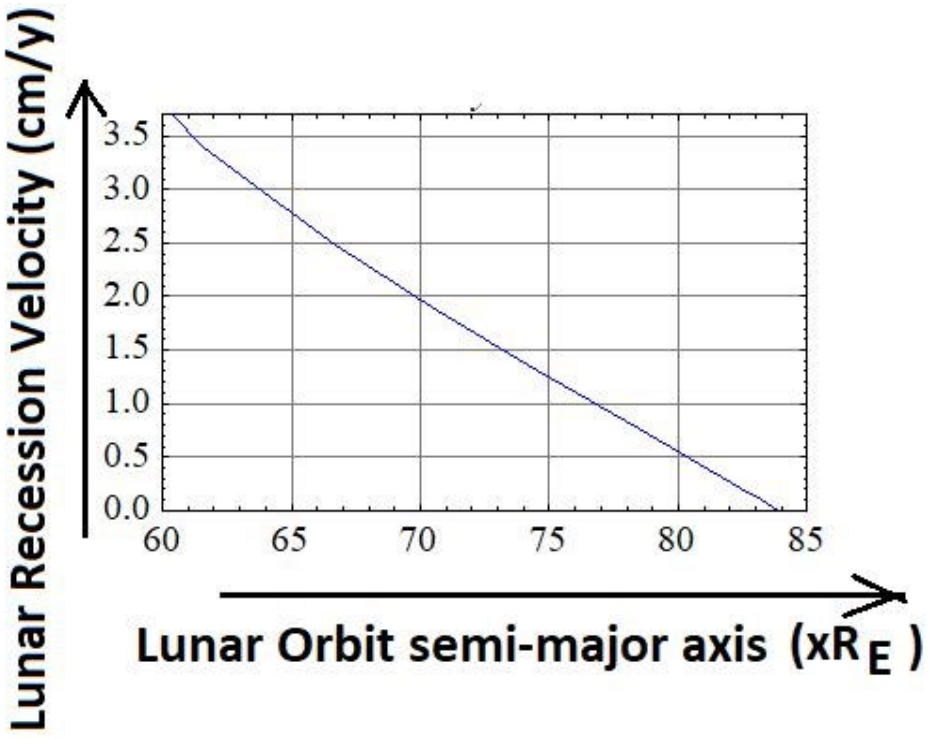


Figure 7. Lunar Recessionary Velocity profile from semi-major axis 3.884x108m (60.336RE) to outer geo-synchronous orbit (83.91RE). [Credit: Author].

Moon recedes from Earth radially in an expanding spiral path and interlocks with outer geo-
synchronous orbit (5.345906x108m = 83.91RE) in unfathomable future 97.304Gy from now.
This completes the evolutionary path of tidally interacting binaries as described in Kinematic
Model [30].

The Moon's orbital plane inclination angles are studied. The results are tabulated in Table 3.
For this futuristic evolution we have chosen the present orbit and 11 futuristic orbits approaching the second geo-synchronous orbit and calculating Eq. (1) in futuristic orbits.

Table 3. Moon's orbital plane inclination (α) in the present lunar orbit and in futuristic 11 orbits.

Semi-major axis a(×10⁸m)	α(radians)	α(degrees)
3.844	0.0897017	5.13953°
3.92	0.0891698	5.10905º
4	0.0887304	5.08388°
4.25	0.0879332	5.0382°
4.5	0.0876678	5.02299°
4.75	0.0876481	5.02187°
5	0.0877082	5.02531°
5.25	0.0877542	5.02795°
5.3	0.0877567	5.028088°
5.345	0.087756	5.028º
5.3459	0.087756	5.028º
5.345906	0.087756	5.028º

It had been noted in, that lunar orbital plane inclination will decrease along with simultaneous increase in terrestrial obliquity [6]. Inspection of Table 3 shows the decreasing trend of Lunar orbital plane inclination up to 4.75×108m and beyond this orbit the trend changes. From 5×108m onward the declining trend is restored. The declining trend of lunar orbital plane inclination is due to constraint of angular momentum conservation. The total angular momentum (the vector sum) of Earth-Moon system should be maintained constant at 3.3749210029333725×1034Kg-m2/s with near perpendicularity with respect to the ecliptic plane. Lunar orbital plane inclination near constancy is corroborated by Conway (1983) and Mignard (1981) [16, 19].

Moon's Obliquity Angle (β) are Studied

Using Eq. (2) the Moon's obliquity angle is calculated. The results are tabulated in Table 3.

Table 4. Moon's spin axis tilt (β) with respect to Ecliptic plane normal in the present lunar orbit and in six futuristic orbits. (From 3.844×108m to 3.92×108m the algorithm given in Sharma 2019 is used for calculating the transit time and beyond 3.92×108m the algorithm given in Materials and Methods of the present text is used.)

Semi-major axis a(×10⁸m)	*β*(radians)	*β*(degrees)	Transit Time
3.844	0.0268833	1.5403°	0
3.87	0.0176152	1.00928°	71.267My
3.89	0.0106075	0.609967°	127.472My
3.91	0.00370563	0.212°	184.92My
3.92	0.000294342	0.0168°	214My
3.9208	0.0000225818	0.0012939°	216.34My
3.92087	0	0	216.54My

As seen from Table 4 Moon's spin axis tilt nullifies with respect to the Ecliptic plane normal and Moon's tidal expansion continues with spin axis tilt of 0º with respect to the Ecliptic Plane Normal. Spin axis of Moon and lunar orbital normal are on the same side of Ecliptic normal hence Moon is permanently locked in Cassini State 1. This Cassini State 1 will persist till the final lock-in at outer geo-synchronous orbit. This transition to Cassini state 1 at a = 3.92 ×108m simplifies the Transit Time analysis and is enumerated in Methods & Materials. This particular aspect has not been pointed out by any previous researcher in the field. This implies that Transit Time calculation up to 3.92×108m is done using the algorithm given in Appy A1. of Sharma 2019 whereas Transit Time

calculation beyond 3.92×108m is done using the algorithm given in Methods and Materials of the present paper.

Moon's Orbit's Eccentricity is Studied in Futuristic Orbits

Using Eq. (3), the eccentricity of Moon's orbit is studied in futuristic orbits. The results are tabulated in Table 5. The anomaly observed by the analysis of multi-decadal record of LLR data points has been kept out of the preview of this paper [31].

Table 5. Moon's orbit eccentricity (e) in the present lunar orbit and in four futuristic orbits. (From 3.844×108m to 3.92×108m the algorithm given in Sharma 2019 is used for calculating the transit time and beyond 3.92×108m the algorithm given in Materials and Methods of the present text is used.)

Semi-major axis a(×108m)	*Eccentricity (e)*	*Transit Time*
3.844	0.0548998	0
3.92	0.0421992	214My
4.0	0.0284268	456.822My
4.15	0.00148841	963My
4.158	0.0000108398	992My

In 992My from NOW at a = 4.158×108m, Moon will be moving in a circularized orbit as well as in synchronous orbit. This had been predicted by Zahn (1992) [2]. Every tidally interacting binaries move towards lower-most energy state with orbits being circularized and synchronized and orbital planes being co-planarized. The time scale will depend on the strength of tidal coupling which in turn will depend inversely on the sixth power of the distance between two bodies. This aspect of futuristic evolution has not been studied before.

Earth's Obliquity Angles

For terrestrial obliquity Eq. (5) is extended into future without any loss of accuracy. Different scenarios were tried but Eq. (5) derived in the prequel paper was found to be the best option hence it has been retained.

Table 6. Terrestrial Obliquity obtained from Eq. (5)

aLunar (x108)	ϕ *radians*	ϕ^o
3.844	0.409113	23.44
3.92	0.431492	24.78
4	0.456	26.148
4.25	0.530657	30.40
4.5	0.604	34.66
4.75	0.679	38.92
5	0.7535	43.17
5.25	0.827	47.43
5.3	0.8427	48.258
5.345	0.85605	49.05
5.3459	0.8563	49.06
5.345906	0.8563	49.06

As Moon's orbit spirally expands from 'a' = 4.25×108m (66.7RE) at 1.3458Gy from NOW to
'a' = 4.5×108m (70.63RE) at 2.5Gy from NOW and length of day lengthens to 1.38 Earth day and 1.61485 Earth day respectively, the current equinoctial precession period of 26,000 years increases to 49,000 years at 'a' = 4.25×108m (66.7RE) and 69,000 years at 'a' = 4.5×108m (70.63RE) respectively [32]. The numerical integration of the differential precession equation of the spin axis of Earth results in a chaotic fluctuation of terrestrial obliquity in this tidally expanded orbit [33]. Obliquity becomes chaotic because of resonance overlapping of Solar System fundamental frequencies s3, s4 and s6 which are the solutions of Laskar Equations [23, 24]. Before the resonance, obliquity varies from 26.6º to 43.4º and after the resonance obliquity varies from 43.4º to 60.2º. At highest obliquity Arctic Circle extends upto latitude 30º. Radical changes in global climate will occur over 105 to 106 years resonance time scales. In the present text, AKM gives terrestrial obliquity of 30.42º and 34.66º at Lunar Orbit of (66.7RE) and (70.63RE) respectively.

Have shown, through numerical simulation of global stability of spin axis orientation (obliquity) against secular orbital perturbations, that none of the inner planet's obliquity are primordial. In case of Mercury and Venus, obliquity has been strongly shaped by tidal dissipation. In case of Earth, Moon has been crucial in determining its eco-friendly obliquity. In case of Mars obliquity is still in chaotic range from 0º to 60º.

4. Results of Futuristic Evolution of E-M System

The length of Day (LOD) and terrestrial obliquity is determined and tabulated in Table 6. The data regarding Lunar orbital plane inclination (α radians) given in Table 2, the data regarding Lunar Obliquity (β radians) given in Table 3, the data regarding lunar orbit eccentricity given in Table 4 and data regarding LOM/LOD given in Table 5 have been combined into one Table 7 for the ease of calculations and for having a bird's eye view of the results. The algorithm for determining these futuristic data is given in Materials and Methods.

Table 7. Earth-Moon system parameters in the present lunar orbit and in ten futuristicorbit.

a(×108m)	α(rad.)	β(rad)	e(ecc.)	ω/	ϕ(radians)	ϕ(degrees)	LOD(d)
3.844	0.0897	0.027	0.055	27.32	0.410007	23.4917	0.99868
4	0.0887	0	0.028	26.225	0.456	26.127	1.10436
4.25	0.0879	0	0	24.2362	0.531	30.42	1.38
4.5	0.0876	0	0	21.4004	0.605	34.66	1.61485
4.75	0.0876	0	0	17.4738	0.679	38.9	2.14981
5	0.0877	0	0	12.0822	0.7535	43.17	3.35
5.25	0.0877542	0	0	4.61707	0.8278	47.43	9.432
5.3	0.0877567	0	0	2.79316	0.8427	48.258	15.786
5.345	0.087756	0	0	1.03656	0.85605	49.05	43.1581
5.3459	0.087756	0	0	1.00024	0.8563	49.06	44.7365
5.345906(aG2)	0.087756	0	0	1.	0.8563	49.06	44.7473

5. Conclusions

The algorithms developed in Sharma,2019, have acted like a time machine using which the Author went back 1.2Gy to determine the orbital-globe dynamics parameters of Earth-Moon system. Using the same very algorithm, the Author has travelled in future up to the very terminal point of E-M system evolution. As predicted by Zahn (1999), E-M system will be circularized and synchronized at a = 4.158x108 in 992My from NOW and move towards a final lock in at triple synchrony state [2]. The new feature which has emerged out of this study is that terrestrial obliquity will progressively increase from 23.44° (at 3.844×108m) to 26.127° (at 4×108m) to 30.42° (at 4.25×108m) to 34.66° (at 4.5×108m) to 38.9° (at 4.75×108m) to 43.17° (at 5×108m) to 47.43° (at 5.25×108m) to 48.28° (at 5.3×108m) to 49.05° (at 5.345×108m) to 49.06° (at 5.3459×108m) to 49.06° (at 5.345906×108m = aG2). This happens solely due to the constraint of Angular Momentum conservation which requires that Total angular momentum of Earth-Moon system is J4 = vector sum of J3 and J2 where J4=3.3749210029333725×1034Kg-m2/s has near perpendicularity with the ecliptic plane. Through visual examination of Figure 7 also it is evident that as Jo increases the terrestrial obliquity will increase to conserve total angular momentum. J4 is inclined with respect to ecliptic normal by a very small angle γ = 0.385445°.

Initially Moon companionship has acted as an anchor which has kept Earth-Moon system at even keel and prevented erratic wobbliness of Earth [23-26]. It was precisely because of this erratic wobbliness that Mars lost all the ingredients for life. But in long run the progressive increasing tilt in Earth's spin axis will mean extreme climates as it exists on Uranus. Earth's obliquity will progressively become eco-hostile. This becomes alarming by the time Moon has reached 4.25×108m at about 1.354Gy from Now. By that time because of changes in Sun as well as due to increased terrestrial obliquity 30.42°, life on Earth will become inhospitable and we will be forced to move to an eco-friendly exo-planet [10]. In 1.1Gy from NOW along the Main Sequence path on H-R diagram Sun will be 10% brighter (S5.3, SOM). This extra heat absorbed by Earth will cause a moist greenhouse planet leading to a hot and dry Venus like environment. Further on in 3.5 Gy, Sun's luminosity will be 40% brighter turning Earth into a runaway greenhouse planet bringing a Solar catastrophe. Oceans will boil off, ice-caps will be permanently lost and atmospheric moisture will be lost to outer space. Earth will become a molten remnant.

This paper stands out as compared to earlier works on futuristic evolution in three aspects:
i. This paper is a detailed and exhaustive study of evolving E-M system orbit elements with all transit times calculated in an accurate manner from AKM.
ii. Lunar Obliquity (β) has not been studied in any earlier work. At a = 3.92×108m, E-M system falls into Cassini State I.
iii. Eccentricity (e) has not been studied in this detail by any earlier work. At a =4.158×108m, E-M system falls into a perfectly circularized and synchronized state as predicted by in about 992.53Gy [2]. The anomalous increase in Moon's orbital eccentricity as reported by has been kept out of the purview of this paper [31]. The Author has not brought this paper under the purview of this paper because the Author want to verify the reliability of the results of the given paper.
This futuristic study clearly points out that conditions on Earth will not remain habitable and we must start packing up lock-stock and barrel and look for an alternative home planet The search for an alternative home planet is urgent on two counts: due to the ever-changing conditions of Sun and also due to the tidally evolving Earth-Moon system with worsening terrestrial obliquity with each passing day and also due to chaotic obliquity at 2Gy from NOW due to resonance passage [27]. The full impact of this deteriorating Earth's axial tilt will have to be assessed in a separate paper.

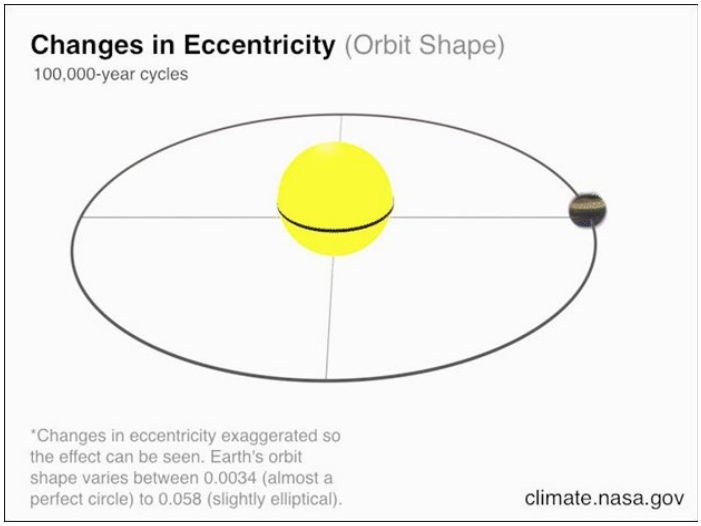
Futuristic Evolution of E-M system is the unfolding of tidally interacting binaries to its Total Energy Minima Valley orbit (see supplementary-on-line materials S1.5) and that is precisely what we find in this research paper. In 214My from NOW Moon's obliquity gets nullified at 3.92x108m = 61.53RE orbital radius. In 992.53My Moon's orbit is completely circularized and continues to be locked in synchrony with Moon's spin as it is today. The Total Angular Momentum constraint forces Earth's obliquity to 30° in 1.3458Gy which triggers the destruction of Habitat Trinity (see supplementary-on-line materials S2) S5.1 as it triggered on Planet Mars. Numerical calculation of precessional rotation differential equation show that through resonance passage, Moon enters obliquity chaotic zone. This will turn Earth non-habitable. So, we must move on to an exo-planet lock-stock and barrel.

Earth-Moon system in unfathomable future 97Gy from NOW will get inter-locked at triple synchrony stage of 44.75 days at orbital radius of outer geo-synchronous orbit 5.345906x108m = 83.91RE and it will remain thus until solar perturbations deflects Moon onto a death spiral into a glancing angle collision with Earth [34]. This is true with the following caveat:
i. If E-M system was to survive the Red-Giant Stage
ii. and survive post-AGB strong winds
iii. and survive Planetary Nebula formation and expansion (which spreads the chemically enriched ashes, including Carbon, of the dying Sun throughout the Cosmos)
iv. and if Earth-Moon system was to survive to the end of the present Big Bang stage of bouncing Universe (Cheung et.al.2019) then only E-M system will reach the fire-wall of outer geo-synchronous orbit [12].

This paper deals with the broader and secular trends and the consequences thereof from the E-M system tidal evolution. In 1.0 Gy from Now, Moon's orbit will be fully circularized and continue to be synchronized as it is today and Length of (Earth) Day would be lengthened from present era 1 day to 1.38 day. The terrestrial obliquity from today's eco-friendly value of ϕ =23.44º will reach an eco-hostile value of ϕ = 30.4º in 1.354Gy as can be seen in Mars. Large obliquity angles early in Mars existence led to irreversible climate collapse and destruction of Habitat Trinity [35, 36]. These secular changes in E-M system in future will be accompanied with the Milankovitch cycle of climate changes which is cyclic and has a definite period of repetition [37]. Milankovitch repetitive cycle occurs due to cyclic repetition in Earth's orbit shape change, orientation change and axial precession of the Earth's spin axis around the Ecliptic normal.

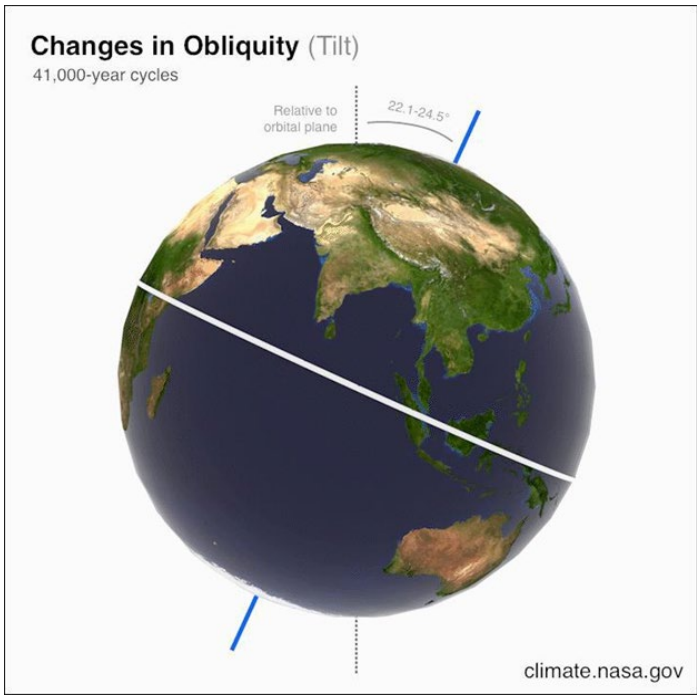
First Milankovitch cycle is eccentricity cycle. The giant gas planets Jupiter and Saturn modulate Earth's orbit's shape from an ellipse to circular and back to ellipse again. This repeats itself in 100,000 years.

Due to eccentricity seasons are not symmetrical in duration. Right now, in Northern Hemisphere Summer is 4 to 5 days longer than Winter and Spring is 3 days longer than Autumn. At mid-latitudes i.e. 30º to 60º N or S of equator receives 23% more insolation when circular orbit turns most eccentric.



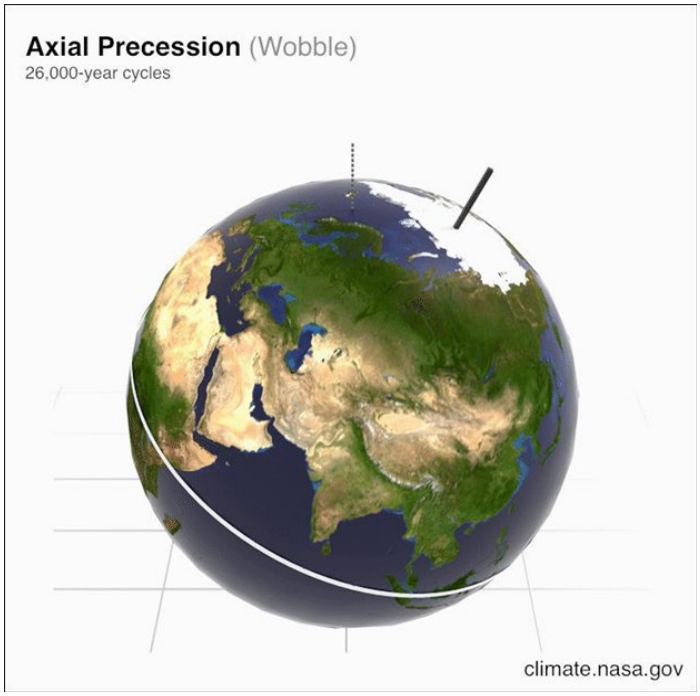
Animation 1. See Animation of Eccentric Orbit

Second Milankovitch cycle is terrestrial obliquity which oscillates between 22.4º to 24.5º over 41,000y cycle. A lower obliquity means milder summer and milder winter whereas higher obliquity means a harsher season. Hotter summers and colder winters.



Animation 2, See Animation of Oscillating Earth's Obliquity

Third Milankovitch cycle is the axial precession with repetition period of 25,771.5 years and apsidal precession with a repetition period of 112,000 years. The two combined have a repetition period of 23,000 years. These precessions are caused due to Sun tides and Moon tides on Earth. In the present times our Earth's spin axis is pointing towards Polaris Star. 13,000 years later spin axis will be pointing towards Vega star. In the present era in Northern Hemisphere, winter falls at the perihelion and summer falls at aphelion. In 13,000 years, this will be reversed.



Animation 3. See animation of precession of the spin axis of Earth around the normal to the Ecliptic Plane. Ecliptic Plane is the mean plane and Earth's spin axis is precessing around the normal of this mean plane. Hence Ecliptic Plane is called the Laplacian Plane of Earth.

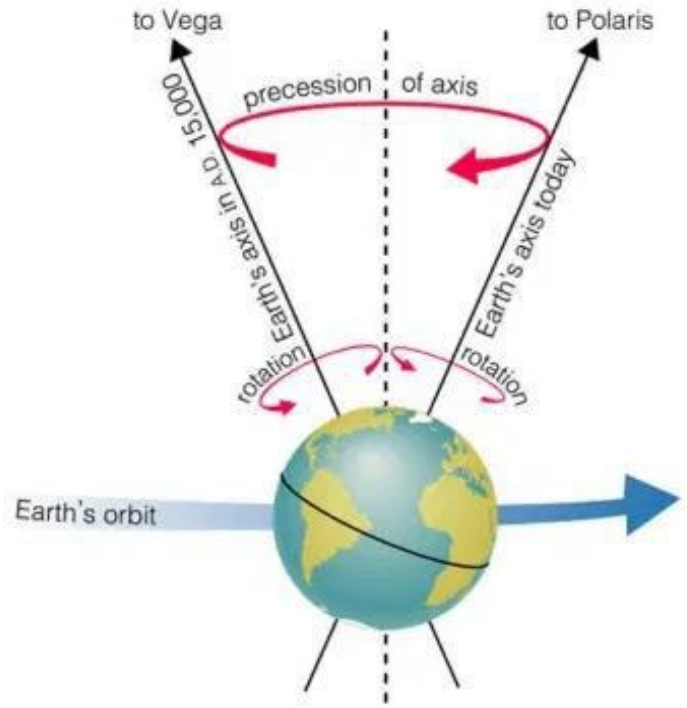


Figure 8. Axial precession of Spin Axis of Earth. The Earth's spin axis is processing around the Ecliptic Normal due to Moon tides and Sun tides.

According to Milankovitch, Ice ages occur at 41,000 yeas inter-glacial interval between 1 t0 3 My ago. Since last 800,000 years ago the interglacial intervals increased to 100,000 years and lasted for 100,000 years. Last Ice Age peaked at 18,000 ya and ended at 11,700ya. A more detailed description is available in S3-SOM.

The same physics is applicable in future as it is today. Angular momentum is constrained to be conserved and constrained to be held constant in future right up to the final interlocking state when E-M system achieves triple synchrony at a = 86.78RE. At this end point, the orbital period = Spin Period of Moon= Spin Period of Earth = 47days. In this paper according to AKM outer geo-synchronous orbit semi major axis is 5.345906x108m = 83.91RE and triple synchrony period is 44.75days.

So, all calculations and predictions will hold good in future. In the future eons Moon will be in synchronous state and Moon's spin period = Orbit period will be greater than Earth's spin period and Moon will be in super-synchronous state hence angular momentum will continue to be transferred from Earth to Orbital System Only after reaching final lock-in position at outer geo-synchronous orbit due to solar perturbation Earth will go in spin-down condition. At this point Earth spin period will become longer than orbital period of E-M system and Moon will fall in sub-synchronous state and Moon will get deflected from outer geo-synchronous orbit (89.54RE/84.162RE) into a collapsing in-spiral orbit. Hence up to the final interlocking state when it is in triple synchrony state the angular momentum will continue to be transferred from Earth to Lunar Orbital System. This is unequivocal.

Acknowledgments, Samples, and Data

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Availability of Data

All data needed for carrying out the futuristic evolution of Earth-Moon system is available in Section 3 of the main text and in supplementary information file. All the algorithm and data are deposited in "Open Topography Community Data Space". My contribution in under review therefore the link is not available.

Conflict of Interest

I have no conflict of interest financial or otherwise whatsoever with anybody.

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