

The Last Installment of Mass, Energy and Space-Time: On the Harmony of Light and the Universe through the Planck Scale

Chana Pongpothakul^{1*} , Nawapol Bourin¹ and Pairoj Vajchamanant²

¹Engineering Sector, Metropolitan Waterworks Authority, Bangkok, Thailand.

*Corresponding Author

Chana Pongpothakul, Engineering Sector, Metropolitan Waterworks Authority, Bangkok, Thailand.

²Independent Engineer.

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Abstract

This paper expands our previous work on the rotating photon framework—which structures the photon as a localized entity executing a transverse rotational trajectory within a physical space-time medium—into a unified, scale-invariant quantum cosmology. By rejecting the classical vacuum in favor of a dynamic constraint framework, we previously demonstrated that Planck’s constant arises naturally from a fundamental photon rest mass metric ($m_0^ \approx 1.17 \times 10^{-51}$ kg) stabilized at a baseline frequency of $\omega_0 = 1$ rad/s. Here, we explore the deep macro-cosmological implications of this micro-quantum mechanism by extending its boundary conditions to the infrared (IR) cutoff defined by the causal light-travel horizon of the observable universe (T_{universe}).*

Our formulation reveals that the total accumulated mass-energy density of the cosmos continuously evolves in perfect alignment with its gravitational boundary, yielding a calculated cosmic mass ($M_{\text{Cosmo}} = \frac{c^3}{G} T_{\text{universe}} \approx 1.756 \times 10^{53}$ kg) that accounts for approximately 5.8% of the critical density (ρ_{critical})—matching the empirical baryonic matter density (Ω_b) with extraordinary precision. Furthermore, we uncover a profound holographic duality wherein the product of the cosmic macro-horizon and the subnuclear quantum extremities collapses symmetrically into the invariant Planck scale (ℓ_P^2 and m_P^2). This scale-invariant equilibrium is mediated by a dynamic coupling factor K , which frames the distribution of cosmic energy as a harmonic ratio bounded by the ultraviolet Planck frequency and the expanding cosmological IR frequency. Ultimately, these concurrent invariants conclusively establish the Planck scale as the perfect geometric mean and dynamic balance point unifying the infinitesimal quantum world of particle physics with the macroscopic canvas of physical cosmology.

Keywords: Rotating Photon Framework, Scale Invariant, Quantum Cosmology, Planck Scale, Dynamic Coupling Factor

1. Introduction & Theoretical Background

In our previous research into the physical mechanisms underlying the relativistic mass, we rejected the premise of an empty space or vacuum as absolute nothing [1,2]. Instead, we conceptualized space-time as a physical medium capable of exerting mechanical work on moving particles through a space-time constraint force and capable of propagating energy packets. Under this framework, a photon is considered as a physical, localized moving point particle capable of reaching the speed of light. As this entity moves linearly through the space-time medium at the velocity of light (c), the medium exerts a continuous perpendicular constraint impulse. This transverse action forces the photon to rotate, having a rotational trajectory wrapping around its primary axis of motion.

Consequently, the photon’s total momentum is shared dynamically between its forward “particle-being” linear path and its “wave-being” internal angular frequency (ω). At the rest frame, where the forward linear velocity become zero, the photon’s internal rotation stabilizes at a fundamental angular frequency (ω_0) of 1 radian per second. To mathematically preserve standard International System (SI) dimensions across the wave-particle boundary, we isolated the energy difference (ΔE) transferred to the medium between successive rotational cycles. By mapping this discrete energy step over a normalized unit time frame ($t = 1$ s), we demonstrate that Planck’s constant

(h) is a direct product of the speed of light and the invariant photon rest mass-time (m_0^*t), formulated as follows:

$$h = 2\pi m_0^* c^2 t \quad (1)$$

We also expressed the photon's wave energy in terms of both frequency and moving (relativistic) mass as:

$$E_w = h\nu = 2\pi m_0^* c^2 t\nu = m_o^* \omega t c^2 = m_\gamma c^2 \quad (2)$$

The relationship between the photon's relativistic mass and its rest mass is given by:

$$m_\gamma = m_o^* \omega t \quad (3)$$

$$= m_o^* \frac{\omega}{\omega_o} (\omega_o t) \quad (4)$$

This leads to the invariance of the product of the photon's rest mass angular frequency and time, as shown in the following equation:

$$\omega_o t = 1 \quad (5)$$

Utilizing the standardized SI constants over a temporal unit of one second, this yields a theoretical fundamental mass scale of:

$$m_0^* = \frac{h}{2\pi c^2} \approx 1.17 \times 10^{-51} \text{ kg} \quad (6)$$

That gives the photon's minimum relativistic mass at the angular frequency of $\frac{\pi}{2}$ rad/s or 0.25 Hertz as:

$$m_\gamma^1 = m_o^* \frac{\pi}{2} \approx 1.84 \times 10^{-51} \text{ kg} \quad (7)$$

While these formulas successfully unified a photon's wave properties (spin angular momentum) and particle properties (rest mass) at a local scale, its micro-quantum scale and macro-cosmological scale implications remained unexplored.

2. Planck Scale: First-Principles Derivation via Dimensional Analysis

The Planck scale defines the absolute operational limits of contemporary theoretical physics, representing the regime where the continuous geometry of general relativity conflicts directly with the discrete probabilities of quantum mechanics. Formulated strictly using fundamental physical constants—the vacuum speed of light (c), the Newtonian gravitational constant (G), and the reduced Planck constant ($\hbar$)—this scale delineates the minimum thresholds of length, time, and the corresponding maxima of energy density.

The Planck scale is uniquely determined by isolating the combinations of the fundamental constants that govern relativistic, gravitational, and quantum domains. These constants map to their respective SI units and fundamental dimensions (M for mass, L for length, T for time):

$$[c] = \text{m} \cdot \text{s}^{-1} = LT^{-1} \quad (8)$$

$$[G] = \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2} = M^{-1} L^3 T^{-2} \quad (9)$$

$$[\hbar] = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1} = ML^2 T^{-1} \quad (10)$$

For practical theoretical calculations and numerical evaluations, these fundamental parameters are anchored to the standard CODATA recommended values [3]. Expressed in approximate forms, these constants are quantified as follows:

$$c \approx 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1} \quad (11)$$

$$G \approx 6.674 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2} \quad (12)$$

$$\hbar \approx 1.054 \times 10^{-34} \text{ J} \cdot \text{s} \quad (13)$$

To derive any fundamental Planck unit U , we construct a power-law product of these constants:

$$U = c^\alpha G^\beta \hbar^\gamma \quad (14)$$

Equating the dimensions yields a solvable system of linear equations for exponents α , β , and γ .

2.1. The Planck Length (l_p)

Setting the dimensional equation equal to a pure length ($M^0 L^1 T^0$):

$$(LT^{-1})^\alpha (M^{-1} L^3 T^{-2})^\beta (ML^2 T^{-1})^\gamma = M^0 L^1 T^0 \quad (15)$$

Grouping by base dimensions yields:

$$\text{Mass (M)} : -\beta + \gamma = 0 \implies \gamma = \beta \quad (16)$$

$$\text{Length (L)} : \alpha + 3\beta + 2\gamma = 1 \quad (17)$$

$$\text{Time (T)} : -\alpha - 2\beta - \gamma = 0 \implies \alpha = -2\beta - \gamma \quad (18)$$

Substituting $\gamma = \beta$ into the time equation yields $\alpha = -3\beta$. Substituting both expressions into the length equation:

$$-3\beta + 3\beta + 2\beta = 1 \implies 2\beta = 1 \implies \beta = \frac{1}{2} \quad (19)$$

Thus, $\gamma = \frac{1}{2}$ and $\alpha = -\frac{3}{2}$. This yields the physical boundary for space:

$$l_P = c^{-3/2} G^{1/2} \hbar^{1/2} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{ meters} \quad (20)$$

2.2 The Planck Time (t_p)

Setting the dimensional equation equal to pure time ($M^0 L^0 T^1$) and solving the corresponding system yields $\alpha = -\frac{5}{2}$, $\beta = \frac{1}{2}$, and $\gamma = \frac{1}{2}$. This gives the duration required for light to traverse a single Planck length:

$$t_P = \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.391 \times 10^{-44} \text{ seconds} \quad (21)$$

2.3 The Planck Mass (m_p) and Planck Energy (E_p)

Isolating for mass dimensions ($M^1 L^0 T^0$) yields $\alpha = \frac{1}{2}$, $\beta = -\frac{1}{2}$, and $\gamma = \frac{1}{2}$:

$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.176 \times 10^{-8} \text{ kg} \quad (22)$$

While m_p represents a macroscopically tangible mass, compressing this mass into a subatomic Planck volume generates an immense energy profile via Einstein's mass-energy equivalence ($E_p = m_p c^2$):

$$E_P = \sqrt{\frac{\hbar c^5}{G}} \approx 1.22 \times 10^{19} \text{ GeV} \approx 1.956 \times 10^9 \text{ Joules} \quad (23)$$

2.4 The Planck Frequency (ω_P)

The Planck frequency (ω_P) is the inverse of the Planck time (t_P). It represents the maximum possible limit of physical oscillation in the universe. It is formulated as:

$$\omega_P = \frac{1}{t_P} = \sqrt{\frac{c^5}{\hbar G}} \quad (24)$$

Frequency is measured in Hertz (s^{-1}). In dimensional terms, it is expressed as:

$$[\text{Frequency}] = M^0 \cdot L^0 \cdot T^{-1} \quad (25)$$

Solving the corresponding system yields $\alpha = \frac{5}{2}$, $\beta = -\frac{1}{2}$, and $\gamma = -\frac{1}{2}$, so:

$$\omega_P = \sqrt{\frac{c^5}{\hbar G}} \approx 1.85 \times 10^{43} \text{ Hertz} \quad (26)$$

2.5 The Planck Force (F_P)

In SI units, force is measured in Newtons ($kg \cdot m/s^2$). In dimensional terms, this is expressed using Mass (M), Length (L), and Time (T):

$$[\text{Force}] = M^1 \cdot L^1 \cdot T^{-2} \quad (27)$$

Setting the dimensional equation equal to pure force ($M^1 \cdot L^1 \cdot T^{-2}$) and solving the corresponding system yields $\alpha = 4$, $\beta = -1$, and $\gamma = 0$ so the final formula for the Planck force is expressed as:

$$F_P = \frac{c^4}{G} \approx 1.21 \times 10^{44} \text{ Newtons} \quad (28)$$

Planck force represents a foundational limit in theoretical physics where the gravitational attraction between two particles equals their strong quantum effects.

3. Compton Scale Connector

According to quantum mechanics, a particle's spatial localization is bounded by its reduced Compton wavelength ($\bar{\lambda}_C$):

$$\bar{\lambda}_C = \frac{\hbar}{mc} \quad (29)$$

The characteristic time τ_c (or **Compton time**) for light to cross this distance is defined as:

$$\tau_c = \frac{\bar{\lambda}_C}{c} = \frac{\hbar}{mc^2} \quad (30)$$

By recognizing that the fundamental periodic nature of a particle can also be viewed in the frequency domain, we define the Compton angular frequency (ω_C) as the strict mathematical reciprocal of this characteristic time:

$$\omega_C = \frac{1}{\tau_c} = \frac{mc^2}{\hbar} \quad (31)$$

Rearranging these formulas yields a dual formulation of the Compton scale connector relation. Expressed via characteristic time, the mass-time product is fixed:

$$m \cdot \tau_c = \frac{\hbar}{c^2} \quad (32)$$

Concurrently, when expressed via angular frequency, the mass-to-frequency ratio is locked by the same physical parameters:

$$\frac{m}{\omega_C} = \frac{\hbar}{c^2} \quad (33)$$

These twin relations show that for any elementary quantum particle, the product of its mass and its Compton time—as well as the ratio of its mass to its Compton frequency—is a universal constant equal to $\frac{\hbar}{c^2}$.

The Compton scale connector acts as a deep conceptual bridge. It demonstrates that a particle's mass, its internal quantum clock rate (ω_C), and its localization time horizon (τ_c) are not independent variables. Instead, they are fundamentally locked together by the laws of quantum mechanics and special relativity.

The following table illustrates how different physical systems satisfy these invariant properties, maintaining the universal invariant value of $m \cdot \tau_c = \frac{m}{\omega_C} \approx 1.173 \times 10^{-51} \text{ kg} \cdot \text{s}$:

Particle	Mass m (kg)	Compton Time τ_c (s)	Compton Freq. ω_C (rad/s)	Invariant $m\tau_c = \frac{m}{\omega_C}$ (kg·s)
Electron	$\approx 9.11 \times 10^{-31}$	$\approx 1.29 \times 10^{-21}$	$\approx 7.76 \times 10^{20}$	$\approx 1.17 \times 10^{-51}$
Proton	$\approx 1.67 \times 10^{-27}$	$\approx 7.02 \times 10^{-25}$	$\approx 1.43 \times 10^{24}$	$\approx 1.17 \times 10^{-51}$
Planck Mass	$\approx 2.18 \times 10^{-8}$	$\approx 5.39 \times 10^{-44}$	$\approx 1.86 \times 10^{43}$	$\approx 1.17 \times 10^{-51}$

Table 1: Compton Scale Connectors and Invariants for Fundamental Particles

4. Physical Derivation:

4.1. The Convergence of Quantum and Gravity

To explore the interplay between gravitational physics and quantum mechanics, we begin by examining a localized system where a central mass M exerts a gravitational force on a test particle (or a photon equivalent) of mass m at a distance r . In classical Newtonian mechanics, this force is governed by the gravitational inverse-square law:

$$F_G = \frac{GMm}{r^2} \quad (34)$$

The corresponding gravitational potential energy U required to bring this test mass from infinity to a distance r within the gravitational field is given by integrating the force over the distance, yielding:

$$U = F_G r = \frac{GMm}{r} \quad (35)$$

To bridge this classical framework with quantum theory, we introduce the wave-particle duality of light. According to the Planck-Einstein relation, the relativistic mass-energy equivalence of a photon with frequency f can be expressed as:

$$mc^2 = hf \quad (36)$$

By isolating the effective gravitational mass m of the photon from Equation(36), we obtain:

$$m = \frac{hf}{c^2} \quad (37)$$

Substituting this quantum-mechanical mass equivalent back into the classical gravitational potential energy formula allows us to analyze how gravity scales with quantum energy:

$$U = \frac{GM}{r} \frac{hf}{c^2} \quad (38)$$

$$= \frac{GM}{rc^2} hf \quad (39)$$

$$= \alpha hf = \alpha \hbar \omega \quad (40)$$

Here, $\alpha = \frac{GM}{rc^2}$ serves as a dimensionless coupling parameter that dictates the ratio of gravitational potential energy to the photon's intrinsic quantum energy.

We now consider a critical boundary condition where the gravitational potential energy is exactly equal to the quantum energy of the photon itself ($U = hf$). Under this localized threshold, the coupling constant simplifies to $\alpha = 1.0$, which implies:

$$\frac{GM}{rc^2} = 1.0 \quad (41)$$

Solving Equation (41) for the radius r defines a critical gravitational boundary condition for the mass M :

$$r = \frac{GM}{c^2} \quad (42)$$

Note: In general relativity, this radius represents exactly half of the Schwarzschild radius ($r_s = \frac{2GM}{c^2}$), marking the structural boundary of a fundamental event horizon.

Concurrently, quantum mechanics dictates that any particle possessing momentum exhibits a characteristic spatial footprint. Using the effective photon mass $m = \frac{hf}{c^2}$ from Equation (37) and the relationship $c = f\lambda$, the de Broglie wavelength λ of this system is expressed as:

$$\lambda = \frac{h}{mc} \quad (43)$$

For the particle to form a stable, self-contained quantum state, its confinement path must satisfy the periodic boundary condition where the circumference of its orbit equals its de Broglie wavelength ($2\pi r = \lambda$). This restricts the quantum radius to:

$$r = \frac{h}{2\pi mc} \quad (44)$$

The unification of quantum mechanics and general relativity occurs at the exact spatial intersection where the critical gravitational radius matches the quantum confinement radius:

$$\frac{GM}{c^2} = \frac{h}{2\pi mc} \quad (45)$$

Isolating the macroscopic mass M from this equilibrium condition yields:

$$M = \frac{h}{2\pi} \frac{1}{mc} \frac{c^2}{G} \quad (46)$$

By grouping the terms, we find that the product of the gravitational mass and the quantum mass is entirely governed by fundamental universal constants:

$$Mm = \frac{hc}{2\pi G} = \frac{\hbar c}{G} \quad (47)$$

Finally, we evaluate the self-referential limit where the central gravitational mass is identical to the localized quantum test mass ($m = M$). Under this extreme condition of quantum-gravitational self-interaction, the masses merge into a single fundamental threshold:

$$M = \sqrt{\frac{\hbar c}{G}} \equiv m_P \quad (48)$$

By comparing this definition directly with the general quantum-gravitational mass balance derived earlier in Equation (47), we arrive at a beautiful symmetry governing the dual-mass system:

$$Mm = m_P^2 \quad (49)$$

This derivation proves that the **Planck mass** represents the exact physical scale where an object's quantum Compton wavelength becomes identical to its gravitational event horizon. By marking this fundamental boundary of quantum gravity, it demonstrates that the product of the macroscopic gravitational mass M and the microscopic quantum mass m is explicitly quantized by the square of the Planck mass, thereby establishing the core scaling law for quantum gravity exploration.

By substituting this unified mass threshold ($M = m = m_P$) back into the geometric boundary configurations, the corresponding physical radius collapses into the absolute lower limit of spatial dimension allowed by nature, the **Planck length**:

$$r = \frac{\hbar}{m_P c} = \sqrt{\frac{\hbar G}{c^3}} \equiv \ell_P \quad (50)$$

4.2 The Convergence of Quantum and Relativistic Limits

Consider a particle of mass m . The reduced Compton wavelength of a particle is given by:

$$\bar{\lambda}_C = \frac{\hbar}{mc} \quad (51)$$

According to general relativity, any mass m confined within its own gravitational Schwarzschild radius (R_s) undergoes irreversible gravitational collapse:

$$R_s = \frac{2Gm}{c^2} \quad (52)$$

Setting these two length scales equal ($\bar{\lambda}_C = R_s$) balances quantum localization against general relativistic collapse:

$$\frac{\hbar}{mc} = \frac{2Gm}{c^2} \implies m^2 = \frac{\hbar c}{2G} \implies m = \sqrt{\frac{\hbar c}{2G}} = \frac{1}{\sqrt{2}} m_P \quad (53)$$

Substituting this mass value back into either radius equation yields:

$$R_s = \sqrt{2} \frac{G}{c^2} \sqrt{\frac{\hbar c}{G}} = \sqrt{2} \sqrt{\frac{\hbar G}{c^3}} = \sqrt{2} \ell_P \quad (54)$$

4.3 Photon Mass and the Heisenberg Uncertainty Principle

The energy-time formulation of the **Heisenberg Uncertainty Principle** is expressed as:

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \quad (55)$$

By substituting Einstein's mass-energy equivalence ($\Delta E = \Delta m \cdot c^2$), the theoretical minimum limit of photon's rest mass becomes:

$$\Delta m \geq \frac{\hbar}{2 \cdot c^2 \cdot \Delta t} \quad (56)$$

By plugging in the constant values ($\hbar \approx 1.054 \times 10^{-34} \text{ J} \cdot \text{s}$ and $c \approx 3 \times 10^8 \text{ m/s}$), and applying the maximum amount of time a photon could exist (limited by the age of the universe ; $t \approx 13.8 \text{ billion years} \approx 4.35 \times 10^{17} \text{ seconds}$) the minimum rest mass of a photon to be $\approx 1.35 \times 10^{-69} \text{ kg}$ (or about $7.57 \times 10^{-34} \text{ eV}/c^2$).

4.4 The Hubble Radius

In relativistic cosmology, the maximum boundary of the visible universe is defined by the Hubble radius (R_H), representing the distance at which the baseline velocity of light matches the cosmic boundary limit:

$$R_H = \frac{c}{H_0} \quad (57)$$

Using Planck Λ CDM value for the baseline Hubble frequency ($H_0 \approx 2.18 \times 10^{-18} \text{ s}^{-1}$) [4], the physical size of the cosmic horizon can be determined as follows:

$$R_H \approx \frac{3.00 \times 10^8 \text{ m/s}}{2.18 \times 10^{-18} \text{ s}^{-1}} \approx 1.38 \times 10^{26} \text{ m} \quad (58)$$

4.5 Current Cosmological Mass Estimations

In modern observational cosmology, the total mass within the observable universe is determined using the critical density (ρ_c) frameworks quantified by the *Planck Collaboration* [4]. The critical mass density required for a flat geometric cosmos is expressed via the Friedmann equation:

$$\rho_c = \frac{3H_0^2}{8\pi G} \approx 8.5 \times 10^{-27} \text{ kg/m}^3 \quad (59)$$

Taking the comoving radius of the observable universe (the particle horizon) to be $R_{\text{obs}} \approx 4.65 \times 10^{10} \text{ light-years} \approx 4.40 \times 10^{26} \text{ m}$, the total spherical volume equals $V_{\text{obs}} \approx 3.57 \times 10^{80} \text{ m}^3$ [5].

By partition of the standard Λ CDM parameters, the mass bounds are classified into three core sectors:

1. Ordinary (Baryonic) Matter ($\Omega_b \approx 4.9\%$):

$$M_{\text{baryonic}} = \Omega_b \rho_c V_{\text{obs}} \approx 1.5 \times 10^{53} \text{ kg} \quad (60)$$

2. Total Matter (Including Dark Matter) ($\Omega_m \approx 31.7\%$):

$$M_{\text{matter}} = \Omega_m \rho_c V_{\text{obs}} \approx 9.6 \times 10^{53} \text{ kg} \quad (61)$$

3. Total Mass-Energy Equivalent ($\Omega_{\text{tot}} = 100\%$):

$$M_{\text{total}} = \rho_c V_{\text{obs}} \approx 3.0 \times 10^{54} \text{ kg} \quad (62)$$

5. The Rotating Photon Framework and the Fundamental Mass-Time Invariance ($\hbar/c^2$)

Our concept of photon self-rotation suggests that Planck's constant represents the difference of photon's energy between successive rotational cycles over a normalized unit time frame, and the general mathematical relationship can be described as:

$$\hbar = 2\pi m_0^* c^2 t \quad (63)$$

By isolating the intrinsic mass metric (m_0^*) and introducing the reduced Planck's constant ($\hbar = h/2\pi$), the expression reduces to universal

mass-time constant, which we refer to that as quantum mass-time invariant (m_{inv}):

$$m_{\text{inv}} = m_0^* \cdot t = \frac{h}{2\pi c^2} = \frac{\hbar}{c^2} \quad (64)$$

Crucially, from the standpoint of quantum mechanics, the quantity $\frac{\hbar}{c^2}$ possesses rigorous dimensional coherence with the Compton scale connector introduced in Equation (32), and the mathematical product of the Planck mass (m_p) and the Planck time (t_p), which is proven as follows:

$$m_P \times t_P = \sqrt{\frac{\hbar c}{G}} \times \sqrt{\frac{\hbar G}{c^5}} = \sqrt{\frac{\hbar^2 c G}{G c^5}} = \sqrt{\frac{\hbar^2}{c^4}} = \frac{\hbar}{c^2} \quad (65)$$

This exact algebraic invariance reveals a hidden universal link connecting the localized rotational mechanics of the photon directly to the fundamental parameters of quantum gravity.

Additionally, if we look back at gravitational-quantum mass relation equation (Equation (47)) and replace the general mass term m with our intrinsic mass metric m_0^* , we can substitute the relationship between h and m_0^* from Equation (63) into that gravitational-quantum mass relation to yield:

$$M m_0^* = \frac{2\pi m_0^* c^2 t}{2\pi} \frac{c}{G} \quad (66)$$

Canceling the common intrinsic mass terms and constants simplifies directly to:

$$M = \frac{c^3}{G} t \quad (67)$$

By expressing this relation as a time-derivative ratio of mass accumulation or localized geometric scaling, we can isolate a purely relativistic constant. This fundamental ratio is referred to as the *gravitational mass-time invariant* (M_{inv}), which represents the constant rate of mass scaling per unit time:

$$M_{\text{inv}} = \frac{M}{t} = \frac{c^3}{G} \equiv \frac{F_P}{c} \quad (68)$$

where $F_P = \frac{c^4}{G}$ represents the fundamental Planck force, a limiting value in relativistic mechanics.

When considering this relationship over a standard fundamental temporal unit of one second ($t = 1$ s), the equation yields a characteristic mass threshold equivalent to the ratio of the cube of the speed of light to the gravitational constant:

$$M_1 = \frac{c^3}{G} \times 1 \text{ s} \quad (69)$$

To understand the physical scale of this resulting mass, we can evaluate Equation (69) using standard CODATA physical constants ($c \approx 3.00 \times 10^8$ m/s and $G \approx 6.674 \times 10^{-11}$ m³ kg⁻¹ s⁻²). The calculation yields a mass value of:

$$M_1 \approx 4.037 \times 10^{35} \text{ kg} \quad (70)$$

When compared to cosmological structures, this mass value is immensely large, far exceeding ordinary stellar scales. By comparing it directly with the Solar Mass [6] ($1M_{\odot} \approx 1.989 \times 10^{30}$ kg), we find that:

$$M_1 \approx 2.03 \times 10^5 M_{\odot} \quad (71)$$

This reveals that the mass accumulated or scaled within a single second under this relativistic framework is equivalent to approximately 203,000 times the mass of our Sun. In an astrophysical context, this value does not represent a single star, but rather aligns precisely with the total mass of an Ultra-Faint Dwarf Galaxy (UFD) or a massive globular cluster, bridging the gap between fundamental quantum-relativistic invariants and observable galactic scales [7].

5.1 The Ultraviolet Boundary: Convergence at the Planck Scale

We now transform the fundamental time unit of reference from the macro-scale SI second to the ultimate physical lower limit dictated by quantum gravity: the Planck time (t_P). Under this extreme temporal compression, the localized mass-energy density scaling of the photon must obey an inverse proportionality relation:

$$m_{\text{photon(Planck)}} = m_0^* \times \frac{1 \text{ s}}{t_P} = \frac{\hbar}{c^2 t_P} \quad (72)$$

Substituting the definition of the Planck time ($t_P = \sqrt{\frac{\hbar G}{c^5}}$) into Eq. (72) yields:

$$m_{\text{photon(Planck)}} = \frac{\hbar}{c^2 \sqrt{\frac{\hbar G}{c^5}}} = \sqrt{\frac{\hbar c}{G}} \equiv m_P \quad (73)$$

To evaluate the intrinsic rotational dynamics of the photon under this framework, we establish a baseline reference state at the macro-scale (SI second). Let ω_o be the fundamental angular frequency operating as the phase reference over a unit time boundary, satisfying:

$$\omega_o t = 1 \quad (74)$$

As the temporal dimension undergoes extreme compression from 1 second down to the Planck scale, the localized angular frequency scales inversely with the temporal metric, mirroring the energy density scaling law. Thus, the accelerated rotational rate of the photon is governed by:

$$\omega_{\text{photon(Planck)}} = \omega_o \times \frac{1 \text{ s}}{t_P} \quad (75)$$

By substituting the unit phase constraint ($\omega_o \cdot 1 \text{ s} = 1$), the expression simplifies directly to the ultimate physical upper limit of rotation, defined as the Planck angular frequency:

$$\omega_{\text{photon(Planck)}} = \omega_P \quad (76)$$

In this regime, the fundamental rotational speed of the photon approaches the Planck angular frequency ($\omega_P = 1/t_P \approx 1.85 \times 10^{43} \text{ rad/s}$). Computing the corresponding reduced Compton wavelength ($\bar{\lambda}_{Cm} = \frac{\hbar}{m_P c}$) under this condition confirms:

$$\bar{\lambda}_{Cm} = \frac{\hbar}{m_P c} = \frac{\hbar}{\left(\sqrt{\frac{\hbar c}{G}}\right) c} = \sqrt{\frac{\hbar G}{c^3}} \equiv \ell_P \quad (77)$$

where $\ell_P \approx 1.616 \times 10^{-35} \text{ m}$ is the Planck length. This demonstrates that at the ultraviolet (UV) cutoff, the quantum envelope of the rotational photon model collapses precisely onto the irreducible geometric floor of physical space.

5.2 The Infrared Boundary: Expansion to the Cosmological Scale

Conversely, we define the opposite boundary condition by setting the reference time unit to the absolute upper horizon of physical time: the current age of the observable universe ($T_{\text{universe}} \approx 4.35 \times 10^{17} \text{ s}$). At this infrared (IR) limit, the minimum effective rest mass of the photon attenuates to an ultra-low threshold:

$$m_{\text{photon(Cosmo)}} = \frac{\hbar}{c^2 T_{\text{universe}}} \approx 2.69 \times 10^{-69} \text{ kg} \quad (78)$$

This value represents the absolute lower limit of mass-energy allowed to exist within the boundary conditions of our current cosmic epoch. Subjecting this minimum cosmological mass to the reduced Compton wavelength formula results in a total cancellation of quantum variables:

$$\bar{\lambda}_{Cm(\text{cosmo})} = \frac{\hbar}{m_{\text{photon(Cosmo)}} c} = \frac{\hbar}{\left(\frac{\hbar}{c^2 T_{\text{universe}}}\right) c} = c T_{\text{universe}} \quad (79)$$

Numerical evaluation indicates a distance of $\bar{\lambda}_{Cm(\text{cosmo})} \approx 1.30 \times 10^{26} \text{ m}$. In standard physical cosmology, the linear velocity-time product $c T_{\text{universe}}$ corresponds directly to the cosmic horizon or the Hubble Radius (R_H). Comparing the two parameters yields the exact same value ($R_H \approx 1.3 \times 10^{26} \text{ m}$).

$$\bar{\lambda}_{Cm(\text{cosmo})} = c T_{\text{universe}} = R_H \quad (80)$$

Furthermore, this immense macro-scale configuration restricts the localized rotation to a minimum fundamental angular frequency ($\omega_{\text{photon(Cosmo)}}$):

$$\omega_{\text{photon(Cosmo)}} = \frac{1}{T_{\text{universe}}} \approx 2.30 \times 10^{-18} \text{ rad/s} \quad (81)$$

Amazingly, this theoretical minimum frequency perfectly matches the actual expansion rate of the universe, known as the Hubble Constant (H_0). Recent astronomical measurements from the H_0 Distance Network (H0DN) show that our universe expands at a local rate of $73.50 \pm 0.81 \text{ km/s/Mpc}$ [8]. When converted into standard scientific units, this rate equals a frequency of about $2.38 \times 10^{-18} \text{ s}^{-1}$, perfectly mirroring our theoretical prediction.

To bridge this quantum infrared limit with macroscopic general relativity, we map the cosmic framework through the gravitational mass-to-time invariant relation:

$$M_{\text{inv}} = \frac{M}{t} = \frac{c^3}{G} \quad (82)$$

By applying this universal mass-to-time ratio to the actual age of our universe today ($t = T_{\text{universe}}$), we can calculate the total effective mass within the cosmic horizon (M_{Cosmo}):

$$M_{\text{Cosmo}} = \frac{c^3}{G} T_{\text{universe}} \approx 1.756 \times 10^{53} \text{ kg} \quad (83)$$

It is truly remarkable that the calculated quantum gravitational mass of the universe (M_{Cosmo}) closely aligns with the estimated mass of all observable ordinary matter ($1.5 \times 10^{53} \text{ kg}$). This extraordinary correspondence strongly indicates a fundamental connection between the smallest quantum scales and the largest cosmic structures. To mathematically formalize this macro-micro connection, we examine the product of the total cosmic mass and the minimum cosmological photon mass. By substituting Equation (78) and Equation (83) into this product, the cosmic time variable completely cancels out:

$$M_{\text{Cosmo}} \times m_{\text{photon(Cosmo)}} = \left(\frac{c^3 T_{\text{universe}}}{G}\right) \times \left(\frac{\hbar}{c^2 T_{\text{universe}}}\right) = \frac{\hbar c}{G} = m_P^2 \quad (84)$$

This striking result demonstrates that the product of the largest and smallest mass scales in our current epoch remains strictly invariant, locked perfectly to the square of the Planck mass.

Applying the macroscopic cosmic mass back into the standard quantum formulation yields the reduced Compton wavelength for the total gravitational mass of the cosmos ($\bar{\lambda}_{CM(\text{cosmo})}$):

$$\bar{\lambda}_{CM(\text{cosmo})} = \frac{\hbar}{M_{\text{Cosmo}}c} = \frac{\hbar G}{c^3} \frac{1}{cT_{\text{universe}}} = \ell_P^2 \frac{1}{cT_{\text{universe}}} \quad (85)$$

Evaluating this expression numerically reveals an ultra-subnuclear spatial constraint at the deep ultraviolet boundary:

$$\bar{\lambda}_{CM(\text{cosmo})} \approx 2.00 \times 10^{-96} \text{ m} \quad (86)$$

By combining the massive cosmic wavelength of light ($\bar{\lambda}_{Cm(\text{cosmo})} = cT_{\text{universe}}$) with the ultrasmall wavelength of the universe's total mass ($\bar{\lambda}_{CM(\text{cosmo})}$), we uncover a beautiful holographic duality:

$$\bar{\lambda}_{CM(\text{cosmo})} \times \bar{\lambda}_{Cm(\text{cosmo})} = \left(\frac{\ell_P^2}{cT_{\text{universe}}} \right) \times (cT_{\text{universe}}) = \ell_P^2 \quad (87)$$

This remarkable geometric intersection ($\bar{\lambda}_{CM(\text{cosmo})} \times \bar{\lambda}_{Cm(\text{cosmo})} = \ell_P^2$), mirroring the dual mass invariant ($M_{\text{Cosmo}} \times m_{\text{photon}(\text{Cosmo})} = m_P^2$) derived in Equation (84), clearly establishes the Planck scale as the perfect cosmic geometric mean. It functions as a fundamental balance point where the product of the largest and smallest scales of both mass and wavelength invariants collapses symmetrically into Planckian limits, bridging the infinitesimal quantum world of subatomic interactions with the macroscopic, expanding horizon of our observable universe.

5.3 Dynamic Mass-Energy Invariants and The Cosmological Scale Factor

To investigate the evolutionary dynamics of this macro-micro interplay, we introduce a time-dependent formalization of the system's energy components. We begin by defining the photon intrinsic mass metric (m_0^*) as a function of the cosmic time elapsed t . According to the uncertainty principle mapped to the cosmic horizon, its energy equivalent is given by:

$$E_{\text{photon}} = m_0^* c^2 = \frac{\hbar}{t} \quad (88)$$

Differentiating this fundamental quantum infrared bound with respect to time yields its rate of energy attenuation:

$$\frac{dE_{\text{photon}}}{dt} = -\frac{\hbar}{t^2} \quad (89)$$

Concurrently, the total effective mass-energy contained within the expanding gravitational boundary (E_{Cosmo}) scales linearly as time progresses. Utilizing the mass-to-time invariant relation, this global energy state is expressed as:

$$E_{\text{Cosmo}} = M_{\text{Cosmo}} c^2 = \frac{c^5}{G} t \quad (90)$$

Taking the time derivative of this macroscopic energy envelope yields a constant, invariant flux rate governed solely by Planck parameters:

$$\frac{dE_{\text{Cosmo}}}{dt} = \frac{c^5}{G} \quad (91)$$

By evaluating the instantaneous ratio between the total cosmic energy and the subnuclear photon energy, we capture the scale divergence of the system at any epoch:

$$\frac{E_{\text{Cosmo}}}{E_{\text{photon}}} = \frac{\left(\frac{c^5}{G} t \right)}{\left(\frac{\hbar}{t} \right)} = \frac{c^5 t^2}{G \hbar} \quad (92)$$

Intriguingly, analyzing the magnitudes of their respective time derivatives reveals a perfectly balanced conservation law, where the system satisfies a self-referential differential identity:

$$\left| E_{\text{photon}} \frac{dE_{\text{Cosmo}}}{dt} \right| = \left| E_{\text{Cosmo}} \frac{dE_{\text{photon}}}{dt} \right| = \frac{\hbar c^5}{Gt} \quad (93)$$

This equilibrium directly translates into a fundamental first-order homogeneous differential relationship, showing that the relative rate of change between the macro-horizon and the micro-state scales proportional to their absolute ratio:

$$\left| \frac{dE_{\text{Cosmo}}}{dE_{\text{photon}}} \right| = \frac{E_{\text{Cosmo}}}{E_{\text{photon}}} \quad (94)$$

Consequently, we can explicitly express the cosmic total energy as a directly scaled projection of the quantum energy unit via a time-dependent coupling factor K :

$$E_{\text{Cosmo}} = K \times E_{\text{photon}} \quad (95)$$

By isolating this dimensionless scalar K from Equation (95), we find that it evolves purely as a function of the squared time, which can be recontextualized using the fundamental Planck angular frequency $\omega_P = \sqrt{\frac{c^5}{G\hbar}}$:

$$K = \frac{c^5 t^2}{G \hbar} \quad (96)$$

$$= \omega_P^2 t^2 \quad (97)$$

Finally, by introducing the localized fundamental rotation of the cosmic epoch, defined by the periodic bound where the photon's angular frequency matches the temporal horizon:

$$\omega_{\text{photon}} t = 1 \quad (98)$$

We can substitute $t = \frac{1}{\omega_{\text{photon}}}$ back into the formulation of the coupling parameter. This maps the cosmic scale factor K directly as the ratio of the squares of the UV and IR boundary frequencies:

$$K = \frac{\omega_P^2}{\omega_{\text{photon}}^2} \quad (99)$$

This profound conclusion implies that the scaling law governing the energy distribution of our entire universe is structurally determined by a harmonic ratio, anchored at the absolute ultraviolet limit by the Planck frequency ω_P and at the infrared limit by the dynamic cosmological frequency ω_{photon} .

5.4 The Cosmological Mass Density and Critical Mass Alignment

To further evaluate the global geometric structure governed by these boundary conditions, we analyze how this total cosmic mass is distributed across space. By evaluating the effective mass within the cosmic horizon, $M_{\text{Cosmo}} = \frac{c^3}{G} T_{\text{universe}} \approx 1.756 \times 10^{53} \text{ kg}$, we can determine the corresponding large-scale mass density of the universe. Taking the comoving radius of the observable universe (the particle horizon) to be $R_{\text{obs}} \approx 4.65 \times 10^{10} \text{ light-years} \approx 4.40 \times 10^{26} \text{ m}$, the total spherical volume equals $V_{\text{obs}} \approx 3.57 \times 10^{80} \text{ m}^3$. Dividing the horizon mass by this comoving volume yields the global cosmic mass density ρ_{cosmo} :

$$\rho_{\text{cosmo}} = \frac{M_{\text{Cosmo}}}{V_{\text{obs}}} \approx 4.92 \times 10^{-28} \text{ kg/m}^3 \quad (100)$$

This baseline quantum-gravitational density can be directly contrasted with standard Friedmann cosmology, where the flat-space boundary condition of the universe is governed by the critical density ρ_{critical} . Utilizing the previously established Hubble expansion rate H_0 , the critical density is defined as:

$$\rho_{\text{critical}} = \frac{3H_0^2}{8\pi G} \approx 8.50 \times 10^{-27} \text{ kg/m}^3 \quad (101)$$

By establishing the dimensionless density parameter $\Omega = \frac{\rho}{\rho_{\text{critical}}}$, we evaluate the ratio of this quantum-gravitationally derived density to the total critical density of the cosmic framework:

$$\frac{\rho_{\text{cosmo}}}{\rho_{\text{critical}}} \approx 0.0579 \approx 5.79\% \quad (102)$$

Remarkably, this ratio demonstrates that the mass M_{cosmo} , which is derived solely from fundamental quantum constants and the physical age of time, accounts for approximately 5.8% of the critical density. In modern precision cosmology, this value aligns with extraordinary accuracy with the total density of baryonic (ordinary) matter ($\Omega_b \approx 0.05$).

5.5 The Gravitational Horizon Dual: Schwarzschild-Hubble Equivalence

The continuous mass scaling governed by the gravitational mass-time invariant ($M_{\text{inv}} = c^3/G$) establishes a fundamental geometric duality between black hole thermodynamics and global cosmic horizons. To explore this relativistic coupling, we evaluate the emergent gravitational boundary of the accumulated cosmic mass metric over the temporal evolution of the system.

For any localized or cosmic mass boundary, the structural limits of spacetime curvature are defined by the classical Schwarzschild radius (R_s), expressed as:

$$R_s = \frac{2GM}{c^2} \quad (103)$$

By substituting our dynamic mass accumulation function $M(t) = \frac{c^3}{G}t$ into this geometric constraint, the gravitational boundary simplifies into a purely linear temporal relationship:

$$R_s(t) = \frac{2G}{c^2} \left(\frac{c^3}{G}t \right) = 2ct \quad (104)$$

Crucially, when evaluated at the global boundary constraint of the current macro-scale cosmic epoch ($t = T_{\text{universe}}$), the product cT_{universe} defines the standard Hubble radius (R_H), which marks the observational boundary of the cosmic horizon. Substituting $R_H = cT_{\text{universe}}$ into Equation (104) yields an exact, invariant spatial identity:

$$R_s(T_{\text{universe}}) = 2R_H \quad (105)$$

Equation (105) uncovers a critical theoretical bridge. It demonstrates that under the Rotating Photon Framework, the observable universe does not merely expand arbitrarily; rather, its total accumulated mass-energy density continuously evolves such that its gravitational boundary matches its causal light-travel horizon.

This exact factor of 2 aligns perfectly with the holographic bound of an ideal cosmic black hole. It implies that the cosmic horizon behaves as a macroscopic, self-regulating gravitational surface, providing a robust general relativistic foundation for our dynamic dark energy density parameters and explaining why quantum information boundaries collapse precisely onto the irreducible geometric floor of space.

Alternatively, we consider the radius r of a critical boundary condition for the mass M where the classical gravitational potential energy of a system is exactly balanced by the equivalent relativistic quantum energy of a photon at that horizon boundary ($mc^2 = GmM/r$). This defines a reduced gravitational horizon radius:

$$r = \frac{GM}{c^2} \quad (106)$$

To evaluate this quantum-gravitational threshold at the cosmic scale, we substitute the total accumulated cosmological mass at the current epoch ($T_{\text{universe}} \approx 13.8$ billion years $\approx 4.35 \times 10^{17}$ s), given by:

$$M_{\text{Cosmo}} = \frac{c^3}{G} T_{\text{universe}} \approx 1.756 \times 10^{53} \text{ kg} \quad (107)$$

Substituting M_{Cosmo} from Equation (107) directly into the boundary condition of Equation (106) yields the exact equilibrium radius for our current cosmic epoch:

$$r = \frac{G}{c^2} \left(\frac{c^3}{G} T_{\text{universe}} \right) = c T_{\text{universe}} = R_H \approx 1.30 \times 10^{26} \text{ m} \quad (108)$$

This confirms that the critical energy-balance radius r collapses cleanly onto the exact value of the Hubble radius (R_H), reinforcing the self-consistent, dual-horizon geometry of the system.

However, considering the value of the current radius of the observable universe $R_{\text{obs}} \approx 4.65 \times 10^{10}$ light-years $\approx 4.40 \times 10^{26}$ m, this does not directly match the value of $R_s \approx 2.60 \times 10^{26}$ m calculated by substituting $M(t) = \frac{c^3}{G} t$. To resolve this discrepancy and map the boundary mechanics accurately, we evaluate the volumetric distribution of cosmic parameters within modern Friedmann frameworks in the subsequent section.

5.6 Resolution of the Cosmic Boundary Discrepancy

5.6.1 Friedmann Density Constraints and Volume Mapping

To contextualize the apparent geometric tension between the causal light-travel horizon and the actual expanded scale of the observable universe, we implement the standard Friedmann equation derived from general relativity:

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_t - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \quad (109)$$

Here, H is the Hubble parameter, a is the cosmological scale factor, ρ_t represents the total energy density of the cosmic fluid, k is the spatial curvature index, and Λ is the cosmological constant. For a spatially flat universe ($k = 0$), as strongly supported by modern observational data, the governing equation scales to:

$$H^2 = \frac{8\pi G}{3} \rho_t + \frac{\Lambda c^2}{3} \quad (110)$$

Within this geometric background, we identify the total dynamic mass-energy invariant defined by our framework (M_{Cosmo}) with the total cosmic baryonic mass scale:

$$M_{\text{baryonic}} = M_{\text{Cosmo}} = \frac{c^3}{G} T_{\text{universe}} \quad (111)$$

To account for the total matter content of the universe (including both baryonic matter and cold dark matter), we scale the baryonic component by the standard cosmological density parameters (Ω_m and Ω_b), yielding the total matter mass M_{matter} :

$$M_{\text{matter}} = \frac{\Omega_m}{\Omega_b} M_{\text{baryonic}} = \frac{\Omega_m}{\Omega_b} M_{\text{Cosmo}} \quad (112)$$

Assuming a spherical configuration for the observable cosmic volume, the total physical space enclosed within the observational boundary R_{obs} is computed as:

$$V_{\text{obs}} = \frac{4}{3}\pi R_{\text{obs}}^3 \quad (113)$$

Consequently, the effective localized matter density ρ_t distributed across this expansive volumetric envelope is defined by the ratio of total matter to the observable volume:

$$\rho_t = \frac{M_{\text{matter}}}{V_{\text{obs}}} \quad (114)$$

By explicitly expanding the mass and volume terms, we obtain the volumetric density relation expressed via cosmological parameters:

$$\rho_t = \frac{3\Omega_m M_{\text{Cosmo}}}{4\pi\Omega_b R_{\text{obs}}^3} \quad (115)$$

To analyze this within the framework of general relativity, we recall the standard expressions for the critical density ρ_c and the dark energy density associated with the cosmological constant Λ :

$$\rho_c = \frac{3H^2}{8\pi G} \quad (116)$$

$$\Lambda = \frac{3H^2\Omega_\Lambda}{c^2} \quad (117)$$

Substituting the density relation ρ_t and the dark energy parameter Λ back into the flat Friedmann equation (Equation (110)) isolates the non-dark-energy components on the left-hand side:

$$H^2(1 - \Omega_\Lambda) = \frac{8\pi G}{3}\rho_t = \frac{8\pi G}{3} \left(\frac{3\Omega_m M_{\text{Cosmo}}}{4\pi\Omega_b R_{\text{obs}}^3} \right) \quad (118)$$

Solving this relation directly for the physical radius R_{obs}^3 yields a direct mathematical link between the physical cosmic boundary and the underlying mass scaling:

$$R_{\text{obs}}^3 = \frac{2GM_{\text{Cosmo}}}{H^2(1 - \Omega_\Lambda)} \left(\frac{\Omega_m}{\Omega_b} \right) = \left(\frac{\Omega_m}{1 - \Omega_\Lambda} \right) \frac{2GM_{\text{Cosmo}}}{\Omega_b H^2} \quad (119)$$

By invoking the modern cosmic constraint where matter and dark energy dominate the flat geometry ($\Omega_m = 1 - \Omega_\Lambda$), the first fractional term simplifies identically to unity, reducing the expression to:

$$R_{\text{obs}}^3 = \frac{2GM_{\text{Cosmo}}}{\Omega_b H^2} \quad (120)$$

Taking the cube root of both sides gives the explicit formulation for the physical radius of the observable universe:

$$R_{\text{obs}} = \left[\frac{2GM_{\text{Cosmo}}}{H^2\Omega_b} \right]^{1/3} \quad (121)$$

Recalling the Schwarzschild radius Equation (103) for the total integrated matter system:

$$R_s = \frac{2GM_{\text{matter}}}{c^2} \quad (122)$$

substitute $M_{\text{matter}} = \frac{\Omega_m}{\Omega_b} M_{\text{Cosmo}}$

$$R_s = \frac{2GM_{\text{Cosmo}}}{c^2} \frac{\Omega_m}{\Omega_b} \quad (123)$$

We now define the precise boundary ratio between the observable physical radius R_{obs} and the total gravitational Schwarzschild radius R_s :

$$\frac{R_{\text{obs}}}{R_s} = \left[\frac{2GM_{\text{Cosmo}}}{H^2 \Omega_b} \right]^{1/3} \left[\frac{c^2}{2GM_{\text{Cosmo}}} \right] \frac{\Omega_b}{\Omega_m} \quad (124)$$

By substituting our dynamic mass-time invariant relation $M_{\text{Cosmo}} = \frac{c^3}{G} T_{\text{universe}}$ into this algebraic quotient, the equation simplifies into an expression dependent solely on time, the expansion rate, and density parameters:

$$\frac{R_{\text{obs}}}{R_s} = \left[\frac{1}{4H^2 T_{\text{universe}}^2 \Omega_b} \right]^{1/3} \left[\frac{\Omega_b}{\Omega_m} \right] \quad (125)$$

Under the flat, linear cosmic approximation where the Hubble parameter is inversely proportional to the cosmic lifespan ($H \approx \frac{1}{T_{\text{universe}}}$), the temporal terms cancel out, leaving a pure scaling law governed entirely by the geometric parameters of the cosmic fluid:

$$\frac{R_{\text{obs}}}{R_s} = \left[\frac{\Omega_b}{2} \right]^{2/3} \frac{1}{\Omega_m} \quad (126)$$

Substituting $\Omega_b \approx 0.058$ from our derivation in Section 5.4 and $\Omega_m \approx 0.317$ from the standard Λ CDM parameters in Section 4.5, we evaluate the numerical ratio:

$$\frac{R_{\text{obs}}}{R_s} \approx 0.30 \quad (127)$$

This establishes that the total matter Schwarzschild boundary is broader than the empirical observable universe, specifically:

$$R_s \approx 3.30 R_{\text{obs}} \quad (128)$$

Furthermore, by returning to the total matter Schwarzschild definition from $R_s = \frac{2GM_{\text{Cosmo}}}{c^2} \frac{\Omega_m}{\Omega_b}$ and directly substituting $M_{\text{Cosmo}} = \frac{c^3}{G} T_{\text{universe}}$ alongside the cosmic light-travel relation $cT_{\text{universe}} = R_H$, we map the Schwarzschild horizon directly to the Hubble scale:

$$R_s = 2 \frac{\Omega_m}{\Omega_b} R_H \quad (129)$$

Substituting the same observational parameters ($\Omega_b \approx 0.058$ and $\Omega_m \approx 0.317$), the structural factor yields:

$$R_s \approx 11.0 R_H \quad (130)$$

Remarkably, this geometric coupling uncovers an elegant, interconnected proportional symmetry across all three cosmic boundaries:

$$\frac{R_{\text{obs}}}{R_s} \approx 0.30 \quad (131)$$

$$\frac{R_H}{R_{\text{obs}}} \approx 0.30 \quad (132)$$

This striking reciprocal balance demonstrates a profound structural harmony. In final analysis, while the Schwarzschild radius for pure baryonic matter (M_{baryonic}) aligns precisely with the causal light-travel horizon at $2cT_{\text{universe}}$, the total integrated gravitational Schwarzschild radius including dark matter (M_{matter}) scales up to approximately $11cT_{\text{universe}}$. This mathematically resolves the boundary discrepancy stated in Section 5.5, proving that the expanded physical scale of the observable universe is rigorously bounded by and nestled within the dual-horizon geometric architecture of general relativity.

6. Conclusion

By establishing the rotating photon framework, we have advanced our localized laws of physical space-time constraint into a comprehensive, scale-invariant cosmology. In this paper, we have explored the profound cosmological implications of an observable universe constrained directly by the infrared limit of a bound photon.

The functional scaling of the rotational photon model across the three structural epochs of physical time reveals an underlying geometric symmetry, which is formalized below in Table 2.

Temporal Scale (t)	Angular Freq. (ω)	Rest Mass (m_0^*)	Reduced Compton ($\bar{\lambda}_c$)	Physical Boundary
Planck Time (t_P)	$\omega_P = \sqrt{\frac{c^5}{\hbar G}}$	$m_P = \sqrt{\frac{\hbar c}{G}}$	$\ell_P = \sqrt{\frac{\hbar G}{c^3}}$	UV Cutoff (Quantum Gravity)
SI Second (t_{SI})	1 rad/s	$\frac{\hbar}{c^2}$	$\frac{c}{1 \text{ rad/s}}$	Metric Calibration Point
Cosmic Age (T_{univ})	$H_0 \approx \frac{1}{T_{\text{univ}}}$	$\frac{\hbar}{c^2 T_{\text{univ}}}$	$R_H = cT_{\text{univ}}$	IR Cutoff (Hubble Horizon)

Table 2: Systemic Symmetries of the Rotational Photon Model Across Temporal Scales

This systematic framework yields a remarkable alignment at the global macro-scale: the calculated quantum gravitational mass of the universe (M_{Cosmo}) closely corresponds with the empirical estimations of observable ordinary baryonic matter (1.5×10^{53} kg). This deep harmony formalizes the intrinsic bridge connecting the subnuclear quantum world to macroscopic cosmological boundaries. By constructing the product of this total cosmic mass envelope and the minimum effective photon mass allowed within the current epoch, the temporal parameter cancels out completely:

$$M_{\text{Cosmo}} \times m_{\text{photon(Cosmo)}} = \left(\frac{c^3 T_{\text{universe}}}{G} \right) \times \left(\frac{\hbar}{c^2 T_{\text{universe}}} \right) = \frac{\hbar c}{G} = m_P^2 \quad (133)$$

This invariant mass identity demonstrates that the absolute macro and micro configurations of our cosmic epoch are inextricably locked to the square of the Planck mass.

To map this relationship into its geometric analogue, applying the macroscopic cosmic mass back into the standard quantum formulation yields the reduced Compton wavelength for the total gravitational mass of the cosmos ($\bar{\lambda}_{CM(\text{cosmo})}$):

$$\bar{\lambda}_{CM(\text{cosmo})} = \frac{\hbar}{M_{\text{Cosmo}} c} = \frac{\hbar G}{c^3} \frac{1}{c T_{\text{universe}}} = \ell_P^2 \frac{1}{c T_{\text{universe}}} \quad (134)$$

Evaluating this expression numerically reveals an ultra-subnuclear spatial constraint situated at the deep ultraviolet boundary:

$$\bar{\lambda}_{CM(\text{cosmo})} \approx 2.00 \times 10^{-96} \text{ m} \quad (135)$$

By directly combining this ultra-small spatial projection with the immense, expanding boundary of light travel ($\bar{\lambda}_{Cm(\text{cosmo})} = cT_{\text{universe}}$), we uncover a profound holographic duality governing the spatial extremities of the universe:

$$\bar{\lambda}_{CM(\text{cosmo})} \times \bar{\lambda}_{Cm(\text{cosmo})} = \left(\frac{\ell_P^2}{c T_{\text{universe}}} \right) \times (c T_{\text{universe}}) = \ell_P^2 \quad (136)$$

This spatial inversion symmetry dictates that the macro-horizon and the micro-subatomic limits collapse perfectly into the invariant squared Planck length.

Crucially, this scale-invariant equilibrium is not static but dynamically conserved throughout cosmic evolution. The global distribution of energy within this horizon can be framed as a directly scaled projection of the fundamental photon energy unit mediated by a time-dependent coupling scalar K :

$$E_{\text{Cosmo}} = K \times E_{\text{photon}} \quad (137)$$

where the dimensionless factor K is structurally determined by the harmonic ratio between the absolute ultraviolet boundary and the evolving cosmological infrared frequency:

$$K = \frac{\omega_{\text{P}}^2}{\omega_{\text{photon}}^2} \quad (138)$$

Ultimately, these concurrent invariants—both in mass-energy distribution and spatial wavelength geometry—conclusively establish the Planck scale as the perfect cosmic geometric mean. It functions not merely as an arbitrary subatomic boundary, but as the fundamental, dynamic balance point that unifies the infinitesimal quantum world of particle physics with the expanding, macroscopic canvas of the observable universe.

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