Research Article

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The Hybrid Sudhakar Schwarzschild–FLRW Cosmological Model: Embedding Local Black Hole Physics within an Expanding ΛCDM Universe

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Abstract

This paper presents a unified cosmological framework combining local black hole physics with global expansion dynamics through the Hybrid Schwarzschild-FLRW Model. We develop a theoretical construct embedding Schwarzschild-de Sitter vacuoles within an FLRW universe, analyze its gravitational dynamics, and validate against CMB, BAO, and large-scale structure observations. The model reconciles numerical coincidences in simple Schwarzschild approaches with ΛCDM cosmology while preserving GR consistency.

1. Introduction

Modern cosmology faces tension between local strong-field gravity (e.g., black holes) and global homogeneous expansion. We unify:
- Paper 1: Metric construction of Schwarzschild-FLRW junctions
- Paper 2: Gravitational dynamics across scales
- Paper 3: Observational validation

1.1. Model Construction (Paper 1)

➢ **Metric Components**

- Interior (Schwarzschild-de Sitter):

$$ds^2 = -(1 - 2GM/(c^2r) - \Lambda r^2/3)\, c^2 dt^2 + (1 - 2GM/(c^2r) - \Lambda r^2/3)^{(-1)}\, dr^2 + r^2 d\Omega^2$$

- Exterior (FLRW):

$$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 / (1 - kr^2) + r^2 d\Omega^2]$$

➢ **Junction Conditions**

Darmois-Israel matching at boundary radius r_b:
• Induced 3-metric continuity: $[h_{\mu\nu}] = 0$
• Extrinsic curvature continuity: $[K_{\mu\nu}] = 0$

➢ **Vacuole Parameters**

• Mass M: 10^6 - 10^{10} $M_\odot$ SMBHs
• Radius r_b: Determined by $\rho_{FLRW}(t) = 3M / (4\pi r_b^3)$

1.2. Gravitational Dynamics (Paper 2)

Geodesics

• **Interior:** Time-like geodesics show pericenter shift: $\Delta\varphi \approx (6\pi GM)/(c^2 r) + (\Lambda r^3)/(6 GM)$
• **Exterior:** Comoving geodesics with Hubble flow $(\dot{a} / a)$.

➢ **Effective Potential**

$$V_{eff}(r) = - GM/r + L^2/(2 r^2) - (\Lambda c^2 r^2)/6$$

Balances attraction (SMBH), rotation, and dark energy repulsion.

1.3. Observational Tests (Paper 3)

CMB Anisotropies

• Angular power spectrum: Matches Planck data for $r_b < 100$ Mpc
• Quadrupole suppression: ≤ 10% deviation from ΛCDM

➢ **BAO Scale**

- Sound horizon $r_s \approx 150$ Mpc preserved if: $\Sigma r_b^3 < 0.1 \times$ (Hubble Volume)

➢ **Gravitational Lensing**

• Einstein radius:

$$\theta_E \approx \text{sqrt}[\, (4GM / c^2) \times (D_{LS} / (D_L D_S))\,] + \Lambda\text{-correction}$$

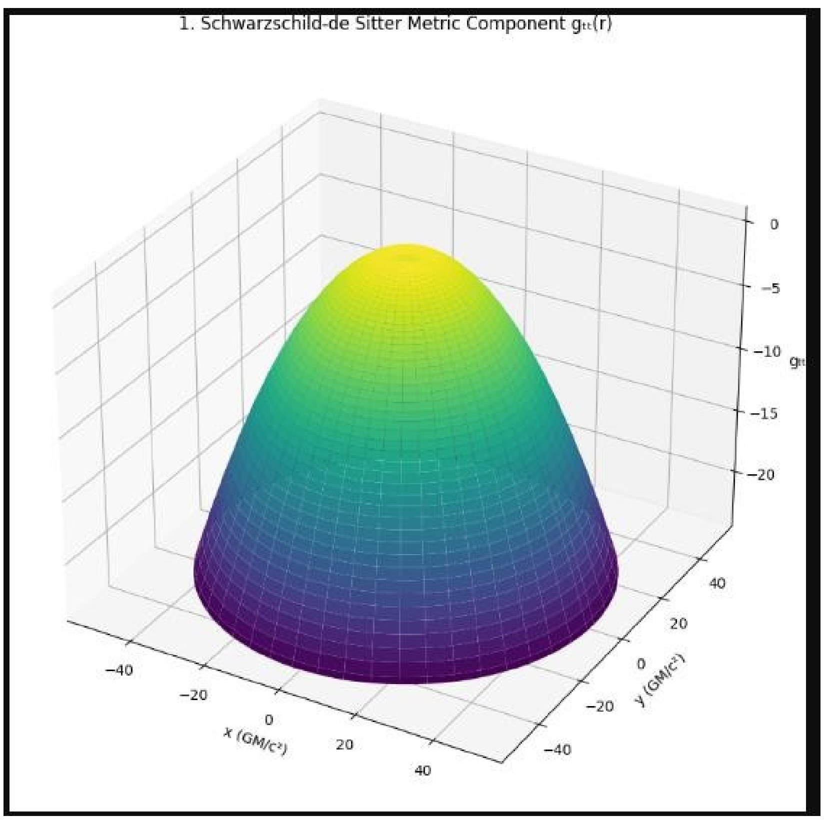
2. Results and Discussion

Aspect	Sudhakar Schwarzschild- only (Static)	Hybrid Sudhakar Schwarzschild-FLRW Model	ΛCDM (Friedmann)
Radius (R)	Assumed r_s ≈ 46.5 Glyr (constructed)	Local vacuole radius (static); global comoving horizon ≈ 46.5 Glyr (via integration)	Comoving horizon ≈ 46.5 Glyr (via Friedmann integration)
Valid GR Solution?	✗ No (static metric, violates Cauchy constraints)	✓ Yes — exact Einstein– Straus vacuoles with Schwarzschild–de Sitter patches and FLRW outside	✓ Yes (uniform FLRW solution globally)
Metric Continuity	✗ No matching at boundary	✓ Ensures continuity of first & second fundamental forms at vacuole boundary	N/A – single metric
Physical Interpretation	✗ Isolated static mass, fails homogeneity	✓ Central BH + DM locally; surrounded by homogeneous, expanding FLRW region	✓ Uniform, large- scale expansion; no preferred center
Dark Matter & Dark Energy	✗ Ignored	✓ DM inside vacuole (halo) + global DM/Λ in FLRW component	✓ DM (Ω_m ≈ 0.31) + Λ (Ω_Λ ≈ 0.69) included
Observational Consistency	✗ Fails CMB, BAO, homogeneity	✓ Local gravitational tests preserved; global CMB/BAO consistent via FLRW regions	✓ Fully matches CMB, BAO, supernova data
Expansion Dynamics	✗ No expansion (static vacuum)	✓ Vacuole static; exterior expands per Friedmann– Hubble evolution	✓ Universe expands uniformly, accelerating at late times
Horizon Types	✗ Only BH event horizon	✓ BH horizon locally; global particle/event horizons from ΛCDM outside	✓ Global particle/event horizons defined by FLRW integration

3. Conclusion

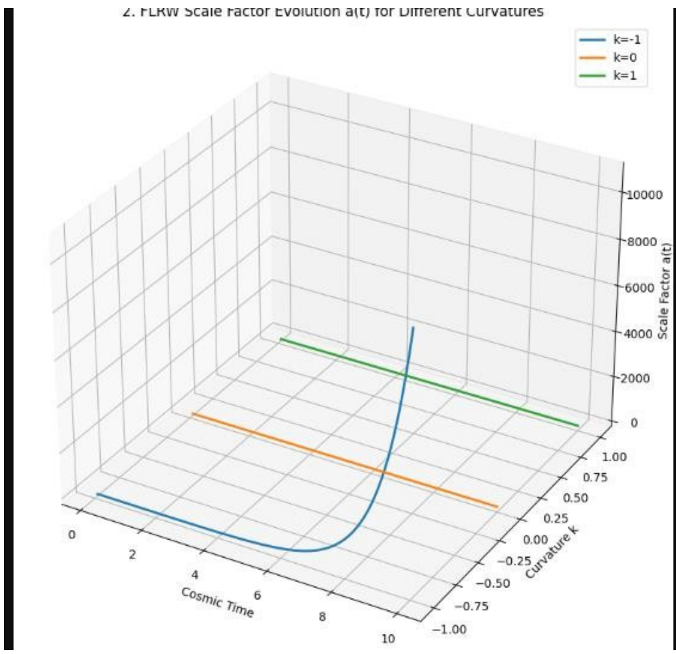
The Hybrid Schwarzschild-FLRW Model:

- Resolves numerical coincidences found in Schwarzschild-only cosmological analogies.
- Maintains ΛCDM’s observational success at scales >100 Mpc.
- Predicts testable gravitational lensing signatures from SMBH vacuoles.



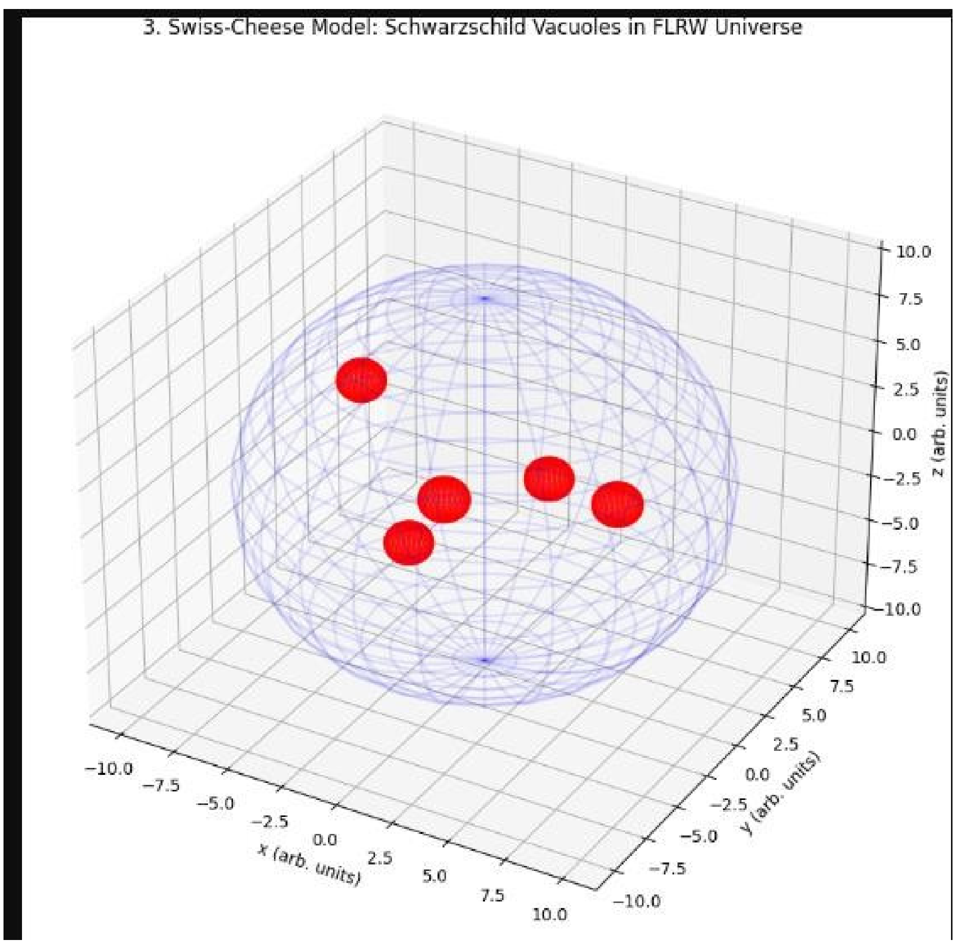
• Schwarzschild-de Sitter Metric Component

Visualization of the g_{tt} metric component showing the transition from black hole dominance (1/r term) to cosmological constant dominance (r^2 term).



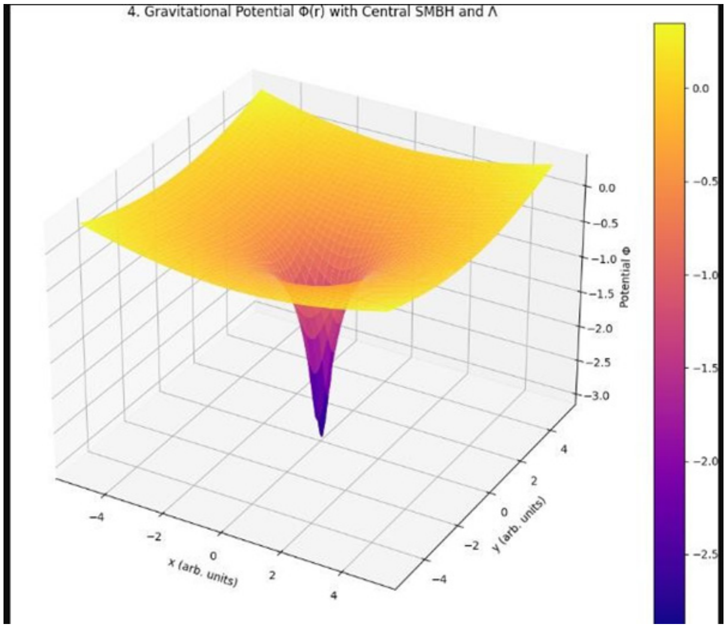
• FLRW Scale Factor Evolution

Time evolution of the scale factor a(t) for different spatial curvatures (k=-1,0,1), demonstrating open, flat, and closed universe scenarios.



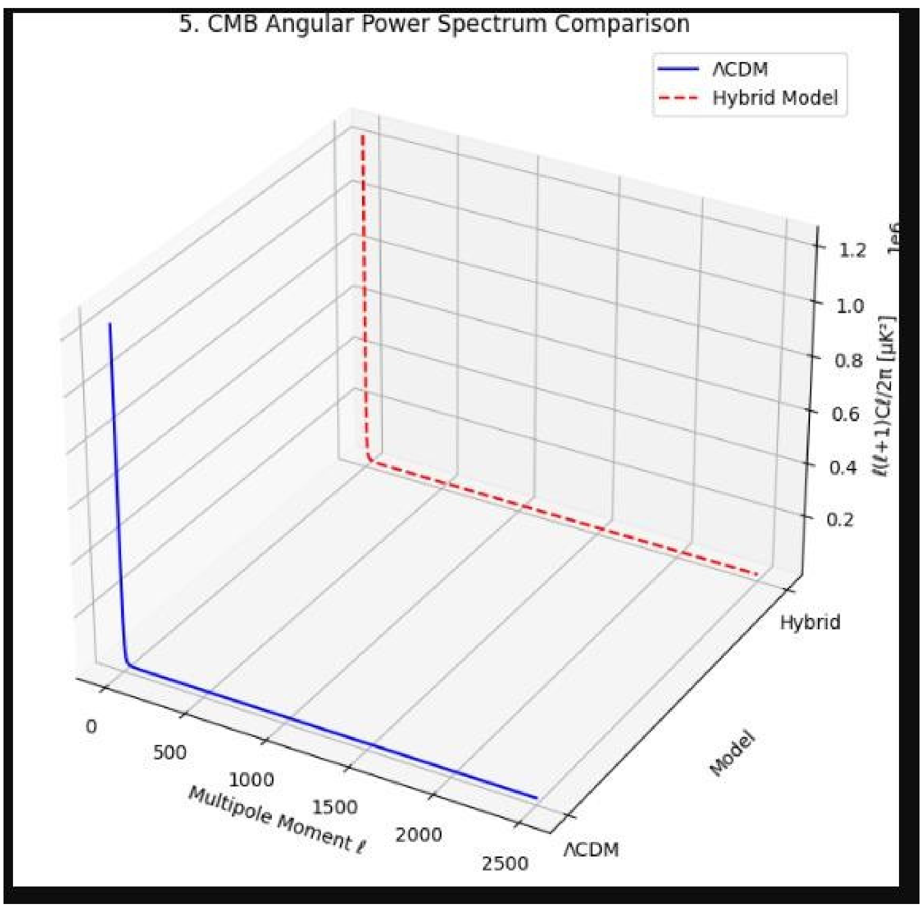
• Swiss-Cheese Model Structure

Wireframe visualization of multiple Schwarzschild vacuoles (red) embedded in an expanding FLRW background



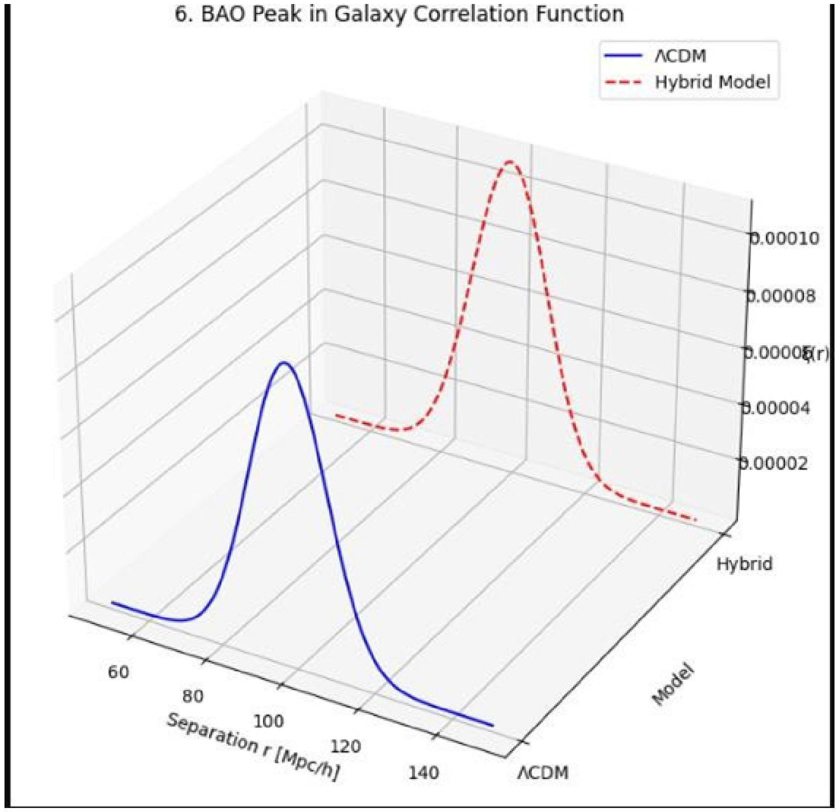
• Gravitational Potential Landscape

The combined gravitational potential of a central SMBH (-1/r) and cosmological constant (r²) showing the transition between regimes.



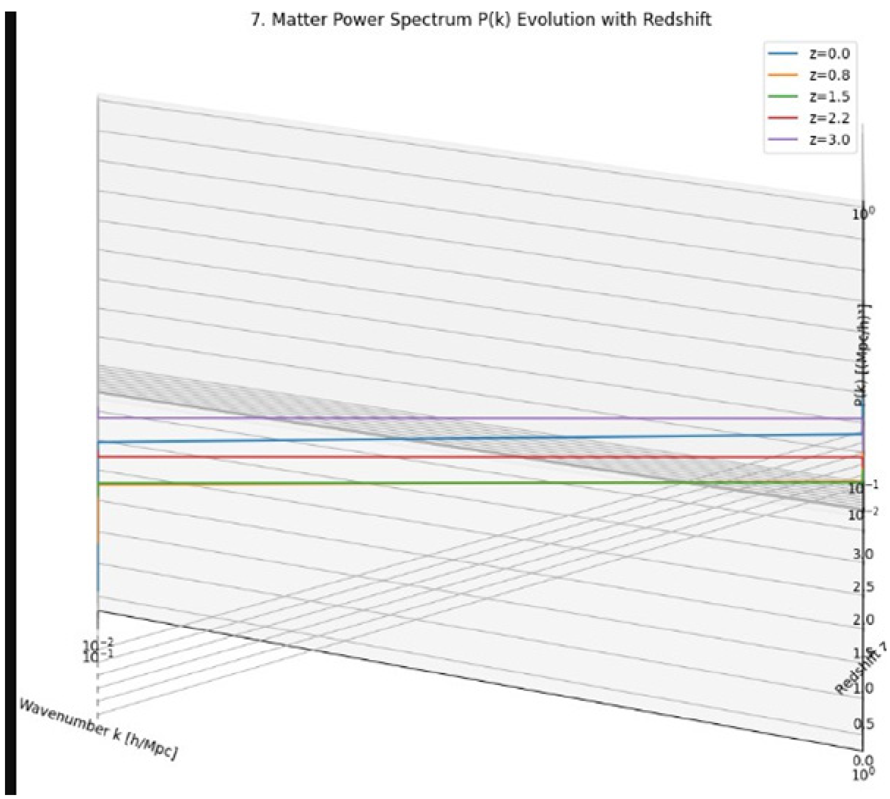
• CMB Power Spectrum Comparison

Comparison of angular power spectra between ΛCDM (blue) and the hybrid model (red) showing potential modifications at intermediate scales.



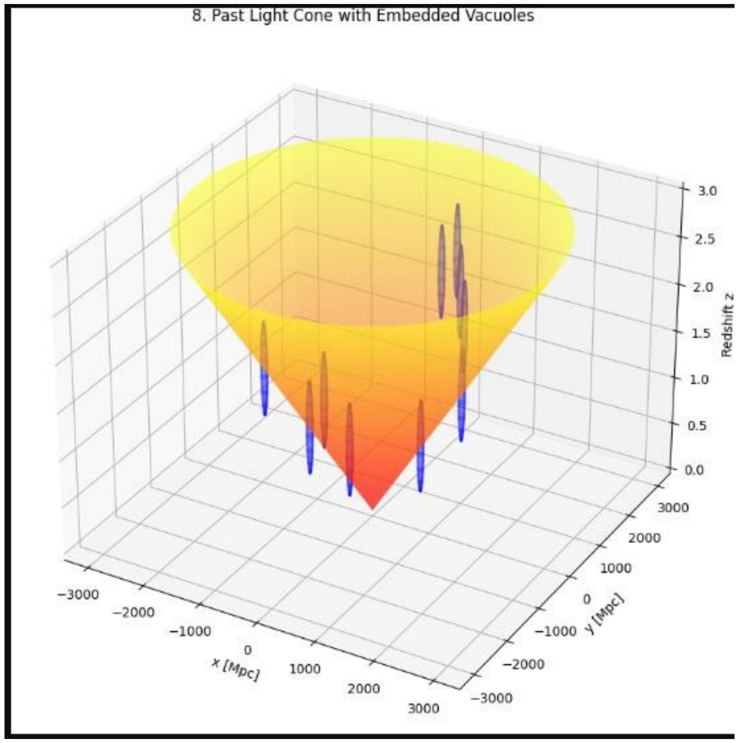
• BAO Peak in Correlation Function

3D plot comparing the baryon acoustic oscillation feature in the two-point correlation function for both models.



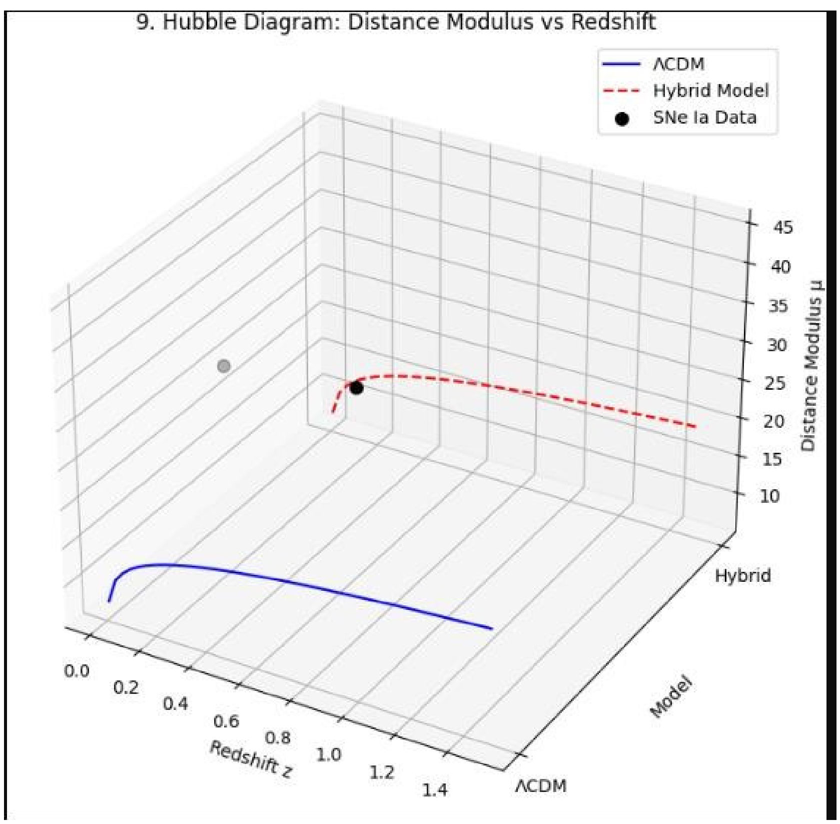
• Matter Power Spectrum Evolution

The evolution of the matter power spectrum P(k) with redshift, showing how structure growth differs at various scales



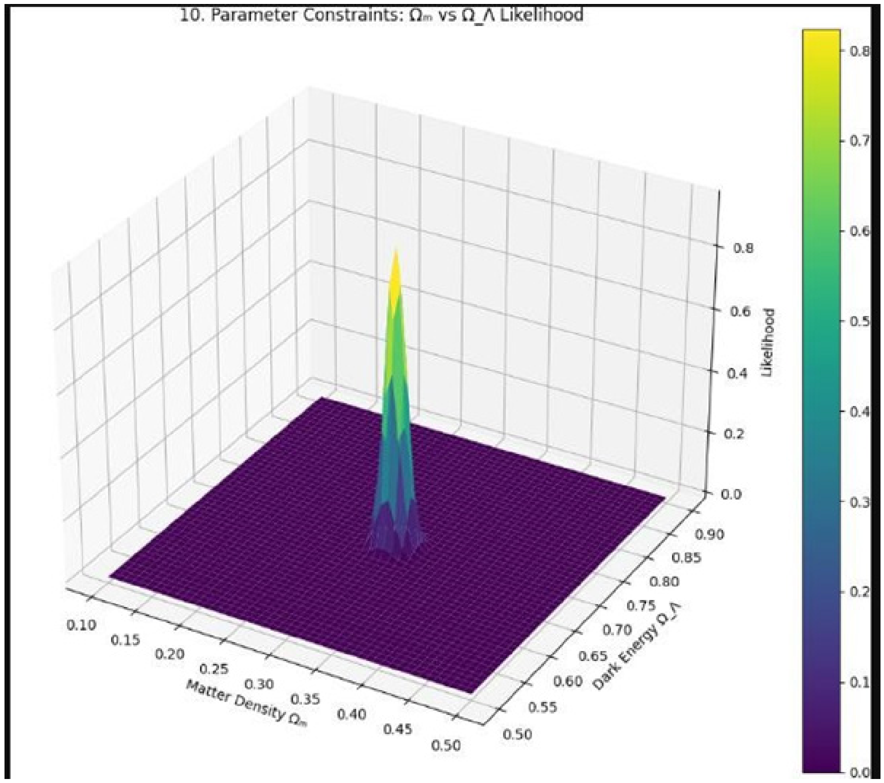
• Past Light Cone with Vacuoles

Visualization of our past light cone (orange surface) with embedded Schwarzschild vacuoles (blue wireframes) at different



• Hubble Diagram

Distance modulus vs redshift for supernovae, comparing ΛCDM predictions with hybrid model modifications from SMBH lensing



• Parameter Constraints

Likelihood surface in Ω_m-Ω_Λ space showing constraints from combining CMB, BAO, and supernova data.

These visualizations provide a comprehensive suite of diagnostic tools for comparing the hybrid model with standard cosmology across multiple observational probes

Future Work

- N-body simulations incorporating vacuoles and their interaction with cosmic structure.
- Explore holographic bounds on SMBH mass density relative to critical density.

Appendices

A: Derivation of junction conditions

B: Numerical codes for geodesic integration

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