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Research Article

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The Expression of Classical Mechanics Laws Using Wave Mechanics Equations Reflects the Compatibility Between Quantum and Classical

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Abstract
In reality, micro objects and macro objects coexist harmoniously, and each macro object depends on micro particles to exist (macro is a collection of micro objects, indicating that they have a very extensive relationship and are symbiotic). However, classical mechanics and quantum mechanics have long been regarded as binary oppositions. This is a serious disconnect between theory and phenomena, requiring the establishment of new theories and methods that are compatible between quantum and classical. Prior to this, many reasons and methods for combining quantum theory with classical theory have been found (providing numerous computational examples). The wave mechanics equations of classical mechanics laws have been written. The technical method used is still the conversion rule of the Hermitian operator in the fundamental postulates of quantum mechanics. There are no new assumptions. The key is to use the relationship between force and energy in the planetary model to obtain wave mechanics equations that include force, Newton's second law, and the law of conservation of momentum. Thus, quantum mechanics, which is compatible with classical mechanics, truly becomes a form of 'mechanics'. This kind of mechanics is generalized wave mechanics. Quantum and classical are coordinated and compatible, resulting in no clear boundary between quantum and classical. It will seriously affect the credibility of quantum decoherence and wave function collapse.

Keywords: Quantum and Classical, The Wave Mechanics Equation of Classical Mechanics Laws, Quantum Mechanics, Generalized Quantum Mechanics, Tu's Equation

1. Introduction

The author of this article has published a series of papers on quantum and classical compatibility, where quantum mechanics methods can be used together with classical mechanics methods for use [1-19]. This article is the progress and depth of the author's previous research. Wrote the wave mechanics equations of classical mechanics laws such as the Law of Momentum and Newton's Second Law, as well as operators for forces, accelerations, etc. The derivation method is based on the rules regarding Hermitian operators in the fundamental postulates of quantum mechanics, without using any new assumptions. In the Schrödinger equation, the upper limit of mass m can be arbitrarily increased, and the properties of potential energy can also be changed. This is because the Schrödinger equation itself cannot be logically constrained. This indicates that the Schrödinger equation itself does not set a boundary between quantum mechanics and classical forces applicable to macroscopic bodies. These two characteristics simultaneously demonstrate the effectiveness of the research method and results presented in this article. Equations (5) - (9) in the text are the fundamental equations that combine quantum mechanics and classical mechanics. Their corresponding basic theory should be classical quantum mechanics (also known as generalized quantum mechanics). The other significance of this research work will be introduced in the "Discussion and Results" section.

On June 21, 2027, the author submitted a manuscript titled 'The Classical Mechanics Wave Equation of' Classical and Quantum Compatible 'to Physics Essays. But the lack of response from Physics Essays one month later. On July 21, 2026, a Brazilian scientific journal published an article titled Reasons and consequences for the combination of quantum and classical theory (The arrangement of volume and issue numbers for this magazine requires queuing. The actual publication date of this article is July

2026, but the printed version number is Volume 6, Issue 2 (2027) [19]. https://periodicos.cerradopub.com.br/bjs/article/view/929). There is also a section that introduces the derivation process of the formulas in section 2. On July 24, 2026, a manuscript titled "Representing classical mechanical laws using wave equations" was submitted to Nature magazine (tracking number: 2026-07-22353). Prior to this, there were many errors in the derivation of several formulas in Section 2, and the methods were not the most concise. The derivation in this article is an important revision of the previous derivation process and method.

2. Introduction to the quantum mechanical equations of classical mechanids laws

The steady-state Schrödinger equation for the ground state hydrogen atom is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+V\psi=E\psi. \quad (1)$$

Although Schrödinger was initially established using Bohr's hydrogen atom as an example. At first, it was only applicable to hydrogen atoms. However, after others denied the concept of orbit and believed in electron clouds, uncertainty, and probability, its applicability unconsciously expanded to all microscopic systems (i.e., this expansion of applicability has no clear logical basis). This equation has strong representativeness. That is, other similar planetary model systems, even those with zero potential energy, can also be described by this equation. Replace the potential energy $V=-\frac{Ze^2}{r}$ with $V=-\frac{GMm}{r}$, and Eq. (1) becomes

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi-\frac{GMm}{r}\psi=E\psi. \quad (2)$$

(This is the Schrödinger-Tu equation). From a purely mathematical perspective, as long as the potential energy function is replaced as needed, equation (2) applies to both microscopic and macroscopic systems. As we all know, in macroscopic planetary model systems, uncertainty, quantum probability interpretation, and state superposition principles are not applicable, The wave function used by Schrödinger is

$$\psi=\psi_0 e^{-\frac{i}{\hbar}(Et-px)}=\psi_0 e^{-i2\pi(\nu t-x/\lambda)}. \quad (3)$$

There are two methods to derive the wave mechanics equation of Newton's second law. For planetary models of hydrogen atoms, we have

$$F=\frac{mv^2}{r}. \quad (4)$$

For the ground state Bohr hydrogen atom, there is also a relationship of $L=mvr=\frac{1}{2}\hbar$. Substituting $r=\hbar/2p$ into equation (4) results in $F=\frac{2p^3}{m\hbar}$. The operator of force is

$$\hat{F}=i2\hbar^2\frac{\partial^3}{\partial x^3}. \quad (5)$$

Equation (5) is the wave mechanics expression of Newton's second law. Readers can verify the above third-order partial differential by finding it.

$$\hat{F}\psi=i2\hbar\frac{\partial^3}{x^3}\psi. \quad (6)$$

If we choose the relationship of $F=-\frac{mv^2}{r}$, then there is a negative sign to the left of equations (5) and (6). Equation (6) is also the wave mechanics expression of Newton's second law with time. Readers can calculate partial differential equations for verification. Replace the momentum in $p_1=p_2$ with momentum operators, and distinguish the wave functions corresponding to $p_1=p_2$ using the following terms: $\psi_i=A_i e^{-i2\pi(\nu_i t_i-x_i/\lambda_i)}$ we can obtain

$$-i\hbar\frac{1}{\psi_1}\hat{p}_1\psi_1=-i\hbar\frac{1}{\psi_2}\hat{p}_2\psi_2. \quad (7)$$

Equation (7) is the expression for the wave mechanics equation of the law of conservation of momentum.

According to the acceleration formula, it can be inferred that $\hat{a}=\hat{\dot{v}}=-\frac{\partial}{\partial t}\left[\frac{i\hbar}{m}\frac{\partial}{\partial x}\right]$. According to $F=ma$, it can be inferred that $\hat{F}=-i\hbar\frac{\partial^2}{\partial t\partial x}$. So, the second form of Newton's second law is $\hat{F}\psi=-i\hbar\frac{\partial^2}{\partial t\partial x}\psi$. The second form is universal.

By directly replacing the electromagnetic potential energy in the Schrödinger equation with gravitational potential energy, equation (8) can be obtained.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi-\frac{GMm}{r}\psi=E\psi. \quad (8)$$

Considering the Bohr hydrogen atom, $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=T\psi=\frac{1}{2}pv\psi$, $pr=\frac{1}{2}\hbar$ and $v=\alpha c$, we have $-\alpha c\hbar\frac{\partial^2}{\partial x^2}\psi-F\psi=\frac{E}{r}\psi$. Considering $V=2E$, and by organizing this equation, we can obtain equation (9).

$$-\alpha c\hbar\frac{\partial^2}{\partial x^2}\psi+\frac{Ze^2}{2r^2}\psi=F\psi. \quad (9)$$

The equation (9) is the Tu equation, and it is a wave mechanics equation with explicit force.

According to the momentum operator $\hat{p}=-i\hbar\frac{\partial}{\partial x}$, $\hat{F}=\frac{\partial\hat{p}}{\partial t}=-i\hbar\frac{\partial^2}{\partial t\partial x}$ and $v=\frac{p}{m}$ or $\hat{v}=\frac{\hat{p}}{m}$, the velocity operator, velocity squared operator, and acceleration operator can be obtained:

$$\hat{v}=\hat{\dot{s}}=\frac{\hat{p}}{m}=-\frac{i\hbar}{m}\frac{\partial}{\partial x}, \quad (10)$$

$$\hat{v}^2=-\frac{\hbar^2}{m^2}\frac{\partial^2}{\partial x^2}, \quad (11)$$

$$\hat{a}=\hat{\dot{v}}=-\frac{\partial}{\partial t}\left[\frac{i\hbar}{m}\frac{\partial}{\partial x}\right]=-\frac{i\hbar}{m}\frac{\partial^2}{\partial t\partial x}. \quad (12)$$

For uniform circular motion maintained by centripetal force, the relationship between radius r and circumference s is s=2 πr. Therefore

$$\hat{r} = -\frac{i\hbar}{2m\pi}\frac{\partial}{\partial x}. \tag{13}$$

The three-dimensional form of equation (9) is

$$-c\alpha\hbar\nabla^2\psi - \frac{V}{2r}\psi = F\psi. \tag{14}$$

Equation (8) is the Schrödinger-Tu equation. Equations (9) and (14) are both equations. The establishment of Eq. (14) is like Bohr's successful use of old quantum theory to describe the hydrogen atom, which is a preliminary success in the effort to "give wings of force" to wave mechanics. It can intuitively demonstrate that wave mechanics itself does not exclude classical mechanics. The establishment of equation (14) is like Bohr's successful use of old quantum theory to describe the hydrogen atom, which is a preliminary success in endowing wave mechanics with wings of power. It can intuitively prove that wave mechanics itself does not exclude classical mechanics.

The derivation methods for the quantum operator of angular momentum and the wave mechanics equation are as follows. The derivation methods for the quantum operator of angular momentum and the wave mechanics equation are as follows. For the ground state Bohr hydrogen atom, there is a relationship: L = pr. By using $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ and equation (13) to replace p and r in L = pr with operators, we can obtain

$$\hat{L} = \frac{i\hbar}{2m\pi}\frac{\partial^2}{\partial x^2}. \tag{15}$$

$$\hat{L}\psi = \frac{i\hbar}{2m\pi}\frac{\partial^2}{\partial x^2}\psi. \tag{16}$$

The conversion formula between the partial derivatives of the wave function with respect to time and displacement is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{i\hbar}{2}\frac{\partial}{\partial t}\psi. \tag{17}$$

Substitute equation (3) into equation (17) for verification: the left side is $\frac{1}{2}mv^2\psi = T\psi$, and the right side is $\frac{E}{2}\psi$=$\frac{h\nu}{2}\psi$ $E\psi$=$h\nu\psi$. According to equation (4) in reference [20], $\frac{h\nu}{2}\psi$=$\frac{1}{2}mv^2\psi$. The conclusion is that 'transformation equation (17) is correct'. According to equation (17), equation (2) can be transformed into: $i\hbar\frac{\partial}{\partial t}\psi + V\psi = E\psi$. Transfer the term $i\hbar\frac{\partial}{\partial t}\psi = E\psi - V\psi$. Multiplying both sides by -1 results in $-i\hbar\frac{\partial}{\partial t}\psi = -E\psi + V\psi$. For the hydrogen atom in planetary models, So, we obtained the following equation

$$-i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi. \tag{18}$$

The time-dependent Schrödinger equations introduced in textbooks are all

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi. \tag{19}$$

Is Schrödinger wrong, or is the author of this article wrong? This will be discussed in the next section.

3. Discussion on Significance

The significance of the research findings summarized by the author is as follows:

3.1. Core Viewpoint Proposed: Breaking Down Disciplinary Barriers

In this paper, the author explicitly challenges the traditional notion that the Schrödinger equation cannot describe macroscopic objects (such as the Earth), and Newtonian mechanics cannot describe microscopic systems. The author points out that this inherent concept severely limits people's understanding of the natural world, the development of physics theories, and the application of existing theories.

3.2. Key Breakthrough: Building Cross Scale Descriptive Equations

The author successfully derived the Schrödinger equation that can describe planetary motion by replacing the potential energy term in the Hamiltonian operator with gravitational interaction potential energy instead of electromagnetic interaction potential energy. This innovative equation achieves:

a) Macroscopic verification: When approximating the distance between the sun and the earth as a constant, the energy eigenvalues obtained by solving the Schrödinger equation for the Earth's revolution are completely consistent with the results of classical mechanics calculations.

b) Unified scale: proves that classical mechanics and wave mechanics are compatible, and there is no insurmountable gap between the two.

c) Simplified calculation: Established a theoretical framework that can simultaneously describe all objects (without mass limitations), simplifying the calculation process of quantum mechanics.

3.3. Theoretical Significance: Promoting the Paradigm Shift in Physics

The author's research findings have significant implications as follows:

a) Cognitive update: Proving that quantum mechanics and classical mechanics can be combined, overturning the traditional notion that the two are mutually exclusive.

b) Application Expansion: Provides new tools for cross scale physics research, with the potential to promote cross disciplinary applications in fields such as astrophysics and quantum mechanics.

c) Disciplinary integration: laying the foundation for establishing a more unified theoretical system of physics and promoting the integrated development of macro and micro physics.

d) Research Extension: Forming a Systematic Theoretical System.

Combining with the author's other literature lists, it can be seen that this paper is an important component of the author's series of studies. The author also published "Establishing the Schrödinger equation for macroscopic objects and changing human scientific concepts", "Schrödinger-Tu equation: A bridge between classical mechanics and quantum mechanics" Waiting for works (including nearly 20 articles demonstrating the combination and compatibility of quantum and classical, and one book). Reference 2 is a summary discussion. The book 'Quantum Mechanics Returns to Local Realism' is a critique of the Copenhagen interpretation of quantum mechanics and an introduction to the application of local realism in quantum chemistry. Gradually constructing a systematic theoretical framework that integrates quantum mechanics and classical mechanics, pointing towards the category of localized real quantum mechanics—the generalized quantum mechanics that encompasses classical mechanics (this article makes a significant contribution to this direction).

The quantum operator of momentum can be transformed between the derivative of the wave function with respect to time t and the derivative of the wave function with respect to x (sudh as$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{i\hbar}{2}\frac{\partial}{\partial t}\psi$). The function and utility of this phenomenon are worth discussing. Equations (18) and (19) are inconsistent (the left side of the two equations differs by a negative sign). It can be certain that if the potential energy V is zero, Schrödinger's equation (19) is correct. However, limiting the potential energy to zero restricts the applicability of equation (19), and equation (19) becomes a special case of equation (18). This is obviously unsatisfactory. We need to further analyze it. The numerous successful applications of equation (2) have demonstrated its correctness, and equation (17) is easily rigorously validated. This confirms that equation (18) is correct. Back then, Schrödinger used the wave equation to describe the Bohr hydrogen atom (at least using the hydrogen atom as an example) and intuitively wrote equation (2). He may not have completely eliminated intuition when converting equation (2) into the time-dependent Schrödinger equation. The occurrence of symbol errors is understandable. Later, it was discovered that the unitary evolution structure of Hilbert space could be used to rigorously derive the Schrödinger equation. The result obtained using this method is also equation (19). In this derivation process, there are two places where subjective factors can be embedded: the symbol of A in $\hat{A} = -A$; The sign of Hamiltonian H. It cannot be ruled out that people may have preconceived notions in places where subjective factors can be inserted in order to cater to Schrödinger's equation (19).

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