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Research Article

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The Differences Between Equivalent Foster and Cauer Circuits and Factorised Impedance

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Abstract

Approximations of non-integer order elements as Warburg impedance or constant phase element (CPE) are frequently made with equivalent electrical circuits. We can show that 5 various models are impedances mathematically equivalent for their frequency behavior. However, if we look closer at these models according to their various properties: model available in all cases, number of optimum, transient simulation, integration time, it seems that only one of the 5 models have all the qualities. This result is fundamental for the good choice of a model used for transient identification or for on-line identification.

Keywords: Equivalent Circuits, Modelling, Component, Transient Simulations, Parameter Identification

Introduction

In the beginning of the XXth century, the electricians have developed different filters, which realize various transfer functions. We can particularly note the structures of Foster and Cauer [1, 2]. O. Brune gives in 1931 a table of different LC circuits which contains four circuit solutions (2 Foster's from and 2 Cauer's from) "To find the necessary and sufficient conditions to be satisfied by the impedance function of a finite passive network, and to construct a network corresponding to any function satisfying these conditions." The modelling by equivalent networks being more and more used in other domains as the electrochemistry, thermal science or in mechanics with linear viscoelastic systems [3-5]. For different boundary conditions for the frequency F = 0 and F = ∞, these models are classified for RC and RL circuits (see Appendix 1 of [6]). So, this paper is the following of [6]. The objective of this paper is to present the differences between equivalent models having exactly the same impedance and boundary conditions.

Equivalent Models of an Impedance

The different models

The four different circuits correspond to different developments of the impedance Z (transfer function), but there is only one factorized expression. For each circuit we can found a set of parameters, which give the same factorized transfer function. Therefore, these four circuits are all equivalent to the factorized impedance (Figure 1).

For different models, the names of components are the same but to have equivalent the values of components must be different, see for example the results of Table 1.

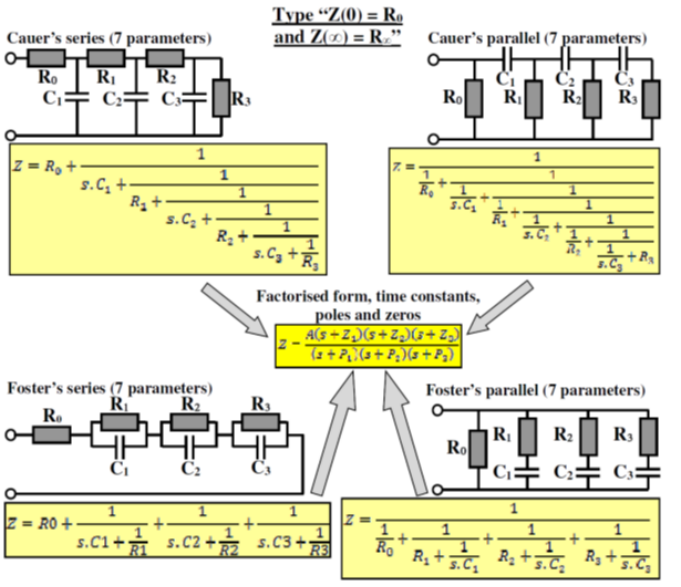


Figure 1: Different equivalent models used in EIS as Cauer's series, Cauer's parallel, Foster's series, Foster's parallel and their same factorised form.

The transformation of the impedance Z from a circuit form to a factorised expression can be easily done with the "factor" function of Maple software.

Identification Procedure

The principle of the parameter optimization is presented on Figure 2. An algorithm of optimization tries to minimize the distance between the measures and the model by adjusting the parameters. The "fminsearch" function in the optimization toolbox of the MatLab software is of good used.

The distance is the RMS value of the difference between the measurements and the model for all frequency points.

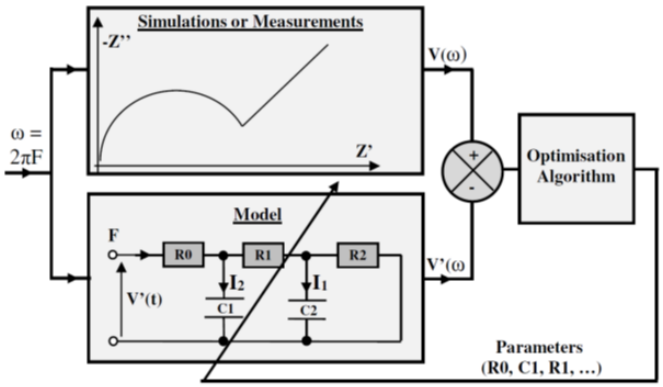


Figure 2: Principe of identification used in electrochemical impedance spectroscopy (EIS).

Identification Procedure

The optimization of the parameters can be made for all the various circuits, even also for factorised impedance. Figure 3 give the results of parameters optimisation on a Nyquist plot. We can see the good accordance between the measures (black) and the 5 models which give the same curve (blue) for N=6 (13 parameters).

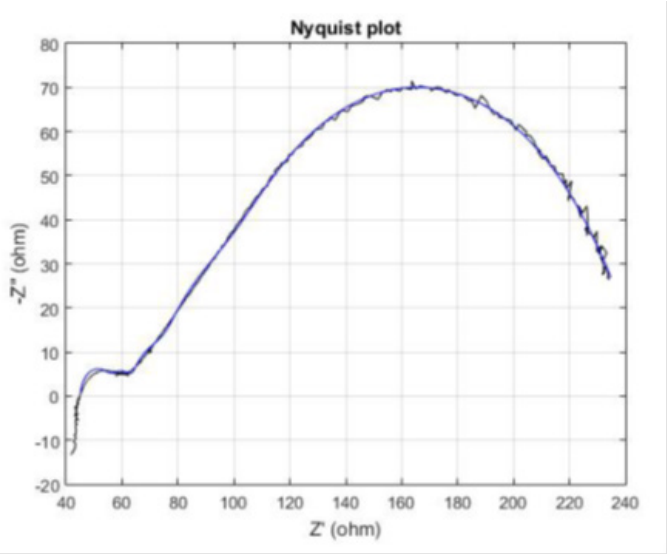


Figure 3: N=6 Comparison of the models obtained by optimization, the 5 same curves superposed (blue) for Cauer' series, Cauer's parallel, Foster's series, Foster's parallel and factorized impedance with regard to measures (Ferri-Ferro solution) (black).

Results

The Table I gives for the criterion and parameters the values obtained by optimisation. Besides stacked curves we obtain exactly the same criterion for the 5 models, which confirm the total equivalence of the 5 models. The data are presented here for N=6 (13 parameters), but we can go easily until N=10 (21 parameters).

Table 1: Results of parameters identification for the 5 models of "R+Tanh" N=6 (7 decades)

	Foster's series	Foster's parallel	Cauer's series	Cauer's parallel		Factorised impedance
	N = 6	N = 6	N = 6	N = 6		N = 6
Criterion	2.5964 E+03	2.5964 E+03	2.5964 E+03	2.5964 E+03	Criterion	2.5964 E+03
ΣR	264.80		264.80		A.ΠZ/ΠP	264.80
1/Σ(1/R)		44.909		44.909		
R_0	44.909	264.80	44.909	264.80	A	44.909
C_1	1.6776 E-03	1.3896 E-04	1.1648 E-06	6.9952 E-04	Z1	1.1180
R_1	142.53	6436.4	14.081	361.58	P1	1.0648
C_2	9.3182 E-04	4.8043 E-04	1.7073 E-05	1.1241 E-03	Z2	9.3738
R_2	28.187	222.04	9.0613	285.63	P2	4.1817
C_3	6.2168 E-02	7.3295 E-05	1.5691 E-04	6.1822 E-05	Z3	53.614
R_3	15.105	254.47	22.347	316.97	P3	38.072
C_4	2.2024 E-04	6.4685 E-06	5.2160 E-04	5.5425 E-06	Z4	421.71
R_4	13.013	366.59	58.354	446.63	P4	348.91
C_5	1.2629 E-06	2.9706 E-07	1.4233 E-03	3.4044 E-07	Z5	7784.85
R_5	12.072	432.42	108.23	338.08	P5	6738.1
C_6	1.6534 E-05	5.8177 E-08	1.1733 E-01	5.5398 E-08	Z6	83564
R_6	8.9760	205.69	7.8131	258.247	P6	65588

The values of R0 for Foster's series, Cauer's series correspond to the value of Z for F=∞ . The values of 1/Σ1/R (parallel resistances) for Foster's parallel and Cauer's parallel correspond to the value of Z for F=∞.

The values of R0 for Foster's parallel and Cauer's parallel correspond to the value of Z for F=0.The values of ΣR (series resistances) for Foster's series, Cauer's series correspond to the value of Z for F=0.

The value of A of factorized impedance correspond to the value of Z for F=∞ (in green). The values of A. (Z1*…*Z6)/(P1*…*P6) correspond to the value of Z for F=0 (inblue). So the values of the boundary conditions appear directly
to the reading of the table of parameters.

We optimize on a window of measures between [Fmin, Fmax]. But if the impedances are mathematically equivalent on the interval between [Fmin, Fmax], there are also for [0, ∞], and thus for the asymptotic behaviors.

It is not easy to give an answer to the question: of these 5 models which converge best? But we already feel that in high orders lake N=10 that it is the series models which have an ease.

Identification of Models from Transient

The identification method used

It is necessary to proceed to an identification of parameters. That corresponds to minimizing the RMS gap from behavior between the transient and the answer of the model obtained by
simulations as presented on the plan of the Figure 4.

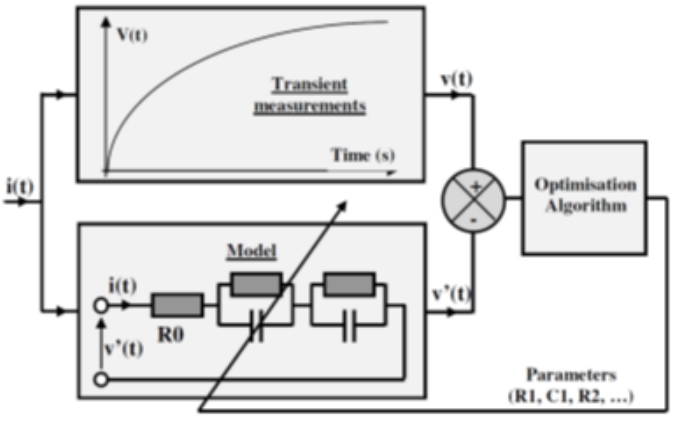
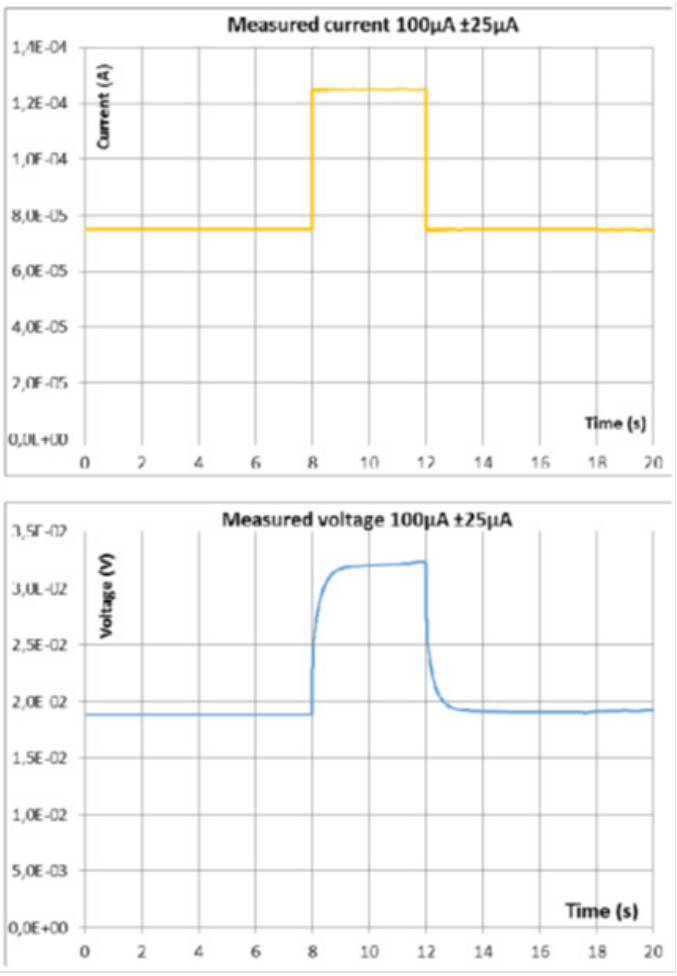


Figure 4: Principle of identification parameters used with transient

Contrary to the spectral measures, which contain rich information with all the frequencies, the transient cannot go beyond half of the frequency of sampling if we apply the theorem of Shannon. Thus, the transient is less rich in high frequencies than the spectroscopy analysis. For this reason, we go only to the order 5 of the models to make an identification of parameters. The programs of simulation for all 5 different models are given in Appendix.

The measures give the current and voltage curves of the Fig. 5. So, this electrochemical system is not linear, and we will just consider the linear part around the polarization point at 100µA. Furthermore, to simulate transients and identify a model, we have the problem with the initial value of the voltage of capacitors. The simplest is to leave with a zero voltage. We shall thus make the translations to arrive at the current and voltage for identification on Figure 5. The identified model being linear it changes nothing the obtained results.

The Appendix shows the simplicity of simulation for certain models (Foster's series and Cauer's series) to see the complexity for others (Foster's parallel, Cauer's parallel and factorized impedance), we shall take in identification only Foster's series and of Cauer's series to compare the obtained results.



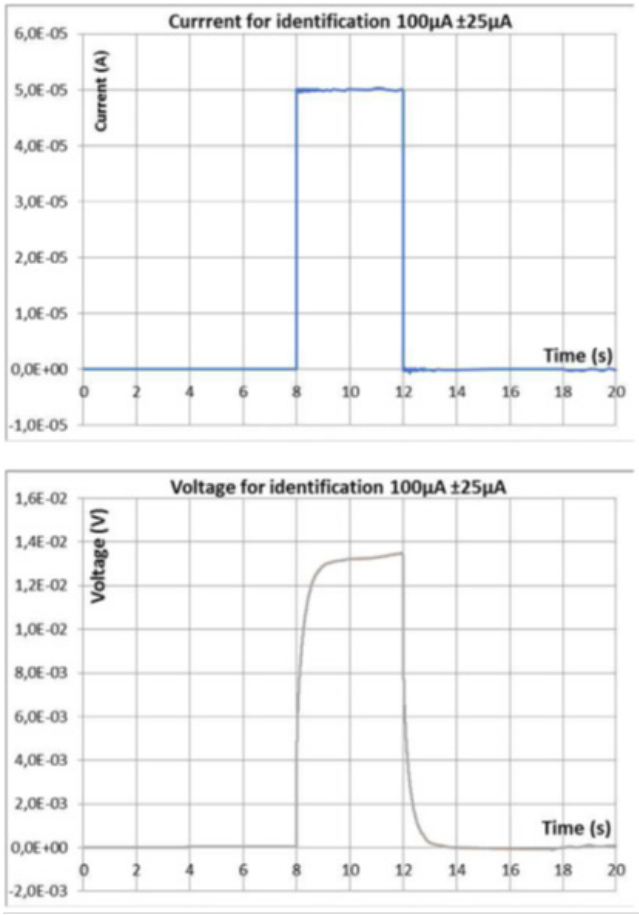


Figure 5: Direct measures and corrected measures for identification (100µA±25µA)

Results for Foster's series models

The results of the identification from a transient are presented in the Table 2 below. We find a value of 265Ω for the total resistance of the solution in direct current. In high frequencies the value of the resistance R0 converges well on 70Ω from N=3 to 5. The value of the criterion does not decrease after N=3.

Table 2: Results of Identification for Foster's Series Models (Transient Data 100µA ±25µA)

100µA ±25µA	N = 0	N = 1	N = 2	N = 3	N = 4	N = 5
Criterion	9.7753 E-08	3.2456 E-08	3.1816 E-08	3.1808 E-08	3.1808 E-08	3.1808 E-08
ΣR	264.85	265.32	265.46	265.46	265.46	265.46
R_0	264.85	121.62	97.162	71.727	69.935	69.507
C_1		2.2291 E-03	7.8425 E-03	1.8256 E-03	1.8256 E-03	1.8256 E-03
R_1		143.70	57.267	110.07	110.08	110.08
C_2			1.7742 E-03	8.0219 E-03	8.0246 E-03	8.0246 E-03
R_2			111.03	56.171	56.156	56.156
C_3				43199.	42220.	42173.
R_3				27.493	27.867	27.885
C_4					1.7308 E-04	1.5540 E-04
R_4					1.4170	1.4490
C_5						5.9339 E-04
R_5						0.37844

We can put these results in the form of factorized impedance. The first time constant (P1) is particularly well identified and have particularly constant whatever is the order N of the model.

Table 3: Poles and Zeros of the Identified Impedance by Foster's Series Models (Transient Data 100µA ±25µA)

	N = 1	N = 2	N = 3	N = 4	N = 5
Criterion	**3.2456 E-08**	**3.1816 E-08**	**3.1808 E-08**	**3.1808 E-08**	**3.1808 E-08**
A	121.62	97.162	71.728	69.935	69.507
P_1	3.1218	2.2265	2.2192	2.2190	2.2190
Z_1	6.8103	2.6172	2.5966	2.5963	2.5963
P_2		5.0761	4.9761	4.9757	4.9758
Z_2		11.7987	11.104	11.103	11.103
P_3			84.186	84.990	85.031
Z_3			119.32	121.07	121.16
P_4				4077.1	4440.7
Z_4				4160.6	4450.2
P_5					4453.0
Z_5					4561.4

We can notice on Figure 6 that we have no asymmetry between the rise and the descent, thus we have a good overlapping between the measures and the identified model.

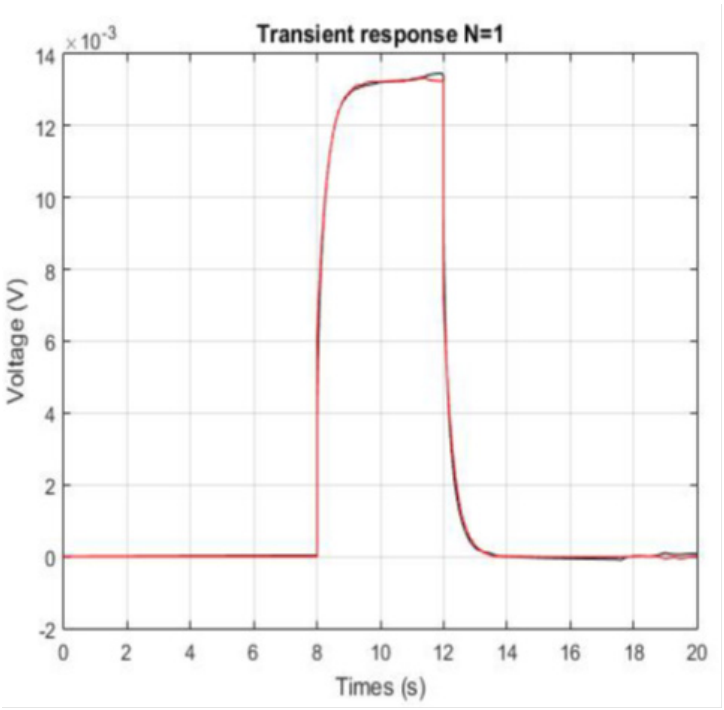


Figure 6: Comparison of transient measures (Black) and simulated models (Red) for different orders of Foster's series models at 100µA ±25µA).

Results for Cauer's Series Models

The results of the identification from a transient are presented in the Table 4 and Table 5 below. We find the same value of 265Ω for the total resistance of the solution in direct current. Contrary to Foster models, the value of the criterion for Cauer's models continues to decrease for N=4 and N=5.

Table 4: Results of Identification for Cauer's Series Models (Transient Data 100µA ±25µA)

100µA ±25µA	N = 0	N = 1	N = 2	N = 3	N = 4	N = 5
Criterion	9.7753 E-08	3.2456 E-08	3.1816 E-08	3,1807 E-08	2.6752 E-08	1.1005 E-08
ΣR	264,85	265.32	265.46	265.46	265.46	265.46
R_0	264,85	121.62	97.162	71.916	67.506	52.145
C_1		2.2291 E-03	1.4468 E-03	3.3757 E-04	3.1313 E-06	1.1772 E-06
R_1		143.70	151.88	45.184	3.7682	10.892
C_2			2.4522 E-02	1.3272 E-04	3.2423 E-04	1.3454 E-05
R_2			16.417	133.68	45.305	8.6649
C_3				2.7421 E-02	1.3302 E-03	3.2613 E-04
R_3				14.678	134.13	24.883
C_4					2.7243 E-02	4.9361 E-04
R_4					14.746	89.113
C_5						3186.7
R_5						79.653

We can put these results in the form of factorized impedance. The first time constant (P1) is particularly well identified and constant for N+2 to N=4. P1 varies for N=5, what can be a sign of over-parameterization.

For N=4, time constant P4 and Z4 is beyond the 5 kHz of Shannon (2π x 5000 Hz = 31415 rad/s) but can have an influence below 5 kHz. It's the same for P5 and Z5 in the case of N=5.

Table 5: Poles and Zeros of the Identified Impedance by Cauer's Series Models (Transient Data 100µA ±25µA)

100µA ±25µA	N = 1	N = 2	N = 3	N = 4	N = 5
Criterion	3.2456 E-08	3.1816 E-08	3,1807 E-08	2.6752 E-08	1.1005 E-08
A	121.621	97.1622	71.9169	67.5064	52.1453
P_1	3.1218	2.2265	2.2153	2.2189	2.9408
Z_1	6.8103	2.6172	2.5905	2.5962	5.2615
P_2		5.0761	4.9655	4.9754	17.1463
Z_2		11.7987	11.0697	11.1023	31.9559
P_3			83.4460	85.1841	209.324
Z_3			118.156	121.642	234.824
P_4				85568.	8173.41
Z_4				90255.	9464.78
P_5					85569.
Z_5					1.0052 E+5

The transient responses for Cauer's series models are identical to that of Fig. 6 in the case of Foster's series models.

Synthesis

From N=0 to N=3 the solutions found by both models arrive everything at the same criterion. The impedance factorization of allows to compare both models which we find identical.

Although the calculations are more or less similar between Foster and Cauer, we notice a faster convergence with Cauer's series with regard to Foster's series. For N=4 and N=5 the model of Cauer's series allows to find a solution with a better criterion while Foster's series who does not progress any more. For N=4 and N=5 Foster's model does not anymore manage to improve the criterion while the Cauer's model still arrives there.

In Foster's models, the different RC cells (Rx, Cx) are in series or in parallel. Therefore, if we make permutations of the positions between RC cells, we do not chance the value of the impedance. We can say that the structures of these circuits are not ordered. We can have different sets of parameters [R0, C1, R1, … , CN, RN] corresponding to the same circuit. Therefore, to compare sets of parameters we must and have to order its.

However, a major problem is that the possible number of permutations increases with the order N as N! (Factorial N). If there is an optimum solution, it exists there then N! other identical solutions by permutations. That can be very disturbing for a program of optimization. Also, if there is local optima, their number is also multiplied by N!. When the order N increase the density of optima also increase.

In Cauer's models, we cannot exchange the elements without

change the impedance. We can say that these structures of circuits are ordered. Therefore, to compare sets of parameters we must not have to order it. The optimum or the local optima remain in this case unique. Which is already the big advantage of the models of Cauer's on Foster's models.

The number of optima does not increase and we have a better sensitivity.

Conclusion

It is necessary to see electrochemical spectroscopy impedance as a tool of metrology of great precision.

The identification from a transient is necessarily less rich than the electrochemical spectroscopy impedance (EIS) in high frequencies. However, the weak point of the EIS is for the very low frequencies, and there the identification from a transient can turn out to be an excellent method. Thus, every method has its domain of preference: the transient measures for the very low frequencies, the EIS for high frequencies.

The financial costs of both methods are not comparable, and the advantage is unmistakably to the transient measures. We can also see the EIS as a tool of laboratory and the analysis of transitory measures as tool of the ground.

This study thus allowed us to see clearly there more between these various models. On five possible equivalent modellings, this study highlights the particular interest of some ones.

The factorized impedance is a very elegant method to obtain a spectral model decomposed into poles and zeros. However, the factorized impedance leads to heavy calculations to obtain the algorithm of simulation by its Z transform. For the factorized impedance, the evolution of the voltage V(n) at the step n is a function only of this previous voltage to the steps n-1, n-2, n-3 and current at the steps n, n-1, n-2 and n-3. There is no internal variable in the calculation. However, with the increase of the order the calculations of simulation quickly become very heavy.

This complexity does not seem to us compatible with an identification of parameters from transient response. A use in parameter identification of the simulations of the factorized impedance model is not a good choice.

For the four models under circuit forms, capacitors are of use as internal variables to the calculations. The calculated voltage V(n) at the step n is a unction only of the current I(n) in the step n and voltage of capacitors at the step n-1.

The transient calculations are more complex for parallel circuits than for series circuits. Therefore, we looked more attentively to the series models.

Among the "series" circuits, Foster's models have the inconvenience to do not exist with all boundary conditions, while the Cauer's models exist with all boundary conditions (see [6]).

In the Foster's models, the different stages RC in series or in parallel can be inverted without changing the behaviors of the impedance or the transfer function. We can say that the structures of these circuits are not ordered. Therefore, to compare a set of parameters we must and have to order it.

For a factorized impedance, the poles can switch between them and the zeros also, without changing the value of the impedance. We can say that the time constants of these functions are not ordered. Therefore, to compare a set of parameters we must and have to order it.

In the Cauer's models, we cannot exchange the elements without change the behaviors or the impedance. We can say that these structures of circuit are ordered. Therefore, to compare a set of parameters we must not have to order it. The Cauer's series circuits are a development from infinite frequency and we improve the model in high frequency by adding elements at the beginning.

Furthermore, in the use it is the Cauer's series models that shows itself the most effective. The Cauer's Series models have proved in optimization more sensitive to find of better optimum with regard to Foster's series models. Thus, for all these reasons, there are Cauer's series models that have our preference.

Table 6: Comparison of Difference Equivalent Models

Models	Available in all cases	Number of optimum	Transient simulation	Integration time	Balance sheet
Foster's series	No	N!	Easy	Variable	
Foster's parallel	No	N!	Difficult	Variable	
Cauer's series	Yes	1	Easy	Variable	The Best!
Cauer's parallel	Yes	1	Difficult	Variable	
Factorized impedance	Yes	N!	Difficult	Constant	

Before this study, we have 4 different equivalent circuit models and their factorized form. Now we can compare these models for the number of optimum and the difficulty to calculate a transient with the Table 6.

The result is that the Cauer's series model has the best properties and can be used easily in all cases of boundary conditions and for impedance and transient identifications.

By finding a good compromise between:

- Levels enough low of steps of 50µA giving variations of voltage of 10mV order, allow to have a linear approximation of the small signals.
- Levels enough brought up compared with the level of the noise allow to the measures to have signals rich in information.

It is possible to obtain results in identification of impedances from transients, which can be compared with the measures by EIS. The obtained results are remarkable and
show all the interest of this method.

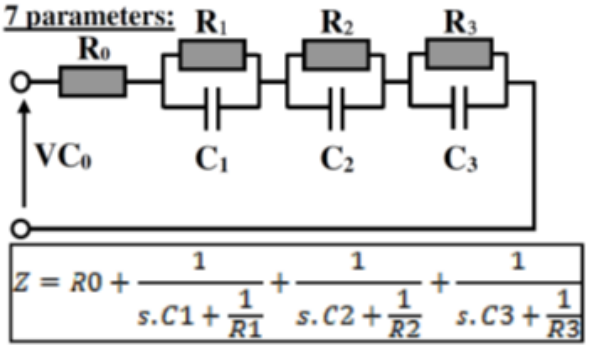
This study thus allows having good bases to go towards the development of the on-line identification for the follow-up of parameters on real electrochemical processes.

Appendix

Comparison of 5 equivalent models for transient simulations

We shall give here as example the various calculations obtained for 5 various models corresponding to boundary conditions Z(0) = R0 Z(∞) = R∞ and for N=3 (7 parameters).

Foster's series model: "Z(0) = R_0 Z(∞) = R_∞"
7 parameters:



Example of lines of program for MatLab calculation

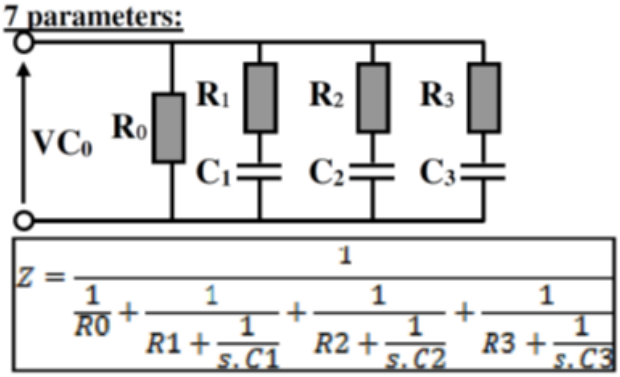
```
% Response to a step of current
% Initialization
 dT = 0.1;
 VC1= 0;
 VC2= 0;
 VC3= 0;
 VC0(1)=V(1) ;
 TC1=0;

% Loop
 for k = 1:500
  IC1 = (I(k)*R1 – VC1)/R1;
  IC2 = (I(k)*R2 – VC2)/R2;
  IC3 = (I(k)*R3 – VC3)/R3;
  VC1 = VC1 + dT*IC1/C1;
  VC2 = VC2 + dT*IC2/C2;
  VC3 = VC3 + dT*IC3/C3;
  TC1 = TC1 + dT;
  VC0(k+1) = R0*I(k) + VC1 + VC2 + VC3;
end;
```

Synthesis

For this Foster's series model, the calculations are simple and the generalization when we increase the order is also very simple.

Foster's parallel model: "Z(0) = R_0 Z(∞) = R_∞"
7 parameters:



Example of lines of program for MatLab calculation

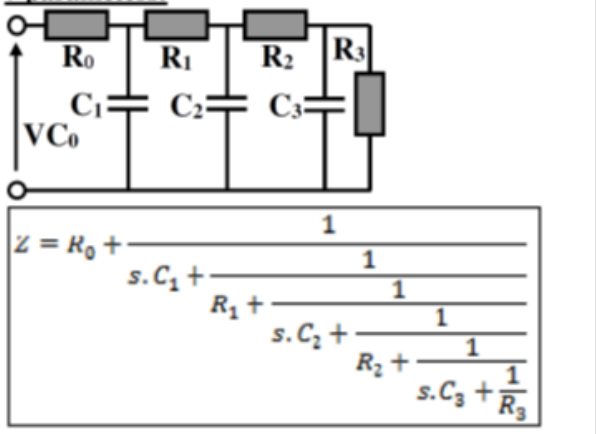
```
% Response to a step of current
% Initialization
  dT = 0.1;
  VC1=V(1);
  VC2=V(1);
  VC3=V(1);
  VC0(1)=V(1);
  TC1=0;
  deno = R0*R1*R2 + R0*R1*R3 + R0*R2*R3 + R1*R2*R3;

% Loop
 for k = 1:500
  I1 = (I(k)*R0*R2*R3 – (R0*R2+R0*R3+R2*R3)*VC1 +
R0*R3*VC2 +
  R0*R2*VC3)/deno;
  I2 = (I(k)*R0*R1*R3 + R0*R3*VC1 –
(R0*R1+R1*R3+R0*R3)*VC2 +
  R0*R1*VC3)/deno;
  I3 = (I(k)*R0*R1*R2 + R0*R2*VC1 + R0*R1*VC2 –
  (R0*R1+R1*R2+R0*R2)*VC3)/deno;
  VC1 = VC1 + dT*I1/C1;
  VC2 = VC2 + dT*I2/C2;
  VC3 = VC3 + dT*I3/C3;
  VC0(k+1) = R0*(I(k)*R1*R2*R3+R1*R2*VC3+R1*R3*VC2 +
  R2*R3*VC1) /deno;
  TC1 = TC1 + dT;
end;
```

Synthesis

For this Foster's parallel model, the calculations quickly become complicated when the order increases. It appears a common denominator, which corresponds to the global matrix resolution.

Cauer's series model: "Z(0) = R_0 Z(∞) = R_∞" 7 parameters:



Example of lines of program for MatLab calculation

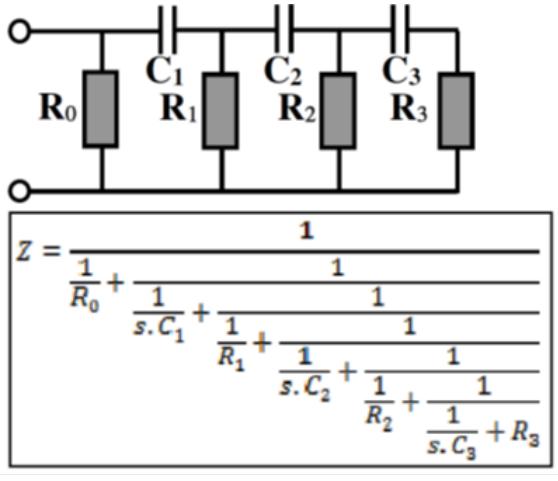
```
% Response to a step of current
% Initialization
dT = 0.1;
 VC3= V(1);
 VC2= V(1) ;
 VC1= V(1);
 VC0(1)= V(1);
 TC1 = 0;

% Loop
 for k = 1:500
  IR1 = (VC1-VC2)/R1;
  IR2 = (VC2-VC3)/R2;
  VC3 = VC3 + dT*((IR3-VC3/R3)/C3);
  VC2 = VC2 + dT*((IR1-IR2)/C2);
  VC1 = VC1 + dT*((I(k)-IR1)/C1);
  TC1 = TC1 + dT;
  VC0(k+1) = VC1 + R0*I(k);
end;
```

Synthesis

For this Cauer's series model the calculations are simple and the generalization when we increase the order is also very simple.

Cauer's parallel model: "Z(0) = R0 Z(∞) = R∞" 7 parameters:



Example of lines of program for MatLab calculation

```
% Response to a step of current
% Initialization
 dT = 0.1;
 VC1 = 0;
 VC2 = 0;
 VC3 = 0;
Deno = (R0*R1*R2 + R0*R1*R3 + R0*R2*R2 + R1*R2*R3);

% Loop
 for k = 1:500
  I1 = (I(k)*R1*R2*R3 + R1*R2*(VC1+VC2+VC3) +
  R1*R3*(VC1+VC2) + R2*R3*VC1)/Deno;
  I2 = (I(k)*R0*R2*R3 + R0*R2*(VC2+VC3) + R0*R3*VC3 –
  R2*R3*VC1) /Deno;
  I3 = (I(k)*R0*R1*R3 + R0*R1*VC3 – R0*R3*VC2 –
  R1*R3*(VC1+VC2)) /Deno;
  VC1 = VC1 + dT*(I(k) – I1)/C1;
  VC2 = VC2 + dT*(I(k) – I1 – I2)/C2;
  VC3 = VC3 + dT*(I(k) – I1 – I2 – I3)/C3;
  VC0(k)= R0*(I(k)*R1*R2*R3 + R1*R2*(VC1+VC2+VC3) +
  R1*R3*(VC1+VC2)+R2*R3*VC1)/Deno;
end;
```

Synthesis

For this Cauer's parallel model, the calculations quickly become complicated when the order increases. It appears a common denominator, which corresponds to the global matrix resolution.

Factorised impedance model: "Z(0) = R0 Z(∞) = R∞"

The impedance is written in its factorized form letting appear the poles and the zeros.

$$Z=\frac{A(s+Z1)(s+Z2)(s+Z3)}{(s+P1)(s+P2)(s+P3)}$$

Example o Lab calculation

```
% Response to a step of current
% Initialization
 dT = 0.1;
    v(1) = A*(1+Z1*dT)*(1+Z2*dT)*(1+Z3*dT)*I(1)
    /((1+P1*dT)*(1+P2*dT)*(1+P3*dT));
    v(2) = (((1+P1*dT)*(1+P2*dT) + (1+P1*dT)*(1+P3*dT) +
    (1+P2*dT)*(1+P3*dT))*v(1) +
    A*(1+Z1*dT)*(1+Z2*dT)*(1+Z3*dT)*I(2) -
   A*((1+Z1*dT)*(1+Z2*dT)+(1+Z1*dT)*(1+Z3*dT)+(1+Z2*
dT)*(1+Z3*
    dT))*I(1)) /((1+P1*dT)*(1+P2*dT)*(1+P3*dT));
    v(3) = (((1+P1*dT)*(1+P2*dT) + (1+P1*dT)*(1+P3*dT) +
    (1+P2*dT)*(1+P3*dT))*v(2) - ((P1+P2+P3)*dT+3)*v(1) +
A * ( ( 1 + Z 1 * d T ) * ( 1 + Z 2 * d T ) * ( 1 + Z 3 * d T ) * I ( 3 ) -
((1+Z1*dT)*(1+Z2*dT)
  +(1+Z1*dT)*(1+Z3*dT) + (1+Z2*dT)*(1+Z3*dT))*I(2) +
   ((Z1+Z2+Z3)*dT+3)*I(1))) / ((1+P1*dT)*(1+P2*dT)*(1+P3
*dT));
% Loop
for n = 4:500
    v(n) = (((1+P1*dT)*(1+P2*dT) + (1+P1*dT)*(1+P3*dT) +
   (1+P2*dT)*(1+P3*dT))*v(n-1) - ((P1+P2+P3)*dT+3)*v(n-2)
+ v(n-3)
```

```
A*((1+Z1*dT)*(1+Z2*dT)*(1+Z3*dT)*I(n)                    -
((1+Z1*dT)*(1+Z2*dT) +
(1+Z1*dT)*(1+Z3*dT) + (1+Z2*dT)*(1+Z3*dT))*I(n-1) +
((Z1+Z2+Z3)*dT+3)*I(n-2) - I(n-3)))
/((1+X(3)*dT)*(1+X(5)*dT)*(1+X(7)*dT));
End
```

Synthesis

The expression of the factorized impedance has certain simplicity. The fact of revealing time constant in the expression is of an attractive elegance.

However, at the third order, the calculations become very complicated and to see inextricable in the order 4 and beyond. That is not compatible with the need of very fast simulations to make of the identification of parameters.

References

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