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Terminal Tidal Evolution of TOI-2431 b in the Primary-Centric Framework

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Abstract

Ultra–short-period exoplanets represent terminal configurations of tidal evolution in close star–planet systems. In the Primary-Centric Framework (PCF), orbital evolution is governed by the angular-momentum ratio $\Lambda(a) = \omega_\star/\Omega(a)$ *which defines a characteristic topological structure with two Clarke radii and a global maximum separating inner and outer evolutionary branches. We apply this framework to the Earth-sized exoplanet TOI-2431b, which orbits its host star with a period of 5.37h. Using solar-approximation parameters, we compute the present orbital radius and the corresponding Clarke radii and show that* $a_{now} < a_{G1}$*, placing the planet deep on the inner branch where* $\Lambda(a) < 1$*. In this regime, inspiral is topologically irreversible: no continuous tidal evolution can restore a torque-balanced configuration without external angular-momentum injection. Dissipation controls the timescale, but the directional inevitability of orbital decay is fixed by the inequality* $\Lambda(a) < 1$*. TOI-2431b therefore provides a concrete realization of terminal tidal evolution in which angular-momentum topology predetermines inward migration and ultimate disruption. The analysis illustrates how PCF supplies a global classification scheme for planetary fates beyond detailed tidal microphysics.Independent tidal-decay modeling suggests a remaining lifetime of* $\sim 3 \times 10^7$ *yr , consistent with the PCF classification of TOI-2431b as a terminal inner-branch system.*

Keywords: Exoplanets, Tidal Evolution, Angular Momentum, Orbital Decay, Irreversibility, Clarke Orbit, Ultra-Short-Period Planets

1. Introduction

The long-term evolution of gravitationally bound two-body systems is governed not only by dissipative microphysics but also by the global structure of their conserved quantities. In close star–planet systems, tidal interactions redistribute angular momentum between orbital motion and stellar spin while simultaneously dissipating mechanical energy. Although detailed tidal models differ in their prescriptions for dissipation, the direction and ultimate fate of orbital evolution are constrained by the total angular-momentum budget of the system. This suggests that tidal decay may be understood not merely as a dynamical process but as a topological consequence of angular-momentum structure.

Ultra–short-period exoplanets provide a particularly clear arena in which to examine this issue. Planets with orbital periods of only a few hours experience extreme tidal forcing, rapid energy dissipation, and strong irradi1ation. Many such objects are believed to be remnant rocky cores, stripped of volatile envelopes by intense stellar radiation. Their continued inward migration raises a fundamental question: is orbital decay in these systems contingent on detailed dissipation parameters, or is it fixed by deeper angular momentum constraints?

In the Primary-Centric Framework (PCF), the evolution of a star–companion system is characterized by the dimensionless ratio

$$\Lambda(a) \equiv \frac{L_{\rm orb}(a)}{L_{\rm spin}}, \tag{1}$$

where L_{orb}(a) denotes the orbital angular momentum of the planet about the primary star, and L_{spin} denotes the spin angular momentum of the primary. For a circular orbit, the orbital angular momentum is

$$L_{\mathrm{orb}}(a) = \mu\sqrt{G(M+m)a}, \tag{2}$$

where M and m are the stellar and planetary masses, $\mu = \frac{Mm}{M+m}$ is the reduced mass, and G is the gravitational constant.

The stellar spin angular momentum is

$$L_{\mathrm{spin}} = C_\star\,\omega_\star, \tag{3}$$

where $C_\star = kMR^2$ is the moment of inertia constant (with k the dimen sionless structure constant and R the stellar radius), and $\omega_\star$ is the stellar rotation angular frequency. The functional form

$$\Lambda(a) = A\,a^{3/2} - F\,a^2 \tag{4}$$

defines a characteristic curve possessing two Clarke radii and a global maximum that separates inner and outer evolutionary branches. The inequality $\Lambda(a) < 1$ identifies configurations for which no torque-balanced state exists on the inner branch, thereby establishing a criterion for irreversible inspiral. Substituting Eqs. (18) and (3) into Eq. (1) yields

$$\Lambda(a) = A\,a^{3/2} - F\,a^2, \tag{5}$$

where the constants A and F depend on the total angular momentum budget and the stellar moment of inertia, as discussed in Sections 2–3. In this work we apply the PCF to the Earth-sized ultra–short-period exoplanet TOI-2431b, which completes an orbit in approximately 5.37h and has a mass of ~ 6.2 Earth masses. Adopting solar-approximation parameters for the host star, we compute the present orbital separation and the corresponding Clarke radii. We show that the system satisfies $a_{\mathrm{now}} < a_{\mathrm{G1}}$ and therefore $\Lambda(a_{\mathrm{now}}) < 1$, placing the planet deep within the inner irreversible branch. The result demonstrates that the inward migration of TOI-2431b is not merely a consequence of strong tidal dissipation, but a manifestation of the global angular-momentum topology of the system.

The broader implication is that ultra–short-period planets represent terminal configurations in angular-momentum phase space. Dissipation determines the rate of evolution, but the directional inevitability of orbital decay is governed by the structure of $\Lambda(a)$. The PCF therefore provides a global classification scheme for planetary fates, complementing detailed tidal micro physics with a topological criterion for irreversibility.

2. Observed System Parameters

TOI-2431b is an ultra-short period rocky exoplanet with orbital period

$$P_{\mathrm{orb}} \approx 5.37\ \mathrm{hr}. \tag{6}$$

The planet has mass

$$m_p \approx 6.2\,M_\oplus \tag{7}$$

and radius

$$R_p \approx 1.5\,R_\oplus. \tag{8}$$

2.1. Orbital Period in SI Units

$$P_{\mathrm{orb}} = 5.37\ \mathrm{hr} = 5.37 \times 3600\ \mathrm{s} \approx 1.93 \times 10^4\ \mathrm{s}. \tag{9}$$

The semi-major axis is obtained from Kepler's third law:

$$a = \left(\frac{GM_* P_{\mathrm{orb}}^2}{4\pi^2}\right)^{1/3}. \tag{10}$$

2.2 Semi-Major Axis

Assuming $M_* \approx M_\odot$,
Using Kepler's third law, the present semi-major axis is obtained as

$$a_{\mathrm{now}} = \left(\frac{G(M_\star + m_p)P_{\mathrm{orb}}^2}{4\pi^2}\right)^{1/3}, \tag{11}$$

where $M_\star$ is the stellar mass, mp is the planetary mass, Porb is the orbital period, and G is the gravitational constant. For P_{orb} = 5h22min = 1.932 ×10^4 s, and adopting $M_\star \simeq M_\odot$ with mp ≪ $M_\star$, we obtain

$$a_{\mathrm{now}} \approx 1.08 \times 10^9\ \mathrm{m}. \tag{12}$$

2.3 Mass Parameters

$M\star$ (solar approximation or observed) mp = 6.2, $M_\oplus$ Reduced mass $\mu = \frac{M_\star m_p}{M_\star + m_p}$. For $M_\star \simeq M_\odot = 1.989 \times 10^{30}$ kg and $m_p = 6.2\,M_\oplus$ = 3.70×10^{25} kg, we obtain

$$\mu \approx 3.70 \times 10^{25}\ \mathrm{kg}. \tag{13}$$

Because $m_p \ll M_\star$, the reduced mass differs from the planetary mass only at relative order $m_p/M_\star \sim 10^{-5}$. For the adopted solar-approximation parameters,

$$M_\star = 1.989 \times 10^{30}\ \mathrm{kg}, \tag{14}$$

$$m_p = 3.70 \times 10^{25}\ \mathrm{kg}, \tag{15}$$

$$\mu = 3.699 \times 10^{25}\ \mathrm{kg}. \tag{16}$$

Because $\mathbf{m_p} \ll M_\star$, the reduced mass satisfies

$$\mu \approx m_p, \tag{17}$$

with relative corrections of order $m_p/M_\star \sim 10^{-5}$. For the adopted parameters, $M_\star = 1.989 \times 10^{30}$ kg, $m_p = 3.70 \times 10^{25}$ kg, and $\boldsymbol{\mu} = 3.699 \times 10^{25}$ kg. Since $m_p \ll M_\star$, we have $\mu \approx m_p$.

3. Angular Momentum Budget

The secular evolution of a close star–planet system is governed by the redistribution of angular momentum between orbital motion and stellar spin through tidal interaction [1,2]. For a two-body system of stellar mass $M_\star$ and planetary mass m_p, the orbital angular momentum for a circular orbit is

$$L_{\text{orb}}(a) = \mu\sqrt{G(M_\star + m_p)a}, \tag{18}$$

where $\mu = \frac{M_\star m_p}{M_\star + m_p}$ is the reduced mass. The stellar spin angular momentum is

$$L_{\text{spin},\star} = C_\star \omega_\star, \tag{19}$$

Where $I_\star = kM_\star R^2$ is the stellar moment of inertia and $\omega_\star$ is the stellar spin angular velocity. The dimensionless structure constant k depends on stellar internal structure [3].

$$L_{\text{spin},\star} = C_\star \omega_\star, \tag{20}$$

where $C_\star = kM_\star R^2_\star$ is the stellar moment of inertia and $\omega_\star$ is the stellar spin angular velocity, and k is the dimensionless moment-of-inertia constant that depends on stellar internal structure [3].

The total angular momentum of the system is therefore

$$J_T = L_{\text{orb}}(a) + L_{\text{spin}*}, \tag{21}$$

where

$$L_{\text{orb}}(a) = \mu\sqrt{G(M_\star + m_p)a}, \tag{22}$$

$$L_{\text{spin}*} = C_\star \omega_\star. \tag{23}$$

Here we neglect the planetary spin angular momentum and any additional companions, so that J_T is well approximated by the sum of stellar spin and orbital angular momentum.

(Planetary spin is tiny compared to orbital AM for close-in planets, so this is standard.)

For fixed J_T, the partition between orbital and spin components con strains the dynamical phase space accessible to the system. The characteristic ratio.

$$\Lambda(a) = \frac{L_{\text{orb}}(a)}{L_{\text{spin}}}, \tag{24}$$

Substituting these expressions yields

$$\Lambda(a) = A\,a^{3/2} - F\,a^2, \tag{25}$$

where A and F are constants determined by the total angular momentum budget and the stellar moment of inertia. For a circular orbit,

$$L_{\text{orb}}(a) = \mu\sqrt{G(M_\star + m_p)a}, \tag{26}$$

where $\mu = \frac{M_\star m_p}{M_\star + m_p}$ is the reduced mass.

The stellar spin angular momentum is

$$L_{\text{spin}} = C_\star \omega_\star, \tag{27}$$

where $C_\star = kM_\star R^2 \star$ and $\omega_\star$ is the stellar spin frequency.
encodes this partition and determines whether a torque-balanced configuration exists at a given orbital separation. The two Clarke radii arise from the condition $\Lambda(a) = 1$ [4,5], while the global maximum of Λ(a) reflects the limited capacity of the orbital reservoir to compensate stellar spin exchange.

Thus, the angular-momentum budget defines not merely the present state of the system but the topology of its tidal evolution.

3.1 LOM/LOD Ratio

$$\Lambda(a) \equiv \frac{L_{\text{orb}}(a)}{L_{\text{spin},\star}}, \tag{28}$$

where $L_{\text{orb}}(a)$ is the orbital angular momentum of the planet about the star and $L_{\text{spin},\star}$ is the stellar spin angular momentum. If $\Lambda(a) \ll 1$, the system lies on the inner branch ($a < a_{G1}$), i.e. inside the inner Clarke orbit. In this region, tidal dissipation can only reduce the orbital separation, so the subsequent evolution is a one-way (topologically irreversible) inspiral in the PCF sense.

3.2 Angular-Momentum Topology

The significance of the angular-momentum budget extends beyond numerical evaluation. For fixed total angular momentum J_T, the partition between orbital and spin components constrains the accessible phase space of the system. The characteristic function

$$\Lambda(a) \equiv \frac{L_{\text{orb}}(a)}{L_{\text{spin},\star}}, \tag{29}$$

encodes this partition and determines whether a torque-balanced configuration exists at a given separation. The two Clarke radii arise from the condition $\Lambda(a) = 1$, while the global maximum of $\Lambda(a)$ reflects the limited capacity of the orbital reservoir to compensate stellar spin exchange.

Thus, the angular-momentum budget does not merely quantify the state of the system; it defines the topological structure of its evolution. In particular, when $a_{\mathrm{now}} < a_{G1}$, the system is confined to the inner branch and the subsequent evolution is irreversible under purely tidal processes.

3.3 Semi-major Axis (Solar Approximation)

For TOI-2431b, the observed orbital period is P = 5.37h. Converting to SI units,

$$P = 1.933 \times 10^4\,\mathrm{s}. \tag{30}$$

Assuming a solar-mass host star, Kepler's third law gives

$$a^3 = \frac{G(M_\star + m_p)P^2}{4\pi^2}. \tag{31}$$

Adopting $M_\star \approx M_\odot$ and mp = $6.2M_\oplus$, we obtain

$$a_{\mathrm{now}} = 1.08 \times 10^9\,\mathrm{m}. \tag{32}$$

This extremely small separation places the planet deep within the regime of strong tidal forcing [6,7].

3.4 Orbital Angular Momentum

Approximating $e \simeq 0$, the orbital angular momentum is

$$L_{\mathrm{orb}} = m_p\sqrt{GM_* a} \approx (6.2\,M_\oplus)\sqrt{GM_\odot a} \approx 1.40 \times 10^{40}\ \mathrm{kg\,m^2\,s^{-1}}. \tag{33}$$

3.5 Stellar Spin Angular Momentum

We write

$$L_{\mathrm{spin},*} = C_*\,\omega_*, \qquad \omega_* = \frac{2\pi}{P_{\mathrm{rot}}}, \qquad C_* \approx k_* M_* R_*^2, \tag{34}$$

We adopt the solar dimensionless moment-of-inertia constant $k_\odot \simeq 0.07$ as an approximation for a solar-type host star, so that $C_\star = k_\odot M_\star R_\star^2$.

For representative stellar rotation periods, the stellar spin angular momentum is

$$P_{\mathrm{rot}} = 10\ \mathrm{d}: \quad L_{\mathrm{spin},\star} \approx 4.90 \times 10^{41}\ \mathrm{kg\,m^2\,s^{-1}}, \tag{35}$$

$$P_{\mathrm{rot}} = 20\ \mathrm{d}: \quad L_{\mathrm{spin},\star} \approx 2.45 \times 10^{41}\ \mathrm{kg\,m^2\,s^{-1}}, \tag{36}$$

$$P_{\mathrm{rot}} = 30\ \mathrm{d}: \quad L_{\mathrm{spin},\star} \approx 1.63 \times 10^{41}\ \mathrm{kg\,m^2\,s^{-1}}. \tag{37}$$

3.6 LOM/LOD Ratio and PCF Regime

Using

$$L_{\mathrm{orb}} \approx 1.40 \times 10^{40}\ \mathrm{kg\,m^2\,s^{-1}},$$

the Primary-Centric control ratio becomes

$$P_{\mathrm{rot}} = 10\ \mathrm{d}: \quad \frac{L_{\mathrm{orb}}}{L_{\mathrm{spin},\star}} \approx 2.86 \times 10^{-2}, \tag{38}$$

$$P_{\mathrm{rot}} = 20\ \mathrm{d}: \quad \frac{L_{\mathrm{orb}}}{L_{\mathrm{spin},\star}} \approx 5.72 \times 10^{-2}, \tag{39}$$

$$P_{\mathrm{rot}} = 30\ \mathrm{d}: \quad \frac{L_{\mathrm{orb}}}{L_{\mathrm{spin},\star}} \approx 8.58 \times 10^{-2}. \tag{40}$$

In all cases, $\Lambda(a_{\mathrm{now}}) \ll 1$, confirming that the system resides deep inside the inner Clarke radius and is therefore committed to irreversible inspiral within the Primary–Centric Framework. placing the system deep inside the irreversible inspiral regime of the Primary-Centric Framework.

4. Inner Clarke Orbit and Topological Irreversibility

In the Primary-Centric Framework (PCF), the long-term tidal evolution of a close-in companion is governed not only by dissipation strength but, more fundamentally, by the angular-momentum topology encoded in the characteristic ratio We define the Primary-Centric characteristic ratio

$$\Lambda(a) \equiv \frac{L_{\mathrm{orb}}(a)}{L_{\mathrm{spin},\star}}, \tag{41}$$

where $L_{\mathrm{orb}}(a)$ is the orbital angular momentum of the planet about the host star and $L_{\mathrm{spin},\star}$ is the stellar spin angular momentum. For a circular orbit

$$L_{\mathrm{orb}}(a) = \mu\sqrt{G(M_\star + m_p)a}, \tag{42}$$

and

$$L_{\mathrm{spin},\star} = C_\star\omega_\star, \tag{43}$$

with $\mu = \frac{M_\star m_p}{M_\star + m_p}$ the reduced mass, $C_\star = k_\star M_\star R_\star^2$ the stellar moment of inertia, and $\omega_\star$ the stellar spin frequency.

For circular or nearly circular configurations (appropriate for ultra-short period planets), the PCF characteristic function may be written in the closed form

$$\Lambda(a) = A\,a^{3/2} - F\,a^2, \tag{44}$$

with constants $A > 0$ and $F > 0$ determined by the system masses, the primary's moment of inertia, and the total angular-momentum budget (Sections 2–3). The two Clarke radii a_{G1} and a_{G2} are defined by $\Lambda(a) = 1$, and the curve reaches a global maximum at amax, separating the inner and outer branches.

4.1 The Irreversibility Condition

The key dynamical discriminator in PCF is the inequality

$$\Lambda(a) < 1. \tag{45}$$

When $\Lambda(a) < 1$, the system lies on the inner inspiral branch. $\Lambda(a) < 1$, the orbital reservoir is insufficient, in a strict angular-momentum sense, to raise the system to a torque-balanced configuration on the outer branch. In this regime the tidal torque produced by the primary's tidal bulge does not admit a stable equilibrium at the current separation: the system is confined to the inner branch and the secular evolution is forced toward decreasing semi-major axis of the exoplanet. The inner Clarke radius aG1 represents a torque-balanced configuration separating dynamically distinct evolutionary branches. However, this equilibrium is only accessible from the outer side. When $a_{\mathrm{now}} < a_{G1}$, the system lies on the inner branch of the characteristic curve where $\Lambda(a) < 1$. In this regime, no continuous tidal evolution can restore the system to aG1 without external angular-momentum injection.

The distinction is therefore structural rather than parametric. Dissipation controls the rate of orbital decay, but the direction of evolution is fixed by the topology of $\Lambda(a)$. The inspiral of TOI-2431b is thus not merely rapid; it is dynamically inevitable.

4.2 PCF Inspiral Law on the Inner Branch

On the inner branch defined by $\Lambda(a) < 1$, the system admits no torque balanced equilibrium accessible from its current state. The secular evolution of the orbital separation may therefore be written in the generic form

$$\frac{da}{dt} = -\mathcal{I}(a), \tag{46}$$

where $\mathcal{I}(a) > 0$ represents an inspiral functional that aggregates the dissipative physics of tidal interaction, including the tidal quality factor, Love number, and frequency dependence of the stellar response.

The essential point is that the negative sign in Eq. (46) is not imposed phenomenologically, but follows from the angular-momentum topology of the system. When $a < a_{G1}$, the characteristic ratio satisfies $\Lambda(a) < 1$, implying that the orbital reservoir is insufficient to sustain outward migration. Energy dissipation therefore manifests as monotonic decrease of a.

In conventional equilibrium-tide models, $\mathcal{I}(a)$ typically scales as a high power of a^{-1}, so that inspiral accelerates as the companion approaches the disruption boundary [2,7]. Within the PCF, however, these details determine only the timescale of decay. The directional inevitability of inward migration is fixed by the inequality $\Lambda(a) < 1$.

Thus, for TOI-2431b, which satisfies $a_{\mathrm{now}} \ll a_{G1}$, the inspiral is not a transient phase approaching equilibrium but a terminal secular trajectory toward tidal disruption or engulfment.

4.3 Physical Interpretation: Terminal Tidal Evolution

The preceding analysis demonstrates that the inspiral of TOI-2431b is not merely rapid but structurally predetermined. Because $a_{\mathrm{now}} < a_{G1}$ and therefore $\Lambda(a_{\mathrm{now}}) < 1$, the system resides on the inner branch of the characteristic curve where no torque-balanced configuration is dynamically accessible.

Physically, tidal dissipation removes mechanical energy while redistributing angular momentum between orbital motion and stellar spin. However, on the inner branch, the orbital angular-momentum reservoir is too small to permit outward migration. Energy loss therefore manifests as monotonic reduction of orbital separation.

As semi-major axis a decreases further, tidal forcing strengthens, dissipation accelerates, and the planet approaches the disruption boundary set by the Roche limit or direct impact with the stellar surface [6]. The final outcome is therefore engulfment or tidal disintegration.

The designation "terminal" is thus not rhetorical. It reflects a topological constraint imposed by the angular-momentum budget. Dissipation controls the rate of approach to the endpoint, but the existence of the endpoint itself follows from the inequality $\Lambda(a) < 1$. TOI-2431b represents a concrete realization of a system whose evolutionary fate is fixed by angular-momentum topology rather than detailed microphysical parameters.

4.4 LOM/LOD Topology Across the Super- to Sub-Synchronous Domain

Caveat. The outer Clarke radius a_{G2} lies in the regime where the stellar gravitational sphere of influence becomes dynamically dominant. In practice, this scale is comparable to the Hill radius of the planet-hosting star in the Galactic tidal field [8,9]. Beyond this domain, the two-body PCF description ceases to be dynamically self-contained.

$a = 1.08 \times 10^9$ m For fixed stellar spin rate ω_*, the orbital mean motion is

$$n(a) = \sqrt{\frac{GM_*}{a^3}}, \tag{47}$$

so the system is super-synchronous when $n > \omega_*$ (small a) and sub-synchronous when $n < \omega_*$ (large a). The co-rotation radius is defined by $n(a_{\mathrm{co}}) = \omega_*$, hence

$$a_{\mathrm{co}} = \left(\frac{GM_*}{\omega_*^2} \right)^{1/3}. \tag{48}$$

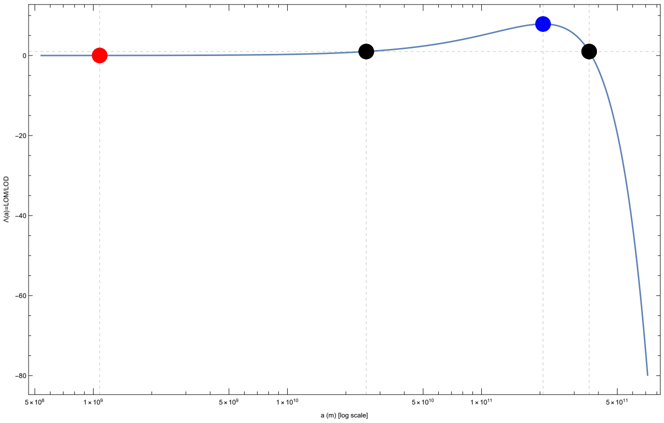

Figure 1: Primary–Centric characteristic curve $\Lambda(a)$ = LOM/LOD for TOI-2431b under the adopted solar-approximation parameters. The red marker denotes the present orbital separation anow, the blue marker indicates the global maximum a_{max}, and the black markers mark the two Clarke radii a_{G1} and a_{G2} defined by $\Lambda(a) = 1$. The system resides deep on the inner branch ($a_{\mathrm{now}} < a_{G1}$), where $\Lambda(a) < 1$ and tidal evolution is topologically irreversible within the Primary–Centric Framework. Figure generated by the author using Wolfram Mathematica.

The PCF control ratio is

$$\Lambda(a) \equiv \frac{L_{\mathrm{orb}}(a)}{L_{\mathrm{spin},\star}} = \frac{\mu\sqrt{G(M_\star + m_p)a}}{C_\star \omega_\star}, \tag{49}$$

which increases monotonically with a as $\propto a^{1/2}$ for fixed ω_*. For $m_p \ll M_*$, we approximate

$$\Lambda(a) \approx \frac{m_p\sqrt{GM_\star a}}{C_\star \omega_\star}. \tag{50}$$

4.5 Angular Momentum Budget

In the Primary–Centric Framework we define the control function as the ratio of the stellar spin frequency to the Keplerian orbital frequency,

$$\Lambda(a) \equiv \frac{\omega_\star}{\Omega(a)} = \frac{P_{\mathrm{orb}}(a)}{P_{\mathrm{rot}}}, \tag{51}$$

where $\omega_\star = 2\pi/P_{\mathrm{rot}}$ is the stellar rotation angular frequency and $\Omega(\mathrm{a}) = 2\pi/P_{\mathrm{orb}}(a)$ is the orbital mean motion.

Neglecting planetary spin, the total angular momentum may be written as

$$J_T = I_\star \omega_\star + \mu a^2 \Omega(a), \tag{52}$$

where $I_\star$ is the stellar moment of inertia and $\mu = \frac{M_\star m_p}{M_\star + m_p}$ is the reduced mass. Using Kepler's law,

$$\Omega(a) = \sqrt{\frac{G(M_\star + m_p)}{a^3}} = \frac{B}{a^{3/2}}, \qquad B \equiv \sqrt{G(M_\star + m_p)}, \tag{53}$$

and dividing Eq. (52) by $I_\star\Omega(a)$ yields

$$\Lambda(a) = \frac{J_T}{BI_\star} a^{3/2} - \frac{\mu}{I_\star} a^2. \tag{54}$$

Here I_* is defined as C_* Defining the constants

$$A \equiv \frac{J_T}{BC_\star}, \qquad F \equiv \frac{\mu}{C_\star}, \tag{55}$$

we obtain the compact polynomial form

$$\Lambda(a) = A\,a^{3/2} - F\,a^2. \tag{56}$$

For reference, the angular-momentum ratio satisfies $L_{\mathrm{orb}}/L_{\mathrm{spin},\star} = Fa^2/(Aa^{3/2} - Fa^2)$.

5. Discussion

The case of TOI-2431b illustrates a broader dynamical category: ultra–short-period planets that occupy the inner branch of the Primary-Centric characteristic curve. In such systems, the inequality $\Lambda(a) < 1$ signals that no torque-balanced configuration is accessible without external angular-momentum injection. The inspiral is therefore structurally irreversible.

This perspective complements conventional equilibrium-tide formulations [2,7], which emphasize dissipation mechanisms and parameter-dependent decay rates. Within the PCF, tidal prescriptions determine the timescale of evolution, but not its direction. The sign of *da/dt* is fixed by angular momentum topology. This separation between rate and direction provides a conceptual clarification: irreversible inspiral is not merely a consequence of strong dissipation, but a manifestation of constrained angular-momentum phase space.

Ultra–short-period planets are likely remnants of previously more extended configurations that have migrated inward over secular timescales. The PCF suggests that once such systems cross the inner Clarke radius, their long-term fate becomes dynamically predetermined. Observationally, this implies that the population of extremely short-period rocky planets may represent a transient but terminal evolutionary phase.

More broadly, the PCF provides a global classification scheme for close star–planet systems. Rather than treating each system as governed solely by dissipation parameters, one may classify systems according to their position relative to the Clarke radii. In this sense, angular-momentum topology plays a role analogous to energy landscapes in other areas of physics, constraining accessible evolutionary pathways.

Future applications of this framework may include systematic classification of ultra–short-period planets, binary stars, and compact-object systems within a unified angular-momentum topology. Such an approach may reveal structural commonalities across gravitationally bound systems that are not immediately apparent from microphysical tidal models alone.

6. Conclusion

We have applied the Primary-Centric Framework (PCF) to the ultra–short-period exoplanet TOI-2431b and demonstrated that its present orbital configuration lies deep within the inner Clarke radius. The condition anow < a_{G1} implies $\Lambda(a_{\mathrm{now}}) < 1$, placing the system on the inner branch of the characteristic curve where no torque-balanced configuration is dynamically accessible.

Within this framework, the inspiral of TOI-1b is not merely a consequence of strong tidal dissipation, but a structural consequence of the angular-momentum budget. Dissipation determines the rate at which the orbit decays, yet the direction of evolution is fixed by the topology of $\Lambda(a)$. The inequality $\Lambda(a) < 1$ establishes a criterion for irreversible inspiral that is independent of detailed tidal microphysics.

TOI-2431b therefore represents a concrete realization of terminal tidal evolution in a star–planet system. Its ultimate fate—tidal disruption or engulfment—follows from angular- momentum constraints rather than from parameter tuning. More broadly, the PCF provides a global classification scheme for planetary fates in which ultra–short-period planets occupy a terminal region of angular-momentum phase space.

The analysis illustrates that tidal evolution can be understood not only as dissipative dynamics but as a manifestation of angular-momentum topology. In this sense, irreversibility in close star–planet systems emerges from structural constraints imposed by conserved quantities, offering a complementary perspective to conventional equilibrium-tide descriptions. The polynomial structure of Eq. (56) reveals that tidal evolution in the Primary–Centric Framework is governed not merely by dissipative microphysics but by the global angular-momentum topology of the system. The two Clarke radii arise as the solutions of $\Lambda(a) = 1$, separating torque-balanced configurations from dynamically committed branches. Because TOI-2431b satisfies $\Lambda(a_{\mathrm{now}}) \ll 1$, it resides deeply within the inner branch ($a < a_{G1}$), where no continuous tidal evolution can restore a torque-balanced state without external angular-momentum injection. Dissipation determines the rate of decay, but the directional inevitability of inward migration is fixed by the structural form of $\Lambda(a)$ itself. In this sense, the inspiral of TOI-2431b is a topological consequence of the angular-momentum budget rather than a contingent outcome of model-dependent tidal prescriptions. Following Taş et al. (2025), the expected remaining time to reach the Roche limit (tidal disruption/engulfment) is of order

$$t_{\mathrm{Roche}} \sim 3.1 \times 10^7 \text{ yr.} \tag{57}$$

This value is model-dependent and primarily reflects assumptions about stellar tidal dissipation; in the PCF interpretation, dissipation sets the timescale while the branch condition fixes the direction of evolution [10].

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Declaration of Interests

The author declares no competing financial or non-financial interests.

Declaration of Generative AI and AI-Assisted Technologies

During the preparation of this work, the author used generative AI tools, including ChatGPT, DeepSeek, and Perplexity, for reasoning and drafting support. After using these services, the author reviewed, edited, and verified all content as needed and takes full responsibility for the accuracy and integrity of the publication.

Ethics, Consent to Participate and Consent to Publish

Not applicable.

Author Contributions

The author collected data regarding the Length of Day (LOD) from popular science literature by Isaac Asimov, George Gamow, and Carl Sagan. Following the NASA press release on the Silver Jubilee Anniversary of the Moon landing (20 July 1994), reporting that the Moon has receded by approximately 1~m in the preceding 25 years, the author reanalyzed the Earth– Moon system and presented the results at the 82nd Session of the Indian Science Congress (Jadavpur University, Kolkata, 1995).

Subsequently, the author presented the Kinematic Model of the Earth–Moon system at the World Science Congress (Houston, 2002). In 2004, at the 35th Scientific Assembly of COSPAR, the author presented a new perspective on the birth and evolution of the Solar System and exoplanetary systems. In 2012, at the 39th Scientific Assembly (Mysore, India), the paper "Iapetus subsatellite revisited and it reveals the celestial body formation in the Primary Centric Framework" (B03-0011-12) was presented. In 2017, at CELMEC VII (Rome), the advanced Kinematic Model of the Earth–Moon system was presented and subsequently published in the Journal of Geography and Natural Disasters, demonstrating a close match between observed and theoretical LOD curves. Further related studies on the Earth–Moon system and its habitability were published in JMTCM.

Subsequent work extended the Primary-Centric Framework to stellar and exoplanetary systems, including stars near Sagittarius A*, the 51 Pegasi system, the WASP-12 system, and several other exoplanetary systems, which are currently under peer review. Another paper continues this research program by applying the framework to early stellar evolution and chemically primitive stellar systems. The research work under peer review studies a scenario where a multiplanetary system is just being born and is in infancy. The present work stydies an exoplanet which is caught in a death spiral.

Data Availability

The data underlying this article are available within the article and in its online supplementary material. All figures in this paper were generated by the author using published observational data and empirical relations.

Clinical Trial Registration

Not applicable.

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