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Research Article

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Study of the Electron Distribution Function Downstream of the Solar Wind Termination Shock: A Touch with the Interstellar Medium

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Abstract
We theoretically describe the evolution of the solar wind electron distribution function downstream of the termination shock under the influence of electron impact ionizations of interstellar H- and He- atoms that enter the heliosheath from the upwind hemisphere and continue to move along the heliotail region. We start from a kinetic phasespace transport equation in the bulk frame of the heliosheath plasma flow that takes into account convective changes, cooling processes, whistler wave-induced energy diffusion, and electron injection into and removal from velocity-space cells due to electron impact ionization processes of interstellar neutral atoms. From this kinetic equation we can ascend to an associated pressure moment of the electron distribution function and there with arrive at a so-called pressure transport equation describing the evolution of the electron pressure in the bulk-velocity frame of the plasma flow. Assuming that the local electron distribution can be represented by a kappa function with a K- parameter that varies with the streamline coordinate s, we obtain an ordinary differential equation for K as function of s. With this result we first gain the heliosheath electron distribution function downstream of the termination shock, and at second, obtain a newly based estimate of the ionization probability of interstellar neutral atoms like H and He at the passage over the heliosheath. The latter information will enable us to quantitatively predict the interstellar inflow of neutral He- and H- atoms into the heliosheath. As we shall show the effect of H- and He impact ionizations especially gives its signature to the electron distribution along the extended down-tail region of the heliosheath. This is why we especially study this 100AU- extended down-tail region here in this article.

Introduction
In several more recent publications the idea has been pushed forward that at the solar wind termination shock, at about a distance of 90 AU from the Sun, the different plasma fluids, like solar wind protons, pick-up protons and electrons, get shock-processed in a fluid-specific way [1-3]. Of special relevance for this paper here is the strong claim in most recent papers by Chalov and Fahr (2013), Fahr and Sievert (2013, 2015), Fahr et al. (2012, 2014), Zieger et al. (2015), Fahr and Verscharen (2016) that solar wind electrons when passing over the shock might experience an extraordinary heating. Fahr, Richardson and Verscharen (2015), in their quantitative theoretical treatment of the electron passage over the electric double layer termination shock, have derived a theoretical basis for a preferential heating of electrons with respect to ions due to conversion of their overshoot kinetic energies into thermal energies in the downstream bulk frame [4-9]. As these authors show, the resulting electron distribution function can well be fitted by a kappa-function with a kappa index of K= 1. 52. This highly suprathermal electron distribution function contains a substantial portion of KeV-energetic electrons that can impact-ionize neutral LISM atoms, like H- and He-atoms, in the heliosheath. This finding opens up quite a new aspect on the passage of LISM (Local InterStellar Medium) neutral atoms over the heliosheath at their approach towards the inner heliosphere and to the downwind heliotail. Up to now it was always tacitly assumed that LISM He-atoms, due to complete absence of ionization sources there, do cross the heliosheath without any influence from the heliosheath plasma [10-12]. While H-atoms undergo charge exchange collisions with the heliosheath protons, but as it was thought up to now in such a negligible manner that the H-density over the upwind heliosheath practically also does not change.

Now, however, since in view of the above mentioned publications it must be assumed that electrons leaving the termination shock in downstream direction are strongly energized, hot and have suprathermal distributions, it has to be seriously reanalyzed whether the earlier assumptions from decades ago with respect to LISM neutral atoms passing untouched over the heliosheath can still be maintained in the form they were published more than 20 years ago [9-11]. This already most recently was a subject of reinvestigation in papers by Scherer et al. (2014) and Gruntman (2015) [13, 14]. Especially under the new aspects mentioned above - showing that the heliosheath electrons have to be expected as a "hot, energetic" particle population, it just now becomes the topic of this paper here to check whether or not it can still be adopted as done in the last three decades that the interstellar gases like H-atoms, and especially He-atoms, can cruise through the heliosheath without undergoing important losses and changes in the moments of their distribution functions like density, bulk velocity and pressure.

Theoretical Representation of the Heliosheath Electron Distribution Function

At the beginning of the theoretical treatment some apriori assumptions with respect to general properties of the heliosheath plasma flow have to be made here. First we want to assume that this downstream plasma flow behaves like an incompressible plasma fluid. This seems to be justified by the fact that due to the dominant, high partial pressures of PUI's and especially electrons [5, 6, 15, 16]. The plasma downstream of the termination shock is a very-low-Mach number flow ($M_{s,2} \leq 0.1$), and thus, with the help of Bernoulli's conservation law, one can easily prove the resulting associated incompressibility of this highly pressurized heliosheath plasma, i.e.$\vec{U} \cdot \vec{grad}\,(p)=0$.

In consequence, for the case of a stationary plasma flow, the mass flow continuity then simply writes in the following form

$$0 = div(\rho\vec{U}) = \vec{U} \cdot \vec{grad}\rho + \rho div\vec{U} = \rho div\vec{U}$$

With ρ and $\vec{U}$ being the mass density and the plasma bulk velocity, respectively. In view of the incompressibility ($\rho = const!$) the above relation consequently simply states that the heliosheath flow is arranged as a divergence-free flow, i.e. governed by: $div.\vec{U}=0$.

This, however, means that for the transported ions and electrons, both assumed to be co-convected with the plasma bulk velocity $\vec{U}$, there does not exist adiabatic heating or cooling

$$\left| \frac{\partial f_{e,i}}{\partial t} \right|_{ad},$$

since as formulated by Fahr and Fichtner (2011) in case given here it turns out that

$$\left| \frac{\partial f_{e,i}}{\partial t} \right|_{ad} = \frac{v}{3} \frac{\partial f_{e,i}}{\partial v} div\vec{U} = 0\,.$$

Another pseudo-polytropic effect may, however, be operating for both species. This is the so-called" magnetic cooling effect"

$$\left| \frac{\partial f}{\partial t} \right|_{mag},$$

connected with the conservation of the ion/electron magnetic moment $\mu_{\perp} = mv_{\perp}^2/B$ at changing magnetic field magnitudes along the flow lines [17, 18]. This effect, termed "magnetic cooling" under decreasing magnetic field magnitudes in the plasma flow direction, we shall investigate further down after study of the gradient of the magnetic field magnitude $B=B(s)$ along flow lines in an incompressible flow configuration.

For the adequate theoretical description of the electron state, expecting isotropic distribution functions in the bulk frame, one can use the following phase-space transport equation [20] that describes the evolution of the isotropic electron distribution function f_e along heliosheath flow lines - taking into account: a) convective changes, b) magnetic cooling, c) velocity diffusion due to non-linear electron-whistler wave interactions, - and not yet taken into account: d) electron impact ionizations - in the following form

$$\frac{\partial f_e}{\partial t} = U\frac{df_e}{ds} = \left| \frac{\partial f_e}{\partial t} \right|_{mag} + \frac{1}{v^2}\frac{\partial}{\partial v}(v^2 D_{vv}\frac{\partial f_e}{\partial v}) + \left| \frac{\delta f}{\delta t} \right|_{ei}$$

Here $\left| \frac{\delta f}{\delta t} \right|_{ei}$ denotes that specific temporal change of the distribution function fe which is due to electron energy losses caused by electron impact ionization processes with ambient H- or He atoms of the LISM inflow cruising through the heliosheath.

The electron impact ionization process

Treating electron impact ionization processes as irreversible, i.e. no recombination processes are considered here, and assuming that the newly appearing secondary impact electrons with probably $\Delta Ei \simeq 15eV\text{-}25eV$ compared to the shock generated electrons with about KeV - energies have vanishing energy, will then in a first view, allow to write this term as a velocity-space exchange term in the following form:

$$\left| \frac{\delta f}{\delta t} \right|_{ei} = n_{H,He} \cdot \sigma_{ei}(E) \cdot [(f_e(E) - \left| \frac{\partial f_e}{\partial E} \right|_E \Delta E_i) - f_e(E)]EdE$$

Instead of further using this above term, we here prefer, however, to formulate this term $\left| \frac{\delta f}{\delta t} \right|_{ei}$ for computational advantages and for the sake of a better generalisation to impact electrons with non-vanishing energies, in the following form as an impact-induced stochastic drift $V_{e,i}$ in the electron velocity space, i.e. the average change per time of the electron velocity due to statistical occurences of impact ionizations of ambient atoms. Or to say it in other words: It denotes the average velocity change per unit of time due to impact ionization processes of electrons with energy E. Using this impact-ionization induced drift $V_{e,i}$ one can write the impact-induced temporal change of the distribution function $\left| \frac{\delta f}{\delta t} \right|_{ei}$ in the following form (see Fahr, 2007, or Fahr and Sievert, 2013)

$$\left| \frac{\delta f}{\delta t} \right|_{ei} = \frac{1}{v^2}\frac{\partial}{\partial v}[v^2 \cdot \dot{V}_{e,i} \cdot f_e]$$

Here by the electron distribution function f_e is assumed to be isotropic with respect to the magnetic field in the bulk frame, e.g. due to effective linear interaction with whistler waves. The above formulation can be derived analogously to the one developed in Fahr (2007). The impact-induced velocity drift $V_{e,i}$ consequently can, along this line and realizing that

$$v_{rel} = |\vec{v} - \vec{v}_a| \simeq v,$$

be calculated as given by:

$$\dot{V}_{e,i} = -\sqrt{\frac{2\Delta E_i}{m}}\, n_H \cdot \sigma_{e,i}(v) \cdot v$$

A More Refined Study

The energy drift, i.e. the change per unit of time of the energy $|dE/dt|_{e,i}$ due to impact ionization, is given by

$$|dE/dt|_{e,i} = \Delta E_i^a \cdot n_a \sigma_{ei}^a(E) v_{rel}(E)$$

Where "a" denotes the LISM atomic species, i.e. H=hydrogen or He=helium, ΔE_i^a is the energy loss of the impacting electron carrying out the impact ionization, i.e. the loss of ionization energy of the atom "a", n_a is the local density of "a"-atoms, $\sigma_{ei}^a(E)$ is the impact ionization cross section, and $v_{rel}(E)$ is the relative velocity between electrons of energy E and neutral atoms of species "a". In the above relation it is assumed that a KeV-energetic heliosheath electron, with the virtue to ionize ambient atoms, i.e. with $E \geq \Delta E_i^a$, produces a practically energy-less secondary impact electron, not sharing an essential fraction of the excess energy $\Delta E = E - \Delta E_i^a$ of the primary impact electron.

If, on the other hand, an energy sharing between the two electrons, i.e. the newly appearing free electron out of the former atomic shell of the ionized atom and the free heliosheath electron that caused the ionization, has to be considered, perhaps with a most probable branching ratio of $\beta = 1/2$, then the most probable value for the energy loss per ionization would, different from the above representation, be given by

$$\langle \Delta E \rangle = E - (1/2)(E - \Delta E_i^a) = \frac{1}{2}(E + \Delta E_i^a)$$

On the other hand, this sharing procedure would imply for the total electron population the appearance of an additional electron with the energy $E'' = 1/2\,(E - \Delta E^{ia})$ which in general for the phase-space balance would in principle imply the need of taking into account an injection term $S^+(E'')$ of secondary electrons at this energy E".

The following helpful simplification may, however, be permitted here, allowing for a much simpler form of balancing the velocity space gains and losses. Namely being aware of the following: The primary electron with an initial energy E after ionization remains with an energy

$$E' = (1/2)[E - \Delta E_i^a]$$

On the other hand also the secondary electron picks up an energy of $E'' = E' = (1/2)[E - \Delta E_i^a]$. Realizing now that the average electron energies in the heliosheath are of the order of *Kev*'s, while ΔE_i^a is of the order of *eV*'s, then clearly reveals that $E'' \simeq E' \simeq (1/2)\,E$, i.e. primary and secondary electrons, originating simultaneously from one ionization process, after impact ionization practically appear at the same energy $(1/2)E$, or say: velocity $v' = \sqrt{E/m}$, as long as $E \gg \Delta E_i^a$ remains valid. The above described net process could thus be described by an adequately formulated injection term.

The Electron Injection Term

Following in an analogous manner the example derived in Fahr (2007) the associated injection term can then be expected in the following representation

$$S^+(E') = n_a \sigma_{ei}^a(E) v_{rel}(E) \cdot f_e(E) \frac{1}{2v'^2} \delta(v') = n_a \sigma_{ei}^a(v) \cdot v \cdot f_e(v) \frac{1}{2v'^2} \delta(v')$$

where δ (v') denotes Dirac's Delta function, and the following relation is used to transform energies into associated velocities

$$v'^2 = (v^2/2 - \Delta E_i^a/m) = (v + \Delta v_i)(v - \Delta v_i)$$

The above expression $|dE/dt|_{e,i}$ can be transformed into an associated electron velocity space drift $\dot{V}_{e,i}$ given by

$$\dot{V}_{e,i} = \Delta v_i n_a \sigma_{ei}^a(v) v_{rel}(v) = \frac{\Delta E_i^a}{mv} n_a \sigma_{ei}^a(v) v = \frac{\Delta E_i^a}{m} n_a \sigma_{ei}^a(v)$$

However, instead of describing this injection into the electron velocity space, one could, as well or even better, describe the effect as an associated drift in velocity space as already formulated before.

The Equivalent, Associated Drift Term

Instead of introducing the above mentioned injection term for secondary electrons, we decide to re-formulate the impact-induced electron drift velocity $\dot{V}_{e,i}$ for the new situation of an energy-sharing between primary and secondary electrons as demanding the following resulting velocity change

$$\frac{1}{2} m \Delta v_{ei}^2 = \Delta E = E - E' = E - \frac{1}{2}(E - \Delta E_{i,a}) = \frac{1}{2}(E + \Delta E_{i,a}) \simeq \frac{1}{2} E$$

delivering the information

$$\Delta v_{ei} = \sqrt{\frac{E}{m}}$$

As we had stated before, connected with this associated velocity-space drift $\dot{V}_{e,i}$, apparently both the primary and the secondary electron are drifting in identical ways, and hence we arrive at the following balance equation:

$$\frac{\partial f_e}{\partial t} = U \frac{df_e}{ds} = \left| \frac{\partial f_e}{\partial t} \right|_{mag} + \frac{1}{v^2} \frac{\partial}{\partial v}\left(v^2 D_w \frac{\partial f_e}{\partial v}\right) - \frac{1}{v^2} \frac{\partial}{\partial v}\left[v^2 \bar{V}_{ei} f_e(v)\right]$$

where in view of the two electrons involved now $\dot{V}_{ei}$ must be defined by

$$\bar{V}_{ei} = 2 \cdot \Delta v_{ei} n_a \sigma_{ei}^a(v) v = 2 \cdot \sqrt{\frac{E}{m}}\, n_a \sigma_{ei}^a(v) v = 2 \cdot \sqrt{\frac{v^2}{2}}\, n_a \sigma_{ei}^a(v) v = \sqrt{2}\, n_a \sigma_{ei}^a(v) v^2$$

With the factor "2" on the right hand side of the above expression the fact is respected that two electrons, i.e. the primary and the secondary one, are appearing which drift with the same velocity in velocity space as long as they have energies large compared to the ionization energy ΔE_i^a.

Thus taking the above derived expression we then obtain the following expression for the impact-induced temporal change of the distribution, i.e. of $\left|\frac{\delta f}{\delta t}\right|_{ei}$ given by

$$\left|\frac{\delta f}{\delta t}\right|_{ei} = \frac{1}{v^2} \frac{\partial}{\partial v}\left[v^2 \sqrt{2}\, n_a \sigma_{ei}^a(v) v^2 f_e\right] = \sqrt{2}\, n_a \frac{1}{v^2} \frac{\partial}{\partial v}\left[v^4 \sigma_{ei}^a(v) f_e(v)\right]$$

Hence the final form of the kinetic electron transport equation attains the following form

$$\frac{\partial f_e}{\partial t} = U \frac{df_e}{ds} = \left| \frac{\partial f_e}{\partial t} \right|_{mag} + \frac{1}{v^2} \frac{\partial}{\partial v}\left(v^2 D_w \frac{\partial f_e}{\partial v}\right) - \frac{\sqrt{2}\, n_a}{v^2} \frac{\partial}{\partial v}\left[\sigma_{ei}^a(v) v^4 f_e(v)\right]$$

Expanding mathematically the last term on the RHS of the above equation, then delivers the following expression

$$-\frac{\sqrt{2}\, n_a}{v^2} \frac{\partial}{\partial v}\left[\sigma_{ei}^a(v) v^4 f_e(v)\right] = -\sqrt{2}\, n_a \left[\frac{\partial}{\partial v}\left[\sigma_{ei}^a(v) v^2 f_e(v)\right] - \sigma_{ei}^a(v) v^4 f_e(v) \frac{\partial}{\partial v}(v^{-2})\right]$$

$$= -\sqrt{2}\, n_a \left[\frac{\partial}{\partial v}\left[\sigma_{ei}^a(v) v^2 f_e(v)\right] + 2\sigma_{ei}^a(v) v f_e(v)\right]$$

Analogously one also finds as a mathematical expansion of the second term on the RHS of the above equation:

$$\frac{1}{v^2}\frac{\partial}{\partial v}(v^2 D_{vv}\frac{\partial f_e}{\partial v}) = \frac{\partial}{\partial v}(D_{vv}\frac{\partial f_e}{\partial v}) - v^2 D_{vv}\frac{\partial f_e}{\partial v}\frac{\partial}{\partial v}(\frac{1}{v^2}) = \frac{\partial}{\partial v}(D_{vv}\frac{\partial f_e}{\partial v}) + 2\frac{1}{v}D_{vv}\frac{\partial f_e}{\partial v}$$

Taking all this together, and since exclusively looking here for a stationary solution (i.e. $\frac{\partial f}{\partial t} = 0$!), one then finally finds the relevant kinetic transport equation in the following form

$$U\frac{df_e}{ds} = \left|\frac{\partial f_e}{\partial t}\right|_{mag} + \frac{\partial}{\partial v}[D_{vv}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a\sigma^a_{ei}(v)v^2 f_e(v)] + 2\frac{D_{vv}}{v}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a\sigma^a_{ei}(v)vf_e(v)$$

As one can see, the above equation is a partial differential equation of second order for $f_e(s,v)$ with respect to v.

The Electron Impact Ionization Process

Looking now at the variation of the impact ionization cross section with energy [21, 22] one finds:

$$\sigma^a_{ei}(v) = \sigma^a_{ei,o}\frac{\ln[E/E_{i,a}]}{E(eV)} = \sigma^a_{ei,o}\frac{\ln[(0.51\cdot10^6/27.2)\cdot(v/c)^2]}{(1/2)(0.51\cdot10^6)(v/c)^2} = \sigma^a_{ei,o}[\frac{\ln[1.875\cdot10^5]+2\ln(v/c)}{0.255\cdot10^6\cdot(v/c)^2}]$$

As can be learned from this expression, the ionization cross section increases from initial impact electron energies of around $10^3 eV$ down to energies around $60 eV$ by a factor of about 7, meaning that also the ionization rate will react accordingly, however, not as strong as by this factor 7, since in the ionization rate there appears the product of cross section and relative velocity taking care that then between $60eV$ and 10^3eV there is only an increase by a factor 1. 55 in the impact ionization rate [23, 24]. Taking the highest impact ionization rate of $\beta_{e,i}=3.10^{-11}s^{-1}$ as a characteristic number valid for the whole heliosheath, - and calculating as travel time of a hydrogen atom over the upwind heliosheath between $130AU$ and $90AU$ the time $\tau_{HS}=40AU/25km/s=2.4.10^8 s$, will then lead to an impact ionization probability in the upwind hemisphere of only $\xi_{e,i} = \tau_{HS}.\beta_{e,i}=7.2.10^{-3}$, expressing the fact that with a maximum probability of about 1 percent an incoming H-atom will suffer an electron impact ionization when cruising over the upwind heliosheath. This practically allows to assume that the H-atom density n_a, at least over the upwind heliosheath, is practically constant. But there may be nonetheless remarkable influences on the distribution function of the upwind heliosheath electrons, especially when looking at the difference in densities, i.e.

$$n_e \simeq 10^{-3}cm^{-3} \simeq n_a \simeq 10^{-1}cm^{-3}.$$

Now we introduce the energy-dependence of the electron impact cross section of atoms ”a” by the formula given by Lotz (1967) for the case $E \geq \Delta E_i^a$ in the following form

$$\sigma^a_{ei}(E) = A_a[1 - B_\alpha\exp(-C_\alpha(E/E_{a,i}-1))]\frac{2\ln(E/\Delta E_i^\alpha)}{E/\Delta E_i^\alpha}[10^{-17}cm^2]$$

where the values of the constants Aα, Bα, Cα for H-atoms or He-atoms, respectively, can be looked up in papers by Kieffer and Dunn (1966) or Lotz (1967, 1970). When expressed as function of the electron velocity, with $v^2_{i\alpha}$ $=2\Delta 2Ei\alpha/m$, one finds the above expression given in the Form

$$\sigma^a_{ei}(v) = A_a[1 - B_\alpha\exp(-C_\alpha\frac{v^2 - v^2_{i\alpha}}{v^2_{i\alpha}})]\frac{4\ln(v/v_{i\alpha})}{(v/v_{i\alpha})^2}[10^{-17}cm^2]$$

As can be seen in the above formula, σeia would become negative, i.e. vanishes, for $v \leq v_{i\alpha}$, and thus can be simplified at values $v \gg v_{i\alpha}$ by

$$\sigma^a_{ei}(v \gg v_{i\alpha}) = A_a\frac{4\ln(v/v_{i\alpha})}{(v/v_{i\alpha})^2}[10^{-17}cm^2]$$

This finally brings us to the following kinetic transport equation:

$$U\frac{df_e}{ds} = \left|\frac{\partial f_e}{\partial t}\right|_{mag} + \frac{\partial}{\partial v}[D_{vv}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a A_a\frac{4\ln(v/v_{i\alpha})}{(v/v_{i\alpha})^2}v^2 f_e(v)] + 2\frac{D_{vv}}{v}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a A_a\frac{4\ln(v/v_{i\alpha})}{(v/v_{i\alpha})^2}vf_e(v)$$

Initial Conditions at the Termination Shock

In Fahr, Richardson, Verscharen (2015) we had shown that the electron distribution function downstream of the termination shock can well be approximated by a kappa distribution f_e^{K0}. Hence here we shall take this suggested distribution as the starting function of electrons, describing their kinetic situation immediately downstream of the shock, at the place where they start moving off along the heliosheath plasma streamlines. This initial electron distribution function hence is given by

$$f^{\kappa_0} = f_{2e}(v,s_0) = \frac{n_{e2}}{(\pi\kappa_2\Theta_2^2)^{3/2}}\frac{\Gamma(\kappa_2+1)}{\Gamma(\kappa_2-3/2)}[1+\frac{v^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)}$$

where n_{e2} is the electron density on the downstream side of the shock, and K_2 and Θ_2 denote the initial kappa index and the thermal core velocity width of the kappa distribution. As shown in Fahr et al. (2015) the initial kappa index at $s = s_0$ (i.e. immediately downstream of the shock) is given by:

$$K_2 = K_{2,0} = 1.522$$

which means that the electron distribution function just downstream of the shock, i.e. at $s=s_0$, is given by a highly suprathermal non-equilibrium function with an extended power-law tail.

The second parameter of the electron kappa distribution is the so-called thermal core velocity spread Θ_{e2} which according to Fahr et al. (2015) should be given by the thermal downstream ion velocities, i.e. given by the following relation

$$\Theta_{e2}^2 \simeq 2P_{2i}/Mn_{2i}$$

Looking for the order of magnitude of the energy of these core electrons, it turns out that

$$E(\Theta_{e2}^2) \simeq (m/2)\Theta_{e2}^2 = (m/M)(P_{i2}/n_{i2}) \leq 1KeV/1840 = 0.54eV$$

i.e. the core electrons are neither able to ionize H-atoms ($\Delta E_{i,H}=13.4eV$) nor He-atoms ($\Delta E_{i,He}=24.6eV$), meaning that the dominant part of the ionization is done by the powerlaw-distributed electrons given by the part for $v \gg \Theta$, i.e. by:

$$f^{\kappa_0} = f_{2e}(v,s_0) \simeq \frac{n_{e2}}{(\pi\kappa_2\Theta_2^2)^{3/2}}[\frac{v^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)}$$

while the core part of the distribution does not directly interfere in the ionization business at all.

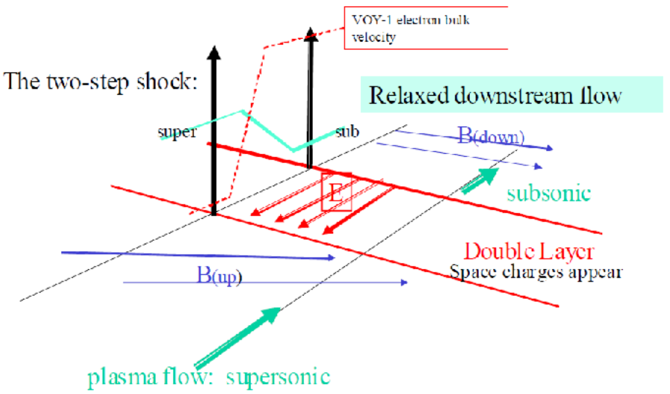

Figure 1: Schematic Representation of the Structure of the Solar Wind Termination Shock

Electron Impact Influences in the Downtail Region

Since the electron impact influence over the upwind hemisphere of the heliosheath practically does not touch the H- and He- atom densities, which also corresponds with results for electron impact ionization rates of $3.10^{-11}s^{-1}$ by Gruntman (2016), meaning for the upwind heliosheath n_H; n_{He} $\simeq$ const, we might be more interested in studying the impact ionization influence on the downwind hemisphere where the solar plasma is flowing along the heliosheath tail which is extended over several $100AU$ giving ample time, about 10^2 years, for impact ionization processes to occur. Here, at this extended, time-consuming tail passage, the electron impact ionization process could show clearer imprints on the electron distribution function, perhaps also showing a clearer influence as well on the atom densities of H- and He- atoms. Concerning the down tail influence on the electron distribution function f_e one would have to study the already developed, kinetic transport equation:

$$U\frac{df_e}{ds} = \left|\frac{\partial f_e}{\partial t}\right|_{mag} + \frac{\partial}{\partial v}[D_{vv}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a A_a \frac{4\ln(v/v_{ia})}{(v/v_{ia})^2}v^2 f_e(v)] + 2\frac{D_{vv}}{v}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a A_a \frac{4\ln(v/v_{ia})}{(v/v_{ia})^2}vf_e(v)$$

To solve the above transport equation along the heliotail axis in the downwind direction has several advantages which we shall discuss here first: In order to treat the first term on the right hand side of the above equation one has to get a handle on the magnitude B of the magnetic field along the streamline, this magnetic cooling term is given by (Fahr et al., 2016) in the form:

$$\left|\frac{\partial f_e}{\partial t}\right|_{mag} = \frac{1}{v^2}\frac{\partial}{\partial v}[v^2(\frac{2}{3}vU\frac{1}{B}\frac{dB}{ds})f_e(v,s)]$$

where s is the streamline coordinate and U is the magnitude of the bulk velocity. Using now for the description of the magnetic fields the theoretical model for stationary field-aligned MHD flows developed by Nickeler, Goedbloed, and Fahr (2006), then one can see that the magnetic field in the heliospheric tail region, especially along the heliotail axis, is constant and hence is no variable quantity with the coordinate s along the tail axis. This has the pleasant advantage that along the tail axis $\left|\frac{\partial f}{\partial t}\right|_{mag}$ vanishes, since here $dB/ds = 0$ is valid, and that the above magnetic cooling term consequently completely dissappears along the heliotail axis.

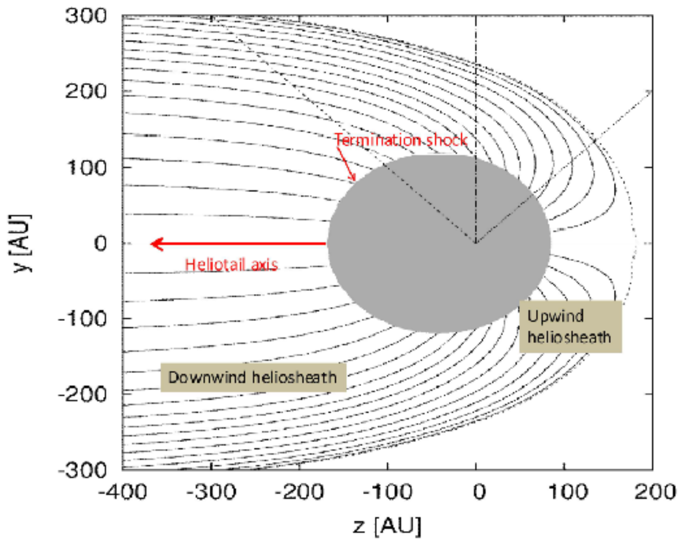

Figure 2: The Heliosheath Flow Lines Originating at the Termination Shock in Upwind and Downwind Directions [25, 26].

One is consequently there left with the following kinetic transport equation:

$$U\frac{df_e}{ds} = \frac{\partial}{\partial v}[D_{vv}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a A_a \frac{4\ln(v/v_{ia})}{(v/v_{ia})^2}v^2 f_e(v)] + 2\frac{D_{vv}}{v}\frac{\partial f_e}{\partial v} - 2\sqrt{2}\,n_a A_a \frac{4\ln(v/v_{ia})}{(v/v_{ia})^2}vf_e(v)$$

Now it may become a question how to treat the electron velocity-space diffusion process down in the heliotail region. This diffusion process is based on electron interactions with the ambient whistler wave turbulence. The latter for instance is thought to be generated by electron heat flux instabilities [27]. Furthermore, it has been shown by observations that the electron heat flux further away from the sun is suppressed to levels given by collisional models [28], though in fact collisions may not be the true reason for it, since suprathermal electron populations are not controlled by Coulomb collisions there. Thus a broad variety of kinetic instabilities may essentially be responsible for controling the shape of the electron distribution function and reducing the skewness of its suprathermal features [29-31]. From this it may be concluded that the appearance of relatively high whistler waves turbulence levels downstream of the termination shock is associated with the appearance of suprathermal electron populations, - like in our case the population of overshooting electrons downstream of the solar wind termination shock [25]. Hence one may be justified to conclude that in case of the heliosheath plasma this shock is the responsible generator of whistler turbulences, and that the amplitudes of these whistler waves may certainly drop down with the distance s from the shock. This may also then allow to further conclude, that the electron velocity diffusion process, which has to be treated here in this paper, may be described as already done in the paper by Fahr and Dutta-Roy (2019), however, with consideration of a fall-off of the turbulence amplitudes increasing with the downstream distance of the heliosheath plasma from the shock front. In conclusion, that advises us here to treat this diffusion process by a dependence of the diffusion coefficient in the form $D_{VV} \sim D_0(s)(v/v_0)^2$, however, taking into account the amplitude fall off perhaps in the form:

$$D_0(s) = D_0 \exp[-\alpha(s - s_0)].$$

However, not knowing appropriate values of D_0 and α, we treat here the kinetic transport equation without taking into account the velocity-space diffusion process, i.e. setting D_0=0!, though keeping in mind that this may be only a crude approximation in the region close to the downwind termination shock.

Consequently, we are now left with the remaining transport equation in the following form:

$$U\frac{df_e}{ds} = -2\sqrt{2}\, n_a A_a \frac{\partial}{\partial v}[\frac{4\ln(v/v_{ia})}{(v/v_{ia})^2} v^2 f_e(v)] - 2\sqrt{2}\, n_a A_a \frac{4\ln(v/v_{ia})}{(v/v_{ia})^2} v f_e(v)$$

and start integrating it with respect to s beginning with the initial distribution function at $s = s_0$ given by

$$f_{2e}(v, s_0) = \frac{n_{e2}}{(\pi\kappa_2\Theta_2^2)^{3/2}} \frac{\Gamma(\kappa_2+1)}{\Gamma(\kappa_2 - 3/2)}[1 + \frac{v^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)}$$

at the downwind termination shock proceeding in downwind direction along the heliotail axis. Reminding the fact that the electron impact due to Θ_2 »v_i^a only influences the power law part, not the core part of $f_{2e}(v, s_0)$, we find when setting $x = v/v_i^a$:

$$U\frac{df_e}{ds} = -\sqrt{8}\, n_a A_a v_{ia}[\frac{\partial}{\partial x}(\frac{4\ln(x)}{x^2} x^2 f_e(v)) + \frac{4\ln(x)}{(x)^2} x f_e(v)]$$

On the basis of the field-aligned MHD flow model by Nickeler, Goedbloed and Fahr (2006) one obtains a constant plasma bulk velocity $U = U_{ht}$ along the heliotail axis which after balancing the fluxes originating in the inner heliosphere (say at the earth orbit $r = r_E$) with the fluxes leaving the heliotail with its cross sectional area of $F_{ht} = 4\pi L^2$, $L=130AU$ being Parker's stand-off distance for the subsonic flow [32], this then leads to a constant heliotail bulk velocity of $U_{ht}=0.\ 2U_E$~$100 km/s$. proceeding now by calculating the changes of the distribution function with the increase of streamline coordinate s by rewriting the above equation in the form:

$$df_e = -4\sqrt{8}\, n_a A_a \frac{v_{ia}}{U_{ht}} \cdot [\frac{\partial}{\partial x}(\ln(x) f_e(x)) + \frac{\ln(x)}{x} f_e(x)] \cdot ds$$

and starting the first step at $s = s_0$ (i.e. at the downwind termination shock!) with

$$f_e(v, s_0) = \frac{n_{e2} v_{ia}^3}{(\pi\kappa_2 x_\Theta^2)^{3/2}} \frac{\Gamma(\kappa_2+1)}{\Gamma(\kappa_2 - 3/2)}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{-(\kappa_2+1)}$$

we arrive at s = s_0+ds with the distribution function:

$$f_e(x, s_0 + ds) = f_e(x, s_0) - ds\zeta \cdot [\frac{\partial}{\partial x}(\ln(x)[\frac{x^2}{\kappa_2 x_\Theta^2}]^{-(\kappa_2+1)}) + \frac{\ln(x)}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{-(\kappa_2+1)}]$$

where ζ is a constant given by:

$$\zeta = \frac{\sqrt{128}\, n_{e2} v_{ia}}{(\pi\kappa_2 x_\Theta^2)^{3/2} U_{ht}} \frac{\Gamma(\kappa_2+1)}{\Gamma(\kappa_2 - 3/2)} n_a A_a$$

Looking at the undeveloped expression in the upper equation we find in several consecutive steps:

$$\frac{\partial}{\partial x}(\ln(x)[\frac{x^2}{\kappa_2 x_\Theta^2}]^{(\kappa_2+1)}) + \frac{\ln(x)}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{(\kappa_2+1)} = \frac{1}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{(\kappa_2+1)} - (\kappa_2+1)$$

$$)[\frac{x^2}{\kappa_2 x_\Theta^2}]^{(\kappa_2+2)} \frac{2x\ln(x)}{\kappa_2 x_\Theta^2} + \frac{\ln(x)}{x}$$

$$\frac{1}{x}|\frac{x^2}{\kappa_2 x_\Theta^2}|^{-(\kappa_2+1)} - (\kappa_2+1)|\frac{x^2}{\kappa_2 x_\Theta^2}|^{-(\kappa_2+1)} \frac{\kappa_2 x_\Theta^2}{x^2} \frac{2x\ln(x)}{\kappa_2 x_\Theta^2} + \frac{\ln(x)}{x}|\frac{x^2}{\kappa_2 x_\Theta^2}|^{-(\kappa_2+1)} =$$

$$\frac{1}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{(\kappa_2+1)} - (2\kappa_2+3)\frac{\ln(x)}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{(\kappa_2+1)} = \frac{1}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{(\kappa_2+1)}[1 - (2\kappa_2+3)\ln(x)]$$

Consequently, from the above expression for $f_e(x, s_0+ds)$ we hence obtain the final relation:

$$f_e(x, s_0 + ds) = f_e(v, s_0) - ds \cdot \zeta \cdot [\frac{1}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{-(\kappa_2+1)}[1 - (2\kappa_2+3)\ln(x)]]$$

Since the electron-impact-induced changes are occuring very moderately, we can dare here to use a linear extrapolation of the upper equation which then leads to the following result:

$$f_e(x, s - s_0) = f_e(v, s_0) - (s - s_0) \cdot \zeta[\frac{1}{x}[\frac{x^2}{\kappa_2 x_\Theta^2}]^{-(\kappa_2+1)}[1 - (2\kappa_2+3)\ln(x)]]$$

In the following Figure 3 we show the electron distribution function at increasing distances s along the heliotail axis, revealing how the electron distribution function over the main part of the power law region is reduced with increasing distances s, while only in the region near the core (at values $x = v/via$ ~ 1 the differential velocity space densities are slightly enhanced which is due to the accumulated effect of impact-generated additional electrons.

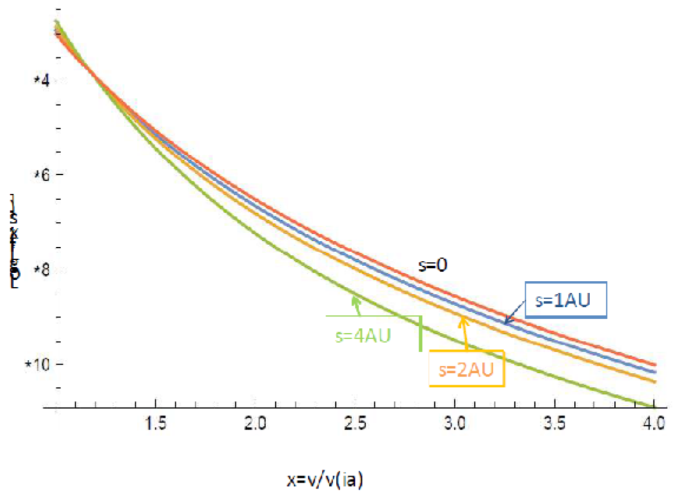

Figure 3: Electron distribution function at different distances si = 0, 1, 2, 4 AU on the heliotail axis. Plotted is the Log of the distribution function as function of the normalized velocity x=v/v(ia) (* should be identified as - signs)

As one can see the distribution function is falling off steeper and steeper with increasing heliotail distances s, while the core density seems to be increasing slightly. This density change is due to the fact that with electron impact processes additional electrons are produced and convected with the plasma flow down the tail towards the core population. These latter density changes one can study more carefully with the change of the first velocity moment of the distribution function as we shall do in the next section.

Density Changes Caused by Electron Impact

We start from the equation for the change of the distribution fucntion due to electron impact processes:

$$\left|\frac{\delta f}{\delta t}\right|_{ei} = \frac{1}{v^2}\frac{\partial}{\partial v}[v^2\sqrt{2}\, n_a \sigma_{ei}^a(v) v^2 f_e] = \sqrt{2}\, n_a \frac{1}{v^2}\frac{\partial}{\partial v}[v^4 \sigma_{ei}^a(v) f_e(v)]$$

Then an integration over the velocity space should deliver us the impact-induced density change per time:

$$\left|\frac{\delta n}{\delta t}\right|_{ei} = \int 4\pi v^2 dv[\sqrt{2}\, n_a \frac{1}{v^2}\frac{\partial}{\partial v}[v^4\sigma_{ei}^a(v) f_e(v)] \;= 4\pi\sqrt{2}\, n_a$$

$$\int dv[\frac{\partial}{\partial v}[v^4\sigma_{ei}^a(v) f_e(v)]$$

leading to:

$$\left|\frac{\delta n}{\delta t}\right|_{ei} = 2^{5/2} n_a \pi v_{ia}^4 \cdot |x^4 \sigma_{ei}^a(x) f_e(x)|_1^\infty = 2^{5/2} n_a \pi v_{ia}^4 \cdot \left|x^4 A_a \frac{4\ln(x)}{(x)^2} f_e(x)\right|_1^\infty$$

$$2^{9/2} n_a \pi A_a v_{ia}^4 \cdot |x^2 \ln(x) f_e(x)|_1^\infty$$

Taking the upper border of the occupied velocity space to be at $x_\infty \simeq 10^2$ one obtains:

$$\left|\frac{\delta n}{\delta t}\right|_{ei} = 2^{9/2} n_a \pi A_a v_{ia}^4 \cdot 10^4 \ln(10^2) f_e(10^2)$$

meaning in fact that one also obtains a space density increase per time, disregarded its magnitude which increases with the travel time of the plasma down the heliotail!

Evaluation of the Impact-Induced Pressure Change

Now we look onto the effect of electron impact ionizations concerning the associated electron pressure changes and start out from the expression for the impact-induced electron pressure change in the form:

$$\frac{\delta P_{e,i}}{\delta t} = \frac{4\pi m}{3}\int \frac{\delta f_{e,i}}{\delta t} v^4 dv = \frac{4\pi m}{3}\int v^2[\frac{1}{v^2}\frac{\partial}{\partial v}(v^2 V_{D_{e,i}} f_e)] v^2 dv$$

where the impact-induced drift, i.e. the average velocity change per unit of time, is given by:

$$V_{D_i} = n_a \sigma_a(v) v \cdot \sqrt{\frac{2\Delta E_i}{m}}$$

From the upper relation we then derive:

$$\frac{\delta P_{e,i}}{\delta t} = \frac{4\pi m}{3}\int[\frac{\partial}{\partial v}(v^2 V_{D_i} f_e)] v^2 dv$$

which leads to

$$\frac{\delta P_{e,i}}{\delta t} = \frac{4\pi m}{3}[\int \frac{\partial}{\partial v}(v^4 V_{D_i} f_e)] dv - 2\int v^3 V_{D_i} f_e dv]$$

Now the first integral when evaluated at the outer borders of the integration region, i.e. at v = 0 and v→∞, is easily shown to vanish completely, and thus one remains with

$$\frac{\delta P_{e,i}}{\delta t} = -\frac{8\pi m}{3}\int v^3 V_{D_i} f_e dv = -\frac{8\pi m n_a}{3}\int \sigma_a(v) v^4 \cdot \sqrt{\frac{2\Delta E_i}{m}} f_e dv$$

which, when paying attention to the fact that

$$\Delta E = \frac{1}{2}(E - \Delta E_{i,a}) = \frac{1}{2}(\frac{m}{2}v^2 + \Delta E_{i,\alpha}) \simeq \frac{m}{4}v^2$$

then simply leads to

$$\frac{\delta P_{e,i}}{\delta t} = -\frac{8\pi m n_a}{3\sqrt{2}}\int_{v_{ia}}^{v_\infty} \sigma_a(v) v^5 f_e dv$$

At the highest velocities $\sigma a \to 0$ suppresses any contribution from the integral. So there does not exist the problem of getting a finite value out of the integral whatever is the form of f_e. We can now further elaborate on this term after introducing the functional dependencies on v by writing

$$\frac{\delta P_{e,i}}{\delta t} = -\frac{8\pi m n_a}{3\sqrt{2}}\frac{n_{e2}}{(\pi\kappa_2\Theta_2^2)^{3/2}}\frac{\Gamma(\kappa_2+1)}{\Gamma(\kappa_2-3/2)}$$

$$\int_{v_{ia}}^{v_\infty} A_a[1 - B_a \exp(-C_a \frac{v^2 - v_{ia}^2}{v_{ia}^2})]\frac{4\ln(v/v_{ia})}{(v/v_{ia})^2}[1 + \frac{v^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)} v^5 dv$$

Now we can evaluate the integral along the following procedure

$$\Pi_{ei} = \int_{v_{ia}}^{v_\infty} A_a[1 - B_a \exp(-C_a \frac{v^2 - v_{ia}^2}{v_{ia}^2})]\frac{4\ln(v/v_{ia})}{(v/v_{ia})^2}[1 + \frac{v^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)} v^5 dv$$

and when keeping in mind that $v^2/\Theta_2^2 \geq vi\alpha^2/\Theta_2^2 \gg 1$ we can obtain, introducing $x = v/v_{ia}$, from the above expression the simplified expression

$$\Pi_{ei} = v_{ia}^6 \int_1^{x_\infty} A_a[1 - B_\alpha \exp(-C_\alpha(x^2-1))]\frac{4\ln(x)}{x^2}[\frac{v_{ia}^2}{\kappa_2\Theta_2^2}x^2]^{-(\kappa_2+1)} x^5 dx$$

and furthermore obtain

$$\Pi_{ei} = 4A_a v_{ia}^6[\frac{v_{ia}^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)}\int_1^{x_\infty}[1 - B_\alpha \exp(-C_\alpha(x^2-1))]$$

$$\ln(x) x^{-2\kappa_2+1} dx$$

Assuming now that the main contribution to the above integral comes from the integrand in the region x » 1 will then allow us to simplify the above integral by:

$$\Pi_{ei} = 4A_a v_{ia}^6[\frac{v_{ia}^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)}\int_1^{x_\infty}\ln(x) x^{-2\kappa_2+1} dx$$

where the remaining integral delivers

$$\int_1^{x_\infty}\ln(x) x^{-2\kappa_2+1} dx = \left|x^{2-2\kappa_2}(\frac{\ln x}{2-2\kappa_2} - \frac{1}{(2-2\kappa_2)^2})\right|_1^{x_\infty} = x_\infty^{2-2\kappa_2}($$

$$(\frac{\ln x_\infty}{2-2\kappa_2} - \frac{1}{(2-2\kappa_2)^2}) + \frac{1}{(2-2\kappa_2)^2})$$

which in view of the fact that x∞»1 can be simplified to yield

$$\int_1^{x_\infty}\ln(x) x^{-2\kappa_2+1} dx = x_\infty^{2-2\kappa_2}\frac{\ln x_\infty}{2-2\kappa_2}$$

Using this result, we thus finally obtain

$$\Pi_{ei} = 4A_a v_{ia}^6[\frac{v_{ia}^2}{\kappa_2\Theta_2^2}]^{-(\kappa_2+1)} x_\infty^{2-2\kappa_2}\frac{\ln x_\infty}{2-2\kappa_2}$$

which brings us to the following expression for the pressure change per time due to impact ionization processes:

$$\frac{\delta P_{e,i}}{\delta t} = -\frac{8\pi m n_a}{3\sqrt{2}} \frac{n_{e2}}{(\pi\kappa_2\Theta_2^2)^{3/2}} \frac{\Gamma(\kappa_2+1)}{\Gamma(\kappa_2-3/2)} 4A_a v_{i\alpha}^6$$

$$_{i\alpha}^{6}\left[\frac{v_{i\alpha}^2}{\kappa_2\Theta_2^2}\right]^{-(\kappa_2+1)} x_\infty^{2-2\kappa_2} \frac{\ln x_\infty}{2-2\kappa_2}$$

Conclusions

We have studied in this article the heliosheath electron distribution function under the influence of electron impact ionization processes of interstellar neutral atoms, especially H- and He- atoms, cruising with the interstellar wind over the heliosheath. Since over the upwind hemisphere of the heliosheath the influences of electron impact ionization are of marginal influences, we here have especially taken care of the downwind heliosheath. Hereby our calculations down the heliotail were based on the assumption that the interstellar atom densities are essentially constant. This is a good approximation in the upwind hemisphere of the heliosheath, while on the downwind side of the heliosheath and down the heliotail this assumption may appear questionable. However, in fact this assumption is not bad outside of a circumsolar region of the order of 15 AU where the solar photoionization produces non-negligible gradients in the neutral atom densities [31, 33, 34].

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