ISSN: 2997-6170

Research Article

Space Science Journal

Structured Residuals in the Radial Acceleration Relation a Multi Variable Analysis of the SPARC Dataset

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Submitted: 2026, Jul 30 **Accepted:** 2026, Aug 19; **Published:** 2026, Sep 08

Citation: Radcliffe, B. K. (2026). Structured Residuals in the Radial Acceleration Relation: A Multi-Variable Analysis of the SPARC Dataset. *Space Sci J, 3*(3), 01-09.

Abstract

The Radial Acceleration Relation (RAR) provides one of the strongest empirical descriptions of galaxy dynamics, linking observed gravitational acceleration to that predicted from baryonic matter. While the relation captures the dominant structure of galaxy rotation curves, small residual deviations remain and are often treated as random scatter. In this study, we investigate whether these deviations contain predictable structure. Using the full SPARC (Spitzer Photometry and Accurate Rotation Curves) dataset comprising 175 galaxies and 3,391 radial measurements, we examine residuals relative to a standard RAR reference model. Predictor variables include local quantities such as baryonic acceleration, normalized radius, and rotation-curve slope, together with global galaxy properties including baryonic mass, gas fraction, surface brightness, flat rotation velocity, and morphology. We find that the signed residual is only weakly predictable, indicating limited deterministic predictability of individual deviations. In contrast, the magnitude and statistical structure of the residuals exhibit clear and reproducible dependence on both local and global galaxy properties. Residual magnitude varies systematically across acceleration regimes and predictive performance improves progressively as additional variables are incorporated. Variables associated with global galaxy state, including flat rotation velocity and morphology, provide independent predictive information beyond local acceleration alone. These results demonstrate that deviations from the RAR are not adequately described as featureless random scatter. Instead, they follow a structured, multi-variable pattern that can be partially predicted from observable galaxy properties. The findings suggest that galaxy dynamics are characterized not only by a leading-order acceleration relation but also by a constrained and informative structure of deviations that warrants further investigation.

Keywords: Galaxy Rotation Curves, Radial Acceleration Relation, Sparc Database, Multi-Variable Dependence, Residual Predictability, Galaxy Dynamics

1. Introduction

Galaxy rotation curves remain one of the most important observational probes of gravitational dynamics on galactic scales. Observations spanning a wide range of galaxy types consistently reveal a close relationship between baryonic matter and observed rotational motion, despite substantial variation in mass, size, morphology, and surface brightness. One of the most significant empirical developments in this area is the Radial Acceleration Relation (RAR), which links the observed gravitational acceleration, (g_{obs}), to the acceleration predicted from baryonic matter, (g_{bar}). Across diverse galaxy populations, the RAR provides a remarkably tight description of the dominant behavior present in galaxy rotation curves and has become a central observational benchmark for studies of galaxy dynamics. Despite its success, the RAR is not exact. Small residual deviations remain between observed and predicted velocities. These deviations are commonly treated as scatter arising from measurement uncertainty, observational limitations, or local astrophysical complexity. However, residuals represent the portion of the data not captured by the leading-order relation and therefore constitute a potentially important source of additional information.

Recent improvements in observational precision have made it increasingly possible to investigate whether these residuals are entirely random or whether they contain systematic structure. If residual behavior depends on dynamical regime or other observable galaxy properties, then the residuals themselves may provide insight into aspects of galaxy dynamics not fully described by the primary acceleration relation. The first step in addressing this question is to determine whether residuals exhibit measurable structure. The next step is to determine whether that structure is predictable. These are distinct questions. A system may contain organized statistical behavior even when individual deviations cannot be predicted deterministically. In this study, we examine residuals relative to a standard RAR reference model using the full SPARC (Spitzer Photometry and Accurate Rotation Curves) dataset, comprising 175 galaxies and 3,391 radial measurements. Rather than focusing exclusively on the signed residual at individual points, we investigate whether the magnitude and statistical structure of the deviations can be predicted from observable galaxy properties. To address this question, we combine local variables, including baryonic acceleration, normalized radius, and rotation-curve shape, with global galaxy properties such as baryonic mass, gas fraction, surface brightness, flat rotation velocity, and morphology. A hierarchy of predictive models is then used to evaluate the extent to which residual structure can be explained by observable quantities. The objective of this work is not to introduce a new theory of galaxy dynamics. Instead, it is to determine whether deviations from the RAR contain predictable multi-variable structure. Establishing the existence and predictability of such structure is a necessary step toward understanding the full informational content of galaxy rotation curve observations.

2. Data and Methods

2.1. Dataset

This study utilizes the SPARC (Spitzer Photometry and Accurate Rotation Curves) database, which provides high-quality rotation curves and baryonic mass models for 175 disk galaxies spanning a broad range of masses, morphologies, surface brightnesses, and dynamical properties. The complete sample contains 3,391 radial measurements and serves as the primary dataset for all analyses. Rotation-curve mass models were obtained from the SPARC Newtonian decomposition files, which provide observed rotational velocities together with the corresponding gas, stellar disk, and bulge contributions at each sampled radius. Additional galaxy-level properties were obtained from the SPARC master catalog, including flat rotation velocity, morphology, disk scale length, surface brightness, and HI mass. Stellar mass estimates were obtained from published WISE photometric measurements.

No galaxies were excluded on the basis of residual behavior, fit quality, or agreement with any hypothesis examined in this study. Analyses were performed using the full available sample, with matched subsets used only when required by predictor availability.

2.2. Derived Quantities

The baryonic rotational velocity was computed from the individual mass-model components according to

$$V^2bar = V^2gas + YdV^2disk + YbV^2bulge$$

where the fiducial SPARC mass-to-light ratios $\Upsilon_d = 0.5$ and $\Upsilon_b = 0.7$ were adopted.

Observed and baryonic accelerations were then calculated as

$$g_{\mathrm{obs}} = \frac{V_{\mathrm{obs}}^2}{r},$$

$$g_{\mathrm{bar}} = \frac{V_{\mathrm{bar}}^2}{r}.$$

The baryonic acceleration serves as the primary regime variable throughout the analysis.

2.3. Reference Model and Residual Definition

Residuals were evaluated relative to the empirical Radial Acceleration Relation (RAR) described by McGaugh et al. (2016),

$$g_{\mathrm{RAR}} = \frac{g_{\mathrm{bar}}}{1 - \exp\left(-\sqrt{g_{\mathrm{bar}}/a_0}\right)}.$$

where:

$$a_0 = 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}}.$$

The corresponding reference velocity was obtained from

$$V_{\mathrm{RAR}} = \sqrt{g_{\mathrm{RAR}}\, r}.$$

The primary signed residual was defined as

$$R = V_{\mathrm{obs}} - V_{\mathrm{RAR}}.$$

Residual magnitude and squared residual:

$$|R|$$

$$R^2,$$

2.4. Predictor Variables

Predictor variables were divided into three categories.

Regime Variable

- $\log_{10}(g_{\mathrm{bar}})$

Local Variables

- Normalized radius ((r/R_d))
- Local rotation-curve slope ((dV/dr))
- Local rotation-curve curvature ((d^2V/dr^2))

Global Galaxy Properties

- Flat rotation velocity ((V_{flat}))
- Morphological type

- Baryonic mass
- Gas fraction
- Central surface brightness

Together, these variables characterize both local dynamical conditions and global galaxy state.

2.5. Predictive Framework

The primary objective of the analysis is to determine whether residual structure can be predicted from observable galaxy properties. A hierarchy of models was constructed beginning with a null model and progressively incorporating acceleration regime, local structural variables, and global galaxy properties. Separate analyses were performed for the signed residual (R), residual magnitude (|R|), and squared residual ((R^2)).

Predictive performance was evaluated using leave-one-galaxy-out cross-validation. In each iteration, all radial measurements from a single galaxy were withheld from model training and used exclusively for testing. This procedure prevents information leakage between training and test sets and provides a conservative assessment of predictive performance across independent galaxies.

2.6. Evaluation Metrics

Model performance was assessed using standard regression metrics, including mean absolute error (MAE), root-mean-square error (RMSE), and variance reduction relative to a null model.
Comparisons among models were used to determine whether additional predictor classes contributed independent information regarding residual structure and whether residual magnitude was more predictable than the signed residual itself.

3. Predictive Framework

3.1. Objective

The primary objective of this study is to determine whether the deviations from the Radial Acceleration Relation (RAR) contain predictable structure that can be inferred from observable galaxy properties. A direct attempt to predict the exact signed residual at each radial point represents the most stringent form of prediction. However, preliminary analysis indicates that the signed residual exhibits only weak deterministic behavior. The present study therefore distinguishes between prediction of the signed residual and prediction of the structure of the deviations. Three related targets are examined:

[$R = V_{obs} - V_{RAR}$]

Signed residual

$|R|$

Residual magnitude
[R^2]

Squared residual
The latter two quantities characterize the size and variance of the deviations and therefore provide a direct measure of residual structure.

3.2. Predictor Classes

To evaluate the origin of residual structure, predictors were grouped into three categories.

- **Acceleration Regime**
  The primary regime variable is
  $\log_{10}(g_{\mathrm{bar}})$,

  which specifies position within acceleration space.
- **Local Variables**
  Local variables characterize the immediate dynamical environment of each radial point and include normalized radius, rotation-curve slope, and rotation-curve curvature.
- **Global Variables**
  Global variables describe the overall dynamical and structural state of the host galaxy and include flat rotation velocity, morphology, baryonic mass, gas fraction, and surface brightness.

This separation allows the relative contributions of local and global effects to be evaluated independently.

3.3. Model Hierarchy

A sequence of nested models was constructed to determine whether additional predictor classes provide independent predictive information.

- **Model 0: Null Model**
  The null model predicts a constant value equal to the mean of the training data and serves as the baseline for all comparisons.

- **Model 1: Regime Model**
  The first structured model uses baryonic acceleration alone to predict residual behavior.

- **Model 2: Local Model**
  Local structural variables are added to determine whether rotation-curve shape and radial position provide predictive information beyond acceleration regime.

- **Model 3: Global Model**
  Global galaxy properties are incorporated to test whether residual structure depends on the overall dynamical state of the host galaxy.

- **Model 4: Full Model**
  The full model combines acceleration regime, local variables, and global galaxy properties.

Progressive improvements across the model hierarchy indicate whether residual structure depends on increasingly complex combinations of observable quantities.

3.4. Cross-Validation

Predictive performance was evaluated using leave-one-galaxy-out cross-validation.

For each validation cycle, all radial measurements from a single galaxy were excluded from model training and used exclusively for testing. This procedure was repeated for every galaxy in the sample. Because radial measurements within a galaxy are not statistically independent, galaxy-level cross-validation provides a more conservative and realistic assessment of predictive performance than random point-level partitioning.

3.5. Performance Metrics

Model performance was quantified using root-mean-square error (RMSE), mean absolute error (MAE), and variance reduction relative to the null model.

Performance was evaluated separately for:

- signed residuals R
- residual magnitude [$|R|$]
- squared residuals [R^2]
- Comparison among these targets allows direct assessment of whether galaxy dynamics are more predictable in their residual structure than in their exact point-by-point deviations.

3.6. Hypothesis Testing

The analysis tests two related hypotheses.

The first hypothesis is that residuals exhibit measurable structure rather than featureless random scatter. The second hypothesis is that this structure is partially predictable from observable galaxy properties. Evidence supporting these hypotheses would imply that deviations from the RAR contain additional information beyond that captured by the leading-order acceleration relation and that this information is systematically related to both local and global properties of galaxies.

4. Results

4.1. Dataset Coverage

The analysis was performed using the SPARC database comprising 175 galaxies and 3,391 radial measurements. The full dataset was used for acceleration-regime analyses, while matched subsets were employed for predictive modeling where additional galaxy-level predictors were required.

The primary objective was to determine whether deviations from the Radial Acceleration Relation (RAR) exhibit predictable structure and whether such structure can be related to observable galaxy properties.

4.2. Residual Magnitude Depends on Acceleration Regime

Residual magnitude was first examined as a function of baryonic acceleration.

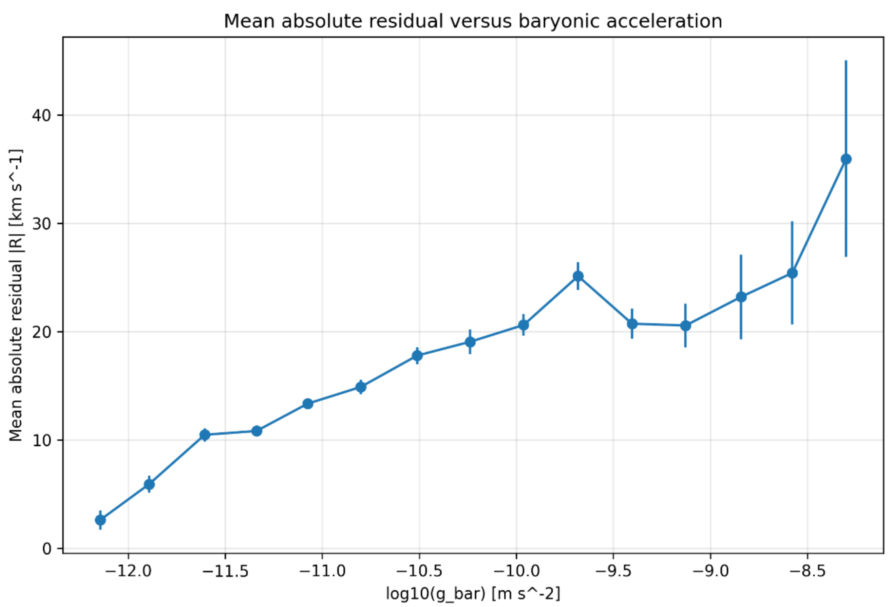

Figure 1: Shows that the Mean Absolute Residual Varies Systematically Across Acceleration Space. Residual Magnitudes are Smallest at both the Highest and Lowest Accelerations and Increase within Intermediate Acceleration Regimes

This behavior demonstrates that the deviation envelope surrounding the RAR is not uniform. Instead, the magnitude of the deviations depends on acceleration regime, indicating the presence of structured behavior beyond the leading-order relation.

4.3. Signed Residuals Remain Weakly Structured

The signed residual was analyzed separately to determine whether the direction of the deviation exhibits similar behavior.

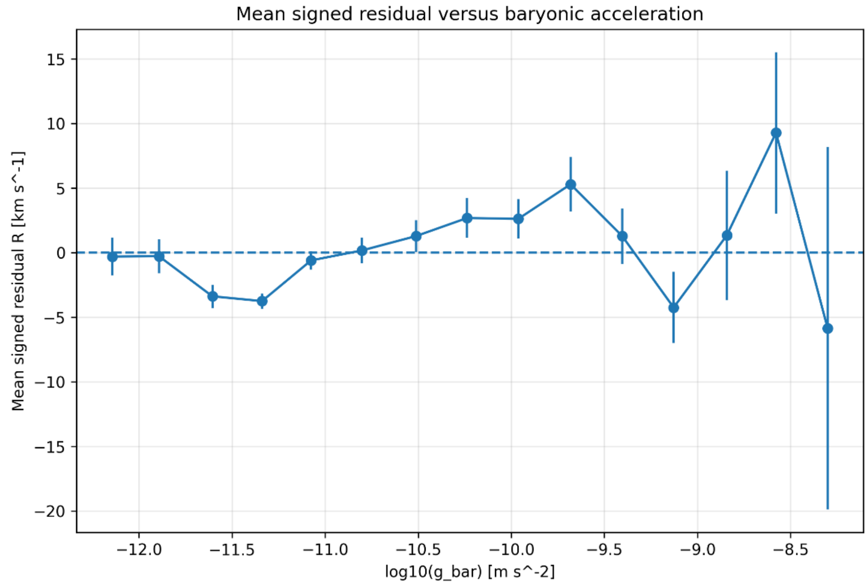

Figure 2: Shows that the Mean Signed Residual Remains Centered Near Zero Across Most Acceleration Regimes. Although Weak Trends are Present, The Systematic Structure is Substantially Smaller than that Observed for Residual Magnitude

The contrast between Figures 1 and 2 indicates that the size of the deviation carries a stronger signal than its direction.

4.4. Predictive Performance Across the Model Hierarchy

Predictive performance was evaluated using the model hierarchy described in Section 3.

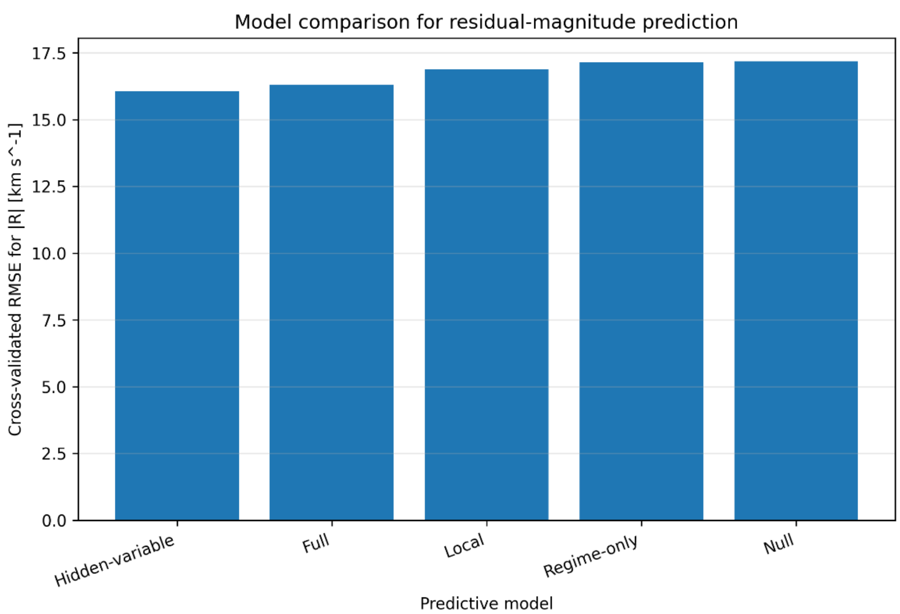

Figure 3: Demonstrates a Progressive Improvement in Predictive Performance as Additional Predictor Classes are Incorporated. the Regime-only Model Improves upon the null Model, while Further Gains are Obtained by Adding Local Structural Variables and Global Gala Properties

The full model achieves the best overall performance, indicating that residual structure depends on a combination of local and global factors rather than on acceleration regime alone.

4.5. Signed Residual Prediction Remains Limited

The same model hierarchy was applied to prediction of the signed residual.

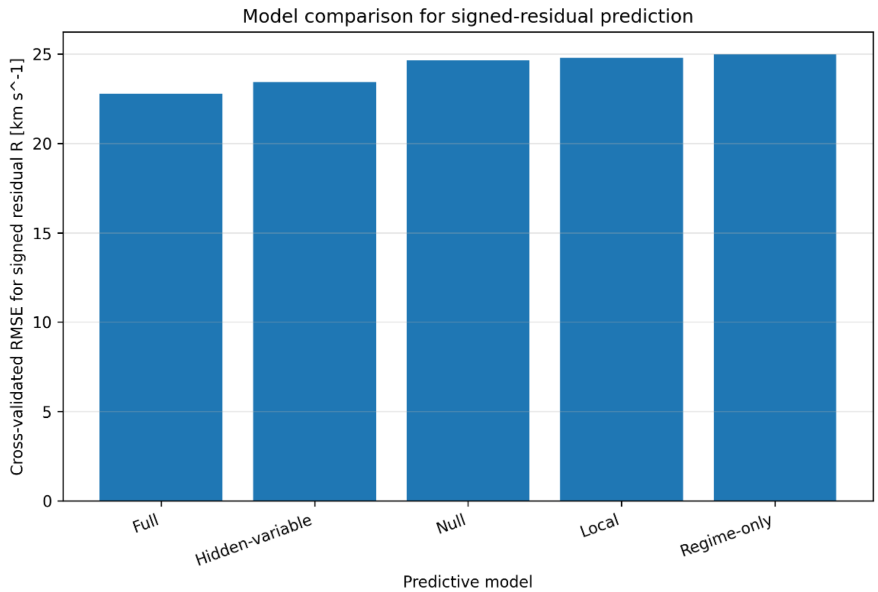

Figure 4: Although Modest Improvements are Observed Relative to the Null Model, the Gains are Substantially Smaller than those Obtained when Predicting Residual Magnitude. This Result Indicates that the exact Sign of the Deviation Remains only Weakly Constrained by the Available Variables, even when Multiple Predictor Classes are Included

4.6. Observed Versus Predicted Residual Magnitude

The performance of the best-performing model was evaluated by comparing predicted and observed residual magnitudes.

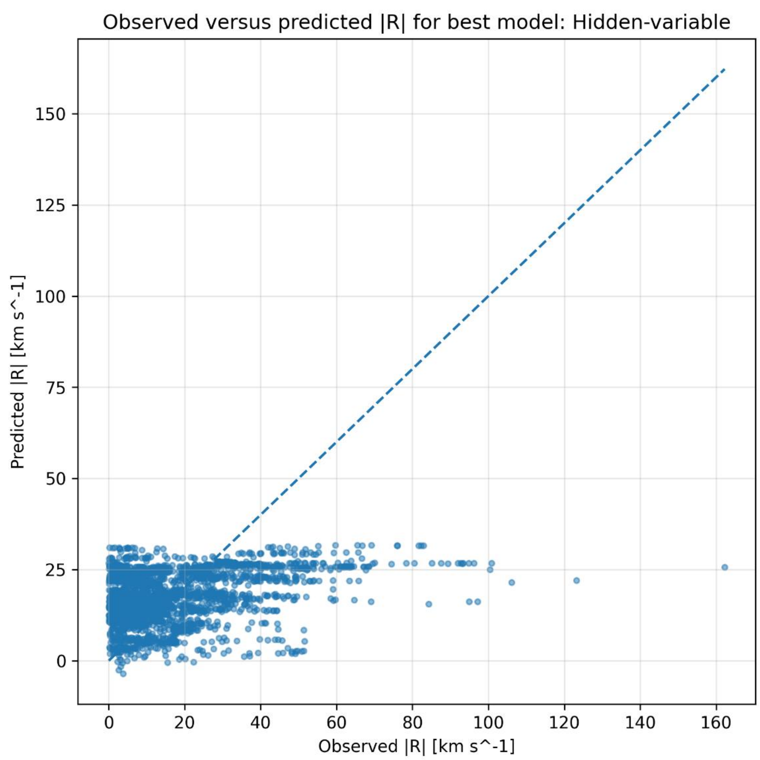

Figure 5: Shows a Positive Correlation Between Predicted and Observed Values, Demonstrating that the Model Captures a Significant Portion of the Structured Variation Present in the Residuals

At the same time, substantial scatter remains, indicating that the residual structure is only partially explained by the variables included in the present analysis.

4.7. Contribution of Global Galaxy Properties

To evaluate the role of galaxy-scale variables, models incorporating flat rotation velocity, morphology, baryonic mass, gas fraction, and surface brightness were compared with models based solely on local quantities. The inclusion of global galaxy properties consistently improved predictive performance. Among these variables, flat rotation velocity and morphology produced particularly strong gains. These results indicate that residual structure depends not only on local acceleration and local curve shape, but also on the broader dynamical state of the host galaxy.

4.8. Residual Structure Across Galaxy Classes

Residual behavior was examined across different galaxy classes.

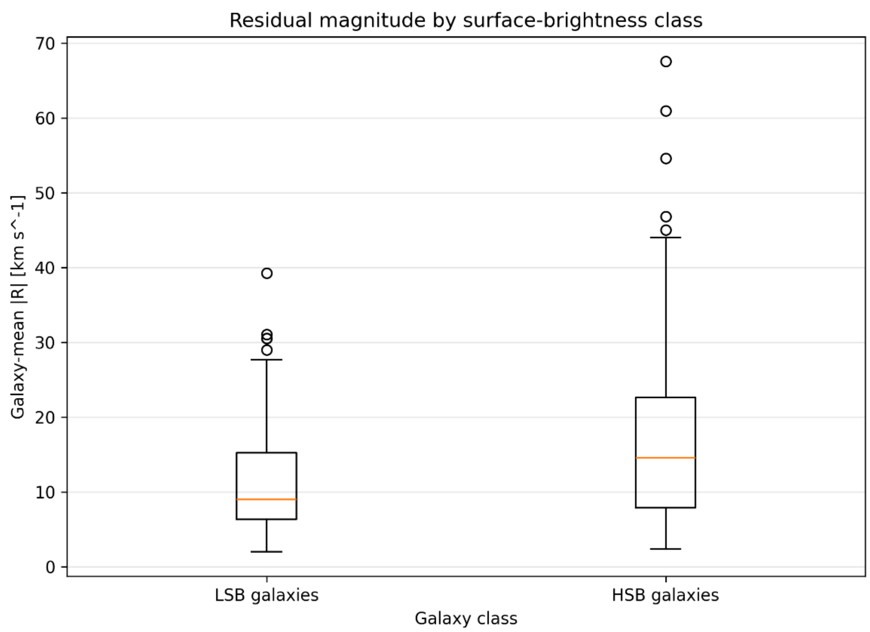

Figure 6: Shows Systematic Differences in Residual Magnitude Between Galaxy Populations. The Observed Variation cannot be Explained Solely by Baryonic Acceleration and Indicates that Galaxy Class Contributes Additional Information Regarding the Structure of the Deviations

5. Summary of Results

The principal findings of the analysis are:

i. Residual magnitude varies systematically with baryonic acceleration.
ii. Signed residuals exhibit substantially weaker structure than residual magnitude.
iii. Predictive performance improves progressively as additional predictor classes areincluded.
iv. Global galaxy properties provide independent predictive information beyond local acceleration and curve shape.
v. Residual magnitude is significantly more predictable than the signed residual.
vi. The deviations from the RAR therefore exhibit a structured, multi-variable pattern rather than featureless random scatter.

Together, these results demonstrate that residual structure constitutes a measurable and partially predictable component of galaxy dynamics.

6. Discussion

6.1. Residuals as an Additional Layer of Structure

The primary result of this study is that deviations from the Radial Acceleration Relation (RAR) exhibit measurable and partially predictable structure. While the RAR remains the dominant empirical description of galaxy dynamics, the residuals are not distributed uniformly across the dataset. Residual magnitude varies systematically with baryonic acceleration and depends on both local and global galaxy properties. This behavior indicates that the residuals contain information beyond that captured by the leading-order acceleration relation. A useful interpretation is that the RAR describes the central tendency of galaxy dynamics, while the residuals encode additional structure associated with the specific dynamical state of individual galaxies.

6.2. Predictability of Structure Versus Predictability of Outcome

One of the most significant findings is the distinction between prediction of residual magnitude and prediction of the signed residual. The signed residual remains only weakly predictable, even when multiple predictor classes are incorporated. In contrast, residual magnitude exhibits clear dependence on observable variables and can be predicted with substantially greater success.

This distinction suggests that the system is more predictable in its statistical structure than in its exact point-by-point behavior. Observable galaxy properties constrain the size of the expected deviation more strongly than they constrain the direction of that deviation. Such behavior is consistent with systems in which variability itself follows organized patterns, even when individual outcomes retain a degree of uncertainty.

6.3. Multi-Variable Dependence

The model hierarchy demonstrates that residual structure cannot be adequately described by baryonic acceleration alone. Although acceleration regime provides substantial predictive information, additional gains are obtained through the inclusion of local structural variables and global galaxy properties. The importance of variables such as flat rotation velocity, morphology, and normalized radius indicates that residual behavior reflects both local conditions and galaxy-wide dynamical organization.

These results imply that galaxy dynamics are inherently multi-dimensional. The leading-order relation captures the dominant behavior, but additional variables contribute systematically to the observed deviations.

6.4. Implications for Galaxy Dynamics

The findings presented here do not favor a specific theoretical framework. Instead, they establish an empirical constraint that any successful description of galaxy dynamics must satisfy. A complete model must account not only for the existence of the RAR itself, but also for the structured behavior of its deviations. The observed dependence of residual magnitude on multiple variables suggests that the dynamics of galaxies cannot be fully characterized by a single-parameter relation. Whether interpreted within dark matter models, modified gravity approaches, or alternative phenomenological frameworks, the existence of predictable residual structure represents an additional observational feature requiring explanation.

6.5. Residuals as Informative Observables

Residuals are often treated primarily as measures of model accuracy or observational uncertainty. The present analysis suggests a broader role. The results indicate that residuals contain systematic information about galaxy dynamics and should be regarded as observables in their own right. Rather than representing merely the failure of a leading-order relation, the deviations provide insight into the conditions under which that relation becomes tighter or looser. From this perspective, residual analysis offers a complementary approach to understanding galaxy dynamics by focusing directly on the structure not captured by first-order empirical relations.

6.6. Limitations and Future Work

Several limitations should be acknowledged.

First, the analysis is restricted to variables available within the SPARC dataset and associated catalogs. Additional environmental or dynamical factors may contribute to residual structure and remain unexplored. Second, the predictive models employed here prioritize interpretability rather than maximum predictive performance. More sophisticated modeling approaches may capture additional structure but would require careful treatment to avoid overfitting. Third, while residual magnitude is partially predictable, a substantial fraction of the observed variation remains unexplained. Future studies should investigate whether additional variables or alternative formulations can account for a larger proportion of the residual structure. Independent validation using other galaxy samples will also be important for determining the generality of the observed behavior.

6.7. Summary

The results demonstrate that deviations from the Radial Acceleration Relation are not adequately described as featureless random scatter. Instead, they exhibit a structured and partially predictable dependence on observable galaxy properties. The analysis shows that residual magnitude carries substantially more predictive information than the signed residual and that both local and global galaxy properties contribute to the observed structure. These findings establish residuals as a meaningful empirical component of galaxy dynamics and provide an additional observational constraint for future theoretical and phenomenological models.

7. Conclusions

This study investigated whether deviations from the Radial Acceleration Relation (RAR) contain predictable structure beyond the leading-order relation between baryonic and observed galaxy dynamics. Using the SPARC database comprising 175 galaxies and 3,391 radial measurements, residuals were analyzed as functions of both local and global galaxy properties. A hierarchy of predictive models was employed to evaluate the extent to which residual behavior could be inferred from observable quantities. Several conclusions emerge from the analysis. First, the RAR remains the dominant first-order empirical relation describing galaxy dynamics across the sample. However, the deviations from this relation are not uniformly distributed throughout the dataset. Second, residual magnitude varies systematically across acceleration regimes and exhibits measurable dependence on additional observable variables.

The structure of the deviations is therefore not adequately described as featureless random scatter. Third, predictive performance improves progressively as local structural variables and global galaxy properties are incorporated. Variables such as normalized radius, flat rotation velocity, morphology, baryonic mass, gas fraction, and surface brightness contribute independent predictive information beyond baryonic acceleration alone. Fourth, residual magnitude is substantially more predictable than the signed residual. This distinction indicates that the statistical structure of the deviations is more strongly constrained than the exact direction of individual deviations. Taken together, these results demonstrate that deviations from the RAR follow a structured, multi-variable pattern that can be partially predicted from observable galaxy properties. The findings suggest that galaxy dynamics are characterized not only by a leading-order acceleration relation but also by an informative and measurable structure of deviations. The results presented here establish residual structure as a meaningful empirical component of galaxy dynamics and provide an additional observational constraint that future theoretical and phenomenological models must explain [1-12].

Author Contributions

The author solely conceived the study, performed the analysis, interpreted the results, and prepared the manuscript.

Funding

This research received no funding.

Data Availability Statement

The data used in this study are publicly available from the SPARC (Spitzer Photometry and Accurate Rotation Curves) database. Rotation-curve mass models and galaxy property tables can be accessed at: http://astroweb.cwru.edu/SPARC/ Additional stellar mass estimates were obtained from publicly available WISE-based galaxy catalogs cited in the References section. All data analyzed during this study are available from the corresponding public sources.

Conflicts of Interest

The author declares no conflict of interest.

Artificial Intelligence Disclosure

Generative artificial intelligence tools (ChatGPT, OpenAI) were used during manuscript preparation for organizational assistance and language refinement. All scientific concepts, analyses, interpretations, conclusions, and final manuscript drafting editing and content are the exclusive work of the author, who assumes full responsibility for the accuracy and integrity of the work.

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