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Research Article

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Stochastic Quantum Probability and Subatomic Particle Experiment Design

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Abstract
This article is the third of a trilogy of articles on the nature of probability in quantum mechanics. The first article [1] began by noting that superposed wavefunctions in quantum mechanics lead to a squared amplitude that introduces interference into the position probability density of an atomic particle, which happens nowhere else in probability theory. It went on to also note that there is an unexplained coincidence in quantum mechanics in that the interference term in the squared amplitude of superposed wavefunctions gives the squared amplitude the form of a variance of a sum of correlated random variables and went on to examine whether there could be an archetypical variable, in the Platonic sense of true form, behind quantum probability that would reconcile quantum probability with classic probability. This examination found such a variable, which can encompass both local and nonlocal quantum events. The second article [2] provided evidence that quantum probability has such a stochastic nature. This evidence was based on the number of electrons that need to be sent through a two-slit interferometer to gain a clear pattern of self-interference, which when compared with the number that would be expected to be sufficient in order for the position probability distribution of the self-interference wavefunction to take clear shape suggests that there is more variability present than that described by the formulation of quantum mechanics, which implies the presence of an underlying and as yet unrecognized physical process. This final article completes the trilogy by considering how a key aspect of experimental design would be affected by the increased variability that would be present if quantum probability is itself stochastic in the manner suggested in the previous two articles.

Keywords: Quantum Probability, Two-Slit Experiment, Self-Interference, Wavefunction, Simulation, Stochastic, Experiment Design

1. Some relevant basics

In the previous articles the Schrödinger picture and the causal interpretation as recounted by Peter Holland in his quantum theory of motion [3] were used, confined to a single spatial dimension to keep a focus on concepts. The wavefunction associated with a particle takes the form $\psi = Re^{iS/(h/2\pi)}$ where R is wave amplitude, S is wave phase, and h is Planck's constant. In one spatial dimension x the squared amplitude R^2 of the wavefunction is $|\psi(x,t)|^2 = \psi(x,t)\ \psi^*(x,t)$, where $\psi^*$is the complex conjugate of ψ, and the probability density $f(x, t)$ associated with the location of the particle being between x and $x + dx$ at time t is given by

$$f(x,t) = \frac{\psi(x,t)\psi^*(x,t)}{C(t)}$$

where $C(t)$ is a normalization constant $\int_{-\infty}^{\infty} \psi\psi^*\, dx$, which varies with t. Provided that $\int_{-\infty}^{\infty} \psi\psi^*\, dx$ is finite, $C(t)$ rescales the squared amplitude to a position probability density by satisfying $\int_{-\infty}^{\infty} f(x,t)\, dx=1$ and reflects that the particle exists somewhere at time t.

2. The first Article

The first article [1] began by noting that a remarkable feature of the interference term in the squared amplitude R^2_{sum} of two superposed wavefunctions, after allowing for the fact that the cosine of the phase difference related angle θ that appears in it is mathematically equivalent to and has all the attributes of a coefficient of correlation, is that from the standard result in statistics for the variance of the sum of two correlated random variables, the squared amplitude R^2_{sum} is mathematically equivalent to this variance, where the two variables have variance R_1^2 and R_2^2 and a correlation coefficient of $\cos\theta$. The article went on to explore what this equivalence could signify. Although Bell's theorem confirmed the view that one cannot reproduce quantum probabilities from classic probability, the article proposed that the possibility nevertheless remains that hidden variables may exist that would bypass this limitation. Indeed, Oaknin [4] has shown that the proof of Bell's theorem involves a subtle, though crucial, assumption that is not required by fundamental physical principles and is not necessarily fulfilled in the experimental set-up that tests the inequality. This assumption is that there exists a preferred absolute frame of reference, supposedly provided by the laboratory. Oaknin shows that this preferred frame of reference, which is required by the proof of Bell's theorem does not necessarily exist and cannot exist under certain circumstances described by Oaknin.

The first article [1] also pointed to what von Weizsacker had said in the conclusion of his 1973 paper on probability and quantum mechanics [5], “there is a quantum theory behind quantum theory, precisely because probabilities can only be defined with the help of probabilities.”

The starting point to explaining the coincidence noted in the article was to suppose that before normalization the squared amplitude is associated with a hidden variable whose mean equals the variance of another closely related and relevant hidden variable, a property called there the *mean/variance property.* Finding a pair of such variables with the properties needed to be consistent with quantum mechanics was the purpose of the first article. The generic forms of such variables were called the unit base variable and the *unit squared amplitude variable,* respectively, with the unit base variable having a mean of zero and variance of one, and the unit squared amplitude variable being the square of the unit base variable in order that its mean equals 1, the variance of the unit base variable (the mean/variance property).

The element of a wavefunction that is directly relevant and capable of having these properties is its real part, the product of its amplitude and the cosine of its argument, which captures its essence. Accordingly, it was proposed that the unit base variable may be the product of two independent variables, an amplitude variable, and a cosine variable, and the unit squared amplitude variable and the related unit base variable would form the pair being searched for.

The search for these variables led to a generic archetypical denoted Z. This result is drawn on later in the section on the effect of this variable on experiment design and is included there rather than here to avoid repetition.

In each instant the generic variable Z was formulated to be simultaneously transformed across the universal set of active quantum processes, wherever and for as long as a process is active, into a corresponding universal set of process-specific squared amplitude variables and their stochastic processes. This transformation involves Z becoming a set of independent and identically distributed variables spanning each spatial point and time at which a quantum process is active, with the realization of the variable at a point being transformed by a scale factor SF, the deterministic squared amplitude there and then of the specific quantum process, into a process specific variable that was denoted Y. The variable Y has a mean at each point and time that equals, and whose average realization over repeated trials of an experiment converges to, the deterministic squared amplitude.

In short, the first article presented a formulation in which behind the squared amplitude of either a superposed or individual wavefunction as formulated in the causal interpretation, there could be an associated specific variable Y at each applicable point in space and time that originates from a universal generic archetypical variable Z.

Importantly, the article showed that despite being developed using classic probability theory, the variable Y and its stochastic process can relate to either a local or a nonlocal quantum mechanical process.

3. The second article

The second article [2] examined and charted the position probability distribution for a two-slit interferometer experiment. It clearly shows a pattern of multiple probability peaks and troughs which will translate, as the cumulative number of electrons sent through the interferometer increases, into increasingly distinct bands of dense detection separated by sparse detection with varying detection density within the bands from peak to trough, and vice versa, taking shape across the entire detection screen.

The article proceeded to consider the celebrated experiment performed by Tonomura [6] in particular Figure 2 from the paper, which illustrates the number of electrons (140,000) that needed to be emitted in order for a relatively clear image (d in Fig. 2) of self-interference to have formed on the detection screen of the interferometer. It is recommended that the reader refer to this paper to see how the process of electron self-interference builds up, showing no discernable pattern with 8 and 200 emissions, just the hint of a pattern with 6000 emissions, and finally a clear pattern with 140,000 emissions.

Before discussing this further, the second article turned to a simulation of the probability density $f(x)$ and cumulative density $F(x)$ of the standard normal distribution to see how many realizations are needed for a relatively clear picture of these distributions to emerge. The result of 5000 realizations of the pdf and of the cdf were charted and these charts gave a relatively sharp picture of the standard normal distribution.

The significance of this exercise was that if 5000 realizations of a simulated standard normal variable are sufficient to provide a relatively clear picture of its distribution then, provided that a degree of relative, rather than absolute, clarity is acceptable, 5000 simulations are also enough to do the same for any other distribution where the probability distribution and the parameters it employs are known, including the interference distribution and with parameters that describe the setup in the Tonomura experiment. Why then, the article asked, does it take 140,000 electron emissions through a two-slit interferometer to gain a relatively clear picture of the self-interference effect?

The second article concluded by suggesting that the answer to this question is that the nature of particle position probability in quantum mechanics is not solely determined by the normalized squared amplitude of the wavefunction of a quantum mechanical process, but rather originates from a random variable whose mean equals that deterministic squared amplitude. The resulting variability around the mean would lead to a much larger number, than the otherwise sufficient 5000 or so electron emissions, to be necessary for a relatively clear picture to form, as was the case in the Tonomura experiment, and for the average of the realizations of the squared amplitude variable at a point and time (x, t) to converge to the deterministic squared amplitude.

This suggestion was of course only a mathematical one which, if valid, would be the manifestation of some underlying and as yet unrecognized physical process that introduces more variability into quantum probability than is described in the existing formulation of quantum mechanics and it is the purpose of the remainder of this final article of the trilogy to consider this increased variability and its effect on atomic particle experiment design.

4. The effect of increased variability on a key aspect of experiment design.

It will be recalled from the earlier discussion of the first paper that the search for a stochastic archetype of quantum probability led to a result that is stated here rather than there to avoid repetition. The result advanced the following hypothesis:

1. *A* generic variable *Z* which is the square of another generic variable *W*, which is the product of two independent seed variables *A* and *C*:
a. Variable $A \sim U\,(0,3)$, which is a generic stochastic analogue of the amplitude of a wavefunction, and
b. Variable $C \sim U\,(-1,1)$, which is a generic stochastic analogue of the cosine of the argument of a wavefunction.

2. The dependent unit base variable *W* is the zero mean/unit variance generic stochastic analogue of the real part of a wavefunction, properties that come from the supports of *A* and *C*.
3. The dependent unit squared amplitude variable *Z* is the unit mean generic stochastic analogue of the square of the real part of a wavefunction.

Our examination of possible increased variability will be based on the unit squared amplitude variable Z hypothesized in the first article [1], as a suitable representative hidden variable that would make quantum probability stochastic, if such were the case, and we examine simulations of the unit squared amplitude variable Z.

It is straightforward using Excel to simulate the variable Z .*A* realization of the variable *A* is generated by RAND()*3 and of the variable *C* by RAND()*2+(-1). A realization of the variable *Z* is then generated by multiplying the realizations of *A* and *C* and squaring the result. The first step was to set up the ability to perform a set of 140,000 simulations of *Z* in Excel. It is shown in [1] that the mean of *Z* is1 and the variance is 2.24, and the adequacy of any number of repeated trials of an experiment such as 5000 or 140,000 is judged in the case of 5000 trials by how well the first 5000 simulations in the set reflected the mean and variance when 200 refreshed sets of 140,000 simulations were performed and in the case of 140,000 simulations by how well the full 140,000 reflected the mean and variance in the 200 refreshed sets. 5000 and 140,000 were chosen because 5000 should be adequate if the probability distribution and its parameters are known and 140,000 because that number appears to have been deemed necessary in the Tonomura self-interference experiment. Although the latter presented only an image of self-interference the experimenters would have had the underlying data and it may have contributed to their decision to fire as many as 140,000 electrons through the interferometer.

Table 1 shows the results for the mean of the unit squared amplitude variable *Z*. These results are particularly important as in practice the mean measurement from repeated experiments is the primary result. Recalling from the opening section on some relevant basics that the probability density $f(x,\,t)$ associated with the location of the particle being between x and $x + dx$ at time t is given by. (x,t)

$$f(x,t) = \frac{\psi(x,t)\psi^*(x,t)}{C(t)} = \frac{R(\mathrm{x,t})^2}{C(t)}$$

where $C(t)$ is the normalization constant $\int_{-\infty}^{\infty} \psi\psi^*\, dx$, so that the expected number of times the particle is observed between x and $x + dx$ at time t is N times $f(x,\mathrm{t})$, where N is the number of trials of the experiment. Conversely, provided that N is an adequate number of repeated trials of the experiment, the number of times the particle is observed between x and $x + dx$ at time t, when divided by N gives the relative frequency of the particle's observation there, in other words the probability of it being there then. The choice of N is therefore a key aspect of subatomic particle experiment design.

However, if $R(x,t)^2$is not deterministic, but rather is stochastic and is the variable *Y*, which is the unit squared amplitude variable *Z* scaled i.e., transformed, by the deterministic $R(x,t)^2$, then any straying of the mean realization of the underlying unit squared amplitude variable *Z* away from 1 will distort the results of the experiment. As explained in the section on the first article this transformation involves the generic universal variable *Z* becoming a set of independent and identically distributed variables spanning each spatial point and time at which a quantum process is active, with the realization of the variable at a point being scaled by the deterministic squared amplitude for the point and time of the specific quantum process that is active there and then. In the case of the Tonomura experiment for example, the distortion would be present across all points on the detection screen and would only be moderated by performing an adequate number of repeated trials of the experiment, as was done in that case.

Count	5000	last Z means	140000	twin	nil	total
solo	0	1.01				
solo		1	85			
nil					0	
twin				115		200
	5000		140000		nil	
	115 57.50%		200 100.00%		0	

Table 1

Counts for solo and twin are for the mean of Z, analytically 1. Only means that when rounded to two decimal places are .99, 1, or 1.01 are counted. Means outside this set are not counted. Twins are means from the first 5000 and the full 140,000 simulations respectively that both qualify and are counted in each. 200 successive measurements of the mean of Z are made in 200 sets of realizations.

The first result to note is that there were no means when rounded to two decimal places that were measured outside the set of .99, 1, and 1.01 for the full set of 140,000 simulations. However, there were 85 means from the first 5000 simulations that fell outside the set, ranging from as low as .93 and as high as 1.07. All of the count of 115, or 57.5% of the 200 qualifying measurements, from the first 5000 simulations come from twin counts, while the count from the full 140,000 simulations come from 85 solo counts and 115 twin counts, a total of 200, or 100% of the 200 measurements.

Supposing, for example, that there are 200 points where the particle could be at the time, and bearing in mind that the realizations of the unit squared amplitude variable at any time are i.i.d, and are transformed into the variable Y at each point, these results suggest that the mean of an experiment with 5000 trials would only closely approximate the deterministic squared amplitude at each of the points about 60% of the time and be distorted about 40% of the time, while the mean of an experiment with 140,000 trials would closely approximate the deterministic squared amplitude at each point about 100% of the time and be distorted almost none of the time.

Table 2 below shows the results of the test of the variance of the unit squared variable. In a similar fashion to Table 1, counts for solo and twin are for the variance of Z, analytically 2.24 as was shown in [1]. Only variances that when rounded to two decimal places are 2.23,2.24, or 2.25 are counted with the rest being classified as "nil". Variances outside this set are not counted. Twins are variances from the first 5000 and the full 140,000 simulations respectively that both qualify and counted in each. 200 successive measurements of the variance of Z are made in 200 sets of realisations.

Counts	5000	last Z var	140000	twin	nil	total
solo	7	2.11				
solo		2.23	128			
nil					53	
twin				12		200
	5000		140000		nil	
	19 9.5%		140 70%		53 26.5%	

Table 2

There is more variability here than for the mean. The Table shows that 53, or 26.5% of the 200 measurements fell outside the qualifying set of 2.23, 2.24 and 2.25. These measurements ranged from a low of 2.11 to a high of 2.42 for the first 5000 of the 140,000 simulations, and from a low of 2.20 to a high of 2.28 for the full 140,000. Of the qualifying measurements, only 19, or 9.5% of the 200 measurements came from the first 5000 of the 140,000 simulations, with 140, or 70% coming from the full 140,000 simulations. As is the case with the mean, 140,000 simulations and hence repeated trials of an experiment are necessary if the squared amplitude is a variable. Indeed, if the squared amplitude is a variable then, as can be seen from the Table, this order of number of trials is even more necessary if it is important to have a proper reading of the variance because when the unit squared amplitude variable is transformed by multiplying it by the deterministic squared amplitude the variance of the resulting variable Y is that of the unit variable multiplied by the square of the deterministic squared amplitude, greatly amplifying any distortion in the variance.

5. Conclusion

This article completes a trilogy of articles devoted to the nature of quantum probability. The first article hypothesized that quantum probability is itself stochastic and uncovered an archetypical hidden variable that could be a suitable candidate for such a variable behind quantum probability if indeed it is itself stochastic. The second article examined evidence provided by the famous Tonomura two-slit electron self-interference experiment that supported the hypothesis of the first article. It claimed that if quantum probability is itself stochastic the resulting variability around the mean would explain the much larger number, than the otherwise sufficient 5000 or so electron emissions, that were necessary for a relatively clear picture to form in the Tonomura experiment, where 140,000 electron emissions were necessary.

This third and final article has confirmed the claim in the second article, showing, for example, that if there are 200 points where a particle could be any time, and bearing in mind that the realizations of the unit squared amplitude variable at any time are i.i.d, the mean of an experiment with 5000 trials would only closely approximate the deterministic squared amplitude at each of the points about 60% of the time and be distorted about 40% of the time, while the mean of an experiment with 140,000 trials would closely approximate the deterministic squared amplitude at each point about 100% of the time and be distorted almost none of the time. It has also shown that a similar result applies to the variance, but with more amplified distortion than happens with the mean.

Finally, the author hopes that the results and suggestions advanced in this trilogy may prove to be helpful in practice.

References

1. Jenkins, T. C. (2021). A Search for A Stochastic Archetype of Quantum Probability, Adv Theo Comp Phy, 4(4), 304-313.
2. Jenkins, T. C. (2022). Report of Evidence That Quantum Probability Is Itself Stochastic. Adv Theo Comp Phy, 5 (3), 560-564.
3. Holland, P. R. (1995). The quantum theory of motion: an account of the de Broglie-Bohm causal interpretation of quantum mechanics. Cambridge university press.
4. Oaknin, D. H. (2020). The Bell theorem revisited: geometric phases in gauge theories. Frontiers in Physics, 8, 142.
5. von Weizsācker, C. F. (1973). Probability and quantum mechanics. The British Journal for the Philosophy of Science, 24(4), 321-337.
6. Tonomura, A. (2008). Development of electron holography and its applications to fundamental problems in physics. Japanese journal of applied physics, 47(1R), 11.