ISSN: 2639-0108

Research Article

Advances in Theoretical & Computational Physics

Statistical Principles of Elastic Particles in Digital Phase Space

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Submitted: 2026, Jun 11; **Accepted:** 2026; Aug 04; **Published:** 2026, Aug 21

Citation: Liang, Z. C. (2026). Statistical Principles of Elastic Particles in Digital Phase Space. *Adv Theo Comp Phy, 9*(3), 01-16.

Abstract

By introducing the path subspace, this study extends the energy phase space to a digital phase space (Λ -space), thus accomplishing the numerical statistics of the elastic particle equilibrium systems. Unlike the Γ-space (6N dimensions) of Gibbs statistical ensemble, the Λ-space (18N dimensions) is divided into three areas: liquid, solid, and gas, and their energy probability distributions have cyclic symmetry. The interaction between particles in Λ-space is characterized by the degree of association/dissociation, thereby avoiding the problem of potential energy integration in Γ-space. This article presents the macroscopic and mesoscopic equilibrium conditions for elastic particle systems and derives the partition functions and state functions for three areas. In addition, the differential equations of thermodynamics are generalized to the entire range of matter states by extending the definition of the entropy function. The results show that the states and changes of the thermodynamic system can be completely expressed by five independent parameters (two digits and three scales), and the first and second laws of thermodynamics can also be naturally derived from the statistical theory of digital phase space.

Keywords: Statistical Physics, Elastic Particles, Statistical Phase Space, Partition Function, Thermodynamic Functions, Thermodynamic Laws

1. Introduction

Unlike point particles, elastic particles possess three modes of motion in the barycenter reference frame: translation, rotation, and vibration. Based on the elastic particle model, we have previously established a theoretical paradigm known as Real Physics [1-7], which includes the theories of particle field [1, 8-10], particle dynamics [2, 11-14], and particle statistics [3, 15-18]. Within particle statistical theory, the statistical quantities of the particle's three motion modes constitute an energy phase space. This phase space encompasses both equilibrium and non-equilibrium states, with the equilibrium states located on three parabolic surfaces. The distribution of the equilibrium states on these surfaces is discrete and non-uniform, making direct integral statistics of the equilibrium states particularly challenging. To address this fundamental challenge, we introduce a mesoscopic statistical approach based on the distribution vector of midsons (clusters). This approach bridges the microscopic dynamics of baseons and the macroscopic thermodynamics of the system, effectively bypassing the intractable problem of direct integration over the curved equilibrium surfaces. This allows us to derive the system's state functions and differential equations rigorously, providing a powerful tool for studying phase transition phenomena [3].

The mathematical foundation of Real Physics is the theory of actual quantities. An actual quantity is the product of a scale factor and a digit factor. Representing physical quantities as actual quantities and determining their scales and digits are referred to as digitization. A complete physical theory must achieve the digitization of all its physical quantities. Energy phase space statistics [3,13,14] is a microscopic statistical method at the level of baseons, which achieves the digitization of the system's macroscopic quantities. To investigate the equilibrium properties of systems, particle statistics introduces parameters at the level of midsons that characterize particle interactions. Determining the numerical values of these mesoscopic quantities to achieve the complete digitization of the particle statistics theory is an anticipated goal.

This research introduces the path subspace and the momentum subspace, expanding the energy phase space into a digital phase space, thereby completing the statistical theory for elastic particle systems. The method of digital phase space statistics is analogous to that of Gibbs' ensemble theory [19]; however, their physical principles and mathematical foundations differ significantly. This article presents the principles of digital phase space statistics, provides the probability density within the digital phase space, and calculates the system's partition function and state functions. By extending the concept of the entropy function, the thermodynamic differential equations applicable to all states of matter (liquid, solid, and gas) are derived. Crucially, this formalism demonstrates that the vast information encoded in a thermodynamic system can be compressed into a minimal set of five independent parameters.

The success of this unified method across the state of matter not only established the position of digital phase space as a tool in statistical physics, but also laid the foundation for exploring the connections between microscopic dynamics and macroscopic thermodynamics. As examples, this paper uses a set of five parameters to derive the state functions of liquid materials, and applies differential equations to formulate the fundamental laws of thermodynamics, thereby demonstrating the nature of statistical mechanics of elastic particles as a meta-theory.

2. Energy Phase Space Statistics

2.1. Motion of Particles

An elastic object is a closed system composed of a conserved number of elastic particles. The spatial states of an elastic particle are described by its Position, Posture, and Profile (the "3P" states). The Position is defined by the location of the particle's barycenter (center of mass), the Profile by the eigenvalues (principal moments of inertia) of its inertia matrix, and the Posture by the eigenvectors (principal axes of inertia) of the inertia matrix. In the barycenter reference frame, the particle's motion consists of three modes: translation, rotation, and vibration, which correspond to changes in its position, posture, and profile, respectively.

At time t, consider an elastic particle with its barycenter at O_c, principal moments of inertia (I_1,I_2,I_3), and principal axes of inertia (e_1,e_2,e_3). Using these, the barycenter frame of reference $[O_c e_1 e_2 e_3]$ at time t is established. After a unit time t_s, the particle's barycenter moves to O_c' the principal moments of inertia change to (I_1', I_2', I_3'), and the principal axes of inertia change to $(\boldsymbol{e}_1', \boldsymbol{e}_2', \boldsymbol{e}_3')$. Consequently, the barycenter frame of reference at time $t' = t + t_s$ becomes $[O_c' \boldsymbol{e}_1' \boldsymbol{e}_2' \boldsymbol{e}_3']$.

Let the displacement vector of the barycenter O_c in the frame $[O_c e_1 e_2 e_3]$ be denoted by $\boldsymbol{\xi}$, the angle change of the principal axes by $\boldsymbol{\vartheta}$, and the cumulative length change of the principal axes by $\boldsymbol{\varrho}$, then:

$$\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3), \xi_\alpha = \tilde{\xi}_\alpha \cdot \xi_s, \xi_s = r_s \left(-\infty < \tilde{\xi}_\alpha < +\infty\right); \tag{1a}$$

$$\boldsymbol{\vartheta} = (\vartheta_1, \vartheta_2, \vartheta_3), \vartheta_\alpha = \tilde{\vartheta}_\alpha \cdot \vartheta_s, \vartheta_s = 1 \left(-\infty < \tilde{\vartheta}_\alpha < +\infty\right); \tag{1b}$$

$$\boldsymbol{\varrho} = (\varrho_1, \varrho_2, \varrho_3), \varrho_\alpha = \tilde{\varrho}_\alpha \cdot \varrho_s, \varrho_s = r_s \ (0 < \tilde{\varrho}_\alpha < +\infty\ ). \tag{1c}$$

Here α =1,2,3. The set $\{\boldsymbol{\xi},\boldsymbol{\vartheta},\boldsymbol{\varrho}\}$ is termed the motion paths of the elastic particle. Specifically, $\boldsymbol{\xi}$ represents the translation path of the barycenter, $\boldsymbol{\vartheta}$ denotes the rotation path of the principal axes, and $\boldsymbol{\varrho}$ signifies the vibration path of the principal axes. These correspond to the three motion modes of the particle, with each mode possessing three degrees of freedom. Thus, a single particle has a total of nine motional degrees of freedom.

According to the calculation rules of actual quantities [1], the time derivatives of the paths are:

$$\dot{\boldsymbol{\xi}} = \left(\dot{\xi}_1, \dot{\xi}_2, \dot{\xi}_3\right), \dot{\xi}_\alpha = \frac{\xi_\alpha}{t_s} = \tilde{\xi}_\alpha \cdot v_s;\ v_s = \frac{r_s}{t_s}. \tag{2a}$$

$$\dot{\boldsymbol{\vartheta}} = \left(\dot{\vartheta}_1, \dot{\vartheta}_2, \dot{\vartheta}_3\right), \dot{\vartheta}_\alpha = \frac{\vartheta_\alpha}{t_s} = \tilde{\vartheta}_\alpha \cdot v_s;\ v_s = \frac{1}{t_s}. \tag{2b}$$

$$\dot{\boldsymbol{\varrho}} = (\dot{\varrho}_1, \dot{\varrho}_2, \dot{\varrho}_3), \dot{\varrho}_\alpha = \frac{\varrho_\alpha}{t_s} = \tilde{\varrho}_\alpha \cdot v_s;\ v_s = \frac{r_s}{t_s}. \tag{2c}$$

Let the mass of the particle be m, its principal moments of inertia be I_α (where $I_s = m_s r_s^2$), and define the particle's translation momentum $\boldsymbol{p}$, rotation momentum $\boldsymbol{q}$, and vibration momentum $\boldsymbol{o}$ as:

$$\boldsymbol{p} = (p_1, p_2, p_3), p_\alpha = m\dot{\xi}_\alpha = \left(\tilde{m}\tilde{\xi}_\alpha\right) \cdot p_s, p_s = m_s v_s. \tag{3a}$$

$$\boldsymbol{q} = (q_1, q_2, q_3), q_\alpha = I_\alpha \dot{\vartheta}_\alpha = \left(\tilde{I}_\alpha \tilde{\vartheta}_\alpha\right) \cdot q_s, q_s = m_s v_s r_s. \tag{3b}$$

$$\boldsymbol{o} = (o_1, o_2, o_3), o_\alpha = m\dot{\varrho}_\alpha = (\tilde{m}\tilde{\varrho}_\alpha) \cdot o_s, o_s = m_s v_s. \tag{3c}$$

The particle's translation energy K, rotation energy L, and vibration energy H can be expressed as:

$$K = \frac{1}{2}\sum_{\alpha=1}^{3}\frac{p_\alpha^2}{m} = \widetilde{K} \cdot K_s;\ \widetilde{K} = \frac{1}{2}\sum_{\alpha=1}^{3}\frac{\tilde{p}_\alpha^2}{\tilde{m}}, K_s = \frac{p_s^2}{m_s} = m_s v_s^2. \tag{4a}$$

$$L = \frac{1}{2}\sum_{\alpha=1}^{3}\frac{q_\alpha^2}{I_\alpha} = \tilde{L} \cdot L_s;\ \tilde{L} = \frac{1}{2}\sum_{\alpha=1}^{3}\frac{\tilde{q}_\alpha^2}{\tilde{I}_\alpha}, L_s = \frac{q_s^2}{I_s} = m_s v_s^2. \tag{4b}$$

$$H = \frac{1}{2}\sum_{\alpha=1}^{3}\frac{o_\alpha^2}{m} = \widetilde{H} \cdot H_s;\ \widetilde{H} = \frac{1}{2}\sum_{\alpha=1}^{3}\frac{\tilde{o}_\alpha^2}{\tilde{m}}, H_s = \frac{o_s^2}{m_s} = m_s v_s^2. \tag{4c}$$

2.2. Energy Phase Space

A closed system (topson) comprises N basic particles (baseons). If the motion energy of the i-th baseon is $\{H_i, K_i, L_i\}$,then the motion energy of the system is:

$$H = \sum_{i=1}^{N} H_i > 0, K = \sum_{i=1}^{N} K_i > 0, L = \sum_{i=1}^{N} L_i > 0. \tag{5}$$

Each baseon possesses 9 degrees of freedom, and a system containing N baseons therefore has $9N$ motional degrees of freedom. The statistical quantities $\{H,K,L\}$ are always greater than zero, a property referred to as the positive definiteness of the motion energy. Furthermore, the full energy of the system is defined as:

$$E := \sqrt{H^2 + K^2 + L^2}. \tag{6}$$

The three-dimensional mathematical space formed by the motion energy $\{H,K,L\}$ is referred to as the energy phase space. Each point in this phase space represents a statistical aggregate of the $9N$ microscopic parameters of the N baseons, with every point corresponding to a distinct macroscopic state of the system.

The energy phase space possesses a unique mathematical structure [3]. Using a pair of angle brackets "⟨ ⟩" to denote an ordered set of elements, the energy phase space is defined as a family of ordered arrays in three zones:

$$\text{Vibration Zone } \mathbb{E}^h := \langle E_1^h, E_2^h, E_3^h \rangle = \langle H^h, K^h, L^h \rangle, \tag{7a}$$

$$\text{Translation Zone } \mathbb{E}^k := \langle E_1^k, E_2^k, E_3^k \rangle = \langle K^k, L^k, H^k \rangle, \tag{7b}$$

$$\text{Rotation Zone } \mathbb{E}^l := \langle E_1^l, E_2^l, E_3^l \rangle = \langle L^l, H^l, K^l \rangle. \tag{7c}$$

Where $z \in \{h,k,l\}$ is referred to as the zone index. Based on their positional order within the zones, we call the first, second, and third element as the main energy, front energy, and back energy, respectively:

$$\text{Main Energy } E_1^z = \{E_1^h, E_1^k, E_1^l\} = \{H^h, K^k, L^l\}, \tag{8a}$$

$$\text{Front Energy } E_2^z = \{E_2^h, E_2^k, E_2^l\} = \{K^h, L^k, H^l\} \leq E_1^z, \tag{8b}$$

$$\text{Back Energy } E_3^z = \{E_3^h, E_3^k, E_3^l\} = \{L^h, H^k, K^l\} \leq E_1^z. \tag{8c}$$

The structural arrangement of motion energies in the sequence ⟨main,front,back⟩ is known as the cyclic symmetry. A fundamental characteristic of the zone is that its main energy possesses the maximum value.

The energy phase space possesses an intuitive geometric structure [3]. The motion energy (*H,K,L*) of the system form a Cartesian vector space. Due to the positive definiteness, the energy space is confined to the first octant (+,+,+). The macroscopic state of the system is characterized by a vector in this Cartesian space:

$$\boldsymbol{E}^z := (H^z, K^z, L^z) = H^z\mathbf{i} + K^z\mathbf{j} + L^z\mathbf{k} = E^z\mathbf{e}_0 . \tag{9}$$

The $\boldsymbol{E}^z$ are referred to as the energy vector, and its components are enclosed in parentheses. The (**i,j,k**) are the unit vectors of the coordinate axes. The endpoints of the vectors $\boldsymbol{E}^h$, $\boldsymbol{E}^k$, and $\boldsymbol{E}^l$ lie within the vibration zone, translation zone, and rotation zone, respectively. The magnitude E^z of the energy vector represents the full energy:

$$E^z = \sqrt{(H^z)^2 + (K^z)^2 + (L^z)^2}. \tag{10}$$

The energy vector (H^z,K^z,L^z) represents the energy in the Cartesian coordinate system, with its components order bound to the unit vectors (**i,j,k**). However, the ordered array ⟨E_1^z,E_2^z,E_3^z ⟩ is arranged based on the zone index, z, following cyclic symmetry; its first element, E_1^z, always represents the main energy of that zone.

2.3. State Functions

The state function of particle system includes energy, pressure, and order degree. Energy includes motion energy and various forms of derived energy. Derived energy is the linear combination of motion energy and is defined according to cyclic symmetry:

$$\text{Entire Energy } E^z := E_2^z + E_3^z , \tag{11a}$$

$$\text{Potential Energy } J^z := E_3^z - E_1^z, \tag{11b}$$

$$\text{Thermal Energy } Q^z := E_1^z + E_2^z, \tag{11c}$$

$$\text{Chemical Energy } G^z := E_2^z - E_3^z, \tag{11b}$$

$$\text{Internal Energy } U^z := Q^z - E_3^z, \tag{11e}$$

$$\text{Enthalpic Energy } Y^z := Q^z + G^z. \tag{11f}$$

The order degree is defined as follows:

$$\text{Front Order Degree } a^z := E_2^z/E_1^z , \tag{12a}$$

$$\text{Back Order Degree } b^z := E_3^z/E_1^z , \tag{12b}$$

Let V^z denote the volume, the motion pressure is defined as the density of motion energy:

$$\text{Main Pressure } P_1^z = E_1^z/V^z , \tag{13a}$$

$$\text{Front Pressure } P_2^z = E_2^z/V^z , \tag{13b}$$

$$\text{Back Pressure } P_3^z = E_3^z/V^z . \tag{13c}$$

If the motion pressure is expressed as a vector:

$$\boldsymbol{P}^z := (P_1^z, P_2^z, P_3^z) = P_1^z\boldsymbol{i} + P_2^z\boldsymbol{j} + P_3^z\boldsymbol{k} = P^z\boldsymbol{e}_0 , \tag{14}$$

then the full pressure is

$$P^z = \sqrt{(P_1^z)^2 + (P_2^z)^2 + (P_3^z)^2}. \quad (15)$$

Both the pressure vector and the energy vector are vectors in the energy phase space, and their directions indicate the degree of order.

2.4. Equilibrium Condition

The motion energy and derived energy at equilibrium are both state functions of the system. Setting the full energy of the system equal to the entire energy among the derived energies, i.e.

$$E^z = \sqrt{(E_1^z)^2 + (E_2^z)^2 + (E_3^z)^2} = E_2^z + E_3^z, \quad (16)$$

the equilibrium condition of the system is determined as:

$$E_1^z = \sqrt{2E_2^z E_3^z}. \quad (17)$$

The above condition is referred to as the equilibrium equation. It can be expressed in terms of pressure and order degree as:

$$P_1^z = \sqrt{2P_2^z P_3^z}, 2a^z b^z = 1. \quad (18)$$

The equilibrium equation defined in the energy phase space does not involve the microscopic structure of particles and belongs to a phenomenological macroscopic equilibrium condition.

The stability condition for system equilibrium is that the energy scale possesses a stable value. Since the main energy attains the maximum value within a zone, the energy scale is determined based on the system's main energy as:

$$E_s^z := \frac{E_1^z}{N}\left\{E_s^h = \frac{H^h}{N} = hD, E_s^k = \frac{K^k}{N} = kT, E_s^l = \frac{L^l}{N} = lB\right\}. \quad (19)$$

E_s^h, E_s^k and E_s^l are applicable to gaseous, liquid, and solid states respectively; D, T, and B represent frequency, thermodynamic temperature, and magnetic induction strength, respectively. $T^* \in \langle D,T,B \rangle$ is known as the motion intensity, while $k^* = \langle h,k,l \rangle$ is the unit transformation coefficient. In the case of using natural units, $k^* = \langle 1,1,1 \rangle$, then $T^* = \langle E_s^h, E_s^k, E_s^l \rangle$. Essentially, the motion intensity represents the scale of energy.

A physical system possesses only three independent scales (termed meta scales). If the meta scales $\{m_s, V_s, E_s\}$ are adopted, other scales can be covariant derived. For example:

$$P_s = E_s/V_s, r_s = \sqrt[3]{V_s}, I_s = m_s r_s^2, v_s = \sqrt{E_s/m_s}\ t_s = \sqrt{I_s/E_s}. \quad (20)$$

The pressure, velocity, and time scales for a liquid are, respectively:

$$P_s = \frac{kT}{V_s}, v_s = \sqrt{\frac{kT}{m_s}}, t_s = \sqrt{\frac{I_s}{kT}}. \quad (21)$$

The equilibrium of a thermodynamic system is a dynamic equilibrium, within which particles undergo perpetual motion. Even an isolated equilibrium system possesses definite scales of velocity and time; it is not a static, time-frozen state of silence.

2.5. Equation of State

Expressing the main energy of the equilibrium state using the energy scale yields:

$$E_1^z = NE_s^z. \quad (22)$$

From the definitions of the entire energy and the order degree, it follows that:

$$E^h = P^h V^h = K^h + L^h = (a^h + b^h) N E_s^h,$$

$$E^k = P^k V^k = L^k + H^k = (a^k + b^k) N E_s^k,$$

$$E^l = P^l V^l = H^l + K^l = (a^l + b^l) N E_s^l.$$

These equations are the equations of state, applicable to gases, liquids, and solids, respectively. Omitting the zone index, the equation of state can be uniformly expressed as:

$$E = PV = (a + b) N E_s. \tag{23}$$

Unlike classical thermodynamics, the equation of state comprises three distinct component equations. Each component equation involves six parameters $\{P,V,a,b,N,E_s\}$, constrained by the equation such that only five independent parameters remain.

The full pressure of an equilibrium system can be calculated as follows:

$$P^h := \frac{E^h}{V^h} = \frac{K^h + L^h}{V^h} = P_2^h + P_3^h,$$

$$P^k := \frac{E^k}{V^k} = \frac{L^k + H^k}{V^k} = P_2^k + P_3^k,$$

$$P^l := \frac{E^l}{V^l} = \frac{H^l + K^l}{V^l} = P_2^l + P_3^l.$$

Therefore, the full pressure equals the sum of the front pressure and the back pressure. Omitting the zone index, it is uniformly expressed as:

$$P = P_2 + P_3. \tag{24}$$

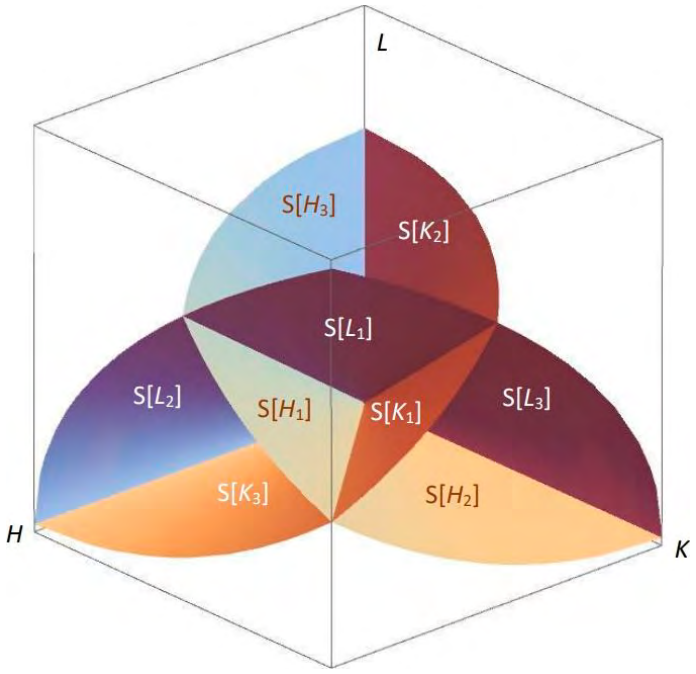

Figure 1: Equilibrium Surfaces in Energy Phase Space. The Equilibrium States on These Surfaces are Discrete Points with Uneven Distribution

In the energy phase space, the equilibrium equations represent three parabolic surfaces [3], as illustrated in Figure 1. The states on these equilibrium surfaces are discrete points with uneven distribution. Statistical analysis of the equilibrium states on the curved surfaces requires the development of specialized mathematical methods.

3. Digital Phase Space Statistics

3.1. Interaction of Particles

The energy statistics of baseons form the foundation of the energy phase space. Within this space, energy, pressure, and order degree are defined phenomenologically. They do not consider the microscopic structure of objects or volume calculations, nor does it elucidate the physical significance of the order degree. Although the energy phase space incorporates particle interactions, these are implicit in the equilibrium constraints of the system and do not address the physical mechanisms.

To investigate interactions, the elastic particle theory employs a hierarchical nesting model [3,13]. This model divides the structure of an object into four levels: hidson, baseon, midson, and topson, where the midson is a mesoscopic particle situated between the baseon and the topson (the system). Baseons attract each other due to mass, combining to form larger midsons---a process termed association. Conversely, baseons repel each other due to motion, resisting the combination of baseons---a process termed dissociation. When association and dissociation reach equilibrium, a relatively stable distribution of midsons is formed. The strength of particle interactions is characterized by the association degree g or the dissociation degree f.

A midson composed of n baseons is denoted as $C(n)$, and the number of $C(n)$ is represented by C_n. An equilibrium system exhibits a stable distribution of midsons, characterized by a distribution vector of midson as:

$$\boldsymbol{C}_\tau = (C_1, C_2, C_3, \cdots, C_N). \tag{25}$$

The modson's distribution vector C_τ possesses the following properties:

1. The sum of the components of the distribution vector equals the total number of midsons

$$C = \sum_{n=1}^{N} C_n \leq N \tag{26}$$

2. The distribution vector is constrained by the conservation condition of the number of baseons

$$\sum_{n=1}^{N} (n \cdot C_n) = N \tag{27}$$

3. The distribution vector determines the probability distribution ρ_n and the dissociation degree distribution f_n

$$\rho_n = \frac{C_n}{C}, f_n = \frac{C_n}{N} = \left(\frac{C}{N}\right)\rho_n. \tag{28}$$

4. The distributions of probability and dissociation degree satisfy the following conditions

$$\sum_{n=1}^{N} \rho_n = 1, \sum_{n=1}^{N} f_n = \frac{C}{N} = f \leq 1. \tag{29}$$

The f is referred to as the dissociation degree of the system. A larger f indicates that the number of midsons is closer to the number of baseons, and f=1 signifies complete dissociation of the midsons.

3.2. Energies of Midson

The vibrational, translational, rotational energies of a midson are denoted by the Greek letters η_n, κ_n, and λ_n, respectively. The corresponding system energies are obtained by statistically summing these over the number of midsons:

$$H = \sum C_n\eta_n = C\sum \rho_n\eta_n = C\eta, \eta = \sum \rho_n\eta_n\,; \tag{30a}$$

$$K = \sum C_n\kappa_n = C\sum \rho_n\kappa_n = C\kappa, \kappa = \sum \rho_n\kappa_n\,; \tag{30b}$$

$$L = \sum C_n\lambda_n = C\sum \rho_n\lambda_n = C\lambda, \eta = \sum \rho_n\lambda_n\,. \tag{30c}$$

The η, κ, and λ represent the average of the motion energy of a midson. Similarly, the ϵ, φ, θ, γ, υ, and ψ are average values of the entire, potential, thermal, chemical, internal, and enthalpic energies of a midson, respectively.

3.3. Statistical Principles

The digital phase space is defined as $\Lambda=\Lambda^k \cup \Lambda^l \cup \Lambda^h$, where the phase space is composed of the path parameters and momentum parameters of the midsons:

$$\Lambda^h = \{\tilde{\boldsymbol{\varrho}}_n, \tilde{\boldsymbol{o}}_n |\, E_s^h = hD\};\ \tilde{\boldsymbol{\varrho}}_n = (\tilde{\varrho}_{n1}, \tilde{\varrho}_{n2}, \tilde{\varrho}_{n3}), \tilde{\boldsymbol{o}}_n = (\tilde{o}_{n1}, \tilde{o}_{n2}, \tilde{o}_{n3}). \tag{31a}$$

$$\Lambda^k = \{\tilde{\boldsymbol{\xi}}_n, \tilde{\boldsymbol{p}}_n |\, E_s^k = kT\};\ \tilde{\boldsymbol{\xi}}_n = (\tilde{\xi}_{n1}, \tilde{\xi}_{n2}, \tilde{\xi}_{n3}), \tilde{\boldsymbol{p}}_n = (\tilde{p}_{n1}, \tilde{p}_{n2}, \tilde{p}_{n3}). \tag{31b}$$

$$\Lambda^l = \{\tilde{\boldsymbol{\vartheta}}_n, \tilde{\boldsymbol{q}}_n |\, E_s^l = lB\};\ \tilde{\boldsymbol{\vartheta}}_n = (\tilde{\vartheta}_{n1}, \tilde{\vartheta}_{n2}, \tilde{\vartheta}_{n3}), \tilde{\boldsymbol{q}}_n = (\tilde{q}_{n1}, \tilde{q}_{n2}, \tilde{q}_{n3}). \tag{31c}$$

A point in the digital phase space represents the motion state of midsons. Λ^h, Λ^k, and Λ^l are referred to as the translation area, rotation area, and vibration area, respectively. They correspond to the three equilibrium surfaces in the energy space, with energy scales taken as E_s^h, E_s^k, and E_s^l, respectively. This paper terms the statistics of midson parameters as mesoscopic statistics, distinguishing it from the microscopic statistics of baseon parameters.

The potential energy, main energy, and back energy of the midson in the phase space are given by:

$$\Lambda^h\colon \tilde{\varphi}_n^h = \tilde{\varphi}_n^h(\tilde{\varrho}_{n1}, \tilde{\varrho}_{n2}, \tilde{\varrho}_{n3})\,, \tilde{\eta}_n^h = \frac{1}{2}\left(\frac{\tilde{o}_{n1}^2 + \tilde{o}_{n2}^2 + \tilde{o}_{n3}^2}{\tilde{m}_n}\right), \tilde{\lambda}_n^h = \tilde{\varphi}_n^h + \tilde{\eta}_n^h. \tag{32a}$$

$$\Lambda^k\colon \tilde{\varphi}_n^k = \tilde{\varphi}_n^k(\tilde{\xi}_{n1}, \tilde{\xi}_{n2}, \tilde{\xi}_{n3})\,, \tilde{\kappa}_n^k = \frac{1}{2}\left(\frac{\tilde{p}_{n1}^2 + \tilde{p}_{n2}^2 + \tilde{p}_{n3}^2}{\tilde{m}_n}\right), \tilde{\eta}_n^k = \tilde{\varphi}_n^k + \tilde{\kappa}_n^k. \tag{32b}$$

$$\Lambda^l\colon \tilde{\varphi}_n^l = \tilde{\varphi}_n^l(\tilde{\vartheta}_{n1}, \tilde{\vartheta}_{n2}, \tilde{\vartheta}_{n3})\,, \tilde{\lambda}_n^l = \frac{1}{2}\left(\frac{\tilde{q}_{n1}^2}{\tilde{I}_{n1}} + \frac{\tilde{q}_{n2}^2}{\tilde{I}_{n2}} + \frac{\tilde{q}_{n3}^2}{\tilde{I}_{n3}}\right), \tilde{\kappa}_n^l = \tilde{\varphi}_n^l + \tilde{\lambda}_n^l. \tag{32c}$$

Here, $\tilde{m}_n$ and $\tilde{I}_{n\alpha}$ represent the mass and moment of inertia of the midson, respectively. Denote the volume elements as:

$$d\tilde{\boldsymbol{\varrho}}_n = d\tilde{\varrho}_{n1}d\tilde{\varrho}_{n2}d\tilde{\varrho}_{n3}, d\tilde{\boldsymbol{o}}_n = d\tilde{o}_{n1}d\tilde{o}_{n2}d\tilde{o}_{n3}\,. \tag{33a}$$

$$d\tilde{\boldsymbol{\xi}}_n = d\tilde{\xi}_{n1}d\tilde{\xi}_{n2}d\tilde{\xi}_{n3}, d\tilde{\boldsymbol{p}}_n = d\tilde{p}_{n1}d\tilde{p}_{n2}d\tilde{p}_{n3}\,. \tag{33b}$$

$$d\tilde{\boldsymbol{\vartheta}}_n = d\tilde{\vartheta}_{n1}d\tilde{\vartheta}_{n2}d\tilde{\vartheta}_{n3}, d\tilde{\boldsymbol{q}}_n = d\tilde{q}_{n1}d\tilde{q}_{n2}d\tilde{q}_{n3}\,. \tag{33c}$$

The volume elements of the subspaces can be expressed as:

$$d\Lambda^h = \prod_{n=1}^{N}(d\tilde{\boldsymbol{\varrho}}_n d\tilde{\boldsymbol{o}}_n)^{C_n}\,, d\Lambda^k = \prod_{n=1}^{N}\left(d\tilde{\boldsymbol{\xi}}_n d\tilde{\boldsymbol{p}}_n\right)^{C_n}\,, d\Lambda^l = \prod_{n=1}^{N}\left(d\tilde{\boldsymbol{\vartheta}}_n d\tilde{\boldsymbol{q}}_n\right)^{C_n}\,. \tag{34}$$

The digital phase space is an equilibrium state space with stable energy scales and a stable midson distribution. Therefore, the energy probability density satisfies the Boltzmann distribution. Specifically, the probability densities of the back energy in the vibrational, translational, and rotational areas are respectively:

$$\rho_L^h = \exp\left(-\tilde{L}^h\right), \rho_H^k = \exp\left(-\tilde{H}^k\right), \rho_K^l = \exp\left(-\tilde{K}^l\right) . \tag{35}$$

The integral of probability density over the phase space is termed the partition function. The partition functions for the translation, rotation, and vibration areas are respectively:

$$Z_L^h := \int_{\Lambda^h} \rho_L^h \, d\Lambda^h = \int_{\Lambda^h} \exp\left(-\tilde{L}^h\right) d\Lambda^h, \tag{36a}$$

$$Z_H^k := \int_{\Lambda^k} \rho_H^k \, d\Lambda^k = \int_{\Lambda^k} \exp\left(-\tilde{H}^k\right) d\Lambda^k, \tag{36b}$$

$$Z_K^l := \int_{\Lambda^l} \rho_K^l \, d\Lambda^l = \int_{\Lambda^l} \exp\left(-\tilde{K}^l\right) d\Lambda^l. \tag{36c}$$

Since the probability distribution of midson satisfies the normalization condition, no additional constraints are imposed on the phase space distribution function. In this context, the system's partition function acquires a specific meaning: the negative logarithm of the partition function equals the system's back energy, that is:

$$\tilde{L}^h = -\ln Z_L^h, \tilde{H}^k = -\ln Z_H^k, \tilde{K}^l = -\ln Z_K^l . \tag{37}$$

Formally, if the partition function is expressed as a product, taking the logarithm transforms the multiplicative result for the system into a summation operation over the midsons.

3.4. Partition Functions

This section illustrates the calculation of the partition function using the translation area as an example. The following summations and products are taken over the range n = 1~ N and α =1,2,3.

$$\begin{aligned} Z_H^k &= \int_{\Lambda^k} \exp\left(-\tilde{H}^k\right) d\Lambda^k = \int_{\Lambda^k} \exp\left(-\sum\nolimits_n C_n \, \tilde{\eta}_n^k\right) \prod\nolimits_n d\Lambda_n^k \\ &= \int_{\Lambda^k} \prod\nolimits_n \exp[-C_n(\tilde{\varphi}_n^k + \tilde{\kappa}_n^k)] \cdot \prod\nolimits_n \left(d\tilde{\boldsymbol{\xi}}_n d\tilde{\boldsymbol{p}}_n\right)^{C_n} \\ &= \prod\nolimits_n \left(\int \exp(-\tilde{\varphi}_n^k) \, d\tilde{\boldsymbol{\xi}}_n\right)^{C_n} \cdot \prod\nolimits_n \left(\int \exp(-\tilde{\kappa}_n^k) \, d\tilde{\boldsymbol{p}}_n\right)^{C_n} \\ &= \prod\nolimits_n \left(\tilde{V}_n Z_{Hn}^k\right)^{C_n} . \end{aligned} \tag{38}$$

In the above expression, $\tilde{V}_n$ is the volume of the midson, and Z_{Hn}^k is the partition function of the midson. Denoting the feature length of midson as $\tilde{\delta}_n$, we have:

$$\tilde{V}_n = \tilde{\delta}_n^3 = \int \exp(-\tilde{\varphi}_n^k) \, d\tilde{\boldsymbol{\xi}}_n = \iiint_{\pm\infty} \exp\left[-\tilde{\varphi}_n^k\left(\tilde{\xi}_{n1}, \tilde{\xi}_{n2}, \tilde{\xi}_{n3}\right)\right] d\tilde{\xi}_{n1} d\tilde{\xi}_{n2} d\tilde{\xi}_{n3} , \tag{39}$$

$$\begin{aligned} Z_{Hn}^k &= \int \exp(-\tilde{\kappa}_n^k) \, d\tilde{\boldsymbol{p}}_n = \iiint_{\pm\infty} \exp\left[-\frac{1}{2}\left(\frac{\tilde{p}_{n1}^2 + \tilde{p}_{n2}^2 + \tilde{p}_{n3}^2}{\tilde{m}_n}\right)\right] d\tilde{p}_{n1} d\tilde{p}_{n2} d\tilde{p}_{n3} \\ &= \prod\nolimits_\alpha \left[\int_{-\infty}^{+\infty} \exp\left(-\frac{\tilde{p}_{n\alpha}^2}{2\tilde{m}_n}\right) d\tilde{p}_{n\alpha}\right] = (2\pi\tilde{m}_n)^{3/2} . \end{aligned} \tag{40}$$

The calculation above utilizes the following integral formula:

$$I(a) = \int_{-\infty}^{+\infty} \exp(-ay^2) \, dy = \left(\frac{\pi}{a}\right)^{1/2} . \tag{41}$$

The integration limits from 0→±∞ cover the accessible range of the particle's translation path. Therefore:

$$Z_H^k = \prod\nolimits_n \left(\tilde{V}_n Z_{Hn}^k\right)^{C_n} = \prod\nolimits_n \left(2\pi\tilde{m}_n\tilde{\delta}_n^2\right)^{3C_n/2}. \tag{42}$$

The calculation procedures for the rotation and vibration areas are analogous, with the results given below:

$$Z_L^h = \prod\nolimits_n \left(\tilde{V}_n Z_{Ln}^h\right)^{C_n} = \prod\nolimits_n \left(0.5\pi\tilde{m}_n\tilde{\delta}_n^2\right)^{3C_n/2},$$

$$Z_K^l = \prod\nolimits_n \left(\tilde{V}_n Z_{Kn}^l\right)^{C_n} = \prod\nolimits_n \left(2\pi\tilde{I}_n\tilde{\delta}_n^2\right)^{3C_n/2}, \tag{43}$$

$$\tilde{V}_n = \tilde{\delta}_n^3, \tilde{I}_n = (\tilde{I}_{n1}\tilde{I}_{n2}\tilde{I}_{n3})^{1/3}.$$

The coefficient 0.5 appears in Z_L^h because the range of the particle's vibration path is from 0→+∞. Since the midson's potential energy $\tilde{\varphi}_n^k$ is a function of the path, the integral $\tilde{V}_n$ represents the space occupied by the motion of midson.

3.5. Energy and Pressure

The calculation of motion energy and pressure is illustrated using the translation area as an example:

$$\tilde{H}^k = -\ln Z_H^k = -\frac{3}{2}\sum C_n \ln\left(2\pi\tilde{m}_n\tilde{\delta}_n^2\right) = C\tilde{\eta}^k, \tag{44a}$$

$$\tilde{\eta}^k = -\frac{3}{2}\sum \rho_n \ln\left(2\pi\tilde{m}_n\tilde{\delta}_n^2\right). \tag{44b}$$

Here $\tilde{\eta}^k$ is the digit of the midson's back energy. According to the definition of the back order degree:

$$b^k := \frac{\tilde{H}^k}{\tilde{K}^k} = \frac{C\tilde{\eta}^k}{N}, \tag{45}$$

Setting $\tilde{\eta}^k = 1$ yields:

$$\tilde{H}^k = C, b^k = \frac{C}{N} = f^k. \tag{46}$$

These two equations indicate that the digit of the system's back energy equals the number of midsons, and the back order degree equals the dissociation degree. Since $\tilde{\eta}^k$ is a statistical result of the midson mass $\tilde{m}_n$ and feature length $\tilde{\delta}_n$, the condition $\tilde{\eta}^k$ =1 is termed the mesoscopic equilibrium condition.

With the back order degree equals the dissociation degree, the front order degree equals the association degree. Applying the macroscopic equilibrium condition $2a^k\, b^k$=1, the association degree is obtained as:

$$g^k = a^k = \frac{1}{2b^k} = \frac{N}{2C}. \tag{47}$$

Since the system energy obtained from the two statistical methods (baseon and midson) are equal, namely:

$$K^k = NE_s^k \equiv C\kappa^k \implies \tilde{K}^k = N \equiv C\tilde{\kappa}^k, \tag{48}$$

the digit of the midson's main energy is:

$$\tilde{\kappa}^k = \frac{\tilde{K}^k}{C} = \frac{N}{C} = \frac{1}{b^k} = 2a^k. \tag{49}$$

From the equilibrium condition: $(\tilde{\kappa^k})^2 = 2\tilde{\lambda^k}\tilde{\eta^k}$, the digit of the midson's front energy is derived as:

$$\tilde{\lambda}^k = \frac{(\tilde{\kappa}^k)^2}{2\tilde{\eta}^k} = 2(a^k)^2. \tag{50}$$

The entire energy of the midson equals the sum of the front and back energies, thus:

$$\tilde{\epsilon}^k = \tilde{\lambda}^k + \tilde{\eta}^k = 2(a^k)^2 + 1. \tag{51}$$

Let the average volume of a midson be V_s, and the digit of the system volume equal the number of midsons, $\tilde{V^k} = C$, then we have:

$$\tilde{K}^k = C\tilde{\kappa}^k = \tilde{V}^k\tilde{P}_1^k, \tilde{L}^k = C\tilde{\lambda}^k = \tilde{V}^k\tilde{P}_2^k, \tilde{H}^k = C\tilde{\eta}^k = \tilde{V}^k\tilde{P}_3^k. \tag{52}$$

From this, we find the digit of the system's motion pressure:

$$\tilde{P}_1^k = \tilde{\kappa}^k = 2a^k, \tilde{P}_2^k = \tilde{\lambda}^k = 2(a^k)^2, \tilde{P}_3^k = \tilde{\eta}^k = 1. \tag{53}$$

It should be noted that the volume scale V_s represents the average volume of a midson and is not equal to the motion volume V_n of the midson.

The calculation methods for the translational, rotational, and vibrational areas are entirely analogous. The mesoscopic equilibrium conditions for the three areas are given below:

$$\Lambda^h: \tilde{\lambda}^h = -\frac{3}{2}\sum \rho_n \ln\left(0.5\pi\tilde{m}_n\tilde{\delta}_n^2\right) = 1, \tag{54a}$$

$$\Lambda^k: \tilde{\eta}^k = -\frac{3}{2}\sum \rho_n \ln\left(2\pi\tilde{m}_n\tilde{\delta}_n^2\right) = 1, \tag{54b}$$

$$\Lambda^l: \tilde{\kappa}^l = -\frac{3}{2}\sum \rho_n \ln\left(2\pi\tilde{I}_n\tilde{\delta}_n^2\right) = 1. \tag{54c}$$

The statistical results for the three areas are summarized as follows:

$$\tilde{P}_3 = 1, \tilde{E}_3 = C, b = \tilde{E}_3/\tilde{E}_1 = C/N. \tag{55}$$

Choosing the baseon's number N and the midson's number C as independent parameters, and using the constraint $2ab = 1$, it is possible to express the energy, pressure, and order degree entirely in terms of N and C, as shown in Table 1.

State Function	Vibration Area	Translation Area	Rotation Area
Energy Scale	$E_s = hD$	$E_s = kT$	$E_s = lB$
Front Order Degree	$a = N/(2C)$	$a = N/(2C)$	$a = N/(2C)$
Back Order Degree	$b = C/N$	$b = C/N$	$b = C/N$
Main Energy	$\tilde{H} = N$	$\tilde{K} = N$	$\tilde{L} = N$
Front Energy	$\tilde{K} = aN = N^2/(2C)$	$\tilde{L} = aN = N^2/(2C)$	$\tilde{H} = aN = N^2/(2C)$
Back Energy	$\tilde{L} = bN = C$	$\tilde{H} = bN = C$	$\tilde{K} = bN = C$

Back Energy	$\tilde{L} = bN = C$	$\tilde{H} = bN = C$	$\tilde{K} = bN = C$
Volume	$\tilde{V} = C$	$\tilde{V} = C$	$\tilde{V} = C$
Main Pressure	$\tilde{P}_1 = \tilde{\eta} = N/C$	$\tilde{P}_1 = \tilde{\kappa} = N/C$	$\tilde{P}_1 = \tilde{\lambda} = N/C$
Front Pressure	$\tilde{P}_2 = \tilde{\kappa} = N^2/(2C^2)$	$\tilde{P}_2 = \tilde{\lambda} = N^2/(2C^2)$	$\tilde{P}_2 = \tilde{\eta} = N^2/(2C^2)$
Back Pressure	$\tilde{P}_3 = \tilde{\lambda} = 1$	$\tilde{P}_3 = \tilde{\eta} = 1$	$\tilde{P}_3 = \tilde{\kappa} = 1$

Table 1: Digitization of Energy and Pressure

4. Differential Equations of State

4.1. Five-Parameter Theorem

TABLE 2 presents the digits of derived energy for the equilibrium system. Since $a = N/((2C)$) and $b = C/N$, all values are functions of N and C

Derived Energy	Energy of System	Energy of Midson
Entire Energy	$\tilde{E} = (a + b)N$	$\tilde{\epsilon} = 2a^2 + 1$
Potential Energy	$\tilde{J} = (b - 1)N$	$\tilde{\varphi} = 1 - 2a$
Thermal Energy	$\tilde{Q} = (1 + a)N$	$\tilde{\theta} = 2a(1 + a)$
Chemical Energy	$\tilde{G} = (a - b)N$	$\tilde{\gamma} = 2a^2 - 1$
Internal Energy	$\tilde{U} = (1 + a - b)N$	$\tilde{\upsilon} = 2a + 2a^2 - 1$
Enthalpic Energy	$\tilde{Y} = (1 + 2a - b)N$	$\tilde{\psi} = 2a + 4a^2 - 1$

Table 2: Derived Energy of the System and the Midson

It can be seen that only two particle numbers, N and C, are sufficient to express all digits of motion energy, derived energy, pressure, and order degree. Meanwhile, all scales of the system can be covariant derived through three meta scales, $\{m_s, V_s, E_s\}$. Therefore, all thermodynamic functions can be fully represented by two digits and three scales. This conclusion is known as the five-parameter theorem of particle statistics.

4.2. Thermal Entropy Function

Using natural unit system where the conversion coefficients are $k^* \in \langle 1,1,1 \rangle$, the motion intensity equals the energy scale, i.e., $T^* \in \langle E_s^k, E_s^l, E_s^h \rangle$. Therefore, a thermal entropy function S can be defined uniformly across the three areas:

$$S \coloneqq \frac{Q}{E_s} = \tilde{S} \cdot S_s \; ; \; \tilde{S} = \tilde{Q} = (1 + a)N \; , S_s = 1. \tag{56}$$

Different from the classical thermodynamic entropy, the essence of thermal entropy is the digit of thermal energy, since the scale of thermal entropy is S_s=1. As an extensive quantity, the thermal entropy per midson is:

$$\sigma = \frac{S}{C} = \frac{Q/E_s}{C} = \frac{Q/C}{E_s} = \frac{\theta}{E_s} \; ; \; S = C\sigma, \theta = \frac{Q}{C} = \sigma E_s. \tag{57}$$

Classical thermodynamics lacks a thermal energy function; the classical entropy function S^* is defined by heat Q^* via $dS^* \geq (\delta Q^*)/T$. Classical entropy uses temperature as a unified motion intensity without distinguishing among liquid, solid, and gas states. Moreover, classical entropy includes the unit conversion coefficient k, whereas thermal entropy is a dimensionless pure digit.

4.3. Fundamental Differential Form

This section derives the fundamental differential form in the rotation area. For a midson in the rotation area, the main, front, and back

energies are λ, η, and κ, respectively. The front energy and thermal energy can be expressed as $\eta = P_2 V_s$ and $\theta = \sigma E_s$. Since the main energy equals the thermal energy minus the front energy, i.e., $\lambda = \theta - \eta$, the differential of the midson's main energy is:

$$d\lambda = d(\theta - \eta) = d(\sigma E_s - P_2 V_s) = (E_s d\sigma - P_2 dV_s) - (-\sigma dE_s + V_s dP_2). \quad (58)$$

Furthermore, since the main energy equals the internal energy minus the chemical energy, i.e., $\lambda = \upsilon - \gamma$, it follows that $d\lambda = d\upsilon - d\gamma$. Thus, the differentials of the midson's internal energy and chemical energy are respectively:

$$d\upsilon = E_s d\sigma - P_2 dV_s, d\gamma = -\sigma dE_s + V_s dP_2. \quad (59)$$

From this, the total differential of the system's chemical energy can be calculated as:

$$dG = d(C\gamma) = Cd\gamma + \gamma dC$$

$$= C(-\sigma dE_s + V_s dP_2) + \gamma dC = -SdE_s + VdP_2 + \gamma dC.$$

That is:

$$dG = -SdE_s + VdP_2 + \gamma dC. \quad (60)$$

This expression is termed the fundamental differential form.

Subtracting the fundamental differential form from $dG = Cd\gamma + \gamma dC$ yields the following identity:

$$SdE_s - VdP_2 + Cd\gamma = 0. \quad (61)$$

Due to cyclic symmetry, the fundamental differential forms in the translation, rotation, and vibration areas share the same mathematical form.

4.4. Total Differentials of Energy

Starting from the fundamental differential form and applying Legendre transformations, the total differentials of various energies can be derived. The results are summarized in Table 3

Energy	Relationship	Total Differential Equation
Main Energy	$E_1 = E_2 - J = Q - E_2 = U - G$	$dE_1 = E_s dS - P_2 dV - Cd\gamma$
Front Energy	$E_2 = G + E_3 = Q - E_1 = P_2 V$	$dE_2 = SdE_s + P_2 dV + Cd\gamma$
Back Energy	$E_3 = J + E_1 = E_2 - G = Q - U$	$dE_3 = SdE_s + P_2 dV - \gamma dC$
Thermal Energy	$Q = E_1 + E_2 = U + E_3 = SE_s$	$dQ = E_s dS + VdP_2 - Cd\gamma$
Chemical Energy	$G = E_2 - E_3 = U - E_1 = C\gamma$	$dG = -SdE_s + VdP_2 + \gamma dC$
Internal Energy	$U = Q - E_3 = E_1 + G = E_1 + E_2 - E_3$	$dU = E_s dS - P_2 dV + \gamma dC$
Enthalpic Energy	$Y = Q + G = U + E_2 = E_1 + 2E_2 - E_3$	$dY = E_s dS + VdP_2 + \gamma dC$
Null	$dE_2 - d(P_2 V) = dQ - d(SE_s) = dG - d(C\gamma) = 0$	$SdE_s - VdP_2 + Cd\gamma = 0$

Table 3: Energy Relations and Total Differential Equations

5. Thermodynamic Application

5.1. State of Liquids

According to the five-parameter theorem, we calculate the state functions of liquids. The energy scale for a liquid is $E_s = kT$, with the main, front, and back energies being K, L, and H, respectively. If the boiling point temperature T_2 of the liquid is known, and the molar volume as a function of temperature $V(T)$ is measured, then the number of baseons (molecules) in the system equals Avogadro's constant N_A:

$$N = N_A = 6.02214076 \times 10^{23} \text{mol}^{-1}. \quad (62)$$

Let the relative molecular mass be μ, and take the molecular mass as the mass scale:

$$m_s = \frac{\mu}{N} \times 10^{-3} \text{ kg}. \tag{63}$$

Taking the molecular volume at the boiling point as the volume scale:

$$V_s = \frac{V(T_2)}{N}. \tag{64}$$

Other scales can be derived, e.g.

$$P_s = \frac{E_s}{V_s} = \frac{kTN}{V(T_2)}, r_s = \sqrt[3]{V_s} = \sqrt[3]{\frac{V(T_2)}{N}}, t_s = \sqrt{\frac{I_s}{kT}}. \tag{65}$$

In addition, the number of midsons in the system is:

$$C(T) = \tilde{V} = \frac{V(T)}{V_s} = \frac{V(T)N}{V(T_2)} \leq N. \tag{66}$$

When $T = T_2$, $C = N$, indicating complete dissociation of the midsons, which corresponds to the point of the liquid-gas phase transition. According to the five-parameter theorem, all the state functions of a liquid can be expressed as functions of temperature using (N, C), as shown in Table 1 and Table 2. For example, the order degree of the system is:

$$a(T) := \frac{L}{K} = \frac{N}{2C} = \frac{V(T_2)}{2V(T)}, b(T) := \frac{H}{K} = \frac{C}{N} = \frac{V(T)}{V(T_2)}. \tag{67}$$

5.2. Laws of Thermodynamics

The laws of classical thermodynamics can be derived from the total differential equations in TABLE 3.

The Second Law. The total differential equation for thermal energy is:

$$dQ = E_s dS + V dP_2 - C d\gamma. \tag{68}$$

Express each item as an actual quantity:

$$dQ = d\tilde{Q} \cdot E_s, S = \tilde{S}, V = C \cdot V_s, P_2 = 2a^2 \cdot P_s, \gamma = (2a^2 - 1) \cdot E_s. \tag{69}$$

Substitute into differential equation to get digital form:

$$d\tilde{Q} = d\tilde{S} + Cd(2a^2) - Cd(2a^2 - 1) \Longrightarrow d\tilde{Q} = d\tilde{S}. \tag{70}$$

For liquids, $E_s = kT$, above equation can be expressed as:

$$dQ = kTd\tilde{S} = Td(\tilde{S}k). \tag{71}$$

By denoting $dQ = \delta Q^*$, and $S^* = \tilde{S}k$, we obtain the relationship between the heat Q^* and the classical entropy S^*:

$$\delta Q^* = TdS^*. \tag{72}$$

This is the equivalent form of the Clausius inequality in reversible processes, $dS^* = \delta Q_{rev}^*/T$, which leads to the core mathematical expression of the second law of thermodynamics.

The First Law. The total differential equation for internal energy is:

$$dU = E_s dS - P_2 dV + \gamma dC. \tag{73}$$

Its digital form is:

$$d\tilde{U} = d\tilde{S} - 2a^2 dC + (2a^2 - 1)dC \implies d\tilde{U} = d\tilde{S} - dC. \tag{74}$$

After combining the energy scale, we get:

$$dU = TdS^* - P_3 dV. \tag{75}$$

This equation is consistent with the mathematical form of the first law of classical thermodynamics. However, here $P_3 = P_s$ represents the back pressure; the back pressure of a liquid corresponds to the density of vibrational energy.

Above, we have recovered the classical laws of thermodynamics from the total differential equation of energy. Consequently, within this theoretical framework, the mathematical formulations of the first law of thermodynamics (conservation of energy) and the second law (the principle of entropy increase) naturally emerge as logical corollaries of statistical equilibrium properties. Specifically, the form of the second law, $dQ = TdS$, follows directly from the definition of thermal entropy $S = Q/E_s$, and the energy scale, $E_s = kT$. Meanwhile, the form of the first law reproduces the classical statement that "the change in internal energy originates from heat transfer and volume work." Since the applicable energy scale E_s varies across different areas, the energy mode conversion channels for solids and gases must be analyzed specifically based on cyclic symmetry.

6. Conclusion and Outlook

Digital phase space statistics and energy phase space statistics jointly constitute a complete statistical theory for elastic particle systems. This represents not merely a technical extension of Gibbs' ensemble theory, but rather a paradigm shift founded upon the concepts of "elastic particles" and "actual quantity mathematics." The Λ-space operates at the mesoscopic level, naturally describing particle interactions through the distribution of midsons (clusters), thereby circumventing the complexities and conceptual dilemmas associated with many-body potentials. The theory's core principle of "cyclic symmetry" provides a universal mathematical framework that elegantly encompasses the gaseous, liquid, and solid states.

The central achievement of this paper is the demonstration that all thermodynamic information of an elastic particle equilibrium system can be compressed into five independent parameters: the number of baseons N, the number of midsons C, the mass scale m_s, the volume scale $V_{s,}$ and the energy scale E_s. This result provides a profound and simple new description framework for understanding the state of matter.

Crucially, building upon this framework, we have systematically derived the total differential equations for all thermodynamic potentials, from which the mathematical forms of the classical first and second laws of thermodynamics naturally emerge. This demonstrates that the two foundational laws of classical thermodynamics are no longer empirical postulates in this theory, but rather logical corollaries of the statistical equilibrium properties of elastic particle systems.

Looking forward, the intrinsic consistency and unified explanatory power exhibited by the real physical paradigm in the realms of particle field theory, dynamics, and statistical mechanics have laid a solid foundation for this theoretical framework. A rigorous direction for future research is to systematically explore the intrinsic connections between this framework and the fundamental principles of quantum mechanics and relativity. Preliminary interpretations of this paradigm regarding phenomena such as quantization and the spacetime background suggest that such explorations are highly likely to unveil a more fundamental physical picture, thereby advancing our unified understanding of the material world.

Acknowledgments

The research leading to these results received funding from the NSFC under Grant No.61775102. The Chinese preprint of this article is available from ChinaXiv via https://chinaxiv.org/abs/202601.00178.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

As a purely theoretical work, all mathematical derivations, models, and findings are fully contained within the article.

AI Usage Statement

AI tools were used only for language editing.

References

1. Liang, Z. C. (2025). A Physical Theory Based on Barycenter Reference Frames I: Principles of Particle Flow Fields. *Adv Theo Comp Phy, 8*(4), 01-17.
2. Liang, Z. C. (2025). A Physical Theory based on the Barycenter Frame of Reference II: Principles of Particle Dynamics. *Adv Theo Comp Phy, 8*(4), 01-19.
3. Liang, Z. C. (2025). A Physical Theory based on the Barycenter Frame of Reference III: Principles of Particle Statistics. *Adv Theo Comp Phy, 8*(4), 01-20.
4. Liang, Z. C., & Dong, K. J. (2025). Return to simplicity and reality: Road to final theory of physics [Preprint, in Chinese]. *ChinaXiv.*
5. Liang, Z. C. (2020). Outline of real physics. *Global Journal of Science Frontier Research: A*, 20(9).
6. Liang, Z. C. (2019). Modeling of real particles. *Journal of Physics: Conference Series, 1391*, 012026.
7. Liang, Z. C. (2015). Physical principles of finite particle system. *Scientific Research Publishing.*
8. Liang, Z. C. (2021). Dark matter and real-particle field theory. *Global Journal of Science Frontier Research: A, 21*, 27.
9. Liang, Z. C. (2019). The origin of gravitation and electromagnetism. *Theoretical Physics, 4,* 85.
10. Liang, Z. C. (2018). Essence of light: Particle, field and interaction. In Proceedings of SPIE 10755: Photonic Fiber and Crystal Devices: *Advances in Materials and Innovations in Device Applications XII (Paper 107550I).*
11. Liang, Z. C., & Chen, N. N. (2023). The distortion and quantization of the earth's orbit. *Journal of Physical Chemistry & Biophysics, 13*, 361.
12. Liang, Z. C., & Xie, L. H. (2023). Wave or particle? Quantum behavior of solar system. *Global Journal of Science Frontier Research: A, 23*(7), 9.
13. Liang, Z. C. (2019). Motion, energy, and state of body particle system. *Theoretical Physics, 4*, 66.
14. Liang, Z. C. (2019). Motion of elastic particles and spectrum of hydrogen atoms. *Global Journal of Science Frontier Research: F, 19*(4), 18.
15. Liang, Z. C. (2019). Cluster ensemble statistics of body particle system. *New Horizons in Mathematical Physics, 3*, 53.
16. Liang, Z. C. (2021). Ion-cluster model and charge-transport properties of ionic liquids [Preprint]. *ResearchGate.*
17. Bi, L., & Liang, Z. C. (2015). Features of velocity-cross-correlations in ionic liquids. *Journal of Molecular Liquids, 212,* 775–781.
18. Bi, L., & Liang, Z. C. (2015). Investigation on dynamics and self-diffusion coefficient of [BMIM][PF6] via molecular dynamics simulations. *Russian Journal of Physical Chemistry B, 9*, 978–984.
19. Lin, Z. H. (2007). *Thermodynamics and statistical physics*. Peking University Press.