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Speed of Light Reflected From or Transmitted Through a Moving Medium: An Explanation of Fizeau's Experiment

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Abstract

The speed of light reflected from or transmitted through a medium (glass or water) moving with speed v in another medium (air or vacuum), are derived in terms of the speed of light c in a vacuum and the refractive indices of the mediums. It is shown that the law of reflection of light applies irrespective of speed of the reflecting surface. A ray of light at normal incidence on a reflecting surface moving in the opposite direction of propagation of the ray, from a stationary source, is reflected with a speed greater than that of light c. The equation for speed of transmitted light in a moving medium, applicable for values of v from -c to c, is found to be different from that of Fresnel's law obtained by relativistic considerations and applicable only if v << c. The non-relativistic equation, for the speed of light in a moving medium, is applied to explain the result of Fizeau's experiment for the speed of light in moving water. It is concluded that the effect on the speed light in a moving medium is due to a media drag on the light ray, as envisaged by Fresnel, but not the result of relativistic addition of velocities.

Keywords: Fizeau's Experiment, Light, Medium, Reflection, Refraction, Refractive Index, Speed

Introduction

Speed of light in a moving medium

Fizeau's experiment was conducted in the 1850s [1] to measure the speed of light, at normal incidence, in moving water. The theory of special relativity gives the speed of light w propagated at normal incidence in a medium of refractive index μ moving with speed v in a vacuum as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right) \qquad 1$$

where c is the speed of light in a vacuum. Equation (1) is Fresnel's law with $(1 - 1/\mu^2)$ as the "Fresnel's ether drag". Equation (1), applicable only if $v << c$, is obtained by considering Einstein's relativistic velocity addition formula for an observer running with speed c/μ in a moving medium of refractive index μ moving with speed v.

In this paper a non-relativistic formula is obtained, for the speed of light in a medium of refractive index μ moving in a vacuum with speed v, as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \qquad 2$$

where $(1 - 1/\mu)$ is the "medium drag". The drag is positive if the light is propagated in the direction of motion of the medium and negative if in the opposite direction.

Equation (2) is applicable for values of v from -c to c. It is simply obtained by considering the ratio of magnitudes of relative velocity between the reflected ray and the moving medium and between the refracted ray and the moving medium.

Equations (1) and (2) are used to respectively give relativistic and non-relativistic explanations of the result of Fizeau's experiment for the speed of light in moving water.

Reflection and Refraction of light

The Greek mathematician, Euclid (330 – 280 B.C.), who discovered the law of formation of images by mirrors, probably knew the law of reflection of light [1]. Snell discovered the law of refraction of light in about 1620 [1]. When a ray of light, propagated in a *medium* 1, reaches the boundary with another *medium* 2, reflection and refraction occur. Figure 1 depicts a ray SP, emitted with velocity s from a stationary source S, incident at a point P on the boundary of two media. The ray SP may be reflected off the boundary or it may be refracted (transmitted) across the boundary.

A reflected ray PR is one propagated with velocity **u** from the boundary in the same medium as the incident ray SP. A refracted ray PT is one transmitted and propagated with velocity **w** in the second medium. The angle of incidence is ι, where (π – ι) is the angle between the directions of propagation of the incident ray SP and the normal OPN to the boundary at the point of incidence P. The angle of reflection ρ is the angle between the directions of propagation of the reflected ray PR and the normal. The angle of refraction is τ, where (π – τ) is the angle between the directions of propagation of the refracted ray PT and the normal at P.

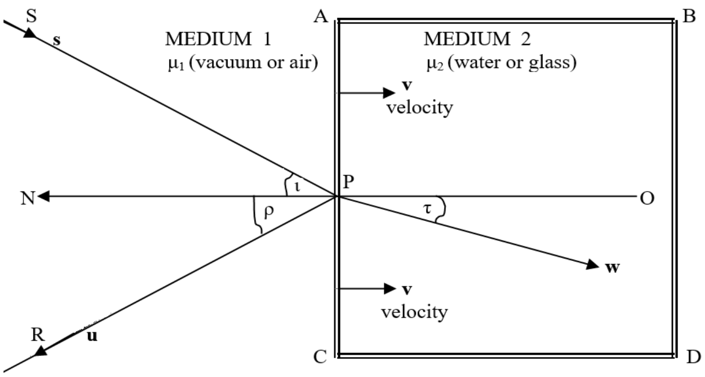

Figure 1: Reflection and refraction of a ray of light emitted from a source S with velocity s in medium 1, incident at a point P on the surface of medium 2 moving with velocity **v**. The ray PR is reflected with velocity **u** and the refracted ray PT is transmitted in medium 2 with velocity **w**. The velocity vector v is in the opposite direction of the normal OPN at P.

A reflected ray PR is one propagated with velocity **u** from the boundary in the same medium as the incident ray SP. A refracted ray PT is one transmitted and propagated with velocity **w** in the second medium. The angle of incidence is ι, where (π – ι) is the angle between the directions of propagation of the incident ray SP and the normal OPN to the boundary at the point of incidence P. The angle of reflection ρ is the angle between the directions of propagation of the reflected ray PR and the normal. The angle of refraction is τ, where (π – τ) is the angle between the directions of propagation of the refracted ray PT and the normal at P.
Yavorsky and Detlaf [2] recapitulated the laws of reflection and refraction of light, for a stationary medium, thus:

(a) The laws of reflection, with reference to Figure 1, are:
1. The incident ray SP, the reflected ray PR and the normal OPN, at the point of incidence P, are coplanar.
2. The angle of reflection ρ is equal to the angle of incidence ι.

(b) The laws of refraction of light, with v = 0 in Figure 1, are:
1. The incident ray SP, the refracted ray PT and the normal OPN, at the point of incidence P, are coplanar.
2. The ratio of the sine of angle of refraction to the sine of angle of incidence, is equal to the ratio of the speeds of light w and s in the media.

The second law of refraction is called after its discoverer, Snell's law, which can be deduced from Fermat's principle [3].

The relative indices of refraction μ_1 and μ_2 of *media* 1 and 2 respectively, are defined, for a stationary medium, and for light of a given wavelength, as the ratio:

$$\frac{w}{s} = \frac{\sin\tau}{\sin\iota} = \frac{\mu_1}{\mu_2} \qquad 1$$

In a vacuum, $\mu = 1$ and $\mu_1 s = \mu_2 w = c$, the speed of light. If medium 1 is a vacuum, $\mu_2 = \mu$ is the absolute index of refraction.

Where $\mu_1 > \mu_2$ (Figure1), that is transmission from a denser medium to a less dense medium, such as glass to air or vacuum, the angle of refraction τ is greater than the angle of incidence ι. Total internal reflection occurs if τ > π/2 radians. The value υ of ι at which τ = π/2 radians is called the *critical* angle, such that sinυ = $\mu_2 / \mu_1 < 1$.

Speed of light reflected from a moving medium

Figure 1, drawn in accordance with the first law of reflection and refraction, shows a monochromatic ray of light in *medium* 1 propagated with velocity s from a stationary source S. The ray is incident at a point P on the plane surface of denser *medium* 2 (ABCD), such as glass or water, moving with velocity **v** (relative to an observer) in a direction opposite to the normal OPN at P. The reflected ray is propagated with velocity **u** in the same *medium* 1 as the incident ray. The angle of incidence is ι and the angle of reflection is ρ.

The *medium 2* may be considered as stationary by giving each of the rays a velocity equal to –v to obtain the relative velocities with respect to the *medium 2*. The ratio of the magnitudes of the relative velocities of the reflected ray and the incident ray, with respect to the *medium 1*, (vacuum or air) is considered and proposed as:

$$\frac{|\mathbf{u}-\mathbf{v}|}{|\mathbf{s}-\mathbf{v}|} = \frac{\mu_1}{\mu_1} = 1 \qquad 2$$

With reference to Figure 1, the ratio is obtained as:

$$\frac{\sqrt{u^2+v^2+2uv\cos\rho}}{\sqrt{s^2+v^2-2sv\cos\iota}} = \frac{u^2+v^2+2uv\cos\rho}{s^2+v^2-2sv\cos\iota} = 1 \qquad 3$$

If $v = 0$, the incident and reflected rays, in the same medium, will have the same speed $u = s$.

At normal incidence and *medium* 2 moving with speed v, ι = ρ = 0, equation (3) gives:

$$\frac{u+v}{s-v} = 1$$

$$u = s - 2v \qquad 4$$

Where the first medium is a vacuum, s is the speed of light c. Equation (4) is exactly as obtained if the ray of light is considered as a "stream of particles" or some kind of effects, propagated with speed c. The "particles" or photons would impinge normally on the moving medium at the point of incidence P, on perfectly elastic collisions, and recoil with speed u given by:

$$u = c - 2v \qquad 5$$

This is "ballistic propagation of light", in accordance with Newton's law of restitution [5].

In equation (5), for a reflecting surface moving in the opposite direction of the ray at normal incidence, the reflected ray is transmitted at speed $c + 2v$, greater than speed of light c.

Reflection angle and speed from a moving medium

Equation (3), with s = c, gives the cosine of the angle of reflection, for a medium moving along the normal with speed v in a vacuum, as:

$$\cos\rho = \frac{1}{2uv}\left(c^2 - u^2 - 2cv\cos\iota\right) \qquad 6$$

Since $\rho = 0$ where ι = 0 and ρ = π/2 radians where ι = π/2 radians, it can be inferred that the law of reflection (ρ = ι) applies irrespective of the speed of the reflecting medium moving along the normal with speed v. At the point of incidence equation (6) then becomes:

$$\cos\rho = \cos\iota = \frac{c-u}{2v} \qquad 7$$

At normal incidence, where ρ = 0 and ι = 0, equation (7) gives equation (5). A mirror moving in a vacuum with speed C/2 in the direction of a ray of light, should show no reflected ray. A mirror moving in a vacuum with speed v in the opposite direction of a ray of light, should reflect it back with speed c + 2v. Is this a situation where the speed of light can be exceeded?

Speed of light transmitted in a moving medium

A ray of light emitted with velocity **s** is incident on the surface of a transparent medium moving with velocity **v** along the normal as in Figure 1. The ray is transmitted through the medium with velocity w at the angle of refraction τ. The refractive indices are defined in terms of the ratio of magnitudes of relative velocities of the refracted ray and the incident ray. The ratio of magnitudes of relative velocity (**w – v**) and relative velocity (**s – v**) is obtained as:

$$\frac{|\mathbf{w}-\mathbf{v}|}{|\mathbf{s}-\mathbf{v}|} = \frac{\mu_1}{\mu_2}$$

With reference to Figure 1, the ratio is obtained as:

$$\frac{\sqrt{w^2 + v^2 - 2wv\cos\tau}}{\sqrt{s^2 + v^2 - 2sv\cos\iota}} = \frac{\mu_1}{\mu_2} \qquad 8$$

At normal incidence, ι = τ = 0 and we obtain

$$\frac{w-v}{s-v} = \frac{\mu_1}{\mu_2}$$

$$w\mu_2 - v\mu_2 = s\mu_1 - v\mu_1$$

$$w = \frac{c}{\mu_2} + v\left(1 - \frac{\mu_1}{\mu_2}\right) \qquad 9$$

where $s\mu_1 = c$ is the speed of light in vacuum. For a medium moving in a vacuum, the refractive index $\mu_1 = 1$, the speed w of transmission, at normal incidence on the surface of the medium, is:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \qquad 10$$

Equation (10) is a significant result of this paper. It is used to give a non-relativistic explanation of the result of Fizeau's experiment, which measured the speed of light in moving water, without recourse to the theory of special relativity.

Fizeau's experiment

A schematic diagram of the apparatus of Fizeau's experiment [6] is shown above in Figure 2. Carried out in the 1850s, it was one of the most remarkable experiments in physics.

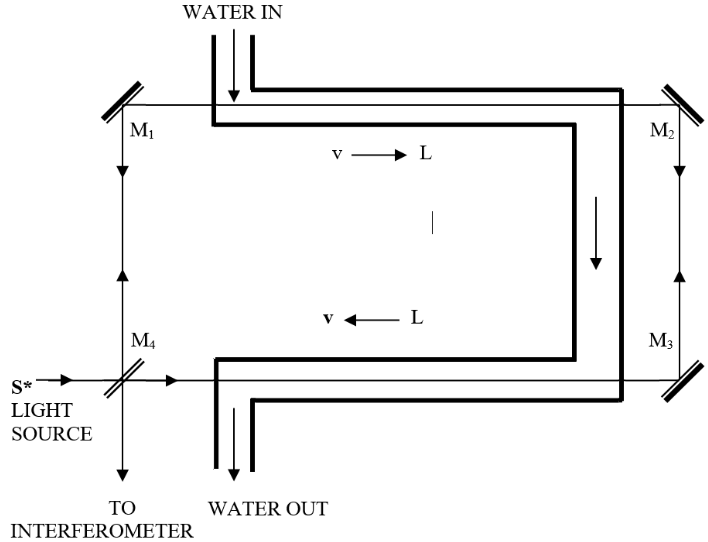

Figure 2: Schematic diagram of the apparatus of Fizeau's experiment

In this experiment, light from a source was sent in two opposite directions through transmission and reflection by four half-silvered mirrors $M_1 - M_4$. One beam travelled downstream (from M_1) through moving water and the other travelled upstream (from M_4) in the same water. By an ingenious arrangement of the mirrors the two beams were made to recombine and be observed in an interferometer. An interference pattern is observed, resulting from the difference in the time taken for the two beams to travel the same path, partly in moving water.

Non-relativistic explanation of the result of Fizeau's experiment

According to the Galilean-Newtonian relativity of classical mechanics, the speed of light in a medium that is moving in a vacuum with velocity v, in the opposite direction of the normal, is given in terms of the refractive index, by equation (10).

Equation (10) gives the speed w of transmission of light (downstream). If the medium is water of index of refraction μ moving with velocity v in a vacuum (Figure 2), the time taken for the beam that is going downstream to cover the distance 2L (where the magnitude v is very small compared with the speed of light c and, therefore, v(squared)/c(squared) can be neglected), is obtained, given by:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \qquad 11$$

For the beam going upstream, with velocity –**v**, the longer transit time is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \qquad 12$$

The time difference (Δt)=($t_2 - t_1$) between the two beams, traversing the same path of length *2L* (downstream or upstream) in moving water, is:

$$t_2 - t_1 = \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} - \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\}$$

$$\Delta t = \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu}\right) \qquad 13$$

The fringe shift δx, for light of wavelength λ, is obtained as:

$$\delta_x = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu}\right) \qquad 14$$

In the experiments performed by Fizeau [6], $L = 3$ *m*, $v = 7$ *m/sec*., $\lambda = 6$ x 10-7 *m* (yellow light), $c = 3$ x 108 *m/sec* and $\mu = 4/3$ (for water). The fringe shift δx is obtained as ≈ 0.2, which was easily observable and measurable in the interferometer.

Michelson and Morley in 1886 and later P. Zeeman and associates in 1915 repeated Fizeau's experiment with greater precision in which the interferometer could measure a fringe shift as low as 0.01. The influence of the motion of the medium on the propagation of light has thus been verified. As to which is the correct explanation, the theory of special relativity or equation (14), remains to be seen.

Relativistic explanation of the result of Fizeau's experiment

According to Galileo's *velocity addition rule,* if you move with velocity u relative to a medium moving with velocity **v** (passenger jogging with velocity **u** in a ship moving with velocity **v** relative to an observer) your velocity relative to the observer is vector sum s, as:

ü= + 15

Here the velocities ***u*** and ***v*** are vectors that can be of any magnitude and in any direction. If the velocities are in the same direction, your speed relative to an observer is the magnitude ***s = u + v.*** This is the Galilean principle of relativity, in agreement with observation and natural sense.

According to Einstein's *velocity addition rule,* if you move with velocity **u** relative to a medium moving with velocity **v** (as a jogger running with velocity **u** in a ship moving with velocity v relative to an observer), the magnitude s of your velocity, relative to an observer, is:

$$s = \frac{u + v}{1 + \frac{uv}{c^2}} \qquad 16$$

How the speed of light *c* in a vacuum comes into equation (16), is perplexing. In vector form, equation (16) may be expressed as:

$$\mathbf{s} = \frac{\mathbf{u} + \mathbf{v}}{1 + \frac{\mathbf{u}.\mathbf{v}}{c^2}} \qquad 17$$

where **u.v** is a scalar product.

In equation (17), **u** and **v** must be collinear to give equation (16). In reality, the jogger should be able to run with velocity **u** in any direction relative to **v**. In equation (16) if $u = c$ (speed of light) or $u = v = c$, the speed s remains as c. Equation (16), more than anything else, had lent support to the principle of constancy of the speed of light, in accordance with the theory of special relativity. For speeds much less than c, or if c is infinitely large, the relativistic formula (equation 16) reduces to the classical formula (equation 15).

According to the theory of special relativity, the velocity of light (jogger), relative to the surface of a medium (ship) moving in a vacuum with velocity **v**, remains as a constant **c**. The velocity of light (jogger) within and with respect to the medium (ship) of refractive index μ is c/μ. Einstein's *velocity addition rule*, with $u = c/\mu$, gives the magnitude of velocity w of light in the moving medium, as:

$$w = \frac{\frac{c}{\mu} + v}{1 + \frac{cv}{\mu c^2}} = \frac{\frac{c}{\mu} + v}{1 + \frac{v}{\mu c}} \approx \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right)$$

$$w \approx \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right) \qquad 18$$

The relativistic equation (18), Fresnel's law, compared with equation (10), is used to obtain the transit time difference between the two beams in Fizeau's experiment, and thereby explain the result from the relativistic point of view.

The transit time of the beam going downstream, with speed v very small compared with *c*, so that v^2/c^2 can be neglected, is:

$$t_1 = \frac{2L}{\frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_1 \approx \frac{2L\mu}{c}\left(1 + \frac{v}{\mu c} - \frac{\mu v}{c}\right)$$

The transit time of the beam going upstream (with speed $-v$) is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_2 \approx \frac{2L\mu}{c}\left(1 - \frac{v}{\mu c} + \frac{\mu v}{c}\right)$$

The time difference between the beam going downstream and the other going upstream is obtained as:

$$\Delta t = t_2 - t_1 \approx \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu^2}\right) \quad 19$$

The fringe shift δ_y, for light of wavelength λ, is obtained as:

$$\delta_y = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu^2}\right) \quad 20$$

This δy is larger than the fringe shift δx as given by equation (14).

Results and discussion

- A ray of light has the effect of a "stream of particles" transmitted at the speed of light c in the direction of propagation. The "stream of particles", which impinge on the surface of a medium exert pressure upon reflection from the surface. So, reflection of light can be treated as the recoil of "moving particles", under perfectly elastic conditions, obeying Newton's law of restitution [5].
- According to equation (5), a ray of light at normal incidence on a mirror moving in the opposite direction of the ray, is reflected with a speed greater than that of light.
- In equations (6) and (7) it is shown that the law of reflection, that is angle of incidence being equal to the angle of reflection, applies irrespective of the speed of the reflecting medium moving along the normal direction. Equation (7) is a simple and new expression subject to experimental verification.
- In equations (10) and (18) the speed v of the moving medium (Figure 1 with light at normal incidence), can take any value between 0 and $\pm c$. For $v = 0$, both equations give the speed of light in the medium $w = c/\mu$, as expected. For $\mu = 1$ both equations give $w = c$, also as expected, For $v = c$, equation (10) gives w = c, again as expected, but equation (18) gives $w = c(1 + 1/\mu - 1/\mu^2)$, which may be greater than c as $\mu > 1$. For $v = -c$, equation (10) gives $w = c(2/\mu - 1)$, which is reasonable as $\mu < 2$, but equation (18) gives $w = c(1/\mu2 + 1/\mu - 1)$, which may be negative as $1 < \mu < 2$, and this is absurd.
- Equation (20) is a good example of Beckmann's *correspondence theory* [9], whereby the desired result is obtained mathematically but based on the wrong underlying principles. Fizeau's experiment might as well have verified the fringe shift in equation (14), rather than the relativistic equation (20), for the transmission of light in a moving medium. Curt Renshaw [10] analysed the results of several experiments conducted to measure the effect of the speed of a medium on the speed of transmission of light in the medium and he concluded that the results could be explained without invoking special relativity.

Conclusions

- The treatment of reflection and refraction of light in this paper, clearly demonstrate the relativity of the velocity of light with respect to a moving medium, in accordance with Galileo's relativity of classical mechanics.
- The result of Fizeau's experiment [6] is not a direct consequence of the relativistic velocity addition rule but due to the effect of motion of the transmission medium on the speed of light.

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