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Revised Edition of Mass, Energy and Space-Time: On the Wave-Particle Duality, Planck's Constant and the Rest Mass of Photon

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Abstract

Based on the concept of space–time constraint that space-time itself imposes a constraint on a particle by producing resistive constraint force during the particle motion, it can be noticed that the impulse of the constraint force is in balance with the impulse in velocity-directed or parallel component of the inertial force. With interpretation of the relationship between impulse and momentum change in the context of wave-particle duality, this work proposes a speculative theoretical framework explaining the behavior of a point-mass particle moving at velocity approaching the speed of light. In this framework, the impulse in the perpendicular direction to the velocity will cause the particle an angular momentum along with wave momentum associated with the space-time entity. This wave momentum represents the wave energy transmitted into space-time. For the particle that can travel at speed of light, such as photon, the wave energy occurs in discrete integer cycles, each rotational cycle corresponds to a quantum of energy, which may be identified with Planck's constant. The coexistence of particle momentum and wave momentum suggests that the particle-like and the wave-like properties manifest simultaneously. In addition, the proposed concept gives new perspectives in quantum physics as follows:

1. *Planck's constant (h) represents the difference in photon energy between successive rotational cycles for unit time, equal to $2\pi m^*_0 c^2$ (Joule) in one second, where m^*_0 is the photon rest mass.*
2. *The photon rest mass is approximately equal to 1.17×10^{-51} kg.*
3. *The fundamental angular velocity of photon is equal to 1 rad/s.*

Keywords: Space-Time Constraint, Wave-Particle Duality, Wave Momentum, Planck's Constant, Photon Rest Mass

1. Introduction

In 1924, Louis de Broglie [1] presented his doctoral thesis entitled *Research on the Theory of Quanta* in fulfillment of his Ph.D. degree. His thesis was based on the conviction that a moving particle must be accompanied by a correlated wave. He explained from Einstein's photoelectric effect that if light waves can exhibit particle properties, particles should also exhibit wave properties. Therefore, he linked Planck's electromagnetic wave energy equation with Einstein's special relativity energy equation, and proposed the relation between wavelength and momentum. This was the foundation of the concept of waveparticle duality, or the matter wave, expressed as:

$$\lambda = \frac{h}{p} \tag{1}$$

Where

$$p = \frac{m_0 v}{\sqrt{1-(v/c)^2}} \tag{2}$$

Here:

- λ is the wavelength,
- p is the particle momentum,
- h is Planck's constant,
- m_0 is the rest mass of the particle,
- v is the velocity of the particle,
- c is the speed of light in vacuum.

Wave–particle duality is one of the most fundamental principles in modern physics. Experimental observations show that entities such as photons and electrons exhibit both wave-like and particle-like properties. Although wave-particle duality has been experimentally confirmed, it is still unclear whether it is two properties appearing simultaneously or a single property depending on observation, that is, when one aspect is observed, the other does not appear.

From the previous article "Mass Energy and Space-Time: On the Mass Increase of Moving Matter and Negative Energy Occurrence"of the authors [2], it is proposed that spacetime is not truly empty, but rather an emptiness that interacts with matter and influences motion through the space-time constraint. This gives rise to a resistive constraint force against the motion. The change in this constraint force at any given time; $d\vec{F}_c$, equals the product of the changing mass and acceleration, expressed as:

$$d\vec{F}_c = (-)\frac{dm_v d\vec{v}}{dt} \tag{3}$$

where:

$$m_v = m_o\gamma$$

- γ is Lorentz's factor which is equal to $\frac{1}{\sqrt{1-(v/c)^2}}$
- m_v is the relativistic mass of the body at time t
- m_o is the rest mass of the body
- dm_v is change in mass between t and $t + dt$
- $d\vec{v}$ is change in velocity between t and $t + dt$
- dt is a small time interval
- $(-)$ means the direction of force is opposite to the change in velocity

By connecting those two concepts altogether, this article aims at proposing a conceptual framework to find a solution for overcoming the motion limitations of massive particles as their velocity approaches the speed of light.

2. When the Speed of Light is the Maximum Velocity in the Universe - The Problem with Massive Particles under Continuous Force

It is well established that the speed of light in vacuum (empty space) is the maximum speed in the universe. Particles with rest mass can never reach, only very close to but never equal to, the speed of light. Light, or photons, which possess both wave and particle properties, are considered to have zero rest mass. This limitation suggests that the velocity of a particle will eventually become nearly constant. As a result, both the change in velocity and the change in mass (relativistic mass) will approach zero. This implies that when a particle's velocity approaches the speed of light, the rate of change of momentum with respect to time will approach zero, meaning the momentum tends to be a constant value as well. But here arises a puzzling question: Why does the velocity or momentum of a particle converge to a constant value even though the apply force, no matter how large it is, continues to act upon it?, apart from the reason that it is due to the increase of particle's mass.

To address this problem, the explanation is built upon the space-time constraint concept under the assumption that when a particle moves in rectilinear motion approaching the speed of light, the particle's translational motion will be in equilibrium, that is the net force in the direction parallel to the motion becomes zero, and the particle will rotate due to the force remaining in the perpendicular direction to the motion.

3. Space-Time Constraint and Impulses

It is stated in [2] that the inertial force; $\vec{F}_i$,which is the resultant of the applied force; $\vec{F}_A$ and the constraint force; $\vec{F}_C$, causes the momentum change of the body with respect to time; that is,

$$\vec{F}_i = \frac{d\vec{p}}{dt} \tag{4}$$

$$= m_v \frac{d\vec{v}}{dt} + \vec{v}\,\frac{dm_v}{dt} \tag{5}$$

$$\vec{F}_i dt = m_v d\vec{v} + \vec{v}\,dm_v \tag{6}$$

$$\int_0^t \vec{F}_i dt = \int_{v=0}^{v} m_v d\vec{v} + \int_{v=0}^{v} \vec{v}\,dm_v \tag{7}$$

Equation (7) is the impulse from the inertial force; $I_i(t)$, so

$$I_i(t) = \int_{v=0}^{v} m_v d\vec{v} + \int_{v=0}^{v} \vec{v}\,dm_v \tag{8}$$

From Equation (3) we obtain the impulse from the constraint force; $I_c(t)$ as follows:

$$d\vec{F}_c\,dt = (-)dm_v\,d\vec{v} \tag{9}$$

$$\iint_0^t d\vec{F}_c\,dt = (-)\iint_{v=0}^{v} dm_v\,d\vec{v} \tag{10}$$

As we assume at the end of section 2 that when the particle's velocity is approaching the speed of light, the translational motion of the particle will be in equilibrium, that means the impulse from the constraint force should be in the direction parallel to the velocity, then

$$I_c(t) = (-)\int_{v=0}^{v} \vec{v}\,dm_v \tag{11}$$

So, the resultant impulse is

$$I_R(t) = \int_{v=0}^{v} m_v d\vec{v} \tag{12}$$

4. Rotational Motion in Space-Time Coupled to Particle Rotating Trajectory

From the resultant impulse in Equation (12) if we substitute m_v with $m_o\gamma$, and $\vec{v}$ with $v\hat{\boldsymbol{i}}_{\boldsymbol{v}}$ where $\hat{\boldsymbol{i}}_{\boldsymbol{v}}$ is the unit velocity vector, then:

$$I_R(t) = \int_{v=0}^{v} m_o\gamma\,d(v\hat{\boldsymbol{i}}_{\boldsymbol{v}}) \tag{13}$$

$$= \int_{v=0}^{v} m_o\gamma v d\hat{\boldsymbol{i}}_{\boldsymbol{v}} + \int_{v=0}^{v} m_o\gamma\hat{\boldsymbol{i}}_{\boldsymbol{v}} dv \tag{14}$$

$$= \int_{v=0}^{v} m_o\gamma v d\hat{\boldsymbol{i}}_{\boldsymbol{v}} + m_o c\arcsin(\frac{v}{c})\hat{\boldsymbol{i}}_{\boldsymbol{v}} \tag{15}$$

The derivative of unit velocity vector in rotating trajectory of the particle can be determined as follows:

$$\frac{d\hat{\boldsymbol{i}}_{\boldsymbol{v}}}{dt} = \frac{d\hat{\boldsymbol{i}}_{\boldsymbol{v}}}{ds}\frac{ds}{dt} \tag{16}$$

$$= \frac{d\hat{\boldsymbol{i}}_{\boldsymbol{v}}}{ds}v \tag{17}$$

$$= \kappa \hat{\boldsymbol{i}}_{\boldsymbol{n}} v \tag{18}$$

$$= \frac{1}{\rho} \hat{\boldsymbol{i}}_{\boldsymbol{n}} v \tag{19}$$

$$d\hat{\boldsymbol{i}}_{\boldsymbol{v}} = \frac{v}{\rho} \hat{\boldsymbol{i}}_{\boldsymbol{n}} dt \tag{20}$$

where:

- κ is the curvature of the particle rotating trajectory.
- ρ is the radius of curvature of the particle rotating trajectory.
- $\hat{i}_n$ is the unit vector normal to the particle rotating trajectory.

The resultant impulse can then be expressed as:

$$I_R(t) = \int_{t=0}^{t} m_o \gamma \frac{v^2}{\rho} \hat{\boldsymbol{i}}_{\boldsymbol{n}} dt + m_o c \arcsin(\frac{v}{c}) \hat{\boldsymbol{i}}_{\boldsymbol{v}} \tag{21}$$

5. Interpretation of the Relationship between Impulse and Momentum Change in the Context of Wave Particle Duality

The relationship between impulse and momentum as shown in Equation (21) can be interpreted as follows:

- **The first term** on the right-hand side of the equation represents the change in **angular momentum** of the particle itself, which will be referred to as the *particle aspect*.
- **The second term** corresponds to the momentum arising within space-time due to the change in the particle's angular momentum. This momentum manifests in the form of a wave, or what can be called **wave-being momentum or wave momentum**, which will be referred to as the *wave aspect*.

Equation (21) itself shows that the particle aspect and the wave aspect manifest simultaneously, wave aspect is the joint component of the particle's rotating trajectory.

The wave momentum; $\vec{p}_w$ can therefore be expressed as:

$$\vec{p}_w = m_o c \arcsin(\frac{v}{c}) \hat{\boldsymbol{i}}_{\boldsymbol{v}} \tag{22}$$

6. From Wave Momentum to Energy Quantum, Planck's Constant, and the Rest Mass of Photon

Equation (22) can be rewritten in another form as:

$$\frac{\vec{p}_w}{m_o c} = \arcsin(\frac{v}{c}) \hat{\boldsymbol{i}}_{\boldsymbol{v}} \tag{23}$$

It can be seen that the wave momentum is a function of arcsin($\frac{v}{c}$). When $v = c$ (that is, when the velocity equals the speed of light), the value becomes:

$$\arcsin(\tfrac{c}{c}) = \arcsin(1) = \tfrac{\pi}{2}, 2\pi + \tfrac{\pi}{2}, 4\pi + \tfrac{\pi}{2}, 6\pi + \tfrac{\pi}{2}, 8\pi + \tfrac{\pi}{2}, \ldots = 2n\pi + \tfrac{\pi}{2} \text{ or } (4n+1)\tfrac{\pi}{2}$$

where n is an integer denoting the cycle number returning to the same trigonometric value.

Therefore, when $v = c$, the wave momentum takes the form:

$$\vec{p}_w = m_o c (4n+1) \frac{\pi}{2} \hat{\boldsymbol{i}}_{\boldsymbol{v}} \tag{24}$$

where $n = 0, 1, 2, 3, 4, \ldots$

The difference in wave momentum between successive cycles is:

$$\Delta p_w = 2\pi m_o c \tag{25}$$

For large *n*, the wave momentum can be expressed as:

$$\vec{p}_w = m_o c(2\pi n)\boldsymbol{\hat{i}_v} \tag{26}$$

Since wave energy equals the product of momentum and velocity, the energy of wave momentum is:

$$E_w = p_w c = m_o c^2(2\pi n) \tag{27}$$

Thus, the difference in wave energy between successive cycles is:

$$\Delta E_w = 2\pi m_o c^2 \tag{28}$$

Because n corresponds to the number of cycles returning to the same trigonometric value, within one unit of time this difference can be interpreted as wave frequency, denoted by ν. Therefore, the magnitude of the wave momentum per unit time and the wave energy per unit time can be written as:

$$\frac{p_w}{t} = m_o c(2\pi\boldsymbol{\nu}) \tag{29}$$

$$\frac{E_w}{t} = (\frac{p_w}{t})c = m_o c^2(2\pi\boldsymbol{\nu}) \tag{30}$$

From Equations (28) and (30), we obtain:

$$E_w = \Delta E_w t\boldsymbol{\nu} \tag{31}$$

Comparing Equation (31) with Planck's energy relation yields:

$$h = \Delta E_w t = 2\pi m_o c^2 t \tag{32}$$

It can be seen from Equation (32) that: **Planck's constant** *h is the difference in "photon"energy in each rotational cycle for unit time, equal to 2πtimes the photon's rest mass energy (Joule) in one second*. Here, the term "**photon**"is used to refer to a particle capable of reaching the speed of light.

If we denote the photon's rest mass by m_o^*, Planck's constant can be expressed as:

$$h = 2\pi m_o^* c^2 t \tag{33}$$

It must be kept in mind that *t* is in one second. Substituting $h = 6.626 \times 10^{-34}\text{kg}.m^2/s$ and $c = 3 \times 10^8 m/s$, the photon's rest mass can be estimated as:

$$m_o^* \approx 1.17 \times 10^{-51}\text{kg}$$

According to Equation (30), the wave energy of photon per unit time:

$$\frac{E_w}{t} = 2\pi m_o^* c^2\boldsymbol{\nu} \tag{34}$$

$$= m_o^*(2\pi\boldsymbol{\nu})c^2 \tag{35}$$

So,

$$E_w = m_o^* \boldsymbol{\omega} t c^2 \tag{36}$$

If we define $m_o^* \boldsymbol{\omega} t$ as the moving mass of the photon, so the relationship between the moving mass of the photon and its rest mass can then be expressed as:

$$\begin{aligned} m_\gamma &= m_o^* \boldsymbol{\omega} t \qquad (37) \\ &= m_o^* \frac{\boldsymbol{\omega}}{\boldsymbol{\omega}_o} (\boldsymbol{\omega}_o t) \qquad (38) \end{aligned}$$

Where m_γ is the moving mass of the photon, and ω is the angular velocity of the wave momentum or the wave energy of the photon, which may also be interpreted as the **photon's angular velocity**. It can be seen that the angular velocity associated with the photon's rest mass, which is the **fundamental angular velocity** of the photon; ω_o is equal to 1 *rad*/s.

According to Equation (23), the first wave momentum magnitude and corresponding energy when the velocity equals the speed of light or the first momentum and energy of a photon can be expressed as:

$$p_w^1 = m_o^* c \frac{\pi}{2} \tag{39}$$

$$\begin{aligned} E_w^1 &= p_w^1 c \qquad (40) \\ &= m_o^* c^2 \frac{\pi}{2} \qquad (41) \end{aligned}$$

So the minimum moving mass of photon can be calculated as:

$$\begin{aligned} m_\gamma^1 &= m_o^* \frac{\pi}{2} \qquad (42) \\ &\approx 1.84 \times 10^{-51} kg. \qquad (43) \end{aligned}$$

Having reached this point, the magnitude of photon's momentum and energy per unit time in terms of angular velocity can then be expressed as:

$$\frac{p_w}{t} = m_o^* c \boldsymbol{\omega} \tag{44}$$

$$\frac{E_w}{t} = m_o^* c^2 \boldsymbol{\omega} \tag{45}$$

From Equation (33), it can be rewritten as:

$$\frac{h}{c} = 2\pi m_o^* c t \tag{46}$$

Substitute the value of $2\pi m_o^* c t$ from Equation (46) into Equation (44) yields:

$$p_w = \frac{h}{c} \nu = \frac{h}{\lambda} \tag{47}$$

This corresponds to the photon momentum equation generally.

7. Matter Wave

For the wave momentum expressed in Equation (22), multiplying the numerator and denominator on the right-hand side of the equation by $2\pi m_o^* ct$ (where m_o^* is the photon rest mass) gives:

$$\vec{p}_w = \frac{m_o}{m_o^*}\frac{2\pi m_o^* c^2 t}{2\pi ct}\arcsin(\frac{v}{c})\hat{\boldsymbol{i}}_{\boldsymbol{v}} \tag{48}$$

Since $2\pi m_o^* c^2 t$ is Planck's constant *h* (from Equation (33)), the magnitude of the wave momentum can then be written as:

$$p_w = \frac{m_o}{m_o^*}\frac{h}{2\pi ct}\arcsin(\frac{v}{c}) \tag{49}$$

Because wave energy equals the product of momentum and velocity, the wave energy equation becomes:

$$E_w = p_w v = \frac{m_o}{m_o^*}\frac{hv}{2\pi ct}\arcsin(\frac{v}{c}) \tag{50}$$

If wave energy is also expressed as the product of Planck's constant and wave frequency;f, then the wave frequency can be expressed as:

$$f = \frac{m_o}{m_o^*}\frac{v}{2\pi ct}\arcsin(\frac{v}{c}) \tag{51}$$

and the angular frequency (or angular velocity) as:

$$\Omega = \frac{m_o}{m_o^*}\frac{v}{ct}\arcsin(\frac{v}{c}) \tag{52}$$

Thus, the magnitude of the wave momentum per unit time can be expressed as:

$$\frac{p_w}{t} = \frac{m_o^* c^2}{v}\Omega \tag{53}$$

Since wave velocity equals wavelength multiplied by frequency, Equation (53) can be rewritten as:

$$\frac{p_w}{t} = \frac{2\pi m_o^* c^2}{\lambda} \tag{54}$$

$$p_w = \frac{h}{\lambda} \tag{55}$$

In the case where *v/c* is very small, arcsin($\frac{v}{c}$) is approximately equal to $\frac{v}{c}$. Therefore, for low velocities relative to the speed of light, the wave momentum becomes equal to the particle momentum as shown in Equation (56):

$$p_w = m_o c\frac{v}{c} = m_o v = p \tag{56}$$

Hence:

$$p = \frac{h}{\lambda} \tag{57}$$

This is precisely the **matter wave equation** proposed by de Broglie.
If we consider Equation (51), the matter wavelength can be obtained directly as follows:

$$f = \frac{v}{\lambda} = \frac{m_o}{m_o^*} \frac{v}{2\pi ct} \arcsin(\frac{v}{c}) \tag{58}$$

$$\lambda = \frac{2\pi m_o^* ct}{m_o \arcsin(\frac{v}{c})};\ t \text{ in one second for SI units.} \tag{59}$$

By calculating the matter wavelength using Equation (59) and comparing it with de Broglie's matter wave equation (as ahown in the examples in Appendix A), the results are nearly identical -essentially yielding the same values. Therefore, it can be concluded that the wave momentum described in Equation (22) can be applied to particle's motion across **all velocity ranges.**

8. Conclusion

By linking the concept of space-time constraint, which generates a resistive constraint force against the motion of matter, with de Broglie's concept that a moving particle is accompanied by a correlated wave (wave-particle duality), and under the assumption that when a particle moves in rectilinear motion with velocity approaching the speed of light, the particle's translational motion will be in equilibrium state, while the particle's rotational motion is taking place, the following understanding can be drawn:

- In such an equilibrium state, the particle's *linear velocity becomes constant*.
- The *impulse* acting on the particle remains only in the direction *perpendicular* to the motion, causing the particle to undergo *rotation*.
- This rotation leads to the emergence of wave momentum in space-time, existing alongside the particle's angular momentum.
- The coexistence of particle momentum and wave momentum suggests that the particle-like and the wave-like properties manifest simultaneously.

The wave momentum (p_w) has a magnitude of:

$$p_w = m_o c \arcsin(\frac{v}{c})$$

and is oriented parallel to the particle's motion.

If the particle continues to be subjected to external force or continuously receives energy **only the wave momentum can increase up to the point where the velocity equals the speed of light**. At this stage, the additional energy received by the particle will be transferred into the energy of wave momentum (wave energy). The values of wave momentum and wave energy are discrete -they occur in cycles or intervals- with the difference between successive cycles remaining constant. Specifically:

- The difference in wave momentum between cycles is $2\pi m_o^* c$.
- The difference in wave energy between cycles is $2\pi m_o^* c^2$.

where m_o^* is the photon's rest mass.

This wave energy is rotational energy quantized in integer multiples of rotational cycles, i.e., the *quantum* of energy.

Furthermore, the proposed concept provides new perspectives in quantum physics as follows:

1. Planck's constant (h) represents the difference in photon energy between successive rotational cycles for unit time, equal to $2\pi m_o^* c^2$ (Joule) in one second.
2. The rest mass of the photon is approximately 1.17×10^{-51} kg.
3. The fundamental angular velocity of the photon is equal to 1 rad/s.

Appendix A: Comparative Calculations for Clear Demonstration

In this section, a comparative calculation of the matter wavelength according to de Broglie's equation and that proposed in this article is performed, by showing two examples as follows:

Example 1: Table Tennis Ball

A ping-pong ball of mass 2.70 grams moving at a velocity of 10 m/s. The matter wavelength according to de Broglie's equation is:

$$\begin{aligned}\lambda &= \frac{h}{p}\\ &= \frac{6.626\times10^{-34}}{2.70\times10^{-3}\times10}\,m.\\ &= 2.454\times10^{-32}\,m.\end{aligned}$$

From equation (59) proposed in this article, by substituting the photon's rest mass 1.17×10^{-51}kg and the speed of light $c = 3\times10^{8} m/s$, the matter wavelength is obtained as:

$$\begin{aligned}\lambda &= \frac{2\pi(1.17\times10^{-51})(3\times10^{8})}{2.7\times10^{-3}\times\arcsin(\frac{10}{3\times10^{8}})}\,m.\\ &= 2.450\times10^{-32}\,m.\end{aligned}$$

Example 2: Electron

An electron of mass $9.11\times10^{-31} kg$ with kinetic energy of 100 eV. The matter wavelength according to de Broglie's equation is:

$$\begin{aligned}\lambda &= \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}\\ &= \frac{6.626\times10^{-34}}{5.40\times10^{-24}}\,m.\\ &= 1.226\times10^{-10}\,m.\end{aligned}$$

From equation (59) proposed in this article, The matter wavelength of the electron is:

$$\begin{aligned}\lambda &= \frac{2\pi(1.17\times10^{-51})(3\times10^{8})}{9.11\times10^{-31}\times\arcsin(\frac{5.930\times10^{6}}{3\times10^{8}})}\,m.\\ &= 1.225\times10^{-10}\,m.\end{aligned}$$

Both examples cleary show that the matter wavelength obtained from Equation (59) gives results that are practically identical to those obtained from de Broglie's matter wave equation. Therefore, it can be concluded that the wave momentum described in Equation (22) can be applied to the motion of particles at all velocity ranges.

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