

# Radiative Models of the Hydrogen Atom outside Quantum Mechanics

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### Abstract

Nuclear and non-nuclear radiative models of the hydrogen atom are devised. The nuclear model consists of  $N$  coplanar orbits each with a particle having the electronic charge  $-e$  and a multiple  $nm$  of the electronic mass  $m$ , revolving in the  $n$ th orbit round a heavy nucleus of charge  $+Ne$  and mass  $N(N + 1)m/2$ , where  $n = 1, 2, 3, \dots, N$ . The non-nuclear model consists of  $N$  orbits each with two particles of the same mass  $nm$  but opposite charges  $+e$  and  $-e$ , revolving in the  $n$ th orbit round a common center of mass. In the stable state a particle revolves with constant angular momentum  $nL$ , in a circle of radius  $nr_1$  at speed  $v_1/n$ , where  $r_1$  is the radius,  $v_1$  the speed and  $L$  the angular momentum in the first orbit. If a particle is dislodged from the circular orbit, it revolves in an unclosed elliptic orbit, with emission of radiation at the frequency of revolution, in many cycles, before reverting into the circular orbit. The radiation frequency is nearly equal to that of revolution in the circular orbit. It is shown that the wave numbers of radiation from the non-nuclear model are in conformity with the Balmer-Rydberg formula for the lines in the emission spectra of hydrogen gas. Radiation is shown to be due a charged particle having a component of its velocity in the direction of an electric field, as in an elliptic orbit. A charged particle moving perpendicular to an electric field, as in Rutherford's nuclear model, does not radiate and there is no need for Bohr's quantum mechanics to stabilize the atom. The non-nuclear model is identified with the atom of hydrogen gas and the nuclear model with the liquid or solid state.

**Keywords:** Angular Momentum, Electric Field, Electron, Hydrogen Atom, Nucleus, Orbit of Revolution, Positron, Spectrum of Radiation

### Introduction

#### Rutherford's nuclear model of the hydrogen atom

In 1904 J.J. Thomson proposed the "plum pudding" model of the atom, soon after his discovery of the electron [1]. In the Thomson's model, the atoms of the elements consisted of a number of negatively charged corpuscles, called electrons, studded in a sphere of uniform positive electrification, like the raisins in a pudding – a tasty English dessert. The negative charges were balanced by the positive sphere to form the neutral atom as a stable entity.

A landmark series of experiments, Geiger-Marsden experiments or Rutherford gold foil experiments, on the scattering of alpha particles shot through a thin gold foil, were conducted between 1904 and 1909 [2-4]. On the basis of these experiments, Lord Ernest Rutherford, in 1911, proposed a nuclear theory of the hydrogen atom, which superseded the Thomson's model [5]. Rutherford devised a model consisting of a heavy positively charged central nucleus around which a cloud of negatively charged electrons revolve in circular orbits. The hydrogen atom is the simplest, consisting of one electron of charge  $-e$  and mass  $m$  revolving in a circular orbit round a much heavier nucleus of charge  $+e$ . This model has sufficed since, although with some difficulties regarding its stability and emitted radiation. The purpose of this paper is to re-examines the Rutherford's nuclear model of the hydrogen atom with a view to

modifying it so as to eliminate some, if not all, of its deficiencies and deduce some incidental properties of its structure and modes of emission of radiation.

According to Abraham-Lorentz formula of classical electrodynamics, where radiation reaction force, due to a charged particle moving in an electric field, is proportional to the rate of change of acceleration, and Lamor formula where radiation power is proportional to the square of acceleration, the electron of the Rutherford's model, in being accelerated towards the positively charged nucleus of the atom, by the centripetal force, should [6]:

1. Emit radiation over a continuous range of frequencies with power proportional to the square of its acceleration.
2. Lose potential energy and gain kinetic energy as it spirals into the nucleus, leading to collapse of the atom.

The second prediction is contradicted by observation as atoms are the most stable objects known in nature. The first effect is contradicted by experiments as a detailed study of the radiation from hydrogen gas, undertaken by J. J. Balmer as early as 1885, as described by Francis Bitter, showed that the emitted radiation had discrete frequencies [7]. The spectral lines in the Balmer series of the hydrogen spectrum satisfy the Balmer-Rydberg formula:

$$\nu_{2q} = \frac{1}{\lambda_{2q}} = R \left( \frac{1}{2^2} - \frac{1}{q^2} \right) \quad (1)$$

where  $\lambda_{2q}$  is the wavelength,  $\nu_{2q}$  is the wave number;  $R$  is the Rydberg constant and  $q$  an integer greater than 2. The first four visible lines of the Balmer Series, with  $q = 3, 4, 5, 6$  are *red, green, blue and violet* respectively [8]. The spectral series limit ( $q \rightarrow \infty$ ), lying in the ultra-violet (not visible) region of the spectrum, is  $\nu_2 = R/4$ .

Niels Bohr, in a brilliant display of original thinking, rescued the atom from radiating and collapsing by invoking the quantum theory and making two, somewhat ad-hoc, postulates that prevented the atom from radiating energy and collapsing. Bohr's postulates are [9]:

1. In those orbits where the angular momentum is  $nh/2\pi$ ,  $n$  being an integer and  $h$  the Planck constant, the energy of the electron is constant.
2. The electron can pass from an orbit of total energy  $E_q$  to an inner orbit of total energy  $E_n$  in a quantum jump, the difference being released in the form of radiation of frequency  $f_{nq}$ , with energy given by  $E_q - E_n = hf_{nq}$

The first postulate quantized the angular momentum with respect to (quantum) number  $n$ . With these postulates Bohr was able to derive a formula for the wave numbers of the lines of the spectrum of the hydrogen atom in exactly the same mathematical form as obtained by Balmer and generalized by J.R. Rydberg in 1889. Bohr's model, for an electron of charge  $-e$  and mass  $m$ , gives:

$$\nu_{nq} = \frac{1}{\lambda_{nq}} = \frac{me^4}{8c\epsilon_0^2 h^3} \left( \frac{1}{n^2} - \frac{1}{q^2} \right) \text{ per meter} \quad (2)$$

$$\frac{1}{\lambda_{nq}} = R \left( \frac{1}{n^2} - \frac{1}{q^2} \right) \text{ per meter} \quad (3)$$

where  $n$  and  $q$  are integers greater than 0, with  $q$  greater than  $n$ ,  $c$  is the speed of light in a vacuum,  $\epsilon_0$  is the permittivity of a vacuum and  $h$  is the Planck constant. Equation (3) is the Balmer-Rydberg formula giving the Rydberg constant  $R$  as:

$$R = \frac{me^4}{8c\epsilon_0^2 h^3} \quad (4)$$

Substituting the values of the physical quantities in equation (4),  $R$  is found as  $1.097 \times 10^7$  per meter, in agreement with observation.

In equation (3), if  $n = 1$  we have the Lyman series, in the far ultra-violet region of the spectrum.  $R$  is the spectral limit for the Lyman series. For  $n = 2$ , we have the Balmer (red to violet lines) series and where  $n = 3$  we get the Paschen series in the near infra-red region. In a series, the lines crowded together, and their intensities should decrease to zero as the series limit ( $q \rightarrow \infty$ ) is approached. Other series, in the infrared region, are obtained for  $n = 4, 5, 6, \dots$

The fact that a purely chance agreement between the physical quantities in equation (4) is highly improbable, lends some plausibility to Bohr's theory of the hydrogen atom. This theory gave great impetus to quantum mechanics and was recognized as a remarkable triumph of the human intellect. However, the quantum jump in zero time and absence of a direct link between the frequency of the emitted radiation and the frequency of revolution of the electron, in its orbit, leaves a question mark on Bohr's quantum theory of the hydrogen atom.

Subsequently, Bohr's quantum theory was modified, notably by Sommerfeld, for elliptic orbits and more complex atoms [10]. This imposed extra ad-hoc conditions justifiable only by the (not always) correctness of their consequences.

This paper introduces two models of the hydrogen atom, a nuclear model for the liquid or solid state of hydrogen and a non-nuclear model for the gaseous state. The nuclear model here is different from the Rutherford's model. Instead of one orbiting electron, the proposed nuclear model has a number of negatively charged particles, revolving in the same sense, in regular concentric coplanar orbits around the nucleus. The sums of magnitudes of the negative charges and masses of the orbiting particles are respectively equal to the magnitudes of the positive charge and mass of the nucleus. The positive and negative charges of the atom balance out exactly to form a neutral particle.

### Proposed Nuclear Model of the Hydrogen Atom

The proposed nuclear model of the hydrogen atom consists of a concentric arrangement of  $N$  coplanar orbits. The orbits are equally spaced, each with a particle of charge  $-e$  and mass  $nm$  revolving in the  $n$ th circular orbit round a nucleus of charge  $+Ne$ . The equation of motion of a particle of mass  $nm$  revolving, in the  $n$ th orbit, with constant angular momentum  $nL$ , at a point distance  $r$  from the nucleus, is derived as [11]:

$$\frac{1}{r} = \frac{A}{n} \exp(-q\psi) \cos(\alpha\psi + \beta) + \frac{m\chi}{nL^2} \quad (5)$$

Where the amplitude of the excitement  $A$  and phase angle  $\beta$  are determined from the initial conditions and  $q$ ,  $\alpha$  and  $\chi = Ne^2/4\pi\epsilon_0$  are constants with  $nL$  as a constant angular momentum in the  $n$ th orbit.

The exponential decay factor  $(-q\psi)$ , in equation (5), is as a result of radiation of energy. An excited negatively charged particle will revolve, round the positively charged nucleus, in an unclosed (aperiodic) elliptic orbit with a few cycles of revolutions, radiating energy at the frequency of revolution, before settling down into the stable circle of radius  $nL^2/m\chi$ , the steady state, until it is disturbed again.

### Proposed Non-nuclear Model of the Hydrogen Atom

The proposed non-nuclear model of the hydrogen atom consists of a concentric arrangement of  $N$  coplanar orbits. The orbits are equally spaced, each with two particles of charges  $-e$  and  $+e$  and same mass  $nm$  revolving in the  $n$ th orbit round an empty centre of mass of the two particles.

The non-nuclear model is different from the Rutherford's nuclear model in that it has no nucleus but an empty center of mass [7]. The equation of the orbit of motion of a particle of mass  $nm$  revolving, in the  $n$ th orbit, at a point distance  $r$  from the centre, is derived [11] as:

$$\frac{1}{r} = \frac{A}{n} \exp(-b\psi) \cos(\alpha\psi + \beta) + \frac{m\kappa}{nL^2} \quad (6)$$

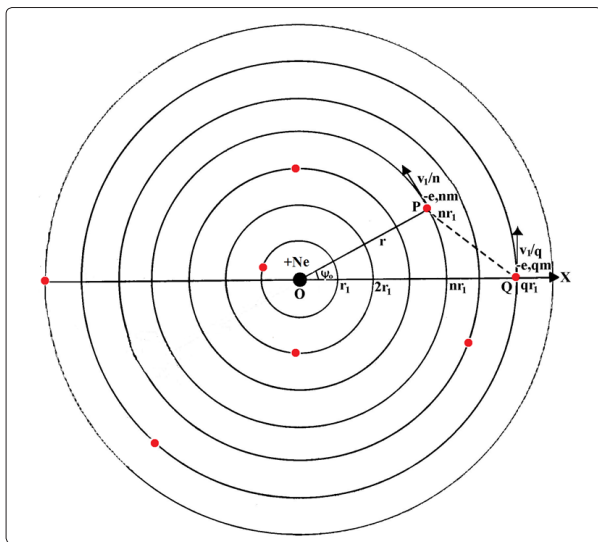
Where the excitement amplitude  $A$  and phase angle  $\beta$  are determined from the initial conditions,  $b$ ,  $\alpha$  and  $\kappa = e^2/16\pi\epsilon_0$  are constants with  $nL$  as constant angular momentum in the  $n$ th orbit.

The exponential decay factor  $(-b\psi)$ , in equation (6), is as a result of radiation of energy. The two charged particles will revolve, round their empty center of mass, in an unclosed (aperiodic) elliptic orbit with some cycles of revolutions, radiating energy, before settling down into the  $n$ th stable orbit, a circle of radius  $nL^2/mk$ .

### Radiative Nuclear Model of the Hydrogen Atom Frequency of radiation

Figure 1 represents the proposed nuclear model of the hydrogen atom. It consists of a concentric arrangement of  $N$  coplanar orbits. The orbits are equally spaced, each with a particle of charge  $-e$  and mass  $nm$  revolving round a nucleus of charge  $+Ne$ ,  $n$  being an integer: 1, 2, 3, and 4... $N$ . A negatively charged particle revolves in the  $n$ th stable circle of radius  $rn = nr_1 = nL^2/m\chi$  with speed  $v_n = v_1/n = \chi/nL$ , where  $\chi = Ne^2/4\pi\epsilon_0$  and  $n = 1$  for the innermost (first) orbit.

Frequency  $f_n$  of revolution of a particle in the  $n$ th stable orbit, at speed  $v_n$ , in a circle of radius  $r_n$ , is:



**Figure 1:** Proposed nuclear model of the hydrogen atom, consisting of  $N$  coplanar orbits each with a negatively charged particle revolving anticlockwise, in angle  $\psi$ , under the attraction of a nucleus of charge  $+Ne$ . A particle in the  $n$ th orbit has a multiple  $nm$  of the electronic mass  $m$  and the electronic charge  $-e$ ,  $n$  being an integer. The particle in the  $n$ th orbit revolves in a circle of radius  $nr_1$  with velocity  $v_1/n$  and angular momentum  $nmv_1r_1 = nL$

$$f_n = \frac{v_n}{2\pi r_n} = \frac{1}{2\pi} \frac{\chi}{nL} \frac{m\chi}{nL^2} = \frac{m\chi^2}{2\pi n^2 L^3} = \frac{mN^2 e^4}{2\pi (4\pi\epsilon_0)^2 L^3 n^2} \quad (7)$$

$$f_n = \frac{mN^2 e^4}{2\pi (4\pi\epsilon_0)^2 L^3 n^2} = \frac{cS}{n^2} \quad (8)$$

$$S = \frac{mN^2 e^4}{2\pi c (4\pi\epsilon_0)^2 L^3}$$

where  $S$  is a constant. If an electron is disturbed or dislodged from the stable circular orbit, it revolves in an unclosed elliptic orbit, emitting a burst of radiation in a narrow band of frequencies nearly equal to the frequency  $f_n$  of the circular revolution (equation 7).

Let us now follow the motion of two particles at positions P and Q with radii  $nr_1$  and  $qr_1$  respectively, revolving in anticlockwise sense round the centre O as in Figure 1. The frequencies of revolution at P and Q are given by equation (7) for the orbital number  $n$  or  $q$ .

In Figure 1, let the particles at positions P and Q be as shown at the initial time  $t = 0$ . The relative positions of the points O, P and Q are as shown, with OP and OQ in an angular displacement  $\psi_0$  at the initial stage. In time  $t$  the line OP moves to  $OP_t$  through an angle of revolution  $\psi_n$  and line OQ moves to  $OQ_t$  through an angle of revolution  $\psi_q$ . The difference in angular displacement, the instantaneous angle  $\psi_t$   $\angle P_t O Q_t$ , is:

$$\psi_t = \psi_0 + \psi_n - \psi_q$$

The angular frequency of oscillation between the particles at P and Q is:

$$\frac{d\psi_t}{dt} = \frac{d\psi_n}{dt} - \frac{d\psi_q}{dt} = \omega_n - \omega_q = 2\pi f_n - 2\pi f_q = 2\pi f_{nq} \quad (9)$$

Combining equation (9) above with equation (7) where  $f_n = cS/n^2$  and  $f_q = cS/q^2$ , gives the frequency  $f_{nq}$ , as:

$$f_{nq} = cS \left( \frac{1}{n^2} - \frac{1}{q^2} \right) \quad (10)$$

$$\frac{f_{nq}}{c} = \frac{1}{\lambda_{nq}} = S \left( \frac{1}{n^2} - \frac{1}{q^2} \right) \quad (11)$$

### Emission or absorption of radiation

Radiation from the proposed nuclear model of the hydrogen atom is of two sources. The first source is interaction between the electrons revolving in the  $n$ th unstable orbit, and the nucleus, producing radiation of frequency  $f_n$  given by equation (7). In the first case there is emission or absorption of radiation only if an electron is dislodged from the stable circular orbit. The second source of radiation is interaction between electrons revolving in the  $n$ th and  $q$ th orbits, producing radiation of frequencies  $f_{nq}$  given by equation (10), with the limit  $(q \rightarrow \infty)$  given by equation (7). In the second case there is emission or absorption of radiation as long as the atom exists.

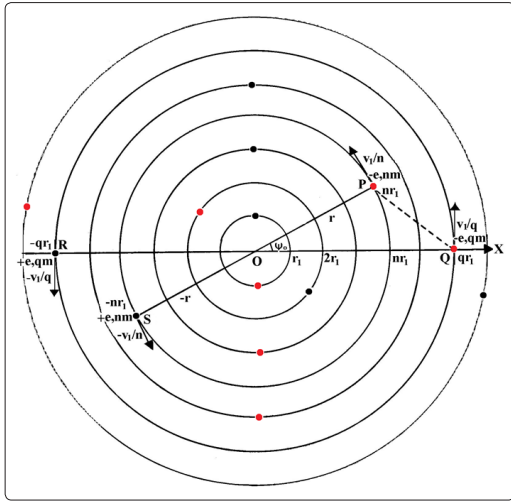
### Number of orbits in the proposed nuclear model

Measurements with a mass spectrometer showed that the hydrogen atom is about 1836 times the mass  $m$  of the electron. The total mass of particles in  $N$  orbits, each orbit containing a particle of mass  $nm$ , is obtained from the sum of the natural numbers:  $n = 1, 2, 3...N$ . This sum, which carries half of the mass of the atom, comes to:  $N(N+1)m/2 = 1836m/2$ . Solving the quadratic equation:  $N^2 + N - 1836 = 0$ , gives  $N = 42.35$ . Obviously,  $N$  should be an integer, 42 or 43 or some other number. However,  $N$  here is a reasonable real number. Further investigation and experimental work are required here to ascertain the actual number  $N$  of orbits.

### Radiative Non-nuclear Model of the Hydrogen Atom Frequency of radiation

In the non-nuclear model, the particle revolves in a stable circular orbit of radius  $r_n = nr_1 = nL^2/mk$  with velocity  $v_n = v_1/n = \chi/nL$  where

$\kappa = e^2/16\pi\epsilon_0$  and  $n = 1$  for the innermost orbit. Such a configuration of  $N$  orbits is shown in Figure 2. The non-nuclear model of the hydrogen atom, in contrast to the nuclear model, has no particle as a nucleus, but an empty common centre of mass for all the particles in  $N$  orbits.



**Figure 2:** Non-nuclear model of the hydrogen atom, consisting of  $N$  coplanar orbits each with two oppositely charged particles revolving, anti-clock-wise, in angle  $\psi$ , under mutual attraction. Each of the two particles in the  $n$ th orbit has mass  $nm$ , one carries charge  $-e$  and the other  $+e$ ,  $n$  being an integer from 1 to  $N$ ,  $m$  is the electronic mass and  $-e$  the electronic charge. The  $n$ th pair of particles revolves in a circular orbit of radius  $nr_1$  with velocity  $v_1/n$  and constant angular momentum  $nmv_1r_1 = nL$

The frequency of revolution of a particle moving with constant speed  $v_n$  in the  $n$ th stable orbit of Figure 2, a circle of radius  $r_n$ , is:

$$f_n = \frac{v_n}{2\pi r_n} = \frac{me^4}{16\pi n\epsilon_0 L} \frac{1}{2\pi(16\pi n\epsilon_0 L^2)} = \frac{me^4}{2\pi(16\pi\epsilon_0)^2 L^3} \frac{1}{n^2} \quad (12)$$

Putting  $L = h/4\pi$ , the angular momentum of the particle in the first orbit, gives equation (12) as:

$$f_n = \frac{me^4}{2\pi(16\pi\epsilon_0)^2 L^3} \frac{1}{n^2} = \frac{me^4}{8\epsilon_0^2 h^3} \frac{1}{n^2} = \frac{cR}{n^2} \quad (13)$$

Where the Rydberg constant  $R = me^4/8c\epsilon_0^2 h^3$ , has the value  $(1.097X 10^7 \text{ per metre})$  as obtained by Bohr (equation 2) and confirmed by observation. The Planck constant  $h$  appears here in a manner reminiscent of Bohr's first postulate which makes  $nL = nh/2\pi$  as the angular momentum in the  $n$ th orbit, in contrast to the angular momentum,  $nL = nh/4\pi$ , as advanced here in order to arrive at equation (13).

### The Balmer-Rydberg formula

Let us now follow the motion of particles in two bipolar orbits with the particles at positions P and Q of radii  $nr_1$  and  $qr_1$  respectively, revolving in anticlockwise sense round the centre O as in Figure 2. The frequencies of revolution at P and Q are given by equation (13) for the orbital numbers  $n$  or  $q$ .

In Figure 2 let the particles at positions P and Q both have negative charges at the initial stage. The relative positions of the points S, R, O, P and Q are as shown, with OP and OQ at an angular displacement  $\psi_o$  at the initial stage, time  $t = 0$ . In time  $t$  let the line OP move to  $OP_t$  through an angle  $\psi_n$ , and let the line OQ move to  $OQ_t$  through an angle  $\psi_q$ . The difference in angular displacement, the instantaneous angle  $\psi_t$  is:

$$\psi_t = \psi_o + \psi_n - \psi_q$$

The angular frequency of oscillation between the particles at P and Q, is:

$$\frac{d\psi_t}{dt} = \frac{d\psi_n}{dt} - \frac{d\psi_q}{dt} = \omega_n - \omega_q = 2\pi f_n - 2\pi f_q = 2\pi f_{nq} \quad (14)$$

Combining equation (14) above with equation (13) where  $f_n = cR/n^2$  and  $f_q = cR/q^2$ , gives:

$$f_{nq} = cR \left( \frac{1}{n^2} - \frac{1}{q^2} \right) \quad (15)$$

The four particles in two radiators, of the hydrogen atom, behave like oscillating pairs, emitting radiation in a narrow band of frequencies, with wave numbers  $\nu_{nq}$  as:

$$\nu_{nq} = \frac{f_{nq}}{c} = R \left( \frac{1}{n^2} - \frac{1}{q^2} \right) \quad (16)$$

The particles revolve in their respective orbits as radiators. Interactions between the  $2N$  particles, in the  $N$  orbits, result in the emission of radiation of discrete frequencies and wave numbers given by equations (15) and (16) respectively. Equation (16) is identical to equation (1), the Balmer-Rydberg formula for the spectral lines of radiation from the atom of hydrogen gas.

### Emission or absorption of radiation

There are two sources of radiation from the proposed non-nuclear model of the hydrogen atom. The first source is due to interactions between different particles revolving in the  $n$ th and  $q$ th orbits of wave number given by equation (14). Here, emission or absorption of radiation takes place as long as the atom exists. In the second case there is radiation as a result of dislodgement of a particle from the stable circular orbit. If two particles, in the  $n$ th orbit, are dislodged from the stable circular orbit, they revolve as radiators in an unclosed (a periodic) elliptic orbit. They emit a burst of radiation in a spread of frequencies very nearly equal to that of revolution of a particle revolving with speed  $v_n$  in a circle of radius  $r_n$ , before reverting back into the  $n$ th stable circular orbit.

### Number of orbits in the non-nuclear model

The hydrogen atom is found to be about 1836 times the mass  $m$  of the electron. The total number  $N$  of orbits, each containing two particles either of mass  $nm$ , is obtained from the sum of the natural numbers:  $n = 1, 2, 3 \dots N$ . Twice this sum, which carries the mass of the atom, gives  $N(N+1)m = 1836m$ . This gives  $N = 42.35$ , same as for the nuclear model. The number  $N$  should be an integer.

## The nuclear model versus the non-nuclear model

The proposed nuclear model of the hydrogen atom has a nucleus of charge  $+Ne$  and  $N$  coplanar orbits in each of which particles of charge  $-e$  and mass  $nm$  revolve. The total charge of the negative particles is  $-Ne$ , equal and opposite of the charge on the nucleus. The total mass of the  $N$  revolving particles is  $\frac{1}{2}N(N+1)m$ , same as the mass of the nucleus.

The proposed non-nuclear model of the hydrogen atom has no nucleus but an empty centre of mass round which particles revolve in  $N$  coplanar orbits. Two positive and negative particles ( $+e$  and  $-e$ ), each bearing a multiple  $nm$  of the electronic mass  $m$ , revolve in the  $n$ th orbit. The total mass of particles in the  $N$  orbits, with two particles in each orbit, is  $N(N+1)m$ . The positive and negative electrical charges, in an atom, balance out exactly to form a stable particle. Thus the nuclear model and the non-nuclear model have the same number of orbits  $N$  and the same mass, equal to  $N(N+1)m$ . The question now is: "What is the significance of these two different models of the hydrogen atom?"

It is suggested here that the non-nuclear model is what obtains with the gas phase of hydrogen while the nuclear model exists with respect to the liquid and solid states, depending on the ambient temperature. Let us now determine the relationship between the constant  $S$  as given by equation (6) for the nuclear model and the Rydberg constant  $R$ , obtained from equation (12), for the non-nuclear model. The expression obtained, equal to the ratio of frequency  $g_n$  in the nuclear model and the frequency  $f_n$  in the non-nuclear nuclear model, is:

$$\frac{S}{R} = \frac{g_n}{f_n} = 16N^2 \quad (17)$$

While the frequencies of radiation from the non-nuclear model are in the red and violet regions, those of the nuclear model are in the x-rays regions.

The ratio  $s_n$  of radius of revolution in the  $n$ th orbit of the nuclear model and the radius  $r_n$  of the non-nuclear model, is obtained as:

$$\frac{s_n}{r_n} = \frac{1}{4N} \quad (18)$$

The ratio  $u_n$  of speed of revolution in the  $n$ th orbit of the nuclear model and the speed  $v_n$  in the non-nuclear obtained, is obtained as:

$$\frac{u_n}{v_n} = 4N \quad (19)$$

It is assumed that the constant angular momentum  $nL$ , with respect to the  $n$ th stable orbit, is the same for the nuclear and non-nuclear models of the hydrogen atom.

## Result and discussions

Expressions for the Balmer-Rydberg formula and the Rydberg constant (equations 3 and 4, for the proposed non-nuclear model of the hydrogen atom, are derived without recourse to Bohr's second postulate but with a modification of the first postulate. The modification is to the effect that *the magnitude of the angular momentum of a particle of mass  $nm$ , revolving in the  $n$ th circular orbit, is equal to  $nL = nh/4\pi$* , where  $h = 6.626 \times 10^{-34}$  J-sec. is the Planck constant. Quantisation of angular momentum ( $nL$ ) and radius of revolution ( $nL^2/m\kappa$ ) and inverse quantisation of velocity ( $\kappa/nL$ ) appear naturally as a consequence of discrete masses ( $nm$ ), being multiples of the electronic mass  $m$ , with  $n$  as the orbital number or

"quantum number", an integer greater than 0.

The angular momentum of a particle in the first orbit of the non-nuclear model, is  $L = h/4\pi$ . This is a fundamental quantity of value equal to  $L = 5.273 \times 10^{-35}$  J-sec. It defines the Planck constant  $h$  in terms of angular momentum rather than "unit of action". Even though the Planck constant is featuring prominently here, Bohr's quantum mechanics is not required in describing the discrete frequencies of radiation from the hydrogen atom.

As a charged particle is disturbed from the  $n$ th stable circular orbit, it revolves in an unclosed elliptic orbit, with increasing or decreasing frequency of revolution and emission of radiation at the frequency of revolution, before reverting into the circular orbit. There should be a narrow spread in the frequencies of emitted radiation, around the frequency of circular revolution. This may explain the "fine structure" of the spectral lines of radiation from the atom of hydrogen gas, as may be observed with a diffraction grating of high resolving power, without recourse to relativistic effects, electron spin or quantum electrodynamics.

The radius of the first orbit ( $n = 1$ ) of the non-nuclear model of the hydrogen atom is obtained as  $r_1 = \epsilon_0 h^2 / \pi m e^2 = 5.292 \times 10^{-7}$  m, the same as the **first Bohr radius** obtained, through quantum mechanics, for the nuclear model. The radius of the first orbit in the nuclear model is about  $s_1 = 3.15 \times 10^{-9}$ , making the particles in the nuclear model much more tightly packed.

The speed of revolution in the first orbit of the non-nuclear model is obtained as  $v_1 = e_z / 4\epsilon_0 h = 1.094 \times 10^6$  m/s. This is different from the speed of revolution in the **first Bohr orbit** of the nuclear model, which is  $u_1 = Ne^2 / \epsilon_0 h$ . A knowledge of the charge  $+Ne$  in the nucleus, as may be obtained from experiment, is required in order to determine the radius  $s_1 = \epsilon_0 h^2 / 4N\pi m e^2$  and speed  $u_1$  of revolution (equations 16 and 17) in the nuclear model.

The Rutherford-Bohr nuclear model of the hydrogen atom does not distinguish between the models in the gaseous state and in the liquid or solid state. Equation (10) is similar to the Balmer-Rydberg formula (equation 3), but the Rydberg constant  $R$  (equation 4) is different from  $S$  (equation 7). The Balmer-Rydberg formula for the emitted radiation from the hydrogen atom, being the result of experimental observations, must stand for something. It is suggested here that the new nuclear model obtains with the liquid or solid phase of hydrogen. The proposed nuclear model is reserved for the liquid or solid state and the non-nuclear model for the gaseous state. The Balmer-Rydberg formula (equation 3.1), giving the frequencies of radiation in the hydrogen atom spectrum, should be for the gaseous state.

The proposed nuclear model of the hydrogen atom is like the solar system with the planets revolving round a much heavier center of attraction, the Sun, held together by gravitational force of attraction. But why should a nucleus, with  $+Ne$  positive charges, not disintegrate due to repulsion between the  $N$  charges? The radiative model of the hydrogen atom provides an answer. While there is no magnetic field at the center of the non-nuclear model, due to the revolution of oppositely charged particles in the same sense, there should be a strong magnetic field at the center of the nuclear model, due to the revolution of the electrons in the same sense. This magnetic field, perpendicular to the plane of the orbit, should constrain a

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rotating or revolving nucleus in a central position by providing an inward force to balance the outward electrostatic and centrifugal force on the nucleus. Should the center fall apart, the particles of the nuclear model would disperse to become the particles in the non-nuclear model.

### Conclusion

The Introduction of radiative nuclear and non-nuclear models of the hydrogen atom with a number  $N$  of coplanar orbits, different from the Rutherford's nuclear model, has been achieved. The nuclear model for the solid state and the non-nuclear model for the gaseous state of hydrogen. The frequency of emitted radiation from the proposed models is related to the frequency of revolution of the charged particle, in the  $n$ th orbit, around a center of force of attraction; something which Bohr's quantum theory failed to do. The source of radiation, from the hydrogen atom, in the solid or state is explained.

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