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Short Communication

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Quantum Mechanics Based on Particles as Mutual Energy Flow Rather Than Self-Energy Flow

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Abstract

According to classical electromagnetic theory, quantum mechanics, and quantum field theory, particles are often associated with self-energy flow. In electromagnetic theory, the so-called self-energy flow corresponds to the Poynting vector of energy flow, and in quantum mechanics, it represents the probability current density. The author believes this is likely incorrect. The author finds that self-energy flow can be replaced by mutual energy flow. Mutual energy flow consists of retarded waves emitted by the source and advanced waves emitted by the sink. The source includes the primary coil of a transformer, the transmitting antenna, radiation atoms, and the cathode emitting electrons. The sink includes the secondary coil of the transformer, the receiving antenna, absorbing atoms, and the anode receiving electrons. Originally, all energy flows are generated by the source. If we assume energy flow is generated jointly by the source and the sink, we first need to compress the field produced by the source to half its original size. In this way, the source produces half of the field, and the sink produces the other half. The mutual energy flow generated by the source and sink together will exactly match the original self-energy flow produced by the source. However, there is still a problem. If the energy flow is mutual energy flow rather than self-energy flow, then self-energy flow should not transfer energy. However, if we do not alter the original electromagnetic theory or quantum theory, self-energy flow still transmits energy. This creates an absurd situation where mutual energy flow particles and self-energy flow particles are two distinct particles, both transferring energy. The author finds that both self-energy flows in electromagnetic theory and quantum theory should be reactive power and therefore do not transfer energy. The author adds a magnetic field, source, and sink to the Schrödinger equation and extends the mutual energy flow law from electromagnetic theory to quantum theory. The author then corrects the phase of the magnetic field in Maxwell's electromagnetic theory and quantum theory by 90 degrees. After the correction, both self-energy flows in the two theories become reactive power. Thus, particles, including photons, electrons, and all other particles, are mutual energy flows, not self-energy flows. For electromagnetic theory, the author also proves that the composite energy flow of many photon mutual energy flows coincides with the self-energy flow corresponding to the Poynting vector calculated from Maxwell's electromagnetic theory. Thus, electromagnetic waves have three different modes: (1) broadcasting mode, where the source and sink simultaneously emit spherical waves at random, which decay with propagation distance. (2) Photon mode, where the retarded wave emitted by the source and the advanced wave emitted by the sink are perfectly synchronized. At this point, the advanced wave forms the waveguide of the retarded wave, and the retarded wave forms the waveguide of the advanced wave. Due to the interference between the retarded and advanced waves, the two waves interfere constructively along the line connecting the source and the sink and weaken in other directions, ultimately turning into quasi-planar waves along the connection line between the source and sink. These retarded and advanced waves constitute the mutual energy flow, which is the photon. (3) The average energy flow consisting of countless photons is consistent with the energy flow calculated according to the Poynting vector. Therefore, Maxwell's electromagnetic theory is still valid for calculating average radiation electromagnetic energy flow. However, calculating photons requires the author's electromagnetic theory. The situation for electrons or other particles is roughly similar.

Keywords: Maxwell, Schrödinger, Silver-Müller, Retarded Wave, Advanced Wave, Retarded Potential, Advanced Potential, Quantum Mechanics, Electromagnetic Wave, Photon, Electron, Electric Field, Magnetic Field

1. Introduction

1.1. Transactional Interpretation

How are particles and waves related? How can the wave-particle duality paradox be explained? This is currently the biggest challenge in quantum mechanics and quantum field theory. A series of quantum mechanical interpretations have been proposed to solve this problem. The most prominent of these is the Copenhagen interpretation of quantum mechanics. This interpretation states that the square of the wave function amplitude gives the probability of a particle appearing. Waves collapse into particles during measurement. There are also many-worlds and many-histories quantum mechanical interpretations. The navigation wave quantum interpretation is one among dozens of different interpretations. Among these, the author notes that the transactional interpretation of quantum mechanics [1,2] is quite reliable.

The transactional interpretation, based on Wheeler and Feynman's absorber theory [3,4], suggests that a current can emit half retarded waves and half advanced waves. Both the source and the sink are currents, so they each emit half retarded waves and half advanced waves. Cramer's transactional interpretation also makes some modifications to Wheeler and Feynman's absorber theory, suggesting that the waves emitted by a current do not need to be omnidirectional; for example, the source can emit a retarded wave forward and an advanced wave backward. Thus, the transactional interpretation tells us that the source emits a retarded wave, and the sink emits an advanced wave. The handshake between the retarded and advanced waves forms the particle. Suppose the source is at x=0 and the sink is at x=L, the source emits a retarded wave to the right. The sink emits an advanced wave to the left. The source emits an advanced wave to the left, and the sink emits a retarded wave to the right. On the left of the source, both the source and the sink emit advanced waves, but they cancel each other out due to a 180-degree phase difference. On the right of the sink, both the source and the sink emit retarded waves, but they cancel each other out due to a 180-degree phase difference. Suppose the source is at x=0 and the sink is at x=L. The wave emitted by the source is:

$$\psi_1 = \begin{cases} -A\exp(jkx) & x < 0 \\ A\exp(jkx) & x \geq 0 \end{cases} \tag{1}$$

The wave emitted by the sink is:

$$\psi_2 = \begin{cases} A\exp(jkx) & x \leq L \\ -A\exp(jkx) & x > L \end{cases} \tag{2}$$

Thus, the sum of the two waves from the source and the sink is:

$$\psi_1 + \psi_2 = \begin{cases} -A\exp(jkx) + A\exp(jkx) & x < 0 \\ A\exp(jkx) + A\exp(jkx) & 0 \leq x \leq L \\ A\exp(jkx) - A\exp(jkx) & L < x \end{cases} \tag{3}$$

$$= 2A\exp(jkx) \begin{cases} 0 & x < 0 \\ 1 & 0 \leq x \leq L \\ 0 & L < x \end{cases} \tag{4}$$

We see that between the source and the sink, the retarded and advanced waves are perfectly synchronized, so they add up. This ensures that energy flows from the source to the sink. Outside this region, the waves emitted by the source and the sink cancel out. Therefore, the energy flow is generated at the source and annihilated at the sink, which suggests that there is a particle between the source and the sink.

The problem with this interpretation is: (1) it does not explain why the two advanced waves have a 180-degree phase difference to the left of the source, and the two retarded waves have a 180-degree phase difference to the right of the sink. (2) Can both the retarded wave and the advanced wave be represented by the same plane wave? (3) The source and sink should emit spherical waves, but in the transactional interpretation, spherical waves are turned into plane waves. Is this possible? Why does the retarded wave emitted by the source exactly synchronize with the advanced wave emitted by the sink? Due to these problems, the transactional interpretation has not become a mainstream interpretation.

1.2. Mutual Energy Flow Theory

In 1987, the author proposed the mutual energy theorem [5]:

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* + \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \quad (5)$$

When $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]^T$ and $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]^T$, if one is a retarded wave and the other is a advanced wave, we have:

$$\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}\, d\Gamma = 0 \quad (6)$$

Here, Γ is a sphere with a radius of ∞. This is because the retarded wave and the advanced wave reach the surface, one in the future and the other in the past. They are not simultaneously zero. Therefore, the surface integral mentioned above is zero.Thus, the mutual energy theorem holds:

$$-\int_{V_1} \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1\, dV = \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*\, dV \quad (7)$$

In 1989, the Huygens' principle was proposed [6], assuming that the current $\boldsymbol{J}_2$ exists outside the surface Γ, we have:

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}\, d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*)\, dV \quad (8)$$

$$\boldsymbol{J}_2 = \delta(\boldsymbol{x} - \boldsymbol{x}')\hat{x} \quad (9)$$

where $\boldsymbol{x} \in V$.

$$-\oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}\, d\Gamma = \int_V (\boldsymbol{E}_1 \cdot \delta(\boldsymbol{x} - \boldsymbol{x}')\hat{x})\, dV = \boldsymbol{E}_1(\boldsymbol{x}) \cdot \hat{x} = \boldsymbol{E}_{1x}(\boldsymbol{x}) \quad (10)$$

Thus, $\boldsymbol{E}_{1x}$ does not need to be computed based on the current $\boldsymbol{J}_1$, but rather from the electric and magnetic fields on surface Γ with $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]^T$. The surface Γ can be any closed surface or infinite open surface that separates the currents $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. Similarly, $\boldsymbol{E}_{1y}$ and $\boldsymbol{E}_{1z}$ can be computed.

In 2017, the author discovered that the mutual energy theorem is a Fourier transform of Welch's Reciprocity Theorem [7]. Welch's Reciprocity Theorem states:

$$-\int_{t=-\infty}^{\infty} dt \int_{V_1} \boldsymbol{E}_2 \cdot \boldsymbol{J}_1\, dV = \int_{t=-\infty}^{\infty} dt \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2\, dV \quad (11)$$

Therefore, the Welch reciprocity theorem and the mutual energy theorem are physically identical formulas, but their interpretations differ. As a reciprocity theorem, one of the two electromagnetic fields, ξ_1, ξ_2, is virtual or mathematical. For example, ξ_1 is a retarded wave, and ξ_2 is a advanced wave. One might consider the advanced wave to be a virtual electromagnetic wave. However, for the energy theorem, both fields ξ_1, ξ_2 must be physically real. In other words, both the retarded and advanced waves must be physically real.

In engineering, it is generally not accepted that the advanced wave is a physically real entity, and it is considered a virtual solution of Maxwell's equations. This solution only has mathematical significance. If this is the case, then the mutual energy theorem and Welch's reciprocity theorem can only be referred to as reciprocity theorems, not energy theorems.

However, the author knows that in circuit theory:

$$P = \mathrm{Re}(VI^*) \quad (12)$$

where P is the average power, and Re represents taking the real part. In electromagnetic field theory, the power should be:

$$P = \mathrm{Re} \int \boldsymbol{E} \cdot \boldsymbol{J}^* \, dV \tag{13}$$

Thus,

$$\mathrm{Re} \int \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 \, dV \tag{14}$$

Clearly, this is the work done by the electric field $\boldsymbol{E}_2$ on the circuit $\boldsymbol{J}_1$.

$$\mathrm{Re} \int \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* \, dV \tag{15}$$

is the work done by the electric field $\boldsymbol{E}_1$ on the current $\boldsymbol{J}_2$. Therefore, the mutual energy theorem is evidently an energy theorem. If the mutual energy theorem is an energy theorem, it requires that the advanced wave must be a physically real entity. The author began searching for literature proving that the advanced wave is physically real, finding Wheeler-Feynman absorber theory, Cramer quantum mechanical transactional interpretation, and the theory of action and reaction [8-10]. Further, the author proved the mutual energy theorem from the Poynting theorem. Since the Poynting theorem is widely accepted as an energy theorem, the mutual energy theorem is indeed an energy theorem, which means that Welch's reciprocity theorem is indeed an energy theorem.

Additionally, the author proposed the law of mutual energy flow:

$$-\int_{t=-\infty}^{\infty} dt \int_{V_1} \boldsymbol{E}_2 \cdot \boldsymbol{J}_1 \, dV = (\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} dt \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 \, dV \tag{16}$$

$$(\xi_1, \xi_2) = \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} \, d\Gamma \tag{17}$$

1.3. Further Development of the Mutual Energy Flow Theory

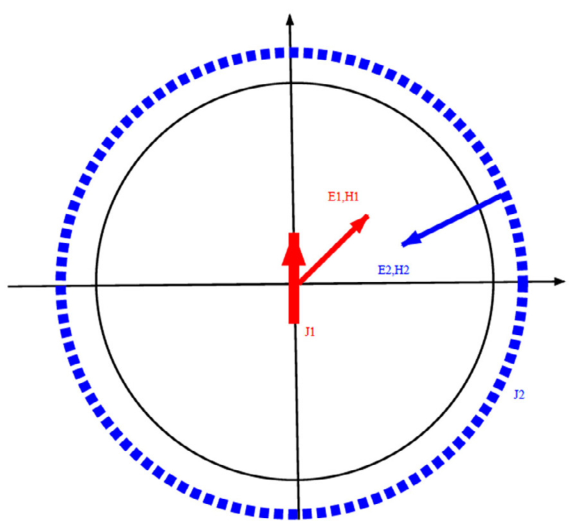

Figure1: Red represents the current source $\boldsymbol{J}_1$, producing the fields $\boldsymbol{E}_1$, $\boldsymbol{H}_1$. To calculate the electromagnetic field at $\boldsymbol{x}$, we assume the presence of an absorbing sink $\boldsymbol{J}_2$ at infinity, shown in blue, which produces the electromagnetic fields $\boldsymbol{E}_2$, $\boldsymbol{H}_2$.

In Maxwell's field theory, the fields at the point $\boldsymbol{x} \in V$ are generated by the source $\boldsymbol{J}_1$, see Figure 1. However, in the mutual energy flow theory, the source generates a retarded wave, and we further assume that the absorbing sink generates a advanced wave. Therefore, it is necessary to arrange the absorbing current source $\boldsymbol{J}_2$ on a spherical surface at infinite radius. Clearly, the field at $\boldsymbol{x}$ is produced by both $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. By initial estimation, the values of $\boldsymbol{E}_1(\boldsymbol{x})$ and $\boldsymbol{E}_2(\boldsymbol{x})$ are equally large, so the total electric field is:

$$\boldsymbol{E}(\boldsymbol{x}) = \boldsymbol{E}_1(\boldsymbol{x}) + \boldsymbol{E}_2(\boldsymbol{x}) \tag{18}$$

$$\boldsymbol{H}(\boldsymbol{x}) = \boldsymbol{H}_2(\boldsymbol{x}) + \boldsymbol{H}_2(\boldsymbol{x}) \tag{19}$$

$$\boldsymbol{E}(\boldsymbol{x}) \times \boldsymbol{H}(\boldsymbol{x}) = (\boldsymbol{E}_1(\boldsymbol{x}) + \boldsymbol{E}_2(\boldsymbol{x})) \times (\boldsymbol{H}_1(\boldsymbol{x}) + \boldsymbol{H}_2(\boldsymbol{x}))$$

$$= \boldsymbol{E}_1(\boldsymbol{x}) \times \boldsymbol{H}_1(\boldsymbol{x}) + \boldsymbol{E}_2(\boldsymbol{x}) \times \boldsymbol{H}_2(\boldsymbol{x}) + (\boldsymbol{E}_1(\boldsymbol{x}) \times \boldsymbol{H}_2(\boldsymbol{x}) + \boldsymbol{E}_2(\boldsymbol{x}) \times \boldsymbol{H}_1(\boldsymbol{x}))$$

$$= \boldsymbol{S}_{11} + \boldsymbol{S}_{22} + \boldsymbol{S}_m \tag{20}$$

where.

$$\boldsymbol{S}_{11} = \boldsymbol{E}_1(\boldsymbol{x}) \times \boldsymbol{H}_1(\boldsymbol{x}) \tag{21}$$

$$\boldsymbol{S}_{22} = \boldsymbol{E}_2(\boldsymbol{x}) \times \boldsymbol{H}_2(\boldsymbol{x}) \tag{22}$$

$$\boldsymbol{S}_m = \boldsymbol{E}_1(\boldsymbol{x}) \times \boldsymbol{H}_2(\boldsymbol{x}) + \boldsymbol{E}_2(\boldsymbol{x}) \times \boldsymbol{H}_1(\boldsymbol{x}) \tag{23}$$

Originally, only $\boldsymbol{S}_{11}$ was responsible for energy transfer. Now, with the addition of $\boldsymbol{S}_{22}$ and $\boldsymbol{S}_m$ contributing to energy transfer, the total calculated energy has certainly increased. Therefore, it is necessary to reduce the electric field generated by the current. This aligns with the principle proposed by Wheeler and Feynman: the idea of "half-retarded wave and half-advanced wave". In other words, both the retarded and advanced potentials generated by the current are halved:

$$\boldsymbol{A}^{(r)} \to \frac{1}{2}\boldsymbol{A}^{(r)}, \quad \boldsymbol{A}^{(a)} \to \frac{1}{2}\boldsymbol{A}^{(a)} \tag{24}$$

$$\boldsymbol{E}_1 \to \frac{1}{2}\boldsymbol{E}_1, \quad \boldsymbol{H}_1 \to \frac{1}{2}\boldsymbol{H}_1 \tag{25}$$

$$\boldsymbol{E}_2 \to \frac{1}{2}\boldsymbol{E}_2, \quad \boldsymbol{H}_2 \to \frac{1}{2}\boldsymbol{H}_2 \tag{26}$$

From this we obtain:

$$-\int_{t=-\infty}^{\infty} dt \int_{V_1} \frac{1}{2}\boldsymbol{E}_2 \cdot \boldsymbol{J}_1 \, dV$$

$$= \int_{t=-\infty}^{\infty} dt \iint_{\Gamma} \left(\frac{1}{2}\boldsymbol{E}_1 \times \frac{1}{2}\boldsymbol{H}_2 + \frac{1}{2}\boldsymbol{E}_2 \times \frac{1}{2}\boldsymbol{H}_1\right) \cdot \hat{n} \, d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \int_{V_2} \frac{1}{2}\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 \, dV \tag{27}$$

Thus, we arrive at:

$$-\int_{t=-\infty}^{\infty} dt \int_{V_1} \boldsymbol{E}_2 \cdot \boldsymbol{J}_1 \, dV$$

$$= \frac{1}{2}\int_{t=-\infty}^{\infty} dt \iint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} \, d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2 \, dV \tag{28}$$

The electromagnetic fields in these expressions are calculated based on Maxwell's electromagnetic theory. After this correction, a factor of $\frac{1}{2}$ appears in the mutual energy flow law.

We know that under these conditions, the retarded and advanced waves are synchronized, which requires that

$$\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1$$

be a real quantity. This implies that both $\boldsymbol{E}_1 \times \boldsymbol{H}_2^*$ and $\boldsymbol{E}_2^* \times \boldsymbol{H}_1$ must also be real. This further requires:

$$E_1 \sim H_2, \quad E_2 \sim H_1$$

where ~ means "proportional to" or "in phase with." Additionally, from Maxwell's theory, we know that the electric and magnetic fields in an electromagnetic wave are in phase, i.e.,

$$E_1 \sim H_1, \quad E_2 \sim H_2$$

From this, it follows that:

$$\boldsymbol{E}_1 = \boldsymbol{E}_2 \tag{29}$$

$$\boldsymbol{H}_1 = \boldsymbol{H}_2 \tag{30}$$

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \tag{31}$$

$$= \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_1 + \boldsymbol{E}_1 \times \boldsymbol{H}_1)$$

$$= \boldsymbol{E}_1 \times \boldsymbol{H}_1 = \boldsymbol{S}_{11} \tag{32}$$

This shows that in this case, the energy exchange calculated by $\boldsymbol{S}_m$ is equal to the mutual energy flow $\boldsymbol{S}_{11}$ obtained by considering only the retarded wave. Thus, according to Maxwell's electromagnetic theory, the radiated energy $\boldsymbol{S}_{11}$ can be replaced by mutual energy flow. Therefore, the energy transfer should be governed by the mutual energy flow, not by self-energy flow corresponding to $\boldsymbol{S}_{11}$, $\boldsymbol{S}_{22}$. Thus, $\boldsymbol{S}_{11}$ and $\boldsymbol{S}_{22}$ should not transfer energy. They should satisfy:

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} \boldsymbol{S}_{11} \cdot \hat{n}\, d\Gamma = \int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} \boldsymbol{S}_{22} \cdot \hat{n}\, d\Gamma = 0 \tag{33}$$

The problem arises because, according to Maxwell's electromagnetic theory, for any antenna, such as a dipole antenna:

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} \boldsymbol{S}_{11} \cdot \hat{n}\, d\Gamma \neq 0 \tag{34}$$

$$\int_{t=-\infty}^{\infty} dt \oiint_{\Gamma} \boldsymbol{S}_{22} \cdot \hat{n}\, d\Gamma \neq 0 \tag{35}$$

This creates a contradiction. This suggests that Maxwell's electromagnetic theory has a problem. At least when Maxwell's theory is combined with the concept of mutual energy flow, the issue becomes more apparent.

1.4. Resolution of the Contradiction

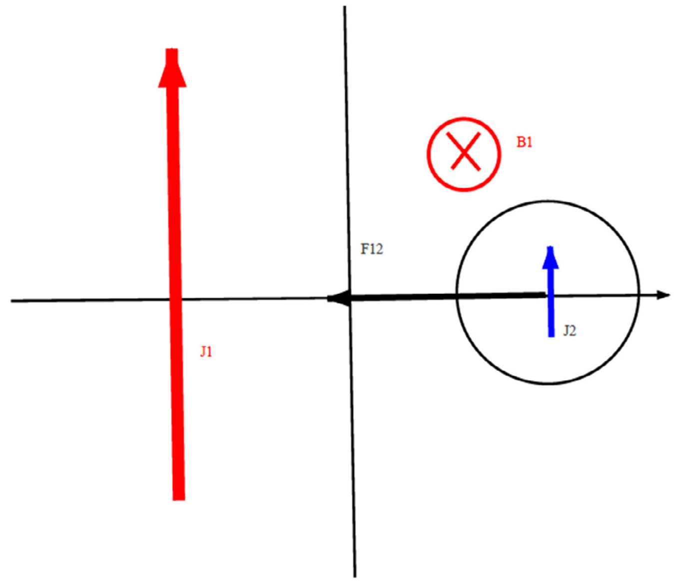

Figure 2: The red part represents the current source $\boldsymbol{I}_1\ d\boldsymbol{l}_1$, which generates the magnetic field $\boldsymbol{B}_1$. To measure the magnetic field at point $\boldsymbol{x}$, we measure the force $\boldsymbol{F}_{1\to2}$. We also know the magnitude of the current element $\boldsymbol{I}_2\ d\boldsymbol{l}_2$, and with this, we can calculate the magnetic field $\boldsymbol{B}_1$. The specific measurement device can be completed by a Hall device. The force $\boldsymbol{F}_{1\to2}$ can be measured by the voltage output from the Hall effect device.

The author found that this contradiction arises from issues in the definition of the magnetic field in Maxwell's electromagnetic theory. The original definition of the magnetic field is based on Ampere's force law, as we know:

$$\boldsymbol{F}_{1\to2} = I_2 d\boldsymbol{l}_2 \times \boldsymbol{B}_1 \tag{36}$$

In this formula, the force $\boldsymbol{F}_{1\to2}$ exerted by the current $\boldsymbol{I}_1\ d\boldsymbol{l}_1$ on the current $\boldsymbol{I}_2 d\boldsymbol{l}_2$ is given by $\boldsymbol{F}_{1\to2}$. The magnetic field $\boldsymbol{B}_1$ is produced by $\boldsymbol{I}_1\ d\boldsymbol{l}_1$. $\boldsymbol{B}_1$ is determined by $\boldsymbol{F}_{1\to2}$ and $\boldsymbol{I}_2\ d\boldsymbol{l}_2$. See Figure 2. The specific measurement device to measure the magnetic field is a Hall effect sensor. The magnitude of the magnetic field defined in this way is given by Biot-Savart's law:

$$\boldsymbol{B}_1 = \frac{\mu_0}{4\pi} I_1 d\boldsymbol{l}_1 \times \frac{\boldsymbol{r}}{r^3} \tag{37}$$

$$\boldsymbol{r} = \boldsymbol{x}_2 - \boldsymbol{x}_1 \tag{38}$$

$$r = ||\boldsymbol{r}|| \tag{39}$$

Considering:

$$I_1 d\boldsymbol{l}_1 \to \boldsymbol{J}_1 dV \tag{40}$$

The magnetic field generated by the current in the region V is:

$$\boldsymbol{B}_1 = \frac{\mu_0}{4\pi} \int_V \boldsymbol{J}_1 \times \frac{\boldsymbol{r}}{r^3} dV \tag{41}$$

Furthermore, we know that under quasi-static conditions, the vector potential is:

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi} \int_V \frac{\boldsymbol{J}_1}{r} dV. \tag{42}$$

$$\nabla \times \boldsymbol{A}_1 = \frac{\mu_0}{4\pi} \int_V \nabla \frac{1}{r} \times \boldsymbol{J}_1 dV$$

$$= \frac{\mu_0}{4\pi} \int_V \boldsymbol{J}_1 \times \frac{\boldsymbol{r}}{r^3} dV \quad (43)$$

We find that:

$$\boldsymbol{B}_1 = \nabla \times \boldsymbol{A}_1 \quad (44)$$

The current issue is whether we can define:

$$\boldsymbol{B}_1 \equiv \nabla \times \boldsymbol{A}_1 \quad (45)$$

and extend this definition to the case of the retarded potential:

$$\boldsymbol{A}_1^{(r)} = \frac{\mu_0}{4\pi} \int_V \frac{\boldsymbol{J}_1}{r} \exp(-jkr) dV \quad (46)$$

Then:

$$\boldsymbol{B}_1^{(r)} \equiv \nabla \times \boldsymbol{A}_1^{(r)} \quad (47)$$

Does this still hold? The author believes that the magnetic field and vector potential are two entirely different quantities. They just happen to have the relationship in equation (44) under quasi-static conditions. However, it cannot be guaranteed that equation (47) will still hold in the case of the retarded potential.

1.5. Average Magnetic Field on a Loop

To measure the average magnetic field $\boldsymbol{B}_1 = B_1\hat{y}$ in the *x-z* plane at $z = 0, x = L$, which is generated by the current element $\boldsymbol{I}_1\, d\boldsymbol{l}_1$, we place several Hall elements on a ring centered at $\boldsymbol{x}(z = 0, x = L)$. See figure 3.

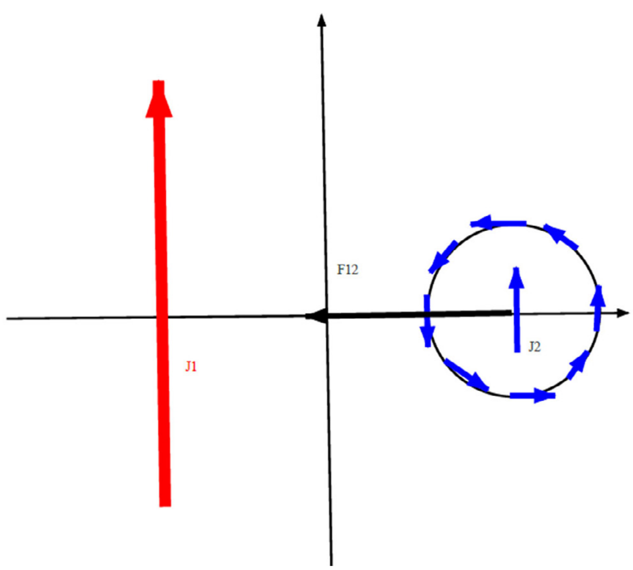

Figure 3: The red part represents the current element $\boldsymbol{I}_1\, d\boldsymbol{l}_1$, which generates the magnetic field $\boldsymbol{B}_1$. To measure the magnetic field at point $\boldsymbol{x}$, we measure the force $\boldsymbol{F}_{1\to 2}$. We also know the magnitude of the current element $\boldsymbol{I}_2\, d\boldsymbol{l}_2$, and to measure the average magnetic field along the circular loop, we arrange $\boldsymbol{I}_2\, d\boldsymbol{l}_2$ along the loop. This allows us to measure the average magnetic field on the loop. The specific implementation is to place multiple Hall devices on the circular loop shown in the figure.

Each Hall device measures a magnetic field, and the average magnetic field is:

$$\overline{\boldsymbol{B}}_1 = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{B}_{1i} \tag{48}$$

If we assume the radius of the loop is very small, under quasi-static conditions:

$$\boldsymbol{B}_{1i} = \boldsymbol{B}_1 \tag{49}$$

Then obviously:

$$\overline{\boldsymbol{B}}_1 = \boldsymbol{B}_1 \tag{50}$$

Thus, we have:

$$\overline{\boldsymbol{B}}_1 = \nabla \times \boldsymbol{A}_1 \tag{51}$$

$\overline{\boldsymbol{B}}_1$ is the average magnetic field on the loop. By comparing equations (44, 47), we know that we can define:

$$\overline{\boldsymbol{B}}_1 \equiv \nabla \times \boldsymbol{A}_1 \tag{52}$$

When dealing with radiated electromagnetic fields, we can also define:

$$\overline{\boldsymbol{B}}_1^{(r)} \equiv \nabla \times \boldsymbol{A}_1^{(r)} \tag{53}$$

If:

$$\overline{\boldsymbol{B}}_1^{(r)} = \boldsymbol{B}_1^{(r)} \tag{54}$$

Then there is no issue whether we use $\overline{\boldsymbol{B}}_1^{(r)}$ or $\boldsymbol{B}_1^{(r)}$. The problem arises in the case of radiated electromagnetic fields, where $\overline{\boldsymbol{B}}_1^{(r)}$ and $\boldsymbol{B}_1^{(r)}$ are different. Thus, we are uncertain about which formula, equation (53) or (47), is correct.

In conclusion, the author discovered that the definition of the magnetic field using the curl of the vector potential is not the same as the magnetic field defined using Ampere's force. When defining the magnetic field using Ampere's force, it can be divided into two cases: the average magnetic field measured along a loop and the magnetic field measured along a straight line. These two cases are consistent under quasi-static conditions. However, for electromagnetic waves, i.e., radiated electromagnetic fields, the two cases differ. Next, we will study the radiated electromagnetic field.

1.6. Electromagnetic Radiation

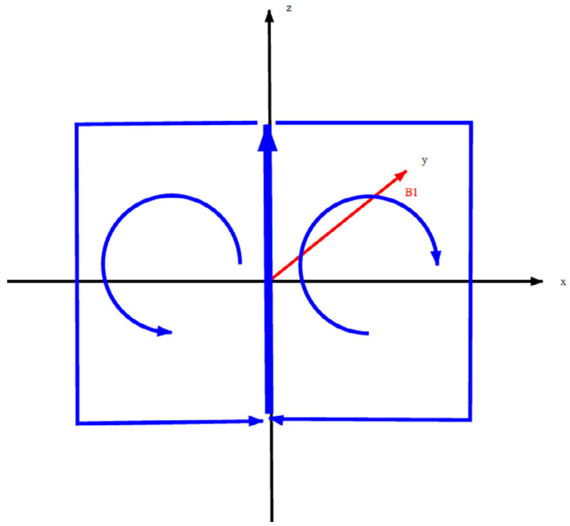

Figure 4: Suppose there is a fluctuating magnetic field along the y-axis. We use a straight current element to measure this magnetic field. However, since it is unclear whether the current element belongs to the left or right loop, it is impossible to measure the induced electromotive force.

In the case of radiated electromagnetic waves, we have Faraday's law:

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} = -j\omega \boldsymbol{B} \tag{55}$$

$$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = -j\omega \iint_\Gamma \boldsymbol{B} \cdot \hat{n} d\Gamma = -j\omega\mu_0 S B_n \tag{56}$$

$$B_n = \frac{1}{-j\omega}\frac{1}{S}\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} \tag{57}$$

Where $S = \iint_\Gamma d\Gamma$, We find that the magnetic field is defined on the loop C, which is a closed loop. The question is whether the magnetic field can be defined on a straight current element?

$$B_n = \frac{1}{-j\omega}\frac{1}{S}\int_L \boldsymbol{E} \cdot d\boldsymbol{l} \tag{58}$$

where L is a straight line segment. We find that this definition is invalid under quasi-static conditions. This is because, for example, with a uniform alternating magnetic field, as shown in figure 4:

$$\boldsymbol{B}_1 = \hat{y}\exp(j\omega t) \tag{59}$$

We use $I_2 d\boldsymbol{l}_2 \, L\hat{z}$ to measure it, but since it is unclear whether this current element surrounds the magnetic field in the left or right loop, the result is opposite in both cases. Therefore, no result can be obtained. Under quasi-static conditions, to measure the magnetic field, it must be done on a loop.

But for radiated electromagnetic fields, the electromagnetic field at any point in space is well-defined. For example, the vector potential is:

$$\boldsymbol{A} = A_0\exp(-jkx)\hat{z} \tag{60}$$

The electric field is:

$$\boldsymbol{E} = -j\omega\boldsymbol{A} = j\omega A_0\exp(-jkx)(-\hat{z}) \tag{61}$$

We can integrate along a straight line:

$$\int_L \boldsymbol{E} \cdot d\boldsymbol{l} = \int_L (j\omega A_0 \exp(-jkx)(-\hat{z})) \cdot d\boldsymbol{l}\hat{z}$$

$$= -Lj\omega A_0 \exp(-jkx)$$

$$= E_0 L \exp(-jkx) \tag{62}$$

where:

$$E_0 = -j\omega A_0 \tag{63}$$

Therefore, for radiated electromagnetic fields, the integral $\int_L \boldsymbol{E} \cdot d\boldsymbol{l}$ has a clear definition, and we can use it to define the magnetic field. This leads to two definitions of the magnetic field: (57) and (58). To distinguish between these two definitions, we use superscripts (C) and (L).

$$B_n^{(C)} = \frac{1}{-j\omega}\frac{1}{S}\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} \tag{64}$$

This is the magnetic field measured along a loop.

$$B_n^{(L)} = \frac{1}{-j\omega}\frac{1}{S}\int_L \boldsymbol{E} \cdot d\boldsymbol{l} \tag{65}$$

This is the magnetic field measured along a straight line, where the normal direction n is $\hat{z}$.

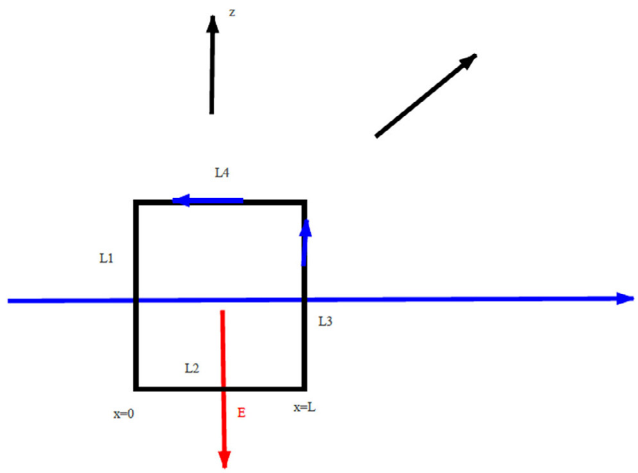

Figure 5: Assume there is a plane electromagnetic wave $\boldsymbol{E} = E_0 \exp(-jkx)(-\hat{z})$. Consider a square loop. We measure the magnetic field using this loop.

$$B_n^{(L)} = \frac{1}{-j\omega}\frac{1}{S}\int_{L_1} (E_0 \exp(-jk0)(-\hat{z})) \cdot d\boldsymbol{l}(-\hat{z})$$

$$= \frac{1}{-j\omega}\frac{1}{S}\int_{L_1} (E_0 \exp(-jk0)) d\boldsymbol{l}$$

$$= \frac{1}{-j\omega}\frac{1}{S} E_0 L$$

$$= \frac{1}{-j\omega}\frac{1}{S}(-j\omega A_0) L$$

$$= \frac{A_0 L}{S} \tag{66}$$

Assume:

$$L = 2\pi R \tag{67}$$

$$S = \pi R^2 \tag{68}$$

Then:

$$B_n^{(L)} = \frac{A_0 2\pi R}{\pi R^2} = \frac{2A_0}{R} \tag{69}$$

$$B_n^{(C)} = \frac{1}{-j\omega}\frac{1}{S}\oint_C \boldsymbol{E}\cdot d\boldsymbol{l}$$

$$= \frac{1}{-j\omega}\frac{1}{S}\left(\int_{L_1} \boldsymbol{E}\cdot d\boldsymbol{l} + \int_{L_2} \boldsymbol{E}\cdot d\boldsymbol{l} + \int_{L_3} \boldsymbol{E}\cdot d\boldsymbol{l} + \int_{L_4} \boldsymbol{E}\cdot d\boldsymbol{l}\right)$$

$$= \frac{1}{-j\omega}\frac{1}{S}\left(\int_{L_1} \boldsymbol{E}\cdot d\boldsymbol{l} + \int_{L_3} \boldsymbol{E}\cdot d\boldsymbol{l}\right)$$

$$= \frac{1}{-j\omega}\frac{1}{S}\left(\int_{L_1} \boldsymbol{E}\cdot d\boldsymbol{l} + \int_{L_3} \boldsymbol{E}\cdot d\boldsymbol{l}\right) \tag{70}$$

and:

$$B_n^{(C)} = \frac{1}{-j\omega}\frac{1}{S}\left(\int_{L_1} (E_0\exp(-jk0)(-\hat{z}))\cdot d\boldsymbol{l}(-\hat{z}) + \int_{L_3} (E_0\exp(-jkL)(-\hat{z}))\cdot d\boldsymbol{l}\hat{z}\right)$$

$$= \frac{1}{-j\omega}\frac{1}{S}\left(\int_{L_1} (E_0 dl) - \int_{L_3} (E_0\exp(-jkL)d\boldsymbol{l}\right)$$

$$= \frac{1}{-j\omega}\frac{1}{S}(E_0 L - LE_0\exp(-jkL))$$

$$= \frac{1}{-j\omega}\frac{L}{S}E_0(1 - E_0\exp(-jkL)) \tag{71}$$

We find that:

$$B_n^{(C)} \neq B_n^{(L)} \tag{72}$$

We can also derive:

$$B_n^{(C)} = \frac{1}{-j\omega}\frac{1}{S}\oint_C \boldsymbol{E}\cdot d\boldsymbol{l} \rightarrow \frac{1}{-j\omega}(\nabla\times\boldsymbol{E})_n$$

$$= \frac{1}{-j\omega}(jk\hat{x}\times(E_0\exp(-jkx)(-\hat{z})))_n$$

$$= \frac{1}{-j\omega}jk(E_0\exp(-jkx))\hat{y})_n$$

$$= \frac{1}{-j\omega}jk(E_0\exp(-jkx))$$

$$= -\frac{k}{\omega}(E_0\exp(-jkx))$$

$$= -\frac{k}{\omega}E_0$$

$$= -\frac{k}{\omega}(-j\omega A_0)$$

$$= jkA_0$$

$$= j\frac{2\pi}{\lambda}A_0 \tag{73}$$

$$B_n^{(L)} \sim A_0 \tag{74}$$

$$B_n^{(C)} \sim jA_0 \sim E \tag{75}$$

It can be seen that the phase of $B_n^{(L)}$ and $B_n^{(C)}$ is different! The average magnetic field measured along the loop is in phase with the electric field E_n. The magnetic field calculated by Maxwell's electromagnetic theory is not considered to have the correct phase by the author. The magnetic field measured along the straight line $B_n^{(L)}$ is in phase with the vector potential A_0, and the author believes that this straight-line measured magnetic field is the correct magnetic field.

2. Left-Going Waves and Right-Going Waves

In this section, we study the radiation of a finite current sheet. Assume the current is in the $\hat{z}$ direction. The electromagnetic wave can either propagate from left to right across the current or from right to left across the current. This forms left-going and right-going waves. For the right-going wave, the wave is a advanced wave to the left of the current and a retarded wave to the right of the current. For the left-going wave, the wave is a advanced wave to the right of the current and a retarded wave to the left of the current. The following examples focus on right-going waves; the reasoning is similar for left-going waves. First, we study the electromagnetic field under quasi-static conditions.

2.1. Magnetic Field Under Quasi-Static Conditions

Assume the current and electromagnetic fields are as shown in Figure 6. The current is in the $\hat{z}$ direction, and the electric field is assumed to be in the -$\hat{z}$ direction. The magnetic field is in the $\hat{y}$ direction.

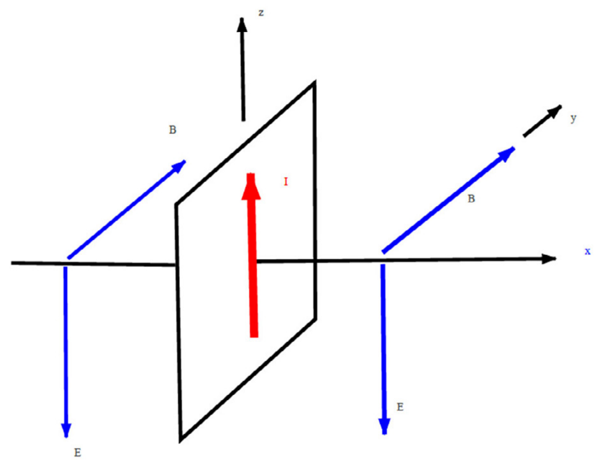

Figure 6: The current element is represented by the thick red arrow, and the electromagnetic field generated on either side is shown by the thin blue arrows.

Suppose there is a current:

$$\boldsymbol{J} = \hat{z}\delta(x)\begin{cases} 1 & |x||y| \le \frac{L}{2} \\ 0 & |x||y| > \frac{L}{2} \end{cases} \tag{76}$$

We calculate the electromagnetic field on either side of the current surface under magnetic quasi-static conditions. The magnetic vector potential is:

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r} dV \sim \hat{z}\int dx \int dy \frac{1}{r}\begin{cases} 1 & |x||y| \le \frac{L}{2} \\ 0 & |x||y| > \frac{L}{2} \end{cases} \tag{77}$$

Assuming the directions of the electric and magnetic fields on both sides of the current are as shown in figure 6, we assume the magnetic field is in the $\hat{z}$ direction and the electric field is in the -$\hat{z}$ direction. Therefore, the vector potential is:

$$\boldsymbol{A} \sim \hat{z}L^2\frac{1}{r} \sim \frac{1}{r}\hat{z} \tag{78}$$

The electric field is:

$$\boldsymbol{E} = -j\omega\boldsymbol{A} \sim -j\omega\frac{1}{r}\hat{z} \sim j(-\hat{z}) \tag{79}$$

Now, consider the magnetic field:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\sim \nabla \times \left(\frac{1}{r}\hat{z}\right) = \nabla\frac{1}{r} \times \hat{z}$$

$$= (-\frac{\boldsymbol{r}}{r^3}) \times \hat{z}$$

$$\sim (-\frac{\boldsymbol{x}}{|x|}) \times \hat{z} \tag{80}$$

If $x > 0$:

$$\nabla \times \boldsymbol{A} \sim (-\frac{x}{|x|}) \times \hat{z} \sim (-\hat{x}) \times \hat{z} = \hat{y} \tag{81}$$

If $x < 0$:

$$\nabla \times \boldsymbol{A} \sim (-\frac{x}{|x|}) \times \hat{z} \sim (\hat{x}) \times \hat{z} = -\hat{y} \tag{82}$$

$$\boldsymbol{B} \sim \hat{y} \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases} \tag{83}$$

In the above equation, the magnetic field on the right side of the current is in the $\hat{y}$ direction, and on the left side of the current, it is in the -$\hat{y}$ direction. This result is consistent with the magnetic field defined by Ampere's law (right-hand screw rule). Under quasi-static conditions, the electromagnetic field is not propagated as a wave but as a static field.

2.2. Right-Going Radiated Electromagnetic Field

2.2.1. Electromagnetic Field Calculated Using Maxwell's Electromagnetic Theory

From figure 6, it can be seen that on the right side of the current surface, the magnetic field is in the $\hat{y}$ direction, and on the left side, it is in the -$\hat{y}$ direction.

$$\exp(-jx) \tag{84}$$

The time factor is:

$$\exp(j\omega t) \tag{85}$$

The vector potential is:

$$\boldsymbol{A} = \frac{\mu_0}{4\pi} \int_V \frac{\boldsymbol{J}}{r} \exp(-jx) dV$$

$$\sim \hat{z} \int dx \int dy \frac{\mathbf{1}}{r} \exp(-jx) \begin{cases} 1 & |x||y| \leq \frac{L}{2} \\ 0 & |x||y| > \frac{L}{2} \end{cases}$$

$$\sim \hat{z} L^2 \frac{\mathbf{1}}{r} \exp(-jx)$$

$$\sim \frac{\mathbf{1}}{r} \exp(-jx) \hat{z} \tag{86}$$

Considering that $\frac{\mathbf{1}}{r}$ can be neglected because we are considering both the retarded and advanced waves simultaneously, this leads to the understanding that the retarded and advanced waves act as waveguides, and spherical waves become plane waves. Thus, the $\frac{\mathbf{1}}{r}$ factor can be neglected. The magnetic vector potential for the electromagnetic wave is:

$$\boldsymbol{A} \sim \exp(-jx) \hat{z} \tag{87}$$

This is the vector potential for a right-going wave, so the electric field for this electromagnetic wave is:

$$\boldsymbol{E} = -j\omega \exp(-jx)\hat{z} \sim j\exp(-jx)(-\hat{z}) \quad (88)$$

The magnitude of the electric field is:

$$E \sim j\exp(-jx) \quad (89)$$

To calculate the magnetic field according to Maxwell's electromagnetic theory:

$$\boldsymbol{B}_{Maxwell} = \nabla \times \boldsymbol{A}$$

$$\sim \nabla \exp(-jx) \times \hat{z}$$

$$= -j\exp(-jx)\hat{x} \times \hat{z}$$

$$= j\exp(-jx)\hat{y} \quad (90)$$

Thus,

$$B_{Maxwell} = j\exp(-jx) \quad (91)$$

It can be seen that, according to Maxwell's electromagnetic theory, the magnetic field B_{Maxwell} and the electric field E are in phase:

$$B_{Maxwell} \sim E \quad (92)$$

2.2.2. Electromagnetic Field Calculated Using the Author's Electromagnetic Theory

When calculating the electromagnetic field using the author's theory, the magnetic field according to Maxwell's theory needs to be corrected.

If $x > 0$, this is a retarded wave, and we consider the correction factor ($-j$):

$$\boldsymbol{B} = (-j)\boldsymbol{B}_{Maxwell} = (-j)j\exp(-jx)\hat{y}$$

$$= \exp(-jx)\hat{y} \quad (93)$$

If $x < 0$, this is an advanced wave, and we consider the correction factor (***j***):

$$\boldsymbol{B} = (j)\boldsymbol{B}_{Maxwell} = jj\exp(-jx)\hat{y}$$

$$= -\exp(-jx)\hat{y} \quad (94)$$

Thus,

$$\boldsymbol{B} = \exp(-jx)\hat{y}\begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases} \quad (95)$$

The electric field is given by equation (88), and

$$\boldsymbol{E} = j\exp(-jx)(-\hat{z}) \tag{96}$$

When $kx \to 0$, i.e., when x is near the current:

$$\lim_{x\to 0}\boldsymbol{E} = j(-\hat{z}) \tag{97}$$

$$\lim_{x\to 0}\boldsymbol{B} = \hat{y}\begin{cases}1 & x > 0\\ -1 & x < 0\end{cases} \tag{98}$$

These equations show that the electromagnetic fields calculated using this method can degenerate into the quasi-static magnetic field (79, 83), thus they are correct. However, if the magnetic field is calculated using Maxwell's electromagnetic theory, it cannot degenerate into the quasi-static electromagnetic field!

2.3. Radiated Electromagnetic Field Propagating to the Left

2.3.1. Electromagnetic Field Calculated Using Maxwell's Electromagnetic Theory

From figure 6, it is clear that on the current surface, the magnetic field to the right is in the $\hat{y}$ direction, and to the left is in the -$\hat{y}$ direction. Now, consider the wave propagating to the left, i.e., the spatial factor is exp(jkx). To simplify, let k=1, so the spatial factor is:

$$\exp(+jx) \tag{99}$$

The time factor is:

$$\exp(j\omega t) \tag{100}$$

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}}{r}\exp(+jx)dV$$

$$\sim \hat{z}\int dx\int dy\frac{\mathbf{1}}{r}\exp(+jx)\begin{cases}1 & |x||y| \le \frac{L}{2}\\ 0 & |x||y| > \frac{L}{2}\end{cases}$$

$$\sim \hat{z}L^2\frac{\mathbf{1}}{r}\exp(+jx)$$

$$\sim \frac{\mathbf{1}}{r}\exp(+jx)\hat{z} \tag{101}$$

Considering that $\frac{\mathbf{1}}{r}$ can be neglected, the magnetic vector potential for the electromagnetic wave is:

$$\boldsymbol{A} \sim \exp(+jx)\hat{z} \tag{102}$$

The electric field for this electromagnetic wave is:

$$\boldsymbol{E} = -j\omega\exp(+jx)\hat{z} \sim j\exp(+jx)(-\hat{z}) \tag{103}$$

The magnitude of the electric field is:

$$E \sim j\exp(+jx) \tag{104}$$

To calculate the magnetic field according to Maxwell's electromagnetic theory:

$$\boldsymbol{B}_{Maxwell} = \nabla \times \boldsymbol{A}$$

$$\sim \nabla\exp(+jx) \times \hat{z}$$

$$= j\exp(+jx)\hat{x} \times \hat{z}$$

$$= j\exp(+jx)(-\hat{y}) \tag{105}$$

For the left-going wave, we still choose the electric field direction to be ($-\hat{z}$), but the magnetic field direction is chosen as ($-\hat{y}$). Thus, the propagation direction of the electromagnetic wave is:

$$\boldsymbol{E} \times \boldsymbol{H} \sim (-\hat{z}) \times (-\hat{y}) = -\hat{x} \tag{106}$$

The propagation direction is $-\hat{x}$, which is correct. Therefore,

$$B_{Maxwell} = j\exp(+jx) \tag{107}$$

It can be seen that, according to Maxwell's electromagnetic theory, the magnetic field $\boldsymbol{B}_{Maxwell}$ and the electric field E are in phase:

$$B_{Maxwell} \sim E \tag{108}$$

2.3.2. Electromagnetic Field Calculated Using the Author's Electromagnetic Theory

In the author's electromagnetic theory, the electromagnetic field should be appropriately corrected based on Maxwell's electromagnetic theory.

If $x > 0$, this is an advanced wave, and we consider the correction factor (j):

$$\boldsymbol{B} = (j)\boldsymbol{B}_{Maxwell} = (j)j\exp(+jx)(-\hat{y})$$

$$= \exp(+jx)(\hat{y}) \tag{109}$$

If $x < 0$, this is a retarded wave, and we consider the correction factor $(-j)$:

$$\boldsymbol{B} = (-j)\boldsymbol{B}_{Maxwell} = -jj\exp(+jx)(-\hat{y})$$

$$= -\exp(+jx)(\hat{y}) \tag{110}$$

$$\boldsymbol{B} = \exp(+jx)(\hat{y})\begin{cases}1 & x > 0\\ -1 & x < 0\end{cases} \tag{111}$$

The electric field is given by equation (103), and

$$\boldsymbol{E} = j\exp(+jx)(-\hat{z}) \tag{112}$$

When $x \to 0$, we have:

$$\lim_{x\to 0}\boldsymbol{E} = j(-\hat{z}) \tag{113}$$

$$\lim_{x\to 0}\boldsymbol{B} = \hat{y}\begin{cases}1 & x > 0\\ -1 & x < 0\end{cases} \tag{114}$$

These two equations show that the electric field and magnetic field calculated using this method (equations 111 and 112) can degenerate into the quasi-static magnetic field (equations 79 and 83). Therefore, this represents the correct left-propagating electromagnetic field.

2.4. Maxwell's Equations are Not Necessary

We have examined the method for determining the magnetic field. The retarded wave uses:

$$\boldsymbol{B}_n^{(r)} = \boldsymbol{B}_{Maxwell\ n}^{(r)},\quad \boldsymbol{B}_f^{(r)} = (-j)\boldsymbol{B}_{Maxwell\ f}^{(r)} \tag{115}$$

Here, the subscript *n* denotes the near field and *f* denotes the far field, with electromagnetic waves belonging to the far field. The superscript *Maxwell* indicates that the quantity is calculated using Maxwell's electromagnetic theory. For the advanced magnetic field:

$$\boldsymbol{B}_n^{(a)} = \boldsymbol{B}_{Maxwell\ n}^{(a)},\quad \boldsymbol{B}^{(a)} = (j)\boldsymbol{B}_{Maxwell}^{(a)} \tag{116}$$

This method is a very effective way to determine the phase of the magnetic field in electromagnetic waves. First, we can calculate the magnetic field phase according to Maxwell's method, where the magnetic field and electric field are in phase. Then, using the above correction method, we obtain the correct phase for the magnetic field. However, we can also determine the phase of the magnetic field using the Ampere's circuital law under magnetic quasi-static conditions. Then, we add the propagation factor for right-going electromagnetic waves:

$$\exp(-jx) \tag{117}$$

For left-going electromagnetic waves, the factor added is:

$$\exp(+jx) \tag{118}$$

We know that:

$$\boldsymbol{B} \sim \hat{y}\begin{cases}1 & x \geq 0\\ -1 & x < 0\end{cases} \tag{119}$$

From this, we can directly obtain the right-going electromagnetic wave as:

$$\boldsymbol{B} \sim \exp(-jx)\hat{y}\begin{cases}1 & x \geq 0\\ -1 & x < 0\end{cases} \tag{120}$$

The left-going electromagnetic wave is:

$$\boldsymbol{B} \sim \exp(+jx)\hat{y}\begin{cases}1 & x \geq 0\\ -1 & x < 0\end{cases} \tag{121}$$

While the electric field is:

$$\boldsymbol{E} = j\exp(+jx)(-\hat{z}) \tag{122}$$

This approach seems more direct and simple. It shows that Maxwell's equations may not be necessary. The electromagnetic waves of photons can be derived by obtaining the electric and magnetic fields under magnetic quasi-static conditions and then adding the plane wave propagation factors.

2.5. Maxwell's Equations Can Be Seen as an Approximation of the Mutual Energy Flow Theory

Assuming $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]^T$ is an electromagnetic wave that propagates to the right from the source, $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]^T$ is an electromagnetic wave that propagates to the right from the sink. The sink is located to the right of the emitter. The two waves have the same magnitude, $|\xi_1| = |\xi_2|$. Since intensity of the mutual energy flow is,

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \tag{123}$$

There is a synchronization relationship between the sink and source:

$$H_2 \sim E_1 \tag{124}$$

Thus:

$$\boldsymbol{H}_2 = \boldsymbol{H}_{Maxwell\ 1} \tag{125}$$

The subscript *Maxwell* indicates that this quantity is derived from Maxwell's electromagnetic theory. $\boldsymbol{H}_{Maxwell\ 1}$ is the retarded magnetic field calculated using Maxwell's electromagnetic theory. The mutual energy flow density has a synchronization relationship:

$$H_1 \sim E_2 \tag{126}$$

Therefore:

$$\boldsymbol{H}_1 = \boldsymbol{H}_{Maxwell\ 2} \tag{127}$$

Thus, we further have:

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_{Maxwell\ 1}^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_{Maxwell\ 2}) \tag{128}$$

Alternatively:

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{S}_{Maxwell\ 11} + \boldsymbol{S}_{Maxwell\ 22}) \tag{129}$$

Since:

$$\boldsymbol{S}_{Maxwell\ 22} = \boldsymbol{S}_{Maxwell\ 11} \tag{130}$$

We obtain:

$$\boldsymbol{S}_m = \boldsymbol{S}_{Maxwell\ 11} \tag{131}$$

This shows that the mutual energy flow density $\boldsymbol{S}_m$ calculated using the author's electromagnetic theory is consistent with the self-energy flow density $\boldsymbol{S}_{Maxwell\ 11}$ calculated using Maxwell's electromagnetic theory. Therefore, we can still use Maxwell's theory to compute the self-energy flow $\boldsymbol{S}_{Maxwell\ 11}$, which is the mutual energy flow. However, this method only provides an equivalence in energy flow between the source and sink. It does not account for the nature of mutual energy flow generated at the source and annihilated at the sink. Moreover, even between the source and sink, the results for electric and magnetic fields from the author's method and Maxwell's theory remain inequivalent.

2.6. Conclusion

According to Maxwell's electromagnetic theory, the retarded wave is generally considered without the advanced wave. Even if the advanced wave is considered as a physical reality, Maxwell's theory is still incorrect. When the magnetic field and the electric field are in phase, this is wrong. The electromagnetic wave's electric field and magnetic field should have a 90-degree phase difference, ensuring the wave is reactive, with no energy transmission. Energy should be transmitted via the mutual energy flow, which will be discussed in the next chapter. The correct electromagnetic field of the wave can be obtained by modifying the magnetic field using Maxwell's theory, or by first calculating the electromagnetic field under quasi-static conditions and then adding the propagation factors to obtain the electromagnetic wave's magnetic field. This method suggests that Maxwell's equations may not be necessary. Specifically, the part of Maxwell's equations that adds the displacement current for the induced electromagnetic field is erroneous, i.e., the Ampere's circuital law:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}(\epsilon_0(-\nabla\phi)) + \frac{\partial}{\partial t}(\epsilon_0(-\boldsymbol{A}))$$

The term $\frac{\partial}{\partial t}(\epsilon_0(-\boldsymbol{A}))$ for calculating the magnetic field is incorrect, but the calculation for the electric field remains valid.

3. Particles as Plane Waves

Quantum mechanics as a whole treats particles as plane waves. However, according to the Schrödinger equation, and the wave derived from Maxwell's equations, the wave is a spherical wave, which decays with distance. Why can we treat particles as plane waves that do not decay? This is because particles consist of mutual energy flow formed by retarded waves emitted by sources and advanced waves emitted by sinks. The advanced waves in this mutual energy flow act as the waveguide for the retarded waves, and the retarded waves act as the waveguide for the advanced waves. In this way, both retarded and advanced waves propagate as though along a waveguide connecting the source and the sink. As a result, the retarded and advanced waves become plane waves, or more accurately, quasi-plane waves, similar to those in waveguides or optical fibers.

3.1. Particles Composed of Self-Energy Flow Plane Waves

In the electromagnetic and quantum theories of today, photons are considered to be plane waves, such as:

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}^* \tag{132}$$

where $\boldsymbol{S}$ is the Poynting vector, representing the energy flow density of the plane wave:

$$\boldsymbol{E} = E(-\hat{z}) \tag{133}$$

$$\boldsymbol{H} = H(\hat{y}) \tag{134}$$

$$\boldsymbol{S} = S\hat{x} \tag{135}$$

$$S = EH^* \tag{136}$$

If:

$$E = \exp(j(\omega t - kx)) \tag{137}$$

The frequency factor $\exp(j\omega t)$ can be omitted, so we have:

$$E = \exp(-jkx) \tag{138}$$

Since our calculations only concern the phase, we can omit k:

$$E = \exp(-jx) \tag{139}$$

According to Maxwell's electromagnetic theory, assuming the electric field and the magnetic field are in phase, and considering the impedance between the electric field and the magnetic field $\eta = 1$, we have:

$$H = \exp(-jx) \tag{140}$$

Thus, the magnitude of the Poynting vector is:

$$S = \exp(-jx)\exp(-jx)^* = 1 \tag{141}$$

The Poynting vector is:

$$\boldsymbol{S} = \hat{x} \tag{142}$$

This provides the theory of energy transfer through self-energy flow, or the Poynting vector, as described by Maxwell's equations. In today's theories, whether electromagnetic or quantum, the assumption is that energy is transferred through self-energy flow. The author, however, believes this to be incorrect. Energy should be transferred via mutual energy flow, which will be discussed in the next sub-section.

3.2. Particles Composed of Mutual Energy Flow Plane Waves

The author's electromagnetic theory differs from Maxwell's electromagnetic theory. The author's theory assumes that energy is transferred through mutual energy flow. This theory posits that mutual energy flow consists of retarded waves $\boldsymbol{E}_1$, $\boldsymbol{H}_1$ emitted by the source and advanced waves $\boldsymbol{E}_2$, $\boldsymbol{H}_2$ emitted by the sink. The mutual energy flow density is:

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \tag{143}$$

Assuming:

$$\boldsymbol{E}_1 = E_1(-\hat{z}), \quad \boldsymbol{E}_2 = E_2(-\hat{z}) \tag{144}$$

$$\boldsymbol{H}_1 = H_1(\hat{y}), \quad \boldsymbol{H}_2 = H_2(\hat{y}) \tag{145}$$

$$\boldsymbol{S}_m = S_m\hat{x} \tag{146}$$

Thus, the mutual energy flow density is:

$$S_m = \frac{1}{2}(E_1 H_2^* + E_2^* H_2) \tag{147}$$

3.3. Scheme 1

This scheme still calculates the electric and magnetic fields according to Maxwell's electromagnetic theory, where the electric field and magnetic field are in phase, so:

$$E_1 = \exp(-jx), \quad H_1 = \exp(-jx) \tag{148}$$

$$E_2 = \exp(-jx), \quad H_2 = \exp(-jx) \tag{149}$$

Thus:

$$S_m = \frac{1}{2}(E_1 H_2^* + E_2^* H_2)$$

$$= \frac{1}{2}(\exp(-jx)\exp(-jx)^* + \exp(-jx)^*\exp(-jx)) = 1 \tag{150}$$

So:

$$\boldsymbol{S}_m = \hat{x} \tag{151}$$

Thus, mutual energy flow can transfer energy. However, if mutual energy flow transfers energy, then self-energy flow can no longer transfer energy. Since mutual energy flow has already transferred all the energy, we have:

$$\boldsymbol{S}_m = \boldsymbol{S}_{11} \tag{152}$$

The left side of the above equation represents the energy flow density transferred by mutual energy flow, and the right side represents the energy flow density calculated using the Poynting vector according to Maxwell's electromagnetic theory, which are now equal. This scheme, however, still has problems. The self-energy flow density is defined as:

$$\boldsymbol{S}_{11} = \boldsymbol{E}_1 \times \boldsymbol{H}_1^* \tag{153}$$

$$\boldsymbol{S}_{22} = \boldsymbol{E}_2 \times \boldsymbol{H}_2^* \tag{154}$$

There should be:

$$\mathrm{Re}(\boldsymbol{S}_{11}) = \mathrm{Re}(\boldsymbol{S}_{22}) = 0 \tag{155}$$

Because without the above condition, both self-energy flow and mutual energy flow will transfer energy, and the total energy transferred will be more than expected. The author first discovered this issue in 2017, and the initial solution was to require these self-energy flows to collapse in reverse through time-reversal waves [11]. However, the reverse collapse introduces Maxwell's equations in time-reversal, making the problem more complicated and unreliable. Therefore, the author later proposed another solution in 2022 [12], which posited that self-energy flow represents reactive power.

3.4. Scheme 2

This scheme follows the author's electromagnetic theory, where the electric and magnetic fields are no longer in phase but are required to satisfy:

$$\mathrm{Re}(E_1 H_1^*) = \mathrm{Re}(E_2 H_2^*) = 0 \tag{156}$$

This condition requires a 90-degree phase difference between H_1 and E_1. Similarly, E_2 and H_2 also maintain a 90-degree phase difference. This approach abandons Maxwell's electromagnetic theory or modifies it. Therefore:

$$H_1 = \pm j E_1 \tag{157}$$

$$H_2 = \pm jE_2 \tag{158}$$

Further, mutual energy flow density is required to satisfy:

$$E_1 H_2^* + E_2^* H_1 = \text{Real}$$

This condition is satisfied when:

$$E_1 H_2^*, \quad E_2^* H_1$$

are both real, meaning:

$$H_2 \sim E_1 \tag{159}$$

$$H_1 \sim E_2 \tag{160}$$

The $\sim$ symbol indicates that both have the same phase. For convenience, we assume:

$$\boldsymbol{A}_1 = \exp(-jx)\hat{z} \tag{161}$$

$$\boldsymbol{E}_1 = -j\omega \boldsymbol{A}_1 = -j\omega \exp(-jx)\hat{z} \sim j\exp(-jx)(-\hat{z}) \tag{162}$$

We assume that:

$$E_1 = j\exp(-jx) \tag{163}$$

Considering the vacuum impedance $\eta = 1$ and the relation in (159), we have:

$$H_2 = j\exp(-jx) \tag{164}$$

Considering (157), there are two possibilities:

$$H_1 = j\big(j\exp(-jx)\big) \;\; or \quad H_1 = -j\big(j\exp(-jx)\big) \tag{165}$$

Considering (160), we have:

$$E_2 = j\big(j\exp(-jx)\big) \;\; \text{or} \quad E_2 = -j\big(j\exp(-jx)\big) \tag{166}$$

3.4.1. First Possibility

$$H_1 = j(j\exp(-jx)), \tag{167}$$

$$E_2 = j(j\exp(-jx)) \tag{168}$$

Thus:

$$S_m = \frac{1}{2}(E_1 H_2^* + E_2^* H_1)$$

$$S_m = \frac{1}{2}((j\exp(-jx))(j\exp(-jx))^* + (jj\exp(-jx))^*(jj\exp(-jx))) = 1 \tag{169}$$

3.4.2. Second Possibility

$$H_1 = -j(j\exp(-jx)), \tag{170}$$

$$E_2 = -j(j\exp(-jx)) \tag{171}$$

$$S_m = \frac{1}{2}(E_1 H_2^* + E_2^* H_1)$$

$$S_m = \frac{1}{2}((j\exp(-jx))(j\exp(-jx))^* + (-j(j\exp(-jx))^*)(-j(j\exp(-jx))) = 1 \tag{172}$$

$$E_1 = j\exp(-jx) \tag{173}$$

$$H_1 = -j(j\exp(-jx)) \tag{174}$$

Thus:

$$H_1 = -jE_1 \tag{175}$$

$$E_2 = -j(j\exp(-jx)) = \exp(-jx) \tag{176}$$

$$H_2 = j\exp(-jx) \tag{177}$$

$$H_2 = jE_2 \tag{178}$$

We find that the magnetic field H_1 and the electric field E_1 differ by -j. However, H_2 and E_2 differ by +j. What does this mean? This is because the quantities with subscript 1, E_1, H_1, are retarded waves, and the quantities with subscript 2, E_2, H_2, are advanced waves.

Among these two possibilities, the author believes the second possibility is correct. This is because E_1 generates I_2, and the direction of I_2 aligns with E_1, so:

$$I_2 \sim E_1 \tag{179}$$

Thus:

$$E_2 = -j\omega I_2 \sim -jI_2 \sim -jE_1 = -j(j\exp(-jx)) = \exp(-jx) \tag{180}$$

Therefore, the second possibility should be adopted.

3.5. Summary of this Chapter

This chapter explains that according to Maxwell's theory, the Poynting energy flow of a plane wave can always be replaced by mutual energy flow. However, this replacement must modify Maxwell's electromagnetic theory; otherwise, self-energy flow and mutual energy flow would both transfer energy, resulting in an excess of transferred energy. In reality, either self-energy flow or mutual energy flow transfers energy, but both cannot transfer energy simultaneously, as this would lead to more energy being transferred than expected. Initially, the author found this issue in 2017, and the proposed solution was to require that these self-energy flows collapse in reverse through time-reversal waves [11]. However, the reverse collapse introduces Maxwell's equations in time-reversal, making the problem more complex and unreliable. Therefore, a new approach was proposed in 2022 [12], which posits that self-energy flow represents reactive power.

4. Constructing Particles from the Schrödinger Equation

4.1. Modified Schrödinger Equation

The author modifies the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + U\psi = i\hbar\frac{\partial}{\partial t}\psi \tag{181}$$

or:

$$-\frac{\hbar}{2im}\nabla\cdot\nabla\psi + \frac{1}{i\hbar}U\psi - \frac{\partial}{\partial t}\psi = 0 \tag{182}$$

Considering that the potential U does not affect energy flow, we remove U and get:

$$-\frac{\hbar}{2im}\nabla\cdot\nabla\psi - \frac{\partial}{\partial t}\psi = 0 \tag{183}$$

The Schrödinger equation is then modified by adding a source term τ:

$$-\frac{\hbar}{2im}\nabla\cdot\nabla\psi - \frac{\partial}{\partial t}\psi = \tau \tag{184}$$

Assuming:

$$\nabla\psi = i\overline{\boldsymbol{h}} \tag{185}$$

We arrive at the modified Schrödinger equation:

$$\begin{cases} -\frac{\hbar}{2im}\nabla\cdot(i\overline{\boldsymbol{h}}) - \frac{\partial}{\partial t}\psi = \tau \\ \nabla\psi = i\overline{\boldsymbol{h}} \end{cases} \tag{186}$$

This is the Schrödinger equation with a source τ and an average magnetic field $\overline{\boldsymbol{h}}$ along the loop. The magnetic field is calculated similarly to the relations in Maxwell's theory. Additionally, we can consider the analogous relationship derived from Maxwell's equations for the Schrödinger equation:

The average magnetic field along the loop in Maxwell's theory and the average magnetic field in Schrödinger's equation are equivalent:

$$\nabla\psi = i\overline{\boldsymbol{h}} \tag{187}$$

This is analogous to Faraday's law in quantum theory.

$$-\frac{\hbar}{2im}\nabla\cdot(i\overline{\boldsymbol{h}}) - \frac{\partial}{\partial t}\psi = \tau \tag{188}$$

This is the Ampere's circuital law in quantum theory.

$$\frac{\partial}{\partial t}\psi \tag{189}$$

This represents the displacement current in quantum theory.

4.2. Calculating Self-Energy Flow Density (Probability Flow Density)

By transitioning from electromagnetic field equations to the analogous Schrödinger equation form, the author closely follows electromagnetic formulas. This method immediately leads to the corresponding Schrödinger equation formulas. Consider:

$$-j \to i \tag{190}$$

This transformation occurs because in electromagnetic field theory, the time factor exp($j\omega t$) is used, whereas in the Schrödinger equation, we use exp($-i\omega t$). The cross product is transformed into regular multiplication:

$$\times \to * \tag{191}$$

The curl and gradient are equivalent:

$$\nabla\times \to \nabla \tag{192}$$

The electric field $\boldsymbol{E}$ and the wave function ψ are equivalent:

$$\boldsymbol{E} \to \psi \tag{193}$$

The average magnetic field in electromagnetic theory corresponds to the average magnetic field in the Schrödinger equation:

$$\overline{\boldsymbol{H}} \to \overline{\boldsymbol{h}} \tag{194}$$

From the Poynting vector, we get the corresponding probability flow density for the Schrödinger equation:

$$\boldsymbol{S} = \mathrm{Re}(\boldsymbol{E}\times\overline{\boldsymbol{H}}^*)$$

This becomes:

$$\boldsymbol{J} = \mathrm{Re}(\psi\overline{\boldsymbol{h}}^*)$$

$$= \mathrm{Re}(\psi(\frac{1}{i}\nabla\psi)^*)$$

$$= \frac{1}{2i}(-\psi(\nabla\psi)^*) + \psi^*(\nabla\psi)$$

$$= \frac{1}{2i}(\psi^*(\nabla\psi) - \psi(\nabla\psi)^*)$$

$$= \mathrm{Im}(\psi^*(\nabla\psi)) \quad (195)$$

Thus, the probability flow density is:

$$\boldsymbol{J}_S = \mathrm{Im}(\psi^*(\nabla\psi)) \quad (196)$$

This is the probability flow density calculated according to Schrödinger's equation. Assuming:

$$\psi = \exp(ix) \quad (197)$$

$$\nabla\psi = i\exp(ix)\hat{x} \quad (198)$$

$$\boldsymbol{J}_S = \mathrm{Im}(\psi^*(\nabla\psi))$$

$$= \mathrm{Im}(\exp(ix)^* i\exp(ix)\hat{x})$$

$$= \mathrm{Im}(i\hat{x}) = \hat{x} \quad (199)$$

This result shows that the self-energy flow density $\boldsymbol{J}_S$ of a plane wave calculated using Schrödinger's equation is a real number $\hat{x}$, indicating that the plane wave in Schrödinger's equation carries energy.

In the author's electromagnetic theory, waves are reactive power, so the magnetic field must be corrected. Considering the retarded wave:

$$\boldsymbol{h} = i\overline{\boldsymbol{h}}, \quad \overline{\boldsymbol{h}} = \frac{1}{i}\boldsymbol{h} \quad (200)$$

Thus:

$$\boldsymbol{J} = Re(\psi\boldsymbol{h}^*)$$

$$= Re(\psi(i\overline{\boldsymbol{h}})^*)$$

$$= Re(\psi(i\frac{1}{i}\nabla\psi)^*)$$

$$= Re(\psi(\nabla\psi)^*)$$

Since:

$$\overline{\boldsymbol{h}} = \frac{1}{i}\nabla\psi$$

Thus:

$$\boldsymbol{J} = \mathrm{Re}(\exp(ix)(\nabla\exp(ix))^*)$$

$$= \mathrm{Re}(\exp(ix)(i\exp(ix)\hat{x})^*)$$

$$= \mathrm{Re}(i)\hat{x}$$

$$= 0$$

This shows that, according to the author's theory, the self-energy flow density is zero.

4.3. Green's Function

4.3.1. Green's Function Corresponding to Maxwell's Equations

Maxwell's two curl equations are:

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}} \tag{201}$$

$$\nabla \times \overline{\boldsymbol{H}} = \boldsymbol{J} + \frac{\partial}{\partial t}\epsilon\boldsymbol{E} \tag{202}$$

or:

$$-\frac{\partial}{\partial t}\epsilon\boldsymbol{E} + \nabla \times \overline{\boldsymbol{H}} = \boldsymbol{J} \tag{203}$$

$$-\nabla \times \boldsymbol{E} - \frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}} = 0 \tag{204}$$

$$L = \begin{bmatrix} -\frac{\partial}{\partial t}\epsilon, & \nabla \times \\ -\nabla \times, & -\frac{\partial}{\partial t}\mu \end{bmatrix}, \quad \xi = [\boldsymbol{E}, \overline{\boldsymbol{H}}]^T, \quad \tau = [\boldsymbol{J}, 0]^T \tag{205}$$

The Maxwell's equation can be written as:

$$L\xi = \tau \tag{206}$$

So:

$$L\xi_1\xi_2^* + \xi_1(L\xi_2)^*$$

$$= (-\frac{\partial}{\partial t}\epsilon\boldsymbol{E}_1 + \nabla\times\overline{\boldsymbol{H}}_1)\cdot\boldsymbol{E}_2^* + \boldsymbol{E}_1\cdot(-\frac{\partial}{\partial t}\epsilon\boldsymbol{E}_2 + \nabla\times\overline{\boldsymbol{H}}_2)^*$$

$$+(-\nabla\times\boldsymbol{E}_1 - \frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}}_1)\cdot\overline{\boldsymbol{H}}_2^* + \overline{\boldsymbol{H}}_1\cdot(-\nabla\times\boldsymbol{E}_2 - \frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}}_2)^*$$

$$= -\frac{\partial}{\partial t}\epsilon\boldsymbol{E}_1\cdot\boldsymbol{E}_2^* + \nabla\times\overline{\boldsymbol{H}}_1\cdot\boldsymbol{E}_2^* - \boldsymbol{E}_1\cdot\frac{\partial}{\partial t}\epsilon\boldsymbol{E}_2^* + \boldsymbol{E}_1\cdot\nabla\times\overline{\boldsymbol{H}}_2^*$$

$$-\nabla\times\boldsymbol{E}_1\cdot\overline{\boldsymbol{H}}_2^* - \frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}}_1\cdot\overline{\boldsymbol{H}}_2^* - \overline{\boldsymbol{H}}_1\cdot\nabla\times\boldsymbol{E}_2^* - \overline{\boldsymbol{H}}_1\cdot\frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}}_2^*$$

$$= -\frac{\partial}{\partial t}\epsilon\boldsymbol{E}_1\cdot\boldsymbol{E}_2^* - \boldsymbol{E}_1\cdot\frac{\partial}{\partial t}\epsilon\boldsymbol{E}_2^* - \frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}}_1\cdot\overline{\boldsymbol{H}}_2^* - \overline{\boldsymbol{H}}_1\cdot\frac{\partial}{\partial t}\mu\overline{\boldsymbol{H}}_2^*$$

$$+\nabla\times\overline{\boldsymbol{H}}_1\cdot\boldsymbol{E}_2^* - \overline{\boldsymbol{H}}_1\cdot\nabla\times\boldsymbol{E}_2^* + \boldsymbol{E}_1\cdot\nabla\times\overline{\boldsymbol{H}}_2^* - \nabla\times\boldsymbol{E}_1\cdot\overline{\boldsymbol{H}}_2^*$$

$$= -\frac{\partial}{\partial t}(\epsilon\boldsymbol{E}_1\cdot\boldsymbol{E}_2^*) - \frac{\partial}{\partial t}(\mu\overline{\boldsymbol{H}}_1\cdot\overline{\boldsymbol{H}}_2^*) - \nabla\cdot(\boldsymbol{E}_2^*\times\overline{\boldsymbol{H}}_1) - \nabla\cdot(\boldsymbol{E}_1\times\overline{\boldsymbol{H}}_2^*)$$

$$= -\frac{\partial}{\partial t}(\epsilon\boldsymbol{E}_1\cdot\boldsymbol{E}_2^* + \mu\overline{\boldsymbol{H}}_1\cdot\overline{\boldsymbol{H}}_2^*) - \nabla\cdot(\boldsymbol{E}_1\times\overline{\boldsymbol{H}}_2^* + \boldsymbol{E}_2^*\times\overline{\boldsymbol{H}}_1) \tag{207}$$

So the Green's function is:

$$L\xi_1\xi_2^* + \xi_1(L\xi_2)^* + \frac{\partial}{\partial t}u + \nabla\cdot\boldsymbol{S} = 0 \tag{208}$$

$$u = \epsilon\boldsymbol{E}_1\cdot\boldsymbol{E}_2^* + \mu\overline{\boldsymbol{H}}_1\cdot\overline{\boldsymbol{H}}_2^* \tag{209}$$

$$\boldsymbol{S} = \boldsymbol{E}_1\times\overline{\boldsymbol{H}}_2^* + \boldsymbol{E}_2^*\times\overline{\boldsymbol{H}}_1 \tag{210}$$

If we substitute Maxwell's equations into equation (208):

$$\tau_1\cdot\xi_2^* + \xi_1\cdot\tau_2^* + \frac{\partial}{\partial t}u + \nabla\cdot\boldsymbol{S} = 0 \tag{211}$$

$$J_1\cdot\boldsymbol{E}_2^* + \boldsymbol{E}_1\cdot\boldsymbol{J}_2^* + \frac{\partial}{\partial t}u + \nabla\cdot\boldsymbol{S} = 0 \tag{212}$$

This equation is derived from Maxwell's electromagnetic theory. For the author's electromagnetic theory, corrections must be made as follows:

$$\overline{\boldsymbol{H}} \to \boldsymbol{H} \tag{213}$$

$$\boldsymbol{S} = \boldsymbol{E}_1\times\boldsymbol{H}_2^* + \boldsymbol{E}_2^*\times\boldsymbol{H}_1 \tag{214}$$

$\overline{\boldsymbol{H}}$ is the average magnetic field along the loop, defined by Maxwell's theory. $\boldsymbol{H}$ is the magnetic field measured by straight current elements (measured by Hall effect devices), defined by the author's electromagnetic theory. From equation (212), we obtain the mutual energy flow law:

$$-\int_{V_1} \boldsymbol{E}_2^* \cdot J_1 = \oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}\, d\Gamma = \int_{V_2} \boldsymbol{E}_1 \cdot J_2^*\, dV \qquad (215)$$

where Γ is any surface that separates the currents $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. This formula should also consider the half-retarded and half-advanced correction. This correction halves the mutual energy flow, and the equation is modified as:

$$-\int_{V_1} \boldsymbol{E}_2^* \cdot J_1 = \frac{1}{2}\oiint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}\, d\Gamma = \int_{V_2} \boldsymbol{E}_1 \cdot J_2^*\, dV \qquad (216)$$

4.3.2. Green's Function of Schrödinger's Equation

In the Schrödinger equation, some constants are omitted:

$$\begin{cases} -\frac{1}{2}\nabla \cdot \overline{\boldsymbol{h}} - \frac{\partial}{\partial t}\psi = \tau \\ \nabla\psi = i\overline{\boldsymbol{h}} \end{cases} \qquad (217)$$

Define:

$$L = \begin{bmatrix} -\frac{\partial}{\partial t}, & -\frac{1}{2}\nabla \cdot \\ -\nabla, & -i \end{bmatrix}, \quad \xi = [\psi, \boldsymbol{h}], \quad \rho = [\tau, 0] \qquad (218)$$

Thus, Schrödinger's equation is:

$$L\xi = \rho \qquad (219)$$

Now compute:

$$L\xi_1 \cdot \xi_2^* + \xi_1 \cdot (L\xi_2)^*$$

$$= (-\frac{\partial}{\partial t}\psi_1 - \frac{1}{2}\nabla \cdot \overline{\boldsymbol{h}}_1) \cdot \psi_2^* + \psi_1 \cdot (-\frac{\partial}{\partial t}\psi_2 - \frac{1}{2}\nabla \cdot \overline{\boldsymbol{h}}_2)^*$$

$$+(-\nabla\psi_1 - i\overline{\boldsymbol{h}}_1) \cdot \overline{\boldsymbol{h}}_2^* + \overline{\boldsymbol{h}}_1 \cdot (-\nabla\psi_2 - i\overline{\boldsymbol{h}}_2)^*$$

$$= -\frac{\partial}{\partial t}\psi_1 \cdot \psi_2^* - \psi_1 \cdot \frac{\partial}{\partial t}\psi_2 - i\overline{\boldsymbol{h}}_1 \cdot \overline{\boldsymbol{h}}_2^* + i\overline{\boldsymbol{h}}_1 \cdot \overline{\boldsymbol{h}}_2^*$$

$$-\frac{1}{2}\nabla \cdot \overline{\boldsymbol{h}}_1\psi_2^* - \frac{1}{2}\psi_1\nabla \cdot \overline{\boldsymbol{h}}_2^* - \nabla\psi_1 \cdot \overline{\boldsymbol{h}}_2^* - \overline{\boldsymbol{h}}_1 \cdot \nabla\psi_2^*$$

$$= -\frac{\partial}{\partial t}(\psi_1 \cdot \psi_2^*) - \frac{1}{2}(\nabla \cdot \overline{\boldsymbol{h}}_1\psi_2^* + \psi_1\nabla \cdot \overline{\boldsymbol{h}}_2^*) - \nabla\psi_1 \cdot \overline{\boldsymbol{h}}_2^* - \overline{\boldsymbol{h}}_1 \cdot \nabla\psi_2^* \qquad (220)$$

$$-\frac{1}{2}(\nabla \cdot \overline{\boldsymbol{h}}_1\psi_2^* + \psi_1\nabla \cdot \overline{\boldsymbol{h}}_2^*) = -\frac{1}{2i}(\nabla \cdot \nabla\psi_1\psi_2^* - \psi_1\nabla \cdot \nabla\psi_2^*)$$

$$= -\frac{1}{2i}(\nabla \cdot \nabla\psi_1\psi_2^* + \nabla\psi_1 \cdot \nabla\psi_2^* - \psi_1\nabla \cdot \nabla\psi_2^* - \nabla\psi_1 \cdot \nabla\psi_2^*)$$

$$= -\frac{1}{2i}(\nabla\cdot(\nabla\psi_1\psi_2^*) - \nabla\cdot(\psi_1\nabla\psi_2^*))$$

$$= -\frac{1}{2i}\nabla\cdot(\nabla\psi_1\psi_2^* - \psi_1\nabla\psi_2^*) = -\nabla\cdot\boldsymbol{J} \tag{221}$$

where:

$$\boldsymbol{J} = \frac{1}{2}(\overline{\boldsymbol{h}}_1\psi_2^* + \psi_1(\overline{\boldsymbol{h}}_2)^*) \tag{222}$$

Now compute:

$$-\nabla\psi_1\cdot\overline{\boldsymbol{h}}_2^* - \overline{\boldsymbol{h}}_1\cdot\nabla\psi_2^* = -\nabla\psi_1\cdot\left(\frac{1}{i}\nabla\psi_2\right)^* - \left(\frac{1}{i}\nabla\psi_1\right)\cdot\nabla\psi_2^*$$

$$= \frac{1}{i}(\nabla\psi_1\cdot\nabla\psi_2^* - \nabla\psi_1\cdot\nabla\psi_2^*) = 0 \tag{223}$$

$$L\xi_1\cdot\xi_2^* + \xi_1\cdot(L\xi_2)^* = -\frac{\partial}{\partial t}(\psi_1\cdot\psi_2^*) - \nabla\cdot\boldsymbol{J} \tag{224}$$

Thus, the Green's function is:

$$L\xi_1\cdot\xi_2^* + \xi_1\cdot(L\xi_2)^* + \frac{\partial}{\partial t}\rho + \nabla\cdot\boldsymbol{J} = 0 \tag{225}$$

where:

$$\rho = \psi_1\cdot\psi_2^* \tag{226}$$

$$\boldsymbol{J} = \frac{1}{2}(\overline{\boldsymbol{h}}_1\psi_2^* + \psi_1(\overline{\boldsymbol{h}}_2)^*) \tag{227}$$

Considering Schrödinger's equation:

$$\tau_1\cdot\psi_2^* + \psi_1\tau_2^* + \frac{\partial}{\partial t}\rho + \nabla\cdot\boldsymbol{J} = 0 \tag{228}$$

This corresponds to stable energy flow:

$$\tau_1\cdot\psi_2^* + \psi_1\tau_2^* + \nabla\cdot\boldsymbol{J} = 0 \tag{229}$$

Thus, we obtain the mutual energy flow law:

$$-\int_{V_1}\tau_1\psi_2^*\,dV = \oiint_{\Gamma}\frac{1}{2}(\overline{\boldsymbol{h}}_1\psi_2^* + \psi_1(\overline{\boldsymbol{h}}_2)^*)\cdot\hat{n}\,d\Gamma = \int_{V_2}\tau_2^*\psi_1\,dV \tag{230}$$

This equation is derived from Schrödinger's equation. If we apply the author's mutual energy flow law, it should be corrected as follows:

$$-\int_{V_1} \tau_1 \psi_2^* \, dV = \oint_\Gamma \frac{1}{2}(\boldsymbol{h}_1 \psi_2^* + \psi_1 (\boldsymbol{h}_2)^*) \cdot \hat{n} \, d\Gamma = \int_{V_2} \tau_2^* \psi_1 \, dV \tag{231}$$

where $\overline{\boldsymbol{h}}$ is the magnetic field in Schrödinger's equation, and $\boldsymbol{h}$ is the magnetic field defined by the author. For retarded waves:

$$\boldsymbol{h} = i\overline{\boldsymbol{h}} = i\frac{1}{i}\nabla\psi = \nabla\psi \tag{232}$$

For advanced waves:

$$\boldsymbol{h} = -i\overline{\boldsymbol{h}} = -i\frac{1}{i}\nabla\psi = -\nabla\psi \tag{233}$$

If we consider half-retarded and half-advanced waves, equation (231) should be multiplied by a factor of $\frac{1}{2}$:

$$-\int_{V_1} \tau_1 \psi_2^* \, dV = \oint_\Gamma \frac{1}{4}(\boldsymbol{h}_1 \psi_2^* + \psi_1 (\boldsymbol{h}_2)^*) \cdot \hat{n} \, d\Gamma = \int_{V_2} \tau_2^* \psi_1 \, dV \tag{234}$$

4.4. Calculating Mutual Energy Flow Density

This result is consistent with the probability flow density obtained from other methods. The mutual energy flow density is:

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1)$$

This leads to:

$$\boldsymbol{J}_m = \frac{1}{4}(\psi_1 \boldsymbol{h}_2^* + \psi_2^* \boldsymbol{h}_1) \tag{235}$$

Thus:

$$\boldsymbol{J}_m = \frac{1}{4}\begin{cases} \psi_1(-i\overline{\boldsymbol{h}}_2)^* + \psi_2^*(-i\overline{\boldsymbol{h}}_1) & x < 0 \\ \psi_1(-i\overline{\boldsymbol{h}}_2)^* + \psi_2^*(i\overline{\boldsymbol{h}}_1) & 0 \le x \le L \\ \psi_1(i\overline{\boldsymbol{h}}_2)^* + \psi_2^*(i\overline{\boldsymbol{h}}_1) & L < x \end{cases}$$

$$= \frac{1}{4}\begin{cases} \psi_1(-\nabla\psi_2)^* + \psi_2^*(-\nabla\psi_2) & x < 0 \\ \psi_1(-\nabla\psi_2)^* + \psi_2^*(\nabla\psi_2) & 0 \le x \le L \\ \psi_1(\nabla\psi_2)^* + \psi_2^*(\nabla\psi_1) & L < x \end{cases} \tag{236}$$

Considering the retarded wave:

$$\boldsymbol{h} = i\overline{\boldsymbol{h}} \tag{237}$$

For the advanced wave:

$$\boldsymbol{h} = -i\overline{\boldsymbol{h}} \tag{238}$$

At $x = 0$, the subscript 1 quantities transition from advanced waves to retarded waves, and at $x = L$, the subscript 2 quantities transition from advanced waves to retarded waves.

$$\psi_1 = -i\exp(ix) \tag{239}$$

$$\nabla\psi_1 = -ii\exp(ix)\hat{x} = \exp(ix)\hat{x} \tag{240}$$

$$\psi_2 \sim \boldsymbol{h}_1 \sim \nabla\psi_1 \tag{241}$$

Hence,

$$\psi_2 = \exp(ix) \tag{242}$$

Hence,

$$\nabla\psi_2 = i\exp(ix)\hat{x} \tag{243}$$

Thus, the mutual energy flow density becomes:

$$\boldsymbol{J}_m = \frac{1}{4}\begin{cases}(-i\exp(ix))(-i\exp(ix)\hat{x})^* + \exp(ix)^*(-\exp(ix)\hat{x}) & x < 0\\ (-i\exp(ix))(-i\exp(ix)\hat{x})^* + \exp(ix)^*(\exp(ix)\hat{x}) & 0 \le x \le L\\ (-i\exp(ix))(i\exp(ix)\hat{x})^* + \exp(ix)^*(\exp(ix)\hat{x}) & L < x\end{cases}$$

$$= \frac{1}{2}\hat{x}\begin{cases}0 & x < 0\\ 1 & 0 \le x \le L\\ 0 & L < x\end{cases} \tag{244}$$

This shows that the mutual energy flow density is generated at $x = 0$ and annihilated at $x = L$.

4.5. Mutual Energy Flow Law

From the mutual energy flow law of electromagnetic fields:

$$-\int_{V_1} \boldsymbol{E}_2^* \cdot \boldsymbol{J}_1\, dV = \frac{1}{2}\oint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n}\, d\Gamma = \int_{V_2} \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^*\, dV \tag{245}$$

This becomes:

$$-\int_{V_1} \psi_2^*\tau_1\, dV = \frac{1}{4}\oint_\Gamma (\psi_1\boldsymbol{h}_2^* + \psi_2^*\boldsymbol{h}_1) \cdot \hat{n}\, d\Gamma = \int_{V_2} \psi_1\tau_2^*\, dV \tag{246}$$

where $\boldsymbol{h}_1$ is the magnetic field calculated according to the author's theory ((237),(237)(238)). In this formula, the energy output by the source τ_1 is converted into the energy flow of the particle, and this energy flow is ultimately absorbed by the sink. The energy absorbed by the sink is:

$$\int_{V_2} \psi_1\tau_2^*\, dV$$

This is the work done by ψ_1 on τ_2. Thus, equation (246) represents the energy flow law of particles corresponding to Schrödinger's equation. The particle is created at the source τ_1 and annihilated at the sink τ_2.
It can be easily proven that:

$$\boldsymbol{J}_m = \boldsymbol{J}_S \tag{247}$$

That is, for the plane wave, the mutual energy flow density calculated using the author's method is consistent with the self-energy flow density calculated using Schrödinger's equation. However, the author's mutual energy flow is emitted at the source and annihilated at the sink, while the probability flow (self-energy flow) calculated using Schrödinger's equation generally has an emission but no annihilation. Therefore, using self-energy flow or probability flow density to describe particles is very limited. Particles should be described using mutual energy flow instead.

5. Conclusion

This paper presents three different modes of electromagnetic waves. (1) The broadcast mode, where the source and sink emit random spherical waves that decay with distance. The source emits retarded waves randomly and the sink emits advanced waves randomly. According to quantum mechanics, this is a superposition state since the source sends signals to all sinks simultaneously. (2) The photon mode or waveguide mode. In this case, there is synchronization between the advanced wave emitted by a sink and the retarded wave emitted by the source. Synchronization can also be referred to as handshake, which means that when a retrded wave hits a certain absorber, the absorber also emits an advanced wave. This advanced wave will synchronize with that retarded wave. Due to interference, both the retarded wave from the source and the advanced wave from the sink interfere constructively along the line between the source and the sink and destructively or weakly in other directions. In this way, the advanced wave becomes the waveguide for the retarded wave, and vice versa. Eventually, both the retarded and advanced waves become quasi-plane waves that do not decay with distance. The mutual energy flow formed by these waves transmits energy. This mutual energy flow has the properties of a particle, which can be regarded as a photon. Thus, the wave collapses from the superposition state into a photon. (3) The average effect of a large number of photons (mutual energy flow) constitutes a macroscopic electromagnetic wave. These macroscopic electromagnetic waves are proven to be identical to the Poynting vector calculated using Maxwell's equations. In this sense, the author's electromagnetic theory can be regarded as a more fundamental theory of electromagnetism than Maxwell's theory. Maxwell's electromagnetic theory agrees with the author's theory in terms of macroscopic radiation patterns but differs significantly in the specific electromagnetic fields. The author found that for waveguide-mode electromagnetic waves, the quasi-static electromagnetic field theory should be used for calculations, and then the retardation or advancement factor can be added. There is no need to use Maxwell's equations. Of course, if Maxwell's equations are used, the magnetic field at the far-field should be corrected. After the correction, the magnetic field and electric field of the electromagnetic wave maintain a 90-degree phase difference instead of being in phase. The author discovered that the error in Maxwell's electromagnetic theory arises because the curl of the magnetic vector potential is the average magnetic field along the loop, not the true magnetic field. The average magnetic field along the loop is equivalent to the magnetic field in quasi-static conditions, but they are no longer equivalent in the case of electromagnetic waves. Maxwell's electromagnetic theory confuses the average magnetic field along the loop with the magnetic field itself. The author found the same error in quantum theory. Therefore, the author applied the correction from electromagnetic theory to quantum theory. The Schrödinger equation was modified to include the source, sink, and magnetic field, and the mutual energy flow law was established. The mutual energy flow is formed by retarded and advanced waves. However, both self-energy flow and mutual energy flow, calculated from Schrödinger's equation, still have issues. The issue is that both self-energy flow and mutual energy flow transfer energy. This issue is identical to that in Maxwell's electromagnetic theory. Therefore, the author applied the same correction to the magnetic field in Schrödinger's equation. After the correction, the magnetic field in Schrödinger's equation and the wave function maintain a 90-degree phase difference. This makes the probability flow density in Schrödinger's equation represent reactive power, no longer transferring energy. Thus, there is no need to call it probability flow anymore. After the correction, mutual energy flow transmits energy, and the amount of energy transmitted by mutual energy flow is exactly the same as the energy transmitted by the original probability flow in Schrödinger's equation. However, mutual energy flow is generated at the source and annihilated at the sink, whereas the probability flow (self-energy flow) calculated by Schrödinger's equation only has an emission point but no annihilation point. Therefore, the corrected quantum theory can use mutual energy flow to describe particles, which provides a good solution to the wave-particle duality problem.

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