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Research Article

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Proof that Fermat's Last Theorem Holds in a Large Range

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Abstract: It is proved that for the equation $x^n + y^n = z^n$, for any positive integer x and n ($n \in N$, $n \geq 3$, $x \geq n$) when $y > [x\sqrt[n-1]{\frac{x}{n}}] + 2$

Fermat's Last Theorem holds.

Keywords: Fermat's Theorem, Mathematical Induction, Sandwich Theorem.

Introduction

Fermat's Last Theorem (sometimes called *Fermat's conjecture,* hereinafter referred to as *FLT*) states that no three positive integers *x, y* and *z* satisfy the equation $x^n + y^n = z^n$ for any positive integer n greater than 2 ($n > 2$). The theorem has been solved by the British mathematician Andrew Wiles in 1994, but his proof method is complex. Whether there is "a wonderful proof" called by Fermat has always inspired people to explore. It is found that there is indeed a simple method to prove that the FLT holds in a large range [1].

Through hundreds of years of in-depth and lasting research, we only need to prove that the proposition is true when *n=4* and n is an odd prime number [2]. The former has been proved by Fermat's infinite descent method, and the latter is the research content of this paper [3]. For the convenience of discussion, suppose that , and *x, y* and *z* are mutually prime, that is, gcd (*x, y, z*) = 1. *FLT* is classified into the following two cases for discussion:

(A) $\forall x\,(x \in N, x \geqslant n)$, it is proved that *FLT* is true when .

$$y > [x\sqrt[n-1]{\frac{x}{n}}] + 2.$$

(B) $\forall x\,(x \in N, x \geqslant n)$, it is proved that *FLT* is true when .

$$x < y \leqslant [x\sqrt[n-1]{\frac{x}{n}}] + 2.$$

This paper proves that (A), and the next step is to prove that (B). First, we give the relationship between exponent *n* and *x, y, z*.

Lemma 1.1: $\forall n\,(n \in N, n \geq 3)$, for the equation $x^n + y^n = z^n$, there is $z - y < \frac{x}{n}$.

Proof. Since $x^n = z^n - y^n = (z-y)\sum_{i=0}^{n-1} y^{n-1-i} z^i > n(z-y)y^{n-1} > n(z-y)x^{n-1}$ so

$$z - y < \frac{x}{n} \qquad (1.1)$$

Lemma 1.1 is proved.

Corollary 1 *For any positive integer x, if , FLT is true.*
Proof. According to Formula (1.1), we have

$$0 < z - y < \frac{x}{n} \qquad (1.2)$$

Therefore

$$z < y + \frac{x}{n} \qquad (1.3)$$

For any positive integers y, z, we have

$$z \geq y + 1 \qquad (1.4)$$

From Formula (1.3) and Formula (1.4), we know that when $x<n$, the two are contradictory. It is easy to know that when $x<n$, FLT is true. Corollary 1 is proved. According to corollary 1, hereinafter we agree that. $x \geqslant n$

2. Notation and sketch of the proof

Notation

$[x\sqrt[n-1]{\frac{x}{n}}]$ - integer part of number . $x\sqrt[n-1]{\frac{x}{n}}$

$\forall$ - arbitrary selection.

First we prove that $\forall n\ (n \in N, n \geq 3)$ and $\forall x\ (x \in N, x \geqslant n)$,

if $x^n + y^n = (y+1)^n$, there must be a positive integer $y_n = [x\sqrt[n-1]{\frac{x}{n}}]$,

let $y^* = y_n + 2$, the equation

$$x^n + (y^*)^n < (y^* + 1)^n \quad (2.1)$$

Then we prove $\forall m\ (m \in N)$, the equation

$$x^n + (y^* + m)^n < (y^* + m + 1)^n \quad (2.2)$$

So, for the equation $x^n + y^n = z^n$, for any positive integers y

with $y > [x\sqrt[n-1]{\frac{x}{n}}] + 2$, we have

$$y^* + m < z < y^* + m + 1 \quad (2.3)$$

Since z is between two positive integers $y^* + m$ and $y^* + m + 1$, it must not be a positive integer. This proves that ($\forall n\ n \in N\ n \geq 3$) and $\forall x(\ x \in N,\ x \geqslant n\ )$, *FLT* is true for all positive integers with .
$y > [x\sqrt[n-1]{\frac{x}{n}}] + 2$

3. Lemmas

Lemma 1. $\forall n\ (\ n \in N,\ n \geq 3)$ *and* $\forall x\ (\ x \in N\ ,\ x \geqslant n\ )$, *for the equation* $x^n + y^n = (y+1)^n$, *there must be a positive integer*

$y_n = [x\sqrt[n-1]{\frac{x}{n}}]$, *with* $y < y_n + 1$

Proof. Since $x^n = (y+1)^n - y^n = \sum_{i=0}^{n-1}(y+1)^i y^{n-1-i}$ so

$$ny^{n-1} < x^n < n(y+1)^{n-1} \quad (3.1)$$

Then we have

$$x\sqrt[n-1]{\frac{x}{n}} - 1 < y < x\sqrt[n-1]{\frac{x}{n}} \quad (3.2)$$

Let the integer part of $x\sqrt[n-1]{\frac{x}{n}}$ be y_n, we write $y_n = [x\sqrt[n-1]{\frac{x}{n}}]$

By (3.2), we have

$$y < y_n + 1 \quad (3.3)$$

Lemma 1 is proved.

Lemma 2. *Let* $y^* = y_n + 2$, *where* $y_n = [x\sqrt[n-1]{\frac{x}{n}}]$, $\forall n\ (n \in N) n \geq 3$ *and* $\forall x\ (\ x \in N,\ x \geqslant n\ )$*, there is* $x^n + (y^*)^n < (y^* + 1)^n$.

Proof. Since $(y^* + 1)^n - (y^*)^n = \sum_{i=0}^{n-1}(y^* + 1)^i (y^*)^{n-1-i}$, *there is*

$$(y^* + 1)^n - (y^*)^n > n\ (y^*)^{n-1} = n\ (y_n + 2)^{n-1}.$$

By (3.1) and (3.3), *we have*

$$n(y_n + 2)^{n-1} > n(y+1)^{n-1} > x^n \quad (3.4)$$

Therefore

$$(y^* + 1)^n - (y^*)^n > x^n \quad (3.5)$$

Lemma 2 is proved.

Lemma 3. $\forall n\ \ n \in N\ \ n \geq 3$, $\forall x\ \ x \in N\ \ x \geqslant n$ and $\forall m\ \ (m \in N)$ *for any positive integers y with*

$y > [x\sqrt[n-1]{\frac{x}{n}}] + 2$, *we have*

$$x^n + (y^* + m)^n < (y^* + m + 1)^n \quad (3.6)$$

Remark. we will prove (3.6) by mathematical induction.
(A) First, we prove that (3.6) holds with $n = 3$, that is

$$x^3 + (y^* + m)^3 < (y^* + m + 1)^3 \quad (3.7)$$

Proof. When $n = 3$, let $y^* = [x\sqrt{\frac{x}{3}}] + 2$, for any positive integer m, we have

$$(y^* + m + 1)^3 - (y^* + m)^3 = (y^* + 1)^3 - (y^*)^3 + 3m(2y^* + 1) + 3m^2 \quad (3.8)$$

Let $n = 3$ in (3.5), we have

$$x^3 + (y^*)^3 < (y^* + 1)^3 \quad (3.9)$$

Substitute formula (3.9) into formula (3.8), then we have

$$(y^* + m + 1)^3 - (y^* + m)^3 > (y^* + 1)^3 - (y^*)^3 > x^3 \quad (3.10)$$

Therefore, when $n = 3$, (3.6) holds.
(B) The following we prove that (3.6) is true when $n > 3$.

Proof. Suppose that $(y^* + m + 1)^n > x^n + (y^* + m)^n$, we have

$$(y^* + m + 1)^{n+1} > (y^* + m + 1)[x^n + (y^* + m)^n] > x^{n+1} + (y^* + m)^{n+1} \quad (3.11)$$

By (3.11), it suffices to show that

$$(y^* + m + 1)^{n+1} - (y^* + m)^{n+1} > x^{n+1} \quad (3.12)$$

Therefore, for index $n + 1$($n \in N$), formula (3.6) holds.
From formula (3.7) and formula (3.12), lemma 3 is proved.

Theorem

Theorem 1. For any positive integer n ($n \in N,\ n \geq 3$) and $x\ (\ x \in N\ ,\ x \geqslant n)$, *for the equation* $x^n + y^n = z^n$, *there must be a positive integer* $y_n = [x\sqrt[n-1]{\frac{x}{n}}]$, *for any positive integer y with* $y > y_n + 2$, *FLT constant holds.*

Proof. For any positive integer

$$y^* = [x\sqrt[n-1]{\frac{x}{n}}] + 2 \quad (n \in N, n \geq 3, x \in N, x \geqslant n),$$

let $x^n + (y^*)^n = z^n$, according to lemma 2, $y^* < z < y^* + 1$. At the same time, for any positive integer m, let $x^n + (y^* + m)^n = z^n$, according to lemma 3, $y^* + m < z < y^* + m + 1$. Easy to know,

$\forall n\,(n \in N, n \geq 3)$ *and* $\forall x\,(x \in N, x \geqslant n)$, for any positive integer y with $y > [x\sqrt[n-1]{\frac{x}{n}}] + 2$, z in equation $x^n + y^n = z^n$ is between two adjacent positive integers y and $y + 1$. Therefore, z must not be a positive integer and *FLT* holds.

Theorem 1 is proved.

Conclusion

This paper proves that for all positive integers $n\,(n \in N, n \geq 3)$ and $x\,(x \in N, x \geqslant n)$, for any positive integer y with $y > [x\sqrt[n-1]{\frac{x}{n}}] + 2$ *FLT* holds. The next step is to prove that y is in the interval $[x+1, x\sqrt[n-1]{\frac{x}{n}} + 2]$ and FLT is correct.

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