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Research Article

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Principles of Classical Mechanics Describe Gyroscopic Effects

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Abstract
The unexplainable motions of the simple spinning top toy, boomerang, and other spinning objects attracted people since the time of ancient civilizations. The intricated motions of the rotating objects and the action of the unknown forces were named gyroscopic effects. The physicists and mathematicians tried to solve them beginning from the Industrial Revolution until our time. One gyroscopic effect which is the side motion of the spinning disc was described by L. Euler and presented in the world encyclopaedias as the fundamental principle of gyroscope theory. The origin of gyroscopic effects is more sophisticated than in published and simplified theories. Analytical solutions to other gyroscopic effects were described only in our times. The system of the interrelated inertial torques generated by the rotating mass acts on the spinning object based on the principle of mechanical energy conservation. This system presents the fundamental principles of gyroscope theory.

Introduction

The phenomenon of gyroscopic effects attracted people since the time of ancient civilizations. The unusual and unexplainable motions of the simple spinning top toy, boomerang and other spinning objects astonished people. With the evolution of science, the intricated behaviour of the rotating objects under the action of the unknown forces named gyroscopic effects did not leave without the attention of the physicists and mathematicians of the Industrial Revolution. However, the solution of the gyroscopic effects dragged to a couple of centuries and proved to be a solid nut for them. One gyroscopic effect is the side motion of the spinning disc under the action of precession torque was described by the mathematician L. Euler in 1765.. His solution as the change in the angular momentum is presented in the contemporary world encyclopaedias as the fundamental principle of gyroscope theory. He could describe the second gyroscopic effect based on the action of the centrifugal forces but did not do it for an unknown reason. Analytical solutions of other gyroscopic effects could not be defined in principle because they were based on the discoveries of the following century. The Coriolis acceleration (1835) and the physical laws of energies were formulated in the middle of the nineteenth century (1847). Finally, in 1905, Albert Einstein established the concept of potential and kinetic energy that was generalized into the form used today.

At the beginning of the twenty century, physicists and mathematicians had all possibilities to describe the gyroscopic effects and explain their physics but it was done after the passing of one century. The known scientific methods developed by physics and mathematics enable formulating and solving of more complex problems than properties of the simple in the design of the rotating disc. As described above historical events and years show the link with the so-called “human factor” and its ability to innovate.

For the time of the twenty century, scientists and researchers published probably tons of manuscripts and dozens of theories dedicated to gyroscopic effects, but the practice did not confirm their mathematical models [1-3]. For practical applications of the gyroscopic devices were elaborated many numerical modeling of gyroscopic effects with software [4-7]. Gyroscopic problems remained without solutions and physical explanation until recent times [8 -12].

Today, gyroscopic effects are formulated by mathematical models, and their physics is described. In reality, the analytical solutions of gyroscopic effects are proved to be more sophisticated than in published and simplified theories. The reason for such a statement is in several physical approaches to the formulation of the mathematical models of the action of the inertial torques on the spinning object. The first is the scientists did not consider the action of the system of torques generated by the centrifugal and Coriolis forces of the distributed mass of the rotating object and the torque of the change in the angular momentum. The second is the mathematical models for gyroscope motions around two axes did not include the interrelated action of all torques.

The origin of gyroscopic effects is the action of the system of inertial torques generated by the rotating mass of the spinning object under the action of external torque. The spin of the object expresses its kinetic energy and the action of the external torque expresses its potential energy These energies produce the two sets of the eight interrelated inertial torques acting about two axes of the gyroscope motions. The set of the inertial torques contains two torques generated by the centrifugal forces around two axes and torques generated by the Coriolis forces and the change in the angular momentum acting around one axis. The equality of the kinetic energies of the gyroscopic motion about axes interrelates the action of the system of inertial torques about two axes of gyroscope motions.

The angular velocities of the gyroscope about axes of rotations and the value of inertial torques are different. The physics of the gyroscopic effects has expressed the principle of the conservation of mechanical energy for spinning objects. The fundamental principles of gyroscopic effects and expressions of mathematical models for gyroscopic inertial torques were published and represented in Table 1 [13-16]. The action of the external and inertial torques and motions of the spinning disc is presented at the Cartesian 3D coordinate system Σoxyz in Fig. 1. The method of coinciding coordinate axis oz and the axle of the disc drastically simplifies the mathematical models for the gyroscope motions and their solutions.

Table 1. Fundamental principles of the gyroscope theory

Fundamental principles of the gyroscope theory		Action	Equation
Inertial torques generated by	centrifugal forces	Resistance	$T_{ct.i} = (4\pi^2/9)J\omega\omega_i$
		Precession	
	Coriolis forces	Resistance	$T_{cr.i} = (8/9)J\omega\omega_i$
	change in angular momentum	Precession	$T_{am.i} = J\omega\omega_i$
Principle of mechanical energy conservation	Dependency of angular velocities of the spinning disc about axes of rotation: $\omega_y = (8\pi^2 + 17)\omega_x$		

Table 1 contains the following symbols: ω_i is the angular velocity of the spinning disc motion about axis *i*; ω is the spinning velocity of the disc about axis *oz; J* is the moment of inertia of the spinning disc. Inertial torques are proportional to the angular velocity of the spinning disc ω, the angular velocity of precession ω_i, and its moment of inertia *J*. The expressions of inertial torques depend on the geometry of the rotating object presented by the different digital coefficients of the inertial torques (Table 1). Engineering practice shows many unique rotating designs such as propellers, cones, paraboloids, spheres, etc., and all of them manifest gyroscopic effects. The expressions of inertial torques for the complex designs of the rotating objects wait for their own solutions and present challenges for engineers and practitioners.

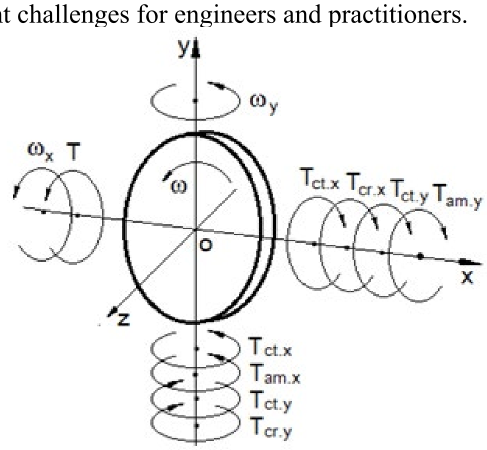

Figure 1. The action of the one external and eight inertial torques on the spinning disc and its motions

Methodology

The gyroscopic effects are the result of the action of the following torques on the spinning disc that manifest their motions. It is considered the spinning disc which rotates with the angular velocity ω in the counter-clockwise direction. The external torque *T* turns the spinning disc in the counter-clockwise direction about axis *ox* with the angular velocity ω_x. The external torque *T* generates the system of the simultaneous action of the inertial torques (Table 1) about two axes. The motions of the spinning disc about axes are implemented by the defined dependency due to the interrelations of the inertial torques (Table 1) [10]. Analysis of the action of torques is implemented by the method of their causal investigatory dependencies about two axes and presented by the following:

The external torque *T* produces

The initial resistance torques ($T_{ct.x}$ and $T_{cr.x}$) generated by the centrifugal and Coriolis forces acting about axis *ox*, and contradict to the action of the external torque *T*;

The initial precession torques ($T_{ct.x}$ and $T_{am.x}$) that are generated by the centrifugal forces and the change in the angular momentum acting about axis *oy*. These precessions torque is load torques for the spinning disc and originated along with axis *ox* but act about axis *oy* in the counter-clockwise direction.

The spinning disc turns in the counter-clockwise direction about axis *oy* under the action of the initial precession torques ($T_{ct.x}$ and $T_{am.x}$). The initial precession torques ($T_{ct.x}$ and $T_{am.x}$) generate the initial resistance torques ($T_{ct.y}$ and $T_{cr.y}$) of centrifugal and Coriolis forces which contradict the action of the initial precession torques.

The initial resulting torque about axis *oy* ($T_{r.y} = T_{ct.x} + T_{am.x} - T_{ct.y} - T_{cr.y}$) is the combination of the initial precession and initial resistance torques, generates the initial precession torques ($T_{ct.y}$ and $T_{am.y}$) of centrifugal forces and the change in the angular momentum acting about axis *ox*. The sum of the initial resistance torques and the initial precession torques ($T_{t.x} = T_{ct.x} + T_{cr.x} + T_{ct.y} + T_{am.y}$) about axis *ox* constitutes the total initial resistance torque $T_{t.x}$ acting opposite to the torque T.
The resulting initial torque acting about axis *ox* is $T_{r.x} = T - (T_{ct.x} + T_{cr.x} + T_{ct.y} + T_{am.y})$. The resulting initial torque acting about axis *ox* $T_{r.x}$ generates the corrected precession torques acting about axis *oy* ($T^*_{ct.x}$ *and* $T^*_{am.x}$) that is not the same as at starting condition (paragraph (a)). The corrected precession torques ($T^*_{ct.x}$ *and* $T^*_{am.x}$)in turn generate the corrected resistance torque (T^*_{cty} *and* $T^*_{am.y}$). The equation of the total corrected resulting torque acting about axis oy is presented by the following equation $T^*_{r.y} = T^*_{ct.x} + T^*_{am.x} - (T^*_{ct.y} + T^*_{cr.y})$ which generates the corrected precession torques ($T^*_{ct.y}$ *and* $T^*_{am.y}$) acting around axis *ox*.

The corrected resulting torque acting about axis *ox* is $T_{c.x} = T - (T_{ct.x} + T_{cr.x} + T^*_{ct.y} + T^*_{am.y})$. The corrected resulting torque acting about axis *ox* $T_{c.x}$ generates the final precession torques acting about axis *oy* ($T_{f.ct.x}$ and $T_{f.am.x}$) that is not the same as the corrected (paragraph (d)). The final precession torques ($T_{f.ct.x}$ and $T_{f.am.x}$) in turn generate the final resistance torque ($T_{f.ct.y}$ *and* $T_{f.cr.y}$). The equation of the final resulting torque acting about axis *oy* is presented by the following equation $T_{f.ct.y} = T_{f.ct.x} + T_{f.am.x} - (T_{f.ct.y} + T_{f.cr.y})$ which generates the final precession torques ($T_{f.ct.y}$ *and* $T_{f.am.y}$) acting around axis *ox*.

The final resulting torque acting about axis *ox* is $T_{f.x} = T - (T_{ct.x} + T_{cr.x} + T_{f.ct.y} + T_{f.am.y})$.
The following modifications of the inertial torques are nuances and demonstrate the firm interrelation of the inertial torques that cannot be separated from their chain because all of them are generated by one rotating mass of the spinning objects. The action of all inertial torques about two axes presents the looped chain with the feedback links.

The action of torques and motions of the gyroscope are described by mathematical models that explain the physics of gyroscopic effects and are validated by practical tests. A new analytical approach to gyroscopic effects shows the new phenomena, which is presented by the deactivation of resistace inertial torques $T_{f.x} = T_{ct.x} + T_{cr.x} + T_{f.ct.y} + T_{f.am.y} = 0$, at the time of blocking the precessed motion ($\omega_y = 0$). The spinning disc rotates around axis *ox* under action only the external torque T. At the first site, these phenomena contradict the principles of Newtonian mechanics but can be described analytically and explained in their physics by the nontraditional method. The deactivation of gyroscopic inertial forces results from the interrelations of the inertial torques and motions of the gyroscope under the condition of blocking its rotation around one axis. The blocking of the gyroscope motion around one axis leads to the absence of the kinetic energy of rotation around the two axes under the action of the inertial torques. All inertial torques generated by the distributed mass ($T_{ct.x}$, $T_{cr.x}$, $T_{f.ct.y}$, $T_{f.am.y}$, acting around axis ox and $T_{ct.x}$, $T_{f.ct.y}$, $T_{f.cr.y}$ acting around axis *oy*) are deactivated. The torque of the change in the angular momentum $T_{am.x}$ is acting, and the angular velocity of the motion of the spinning disc around axis *ox* is increased by the principle of mechanical energy conservation. This sophisticated interrelation of the inertial forces, motions around axes, and the spinning of the gyroscope's disc are perceived with difficulty by physicists. The complex phenomenon of multifunctional processes does not have a simple explanation.

Results and Discussion

A described peculiarity of the action of inertial torques on the spinning disc expresses the physics of their origination. The action of these torques is presented in the equations of gyroscope motions around axes at the accepted coordinate system and depends on the consideration of the gyroscopic problems. The presentation of the action of these torques allows describing the physical processes that result in the gyroscopic effects. All known gyroscopic effects so-called "antigravity effect", "non-inertial", and other artificial terms and anti-scientific statements must be removed from terminology because all of them are described by the principles of classical mechanics and the theory of gyroscopic effects.

The methods of the theory of gyroscopic effects describe all complex motions of the gyroscopic devices. There are presented by the inversion of the Tippe top, the nutation of the gyroscope, and the cyclic inversions of the rotating objects at the orbital free flight that is gyroscopic effects at the weightless condition.

Conclusion

The physics of gyroscope inertial torques and motions are explained in detail and confirmed by practice. The fundamental principles of gyroscope theory are inertial torques and the dependency of the angular velocities of gyroscope motions about axes of rotation based on the potential and kinetic energy of the spinning disc. Classical mechanics receive a new chapter on the dynamic of rotating objects and new methods for the analysis of the inertial torques and kinetic dependency of rotation of gyroscopes. Today, all gyroscope problems can be solved manually without numerical modelling with software. The phenomena of gyroscopic effects have been resolved finally and irrevocably.

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