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Physics and Mechanizm of Creation, Separation and Orbiting of the Solar System Bodies Based on Jacobi Dynamics

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Introduction

Our work sets forth and builds upon the fundamentals of the dynamics of natural systems in formulating the problem presented by Jacobi in his famous lecture series "Vorlesung über Dynamik". In the dynamics of systems described by models of discrete and continuous media, the many-body problem is usually solved in some approximation, or the behavior of the medium is studied at each point of the space it occupies. Such an approach requires the system of equations of motion to be written in terms of space co-ordinates and velocities, in which case the requirements of an internal observer for a detailed description of the processes are satisfied.

In the dynamics discussed here we study the time behavior of the fundamental characteristics of the physical system, i.e. the Jacobi function (polar moment of inertia) and energy (potential, kinetic and total), which are functions of mass density distribution, and the structure of a system. This approach satisfies the requirements of an external observer. It is designed to solve the problem of global dynamics and the evolution of natural systems in which the motion of the system's individual elements written in space co-ordinates and velocities is of no interest. It is important to note that an integral approach is made to internal and external interactions of a system which results in radiation and absorption of energy. This effect constitutes the basic content of global dynamics and the evolution of natural systems.

It was shown in our publications [1-15] that the needs of farther development of fundamentals in physics and mechanics have appeared for interpretation of new experimental facts obtained by the artificial satellites in the cosmic space. In fact, it was found by analysis of the artificial satellite orbits that the Earth and the Moon do not stay in hydrostatic equilibrium accepted as a basic postulate in the existing theories of their motion and the inner structure. That result means that the applied model of the hydrostatic equilibrium for celestial bodies in the uniform force field is not proved by the observed effects of the gravitational mass interaction. The theory of the Earth's and other planets' figure is also based on the hydrostatics. In this case, because the sum of the inner forces and moments are equal to zero, the bodies are considered as solid objects and their rotation is accepted as inertial, which also does not proved by the observation.

A serious discrepancy was found in the motion of Earth and other planets related to the ratio between the potential and kinetic energy. It is well known that the potential energy of the Earth almost by 300 times more the kinetic energy presented by inertial rotation of the planet. The same and even higher ratio is valid for the other planets, the Sun and the Moon. But according to the virial theorem the potential energy should be twice as much of kinetic energy. It means that the Earth and other planets exist without kinetic energy. An idea has appeared that there is some latent form of motion of the particles constituting the bodies, which has not taken into account and is not considered in the existing theories.

Taking into account relationship between the gravitational moments and gravitational field of the Earth observed by study of the artificial satellites of the Earth, we come back to derivation of the virial theorem in classic mechanics. Replacing the vector forces and moments by their volumetric values we obtained for n particle system the condition of its dynamical equilibrium in the own force field. The new generalized form of the virial theorem remains in framework of the Newtonian laws of motion but with periodic component expressed by the second derivative from the polar moment of inertia. Thus, for study of a body dynamics in its own force field the condition of hydrostatic equilibrium by dynamic (periodically oscillating) equilibrium is replaced. In this case the planet's kinetic energy by oscillating motion of the interacting particles is reanimated. And the ratio between the potential and kinetic energy to the classic virial theorem condition has reverted. In addition, a new phenomenon of the nature of gravitation as a dynamical effect of the innate energy of the interacted elementary particles appears. On the basis of the obtained results we found that gravitation, inertia and weightlessness have common innate nature in the form of elementary particles interaction energy which determines all the dynamical processes in creation and decay of natural systems.

Astrophysical science evidences that the observed in nature forms of motion of material particles and objects are determined by interaction of their constituting elementary particles. More than 300 sub-nuclear particles have been discovered up to now. But the strict definition of the term "elementary particle of matter" up to now does not exist because such a particle has not been identified experimentally and theoretically. W. Heisenberg (1968) in its work "Introduction to the unified theory of elementary particles" notes that according to his and other researcher experimental data, at collision of two particles of high energy multiple other elementary particles appear. But they are not necessarily appeared smaller of the collided particles. Moreover, it appears that the new particles are always born of the same type independently of the nature of the collided. And also, the excess of the kinetic energy of the collided particles is converted into the matter of the created particles. It follows from the observation that the produced different elementary particles can be considered as various forms of existing matter or energy. The size of the new particles remains the same.

Taking into account the above experimental results the elementary particles in this work are understand as the sub-nuclear material particles which form basis of all variety of objects of the material world.

The most valuable result of our studies is discovery of the new law of planets and their satellites orbiting in the Solar System [14]. In this discovery the astrophysics postulate about relationship of motion of the natural objects with interaction of their elementary particles has proved. The law demonstrates that all the planets and satellites have been orbited by the first cosmic velocity of their protoparents. Namely, the planets move in orbits with the first cosmic velocity of the protosun, the radius of which was equal to the semi-major axis of modern orbit of each planet. The satellites of each planet move with mean orbital velocity equal to the first cosmic velocity of the corresponding planet having radius equal to the semi-major axis of modern orbit of each satellite. This law holds for all the small planets of the asteroid belt and for all the comets. Theoretically the law follows from solution of the Jacobi's virial equation and proved by the astronomical observation. It follows from the discovered law that the accepted up to now postulate on gravitational attraction of two interacted bodies appears to be a speculation. In fact, the orbital motion initiates by the outer gravitational field of the central parental body. And the direction of the orbiting is determined by the Lenz's rule. Thus, the gravitational field of a celestial body is the centrifugal effect of the body's interacting elementary particles energy and the matter and its energy are the innate natural discrete-wave phenomenon. On this basis we conclude that the gravitation and inertia are the centrifugal and equal to it centripetal effects of the elementary particles interaction energy leading to redistribution of the particles energy and changes in the body's mode motion. All the other dynamical processes should follow from that effect. A self-gravitating body is an excellent example of the natural centrifuge.

From the standpoint of methodology, the integral approach has an important advantage. In this approach the integral character of the principle of least action – the basic philosophical principle of mechanics and physics – is fully realized. It is achieved by using a canonical pair consisting of the Jacobi function and frequency in writing the basic equation of global dynamics. The practical use of this pair in Jacobi's virial equation made it possible to farther generalize the forms of motion and to show that the non-linear oscillations of a system is such a generalization.

We note that the ten well-known integrals of motion in the many-body problem in its classical formulation should be regarded as historically the earliest equations of the integral type. These integrals, however, reflects not the specific nature of a system under consideration but the general properties of space and time, i.e. homogeneity of space and time and isotropicity of space.

The first non-trivial equation of dynamics in terms of the integral characteristics of a system is Jacobi's virial equation, which describes changes in the moment of inertia (Jacobi function) as a function of time. The next step in this direction was taken by Chandrasekhar (1969). He used and developed for solution of problems in mechanics the method of moments, so called in analogy to the method well known in mathematical physics. However, the problem of non-trivial solution of the non-linearized equations in terms of integral characteristics was not solved in either of these cases.

Our work began in 1974. As a result, a number of articles on the theory of virial oscillations of celestial bodies were published in the journal Celestial Mechanics and other periodicals [1-11]. The theory was based on solution of Jacobi's virial equation for conservative and dissipative systems.

To solve Jacobi's initial equation, we first used the heuristically found relationship between the potential energy and moment of inertia of a system, which was expressed in terms of the product of the corresponding form factors. It was found that the product depends little on the law of distribution of mass density for a wide range of formal, non-physical systems. It was then demonstrated that in the asymptotic limit of simultaneous collision of the particles constituting a system, the observed constancy of the product of form factors remained valid without any restrictions within the framework of the Newton and Coulomb interaction laws. The invariant found was also demonstrated to be valid for the widely used relativistic and non-relativistic physical models of natural systems. It enabled us to derive from Jacobi's equation a simple form of the equation of virial oscillations with one unknown function and to find its rigorous solution. The equation obtained describes the dynamics of a wide class of physical systems ranging from empty space-time and collapsing stars to the atom. Thus, it was established that the theory of virial oscillations of celestial bodies was valid far beyond the limits of celestial mechanics based on Newton's law of equations.

The main natural mechanism, which is used for controlling physical processes of the solar system bodies are gravitational, electromagnetic, weak and strong interactions of the matter. Application of those interactions for solution of practical problems creates some difficulties and reciprocal contradictions. Therefore the old dream of physicists is to unify all the interactions by a common theory. The first success in that direction was achieved in 1967 by S. Weinberg and A. Salam who unify electromagnetic and weak interactions into the electroweak interaction. The next step should be done in the same direction. It is assumed that the problem can be solved by discovery of a common natural process where all the physical interactions act as its components. We found that such a process is really exists. It appears to be the process of interaction of n-particle system. Dynamical effects of the n-particle system's interaction energy are really differs in particle frequencies which we perceive as interaction of natural forces.

Analyses of the physical meaning of dynamical effects related to n-particle system's interaction energy were developed in our works [10-15], where the problem of the Solar System formation and the nature of gravitation, inertia and weightlessness were presented. At present the analysis is continued and the conditions of gravitation and electromagnetic interactions unity are discussed. Physical basis for such a unity is served by the energy unity of both interactions. The modern technologies of the electricity generation prove the idea. The process of electricity generation itself in fact represents transformation of mechanical (gravitational) energy of river water or water vapor flow to the ordered electrons flow in conductor. In addition, the modern electric machines are reversible. They work in regime of both as electricity generator and mechanical engine in frame work of energy conservation law. The energy is the quantitative measure of motion in all forms of interaction. The natural energy is not disappeared and appeared from nothing. It is only transformed from one form to another. The law of energy conservation is kept by all the natural processes and fundamental interactions. It is found at solution of the n-body problem in frame work of Jacobi Dynamics that the forces of gravitation and inertia appears to be effects of centrifugal energy of a rotating system's matter In accordance with classical physics the energy of a system can be changeable value. But it keeps constant value in modulus.

It was found in our previous works devoted to solution of Jacobi's virial equation that the common continuously acting process is interaction of elementary matter particles which possess natural energy. Here all physical interactions are developed in the form of n-particle system. The effect was studied with respect to the unity of corpuscular-wave motion of the n-particle system and its potential field.

Our study on Jacobi dynamics opened the mysterious gravitation and inertia [15]. It is appeared that the Newtonian gravitation and Galilean inertia hide in the centripetal and centrifugal effects of the n-particle system's interaction and potential field energy. A self-gravitating celestial body represents a natural centrifuge rotating by the energy of its interacting elementary particles. The dynamical effects of such a centrifuge appear to be the centripetal and centrifugal forces in which roles the inertial and equal by modulus gravitational forces play. In practice those forces are used in different construction of centrifuges for separation of the components of matter in gaseous, liquid and solid states with respect to their density (force of weight). In nature the same forces separate the shells and elementary particles of bodies and their systems. Fundamentals of Jacobi dynamics completely correspond to the conditions of natural centrifuge. The centrifuge is an excellent experimental model for the study of dynamical effects in solving the many-body problem.

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A number of discrepancies were found in the Earth and planets motion related to the ratio between the potential and kinetic energy. Taking into account relationship between the gravitational moments and gravitational field of the Earth observed by study of the artificial satellites of the Earth, we come back to derivation of the virial theorem in classic mechanics. Replacing the vector forces and moments by their volumetric values we obtained for n-particle system the condition of its dynamical equilibrium in the own force field. The new generalized form of the virial theorem remains in framework of the Newtonian laws of motion but with periodic component expressed by the second derivative from the polar moment of inertia. Thus, for study of a body dynamics in its own force field the condition of hydrostatic equilibrium by dynamic (periodically oscillating) equilibrium is replaced. In this case the planet's kinetic energy by oscillating motion of the interacting particles is reanimated. And the ratio between the potential and kinetic energy to the classic virial theorem condition has reverted. In addition, a new phenomenon of the nature of gravitation as a dynamical effect of the innate energy of the interacted elementary particles appears. On the basis of the obtained results we found that gravitation, inertia and weightlessness have common innate nature in the form of elementary particles interaction energy which determines all the dynamical processes in creation and decay of natural systems.

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Imbalance between the Earths' Potential and Kinetic Energy and Generalization of Classical Virial Theorem

We discovered the most likely serious cause, for which even formulation of the problem of the Earth's dynamics based on the hydrostatic equilibrium is incorrect. The point is that the ratio of kinetic to potential energy of the planet is equal to ~1/300, i.e. the same as its oblateness. Such a ratio does not satisfy the fundamental condition of the virial theorem, the equation of which expresses the hydrostatic equilibrium condition. According to that condition the considered energies' ratio should be equal to 1/2. Taking into account that kinetic energy of the Earth is presented by the planet's inertial rotation, then assuming it to be a rigid body rotating with the observed angular velocity $\omega r = 7{,}29.10^{-5}$ s^{-1}, the mass M = 6.10^{24} kg, and the radius R = $6{,}37{\cdot}10^{6}$ m, the energy is equal to: $Te = 0.6MR^2\ \omega_r^2 = 0.6{\cdot}6{\cdot}10^{24}{\cdot}(6.37{\cdot}10^6)^2{\cdot}\ (7.29{\cdot}10^{-5})^2 = 7.76{\cdot}10^{29}$ J =7.76·1036 erg.

The potential energy of the Earth at the same parameters is

$U_e = 0.6{\cdot}GM^2/R = 0.6{\cdot}6.67{\cdot}10^{-11}{\cdot}\ (6{\cdot}1024)^2/6.37{\cdot}10^6 = 2.26{\cdot}10^{32}$ J = $2.26{\cdot}10^{39}$ erg.

The ratio of the kinetic and potential energy comprises

$T_e / U_e = 7.76{\cdot}10^{29} / 2.26{\cdot}10^{32} = 1/291.$

One can see that the ratio is close to the planet's oblateness. It does not satisfy the virial theorem and does not correspond to any condition of equilibrium of a really existing natural system because, in accordance with the third Newton's law, equality between the acting and the reacting forces should be satisfied. The other planets, the Sun and the Moon, the hydrostatic equilibrium for which is also accepted as a fundamental condition, stay in an analogous situation. Since the Earth in reality exists in equilibrium and its orbital motion strictly satisfies the ratio of the energies, then the question arises where the kinetic energy of the planet's own motion has disappeared. Otherwise the virial theorem for the Earth is not valid. Moreover, if one takes into account that the energy of inertial rotation does not belong to the body, then the Earth and other celestial bodies' equilibrium problem appears to be out of discussion.

Thus, we came to the problem of the Earth equilibrium from two positions. From one side, the planet by observation does not stay in hydrostatic equilibrium, and from the other side, it does not stay in general mechanical equilibrium because there is no reaction forces to counteract to the acting potential forces. The answer to both questions is given below while deriving an equation of the dynamical equilibrium of the planet by means of generalization of the classical virial theorem.

The main methodological question arises: in which the state of equilibrium the Earth exists? The answer to the question results from the generalized virial theorem for a self-gravitating body, i.e. the body which itself generates the energy for its own motion by interaction of the constituent particles having innate moments. The guiding effect which we use here is the observed by artificial satellite functional relationship between changes in the outer gravity field of the Earth and its mean (polar) moment of inertia. The deep physical meaning of this relationship is as follows. The observed planet's polar moment of inertia is an integral (volumetric) parameter, which represents not fixed interacted mass particles, but expresses changes in their motion under the inner body's energy. The Clausius' virial theorem represents relationship between the potential and kinetic energy in averaged form for a non-interacted (ideal) gaseous cloud of particles or a uniform body which stay in the outer force field. In order to generalize the theorem for a uniform or non-uniform body staying in the own force field we introduce there the volumetric moments of interacted particles, taking into account their volumetric nature. Moreover, the interacted mass particles of a continuous medium generate volumetric forces (pressure or capacity of energy) and volumetric moments, which, in fact, produce the motion in the form of oscillation and rotation of matter. The oscillating form of motion of the Earth and other celestial bodies is the dominating part of their kinetic energy which up to now has not been taken into account. We wish to fill in this gap in dynamics of celestial bodies.

The classic virial theorem is the analytical expression of the hydrostatic equilibrium condition and follows from Newton's and the Euler's equations of motion. Let us recall its derivation in accordance with classical mechanics.

Consider a system of mass points, the location of which is determined by the radius vector ri and the force Fi including the constraints. Then equations of motion of the mass points through their moments pi can be written in the form

$$\dot{\mathbf{p}}_i = \mathbf{F}_i\,, \qquad (2.1)$$

The value of the moment of momentum is

$$Q = \sum_i \mathbf{p}_i \cdot \mathbf{r}_i\,,$$

where the summation is done for all masses of the system. The derivative with respect to time from that value is

$$\frac{dQ}{dt} = \sum_i \dot{\mathbf{r}}_i \cdot \mathbf{p}_i + \sum_i \dot{\mathbf{p}}_i \cdot \mathbf{r}_i\,. \qquad .2)$$

The first term in the right hand side of (2.2) is reduced to the form

$$\sum_i \dot{r}_i \cdot p_i = \sum_i m_i \cdot \dot{r}_i \cdot \dot{r}_i = \sum_i m_i v_i^2 = 2T$$

where T is the kinetic energy of particle motion under action of the forces Fi. The second term in the Equation (2.2) is

$$\sum_i \dot{\mathbf{p}}_i \cdot \mathbf{r}_i = \sum_i \mathbf{F}_i \cdot \mathbf{r}_i\,. \qquad (2.3)$$

Now Equation.2.22 can be written as

$$\frac{d}{dt} \sum_i \mathbf{p}_i \cdot \mathbf{r}_i = 2T + \sum_i \mathbf{F}_i \cdot \mathbf{r}_i\,.$$

The mean values in (2.3) within the time interval τ are found by their integration from 0 to t and division by τ:

$$\frac{1}{\tau}\int_0^t \frac{dQ}{dt}dt = \overline{\frac{dQ}{dt}} = \overline{2T} + \overline{\sum_i F_i \cdot r_i}$$

or

$$\overline{2T}+\overline{\sum_i \mathbf{F}_i\cdot\mathbf{r}_i}=\frac{1}{\tau}\left[Q(\tau)-Q(0)\right]. \quad (2.4)$$

For the system, in which the co-ordinates of mass point motion are repeated through the period τ, the right hand side of Equation (2.4) after its averaging is equal to zero. If the period is too large, then the right hand side becomes a very small quantity. Then, the expression (2.4) in the averaged form gives the following relation

$$-\overline{\sum_i \mathbf{F}_i\cdot\mathbf{r}_i}=2\overline{T}, \qquad (2.5)$$

or in mechanics it is written with potential energy U in the form

2T = – U.

Equation (2.5) is known as the virial theorem, and its left hand side is called the virial of Clausius (German virial is from the Latin vires which means forces). The virial theorem is a fundamental relation between the potential and kinetic energy and is valid for a wide range of natural systems, the motion of which is provided by action of different physical interactions of their constituent particles. Clausius proved the theorem in 1870 when he solved the problem of work of the Carnot thermal machine, where the final effect of the water vapor pressure (the potential energy) was connected with the kinetic energy of the piston motion. The water vapor was considered as a perfect gas. And the mechanism of the potential energy (the pressure) generation at the coal burning in the firebox was not considered and was not taken into account.

The starting point in the above-presented derivation of virial theorem in mechanics is the moment of the mass point system, the nature of which is not considered both in mechanics and by Clausius. By Newton's definition the moment "is the measure of that determined proportionally to the velocity and the mass". The nature of the moment by his definition is "the innate force of the matter". By his understanding that force is an inertial force, i.e. the motion of a mass continues with a constant velocity.

The observed (by satellites) relationship between the potential and the kinetic energy of the gravitation field and the Earth's moment of inertia evidences, that the kinetic energy of the interacted mass particle motion, which is expressed as a volumetric effect of the planet's moment of inertia, is not taken into account. The evidence of that was given in the previous Section in the quantitative calculation of a ratio between the kinetic and potential energies, equal to ~1/300.

In order to take off that contradiction, the kinetic energy of motion of the interacted particles should be taken into account in the derived virial theorem. Because of any mass has volume the momentum p should be written in volumetric form:

$$\mathbf{p}_i=\sum_i m_i\dot{\mathbf{r}}_i \qquad (2.6)$$

Now the volumetric moment of momentum acquires the wave nature and is presented as

$$Q=\sum_i p_i\cdot r_i=\sum_i m_i\cdot\dot{r}_i\cdot r_i=\frac{d}{dt}\left(\sum_i\frac{m_i r_i^2}{2}\right)=\frac{1}{2}\dot{I}_p \qquad (2.7)$$

Where Ip is the polar moment of inertia of the system (for the sphere it is equal to 3/2 of the axial moment). The derivative from that value with respect to time is

$$\frac{dQ}{dt}=\frac{1}{2}\ddot{I}_p=\sum_i\dot{\mathbf{r}}_i\cdot\mathbf{p}_i+\sum_i\dot{\mathbf{p}}_i\cdot\mathbf{r}_i. \qquad (2.8)$$

The first term in the right hand part of (2.8) remains without change

$$\sum_i\dot{\mathbf{r}}_i\cdot\mathbf{p}_i=\sum_i m_i\cdot\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_i=\sum_i m_i\mathbf{v}_i^2=2T \qquad (2.9)$$

The second term represents the potential energy of the system

$$\sum_i\dot{p}_i\cdot r_i=\sum_i F_i\cdot r_i=U \qquad (2.10)$$

Equation (2.8) is written now in the form

$$\frac{1}{2}\ddot{I}_p=2T+U. \qquad (2.11)$$

Expression (2.11) represents a generalized equation of the virial theorem for a mass point system interacted by the Newton's law. Here in the left hand side of (2.11) the ignored up to now inner kinetic energy of interaction of the mass particles appears. Solution of Equation (2.11) gives a variation of the polar moment of inertia within the period τ. For a conservative system averaged expression (2.8) by integration from 0 to t within time interval τ gives

$$\frac{1}{T}\int_0^t\frac{dQ}{dt}dt=\overline{\frac{dQ}{dt}}=2\overline{T}+\overline{U}=\dot{I}_p. \qquad (2.12)$$

Equation (2.12) at $\ddot{I}_p=0$ (conservative system) gives where I_p=E=const, E is the total system's energy. It means that the interacted mass particles of the system move with constant velocity. In the case of dissipative system, equation (2.12) is not equal to zero and the interacted mass particles move with acceleration. Now the ratio between the potential and kinetic energy has a value in strict accordance with the equation (2.11). Kinetic energy of the interacted mass particles in the form of oscillation of the polar moment of inertia in that equation is taken into account. And now in the frame of the law of energy conservation the ratio of the potential to kinetic energy of a celestial body has a correct value.

Expression (2.11) appears to be an equation of dynamical equilibrium of the self-gravitating body (a star, planet, satellite) inside and between the bodies. The hydrostatic equilibrium presents here as a particular case at and does not take into account the kinetic energy of the continuous motion of the interacted particles which use inner energy as reaction on the outer force action (the Hooke's problem) . Integral effect of the moving particles is fixed by the satellite orbits in the form of changing zonal, sectorial and tesseral gravitational moments. We used the generated, thus, energy of the initial moment (2.6) for derivation of the generalized virial theorem. The initial moments form the inner or "innate by Newton's definition, energy of the body which has an inherited origin. The nature of the Newton's centripetal forces and the mechanism of their energy generation will be discussed in some details later on...

Thus, we obtained a differential equation of the second order (2.11), which describes the body dynamics and its dynamical equilibrium.

The virial equation (2.11) was obtained by Jacobi already one and a half century ago from the Newton's equations of motion in the form (Jacobi, 1884)

$\Phi = U + 2T$, (2.13)

wher где Φ is the Jacobi's function (the polar moment of inertia).

At that time Jacobi was not able to consider the physical meaning of the equation. For that reason he assumed that as there are two independent variables Φ and U in the equation, then it cannot be resolved. We succeeded to find an empirical relationship between the two variables and at first obtained an approximate and later on rigorous solution of the equation [1, 10, 11]. The relationship is proved now by means of the satellite observation.

We can explain now the cause of discrepancy between the geometric (static) and dynamic oblateness of the Earth. The reason is as follows. The planet's moments of inertia with respect to the main axes and their integral form of the polar moment of inertia do not stay in time as constant values. The polar moment of inertia of a self-gravitating body has a functional relation with the potential energy, the generation of which results by interaction of the mass particles in regime of periodic oscillations. The hydrostatic equilibrium of a body does not express the picture of the dynamic processes from which, as it follows from the averaged virial theorem, the energy of the oscillating effects was lost from consideration. Because of that it was not possible to understand the nature of the energy. The main part of the body's kinetic energy of the body's oscillation was also lost. As to the rotational motion of the body shells, it appears only in the case of the non-uniform distribution of the mass density. The contribution of rotation to the total body's kinetic energy compiles a very small part.

The cause of the accepted incorrect ratio between the Earth potential and kinetic energy is the following. Clairaut's equation derived for the planet's hydrostatic equilibrium state and applied to determine the geometric oblateness, because of the above discussed reason, has no functional relationship between the force function and the moment of inertia. Therefore for the Earth dynamical problem the equation gives only the first approximation. In formulation of the Earth oblateness problem, Clairaut accepted the Newton's model of action of the centripetal forces from surface of the planet to its geometric center. In such a physical conception the total effect of the inner force field becomes equal to zero. Below it will be shown, that the force field of the continuous body's interacted masses represents volumetric pressure, but not a vector force field. That is the cause, why the accepted postulate related to the planet's inertial rotation appears to be physically incorrect.

The question was raised about how was it happened, that geodynamic problems and first of all the problem of stability of the Earth motion up to now were solved without knowing the planet's kinetic energy. The probable explanation of that seems to lie in the history of the development of science. In Kepler's problem and in the Newton's two body problem solution the transition from the averaged parameters of motion to the real conditions is implemented through the mean and the eccentric anomalies, which by geometric procedures indirectly take into account the above energy of motion. In the Earth figure problem this procedure of Kepler is not applied. Therefore, the so called "inaccuracies" in the Earth motion appear to be the regular dynamic effects of a self-gravitating body, and the hydrostatic model in the problem is irrelevant. The hydrostatic model was accepted by Newton for the other problem, where just this model allowed discovery and formulation the general laws of the planets motion around the Sun. The Newton's centripetal forces in principle satisfy the Kepler's condition when the distance between bodies is much more than their size accepted as mass points. Such model gives the first approximation in the problem solution. Kepler's laws express the real picture of the planets and satellites motion around their parent bodies in averaged parameters. All the deviations of those averaged values related to the outer perturbations are not considered as it was done in the Clausius' virial theorem for the perfect gas.

Newton solved the two body problem, which has been already formulated by Kepler. The solution was based on the heliocentric world system of Copernicus, on the Galilean laws of inertia and free fall in the outer force field and on Kepler's laws of the planet's motion in the central force field considered as a geometric plane task. The goal of Newton's problem was to find the force by which the planet's motion is resulted. His centripetal attraction and the inertial forces in the two body problem satisfy Kepler's laws.

As it was mentioned, Newton understood the physical meaning of his centripetal or attractive forces as a pressure, which is accepted now like a force field. But by his opinion, for mathematical solutions the force is a more convenient instrument. And in the two body problem the force-pressure is acting from the center (of point) to outer space.

It is worth to discuss briefly the Newton's preference given to the force but not to the pressure. In mechanics the term "mass point" is understood as a geometric point of space, which has no dimension but possesses a finite mass. In physics a small amount of mass is called by the term "*particle*", which has a finite value of size and mass. But very often physicists use models of particles, which have neither size nor mass. A body model like mass point has been known since ancient times. It is simple and convenient for mathematical operations. The point is an irreplaceable geometric symbol of a reference point. The physical point, which defines inert mass of a volumetric body, is also suitable for operations. But the interacted and physically active mass point creates a problem. For instance, in the field theory the point value is taken to denote the charge, the meaning of which is no better understood than is the gravity force. But it is considered often there, that the point model for mathematical presentation of charges is not suitable because operations with it lead to zero and infinite values. Then for resolution of the situation the concept of charge density is introduced. The charge is presented as an integral of density for the taken volume and by this way the solving problem is resolved.

The point model in the two-body problem allowed reduction of it to the one-body problem and for a spherical body of uniform density to write the main seven integrals of motion. In the case when a body has a finite size, then not the forces but the pressure becomes an effect of the body particle interaction. The interacted body's mass particles form a volumetric gravitational field of pressure, the strength of which is proportional to the density of each elementary volume of the mass. In the case of a uniform body, the gravitation-

al pressure should be also uniform within the whole volume. The outer gravitational pressure of the uniform body should be also uniform at the given radius. The non-uniform body has a non-uniform gravitational pressure of both inner and outer field, which has been observed in studying the real Earth field. Interaction of mass particles results appears in their collision, which leads to oscillation of the whole body system. In general if the mass density is higher than the frequency of body oscillation is also higher.
It was known from the theory of elasticity, that in order to calculate the stress and the deformation of a beam from a continuous load, the latter can be replaced by the equivalent lumped force. In that case they found solution will be approximate because the beam's stress and deformation will be different. The question is what degree of approximation of the solution and what kind of the error is expected. Volumetric forces are not summed up by means of the parallelogram rule. Volumetric forces by their nature cannot be reduced for application either to a point, or to a resultant vector value. Their action is directed to the 4π space and they form inner and outer force field. The force field by its action is proportional to action of the energy. This is because the force is the derivative of the energy.

The centrifugal and Coriolis' forces are also proved to be inertial forces as a consequence of inertial rotation of the body. And the Archimedes force has not found its physical explanation, but it became an observational fact of hydrostatic equilibrium of a body mass immersed into a liquid.

Such is the short story of appearance and development of the hydrostatic equilibrium of the Earth in the outer uniform gravity field. The force of gravity of a body mass is an integral value. In this connection Newton's postulate about the gravity center as a geometric point should be considered as a model for presentation of two interacted bodies, when their mutual distance is much more of the body size. It is shown in the next section, that the reduced physical, but not geometrical, gravity center of a volumetric body is represented by an envelope of the figure, which draws averaged value of radial density distribution of the body.

Because of the above discussed the theorem of classical mechanics that if a body is found in the central force field, then the sum of their inner forces and torques are equal to zero, from mathematical point of view is correct in the frame of the taken initial conditions. Like in case of the derived virial theorem, the moment of momentum can be presented by the first derivative from the polar moment of inertia. And then the torque equal to zero in the central field will be presented by oscillation of the polar moment of inertia not equal to zero.

The problem of dynamics of the Earth as a self-gravitating body, including its figure problem in its formulation and solution needs for a higher degree of approximation. Generalized virial theorem (2.13) satisfies the condition of the Earth dynamical equilibrium state and creates a physical and theoretical basis for farther development of theory. It follows from the theorem that hydrostatic equilibrium state there is the particular case of the dynamics. Solution of problem of the Earth dynamics based on the equation of dynamical equilibrium appears to be the next natural and logistic step from the hydrostatic equilibrium model to a more realistic method without loss of the previous preference.

Derivation of Jacobi's Virial Equation for Different Physical Interactions

We derived Jacobi's virial equation from the equations of Newton, Euler, Hamilton, and Einstein and also from equations of quantum mechanics, by doing so we show that Jacobi's virial equation appears to be a unified instrument for the description of the dynamics of natural systems in the framework of the various physical models of the matter interaction employed. Jacobi virial equation for a system moving in its own force field and establishing a relationship between the potential and kinetic energy of the oscillating polar moment of inertia is defined as the generalized (non-averaged) virial theorem or the equation of dynamical equilibrium of a body. In this paper a number of examples are presented for illustration. The others are presented in [13, 15].

The Generalized Virial Theorem as the Equation of Dynamic Equilibrium of Body's Oscillating Motion

We have defined the classical virial theorem for a system moving in the outer uniform force field, which determines the relationship between mean values of the potential and kinetic energy within a certain period of time to be the averaged virial theorem. Conversely, the virial theorem for a system moving in its own force field and establishing a relationship between the potential and kinetic energy of the oscillating polar moment of inertia is defined as the generalized (non-averaged) virial theorem or the equation of dynamical equilibrium of a body.

We come to the conclusion that the physical basis of hydrostatic equilibrium does not satisfy the demands of general equilibrium of a body motion. As it was shown in the previous chapter, hydrostatic equilibrium, expressed by the averaged virial theorem, does not take into account kinetic energy of the interacting mass particles of a self-gravitating body and does not provide fundamentals for study of its dynamics. The dynamic equilibrium state based on the generalized virial theorem is however ensured by study of the following physical and dynamical problems: body's shell oscillation and rotation; interpretation of satellite data with respect to the body's precession, nutation and pole wobbling, non-tidal variation in the body's angular velocity, potential, changes etc.; perturbation effects based on analysis of the dynamical equilibrium of the interacting outer force fields of bodies (the Sun, the Earth and the Moon);
Relationship between the gravitational and electromagnetic interaction of the mass particles and the nature of the gravity forces; other dynamical effects arising from action of the inner force field, which earlier was not taken into account.

The theory presented in this paper is based on solution of the equation of dynamical equilibrium. It can be applied to study the body which by its structure presents a system that includes gaseous, liquid and solid shells. For this purpose derivation of the equation of Jacobi's virial equation (equation of dynamical equilibrium) from the equations of Newton, Euler, Hamilton, and Einstein and also from the equations of quantum mechanics is presented. In this part of the work we justify physical applicability of the above fundamental equation for study of the dynamics and structure of stars, planets, satellites and their shells. The main idea of the above derivations, done by means of introduction of volumetric forces and moments into the transformed equations, as was done in Equations (2.7) and (2.8), is to show that the dynamical effects of any interacting matter in nature is unique.

Derivation of Jacobi's Virial Equation from Newtonian Equations of Motion

Throughout this section the term 'system' is defined as an ensemble of material mass points mi (i = 1, 2, 3… n) which interact by Newton's law of universal attraction. This physical model of a natural system forms the basis for a number of branches of physics, such as classical mechanics, celestial mechanics, and stellar dynamics.

We shall not present the traditional introduction in which the main postulates are formulated; we shall simply state the problem (see, for example, Landau and Lifshitz, 1973a). We start by writing the equations of motion of the system in some absolute Cartesian co-ordinates ξ, η, ζ. In accordance with the conditions imposed, the mass point mi is not affected by any force from the other n–1 points except that of gravitational attraction. The projections of this force on the axes of the selected co-ordinates ξ, η, ζ can be written (Figure 4):

$$\Xi_i = Gm_i \sum_{1\le j\le n, i\ne j} \frac{m_j(\xi_j - \xi_i)}{\Delta_{ij}^3},$$

$$H_i = Gm_i \sum_{1\le j\le n, i\ne j} \frac{m_j(\eta_j - \eta_i)}{\Delta_{ij}^3}, \qquad (3.1)$$

$$Z_i = Gm_i \sum_{1\le j\le n, i\ne j} \frac{m_j(\zeta_j - \zeta_i)}{\Delta_{ij}^3},$$

where G is the gravitational constant and

$$\Delta_{ji} = \sqrt{(\xi_j - \xi_i)^2(\eta_j - \eta_i)^2 + (\zeta_j - \zeta_i)^2}$$

is the reciprocal distance between points i and j of the system.

It is easy to check that the forces affect the i-th material point of the system and are determined by the scalar function U, which is called the potential energy function of the system, and is given by

$$U = -G \sum_{1\le i<j\le n} \frac{m_i m_j}{\Delta_j} \qquad (3.2)$$

Now Equation (3.1) can be rewritten in the form

$$\Xi_i = -\frac{\partial U}{\partial \xi_i},$$

$$H_i = -\frac{\partial U}{\partial \eta_i},$$

$$Z_i = -\frac{\partial U}{\partial \zeta_i}.$$

Then Newton's equations of motion for the i-th point of the system take the form

$$m_i\ddot{\xi}_i = \Xi_i, \qquad (3.3)$$

$$m_i\ddot{\eta}_i = H_i,$$

$$m_i\ddot{\zeta}_i = Z_i,$$

or

$$m_i\ddot{\xi}_i = -\frac{\partial U}{\partial \xi_i},$$

$$m_i\ddot{\eta}_i = -\frac{\partial U}{\partial \eta_i}, \qquad (3.4)$$

$$m_i\ddot{\zeta}_i = -\frac{\partial U}{\partial \zeta_i},$$

Where dots over co-ordinate symbols mean derivatives with respect to time.

The motion of a system is described by Equations (3.3) and (3.4) and is completely determined by the initial data. In classical mechanics, the values of projections ξ_{i0}, η_{i0}, ζ_{i0} and velocities , , at the initial moment of time $t = t_0$ may be known from the initial data.

The study of motion of a system of n material points affected by self-forces of attraction forms the essence of the classical many-body problem. In the general case, ten classical integrals of motion are known for such a system, and they are obtained directly from the equations of motion.

Summing all the equations (3.3) for each co-ordinate separately, it is easy to be convinced of the correctness of the expressions:

$$\sum_{1\le i\le n} \Xi_i = 0,$$

$$\sum_{1\le i\le n} H_i = 0,$$

$$\sum_{1\le i\le n} Z_i = 0.$$

From those equations it follows that

$$\sum_{1\le i\le n} m_i\ddot{\xi}_i = 0,$$

$$\sum_{1\le i\le n} m_i\ddot{\eta}_i = 0, \qquad (3.5)$$

$$\sum_{1\le i\le n} m_i\ddot{\zeta}_i = 0.$$

Equation (3.5), appearing as a sequence of equations of motion, can be successively integrated twice. As a result, the first six integrals of motion are obtained:

$$\sum_{1\le i\le n} m_i\dot{\xi}_i = a_1,$$

$$\sum_{1\le i\le n} m_i\dot{\eta}_i = a_2,$$

$$\sum_{1\le i\le n} m_i\dot{\zeta}_i = a_3.$$

$$\sum_{1\le i\le n} m_i(\xi_i - \dot{\xi}_i t) = b_1, \qquad (3.6)$$

$$\sum_{1\le i\le n} m_i(\eta_i - \dot{\eta}_i t) = b_2,$$

$$\sum_{1\le i\le n} m_i(\zeta_i - \dot{\zeta}_i t) = b_3,$$

where a_1, a_2, a_3, $в_1$, $в_2$, $в_3$ are integration constants.

These integrals are called integrals of motion of the center of mass. The integration constants a_1, a_2, a_3, $в_1$, $в_2$, $в_3$ can be determined from the initial data by substituting their values at the initial moment of time for the values of all the co-ordinates and velocities.

Let us obtain one more group of first integrals. To do this, the second of Equation (3.3) can be multiplied by $-\zeta_i$, and the third третье by η_i. Then all expressions obtained should be added and summed over the index i. In the same way, the first of Equation (3.3) should be multiplied by ζ_i, and the third by $-\xi_i$ added and summed over index I. Finally, the second of Equation (3.3) should be multiplied by ξ_i, and the first by $-\eta_i$ added and summed over index I. It is easy to show directly that the right-hand sides of the expressions obtained are equal to zero:

$$\sum_{1\le i\le n} (Z_i\eta_i - H_i\zeta_i) = 0,$$

$$\sum_{1\le i\le n} (\Xi_i\zeta_i - Z_i\xi_i) = 0,$$

$$\sum_{1\le i\le n} (H_i\xi_i - \Xi_i\eta_i) = 0.$$

Consequently their left-hand sides are also equal to zero:

$$\sum_{1\le i\le n} m_i(\ddot{\zeta}_i\eta_i - \ddot{\eta}_i\zeta_i) = 0,$$

$$\sum_{1\le i\le n} m_i(\ddot{\xi}_i\zeta_i - \ddot{\zeta}_i\xi_i) = 0, \qquad (3.7)$$

$$\sum_{1\le i\le n} m_i(\ddot{\eta}_i\xi_i - \ddot{\xi}_i\eta_i) = 0.$$

Integrating Equation (3.7) over time, three more first integrals can be obtained:

$$\sum_{1\le i\le n} m_i(\dot{\zeta}_i\eta_i - \dot{\eta}_i\zeta_i) = c_1,$$

$$\sum_{1\le i\le n} m_i(\dot{\xi}_i\zeta_i - \dot{\zeta}_i\xi_i) = c_2, \qquad (3.8)$$

$$\sum_{1\le i\le n} m_i(\dot{\eta}_i\xi_i - \dot{\xi}_i\eta_i) = c_3.$$

The integrals (3.8) are called area integrals or integrals of moments of momentum. Three integration constants c1, c2, c3 are also determined from the initial data by changing over from the values of all the co-ordinates and velocities to their values at the initial moment of time.

The last of the classical integrals can be obtained by multiplying the three Equation (3.4) by ξ_i η_i, and ζ_i respectively, and adding and summing all the expressions obtained. As a result, the following equation is obtained:

$$\sum_{1\le i\le n} m_i\left(\ddot{\xi}_i\dot{\xi}_i + \ddot{\eta}_i\dot{\eta}_i + \ddot{\zeta}_i\dot{\zeta}_i\right) = -\sum_{1\le i\le n}\left(\frac{\partial U}{\partial \xi_i}\dot{\xi}_i + \frac{\partial U}{\partial \eta_i}\dot{\eta}_i + \frac{\partial U}{\partial \zeta_i}\dot{\zeta}_i\right) \qquad (3.9)$$

It is not difficult to see that the right-hand side of Equation (3.16) is the complete differential over time of the potential energy function U of the system as a whole. The left-hand side of the same equation is also the complete differential of some function T called the kinetic energy function of the system, and equal to

$$T = \frac{1}{2}\sum_{1\le i\le n} m_i\left(\dot{\xi}_i^2 + \dot{\eta}_i^2 + \dot{\zeta}_i^2\right) \qquad (3.10)$$

Equation (3.16) can then be written finally in the form

$$\frac{d}{dt}(T) = -\frac{d}{dt}(U)$$

from which, after integration, one finds that

$$E = T + U, \qquad (3.11)$$

where *E* is the integration constant, determined from the initial conditions

Equation (3.18) is called the integral of motion or the integral of living (kinetic) forces.

To derive the equation of dynamic equilibrium, or Jacobi's virial equation, each of the equation (3.11) should be multiplied by ξ_i, η_i and ζ_i respectively; then, after summing all the expressions, one can obtain

$$\sum_{1\le i\le n} m_i(\xi_i\ddot{\xi}_i + \eta_i\ddot{\eta}_i + \zeta_i\ddot{\zeta}_i) = -\sum_{1\le i\le n}\left(\xi_i\frac{\partial U}{\partial \xi_i} + \eta_i\frac{\partial U}{\partial \eta_i} + \zeta_i\frac{\partial U}{\partial \zeta_i}\right) \qquad (3.12)$$

We can take farther advantage of the obvious identities:

$$m_i\xi_i\ddot{\xi}_i = \frac{1}{2}\frac{d^2}{dt^2}\left(m_i\xi_i^2\right) - m_i\dot{\xi}_i^2,$$

$$m_i\eta_i\ddot{\eta}_i = \frac{1}{2}\frac{d^2}{dt^2}\left(m_i\eta_i^2\right) - m_i\dot{\eta}_i^2,$$

$$m_i\zeta_i\ddot{\zeta}_i = \frac{1}{2}\frac{d^2}{dt^2}\left(m_i\zeta_i^2\right) - m_i\dot{\zeta}_i^2|$$

From the Eulerian theorem concerning the homogenous functions. For the interaction of the system points, according to Newton's law of universal attraction, the degree of homogeneity of the potential energy function of the system is equal to –1, and hence

$$-\sum_{1\le i\le n}\left(\xi_i\frac{\partial U}{\partial \xi_i} + \eta_i\frac{\partial U}{\partial \eta_i} + \zeta_i\frac{\partial U}{\partial \zeta_i}\right) = U.$$

Substituting the above expressions into the right- and left-hand side of Equation (3.12), one obtains

$$\frac{d^2}{dt^2}\left[\frac{1}{2}\sum_{1\le i\le n} m_i\left(\xi_i^2 + \eta_i^2 + \zeta_i^2\right)\right] - 2\sum_{1\le i\le n}\frac{1}{2}m_i\left(\dot{\xi}_i^2 + \dot{\eta}_i^2 + \dot{\zeta}_i^2\right) = U.$$

For a system of material points we now introduce the Jacobi function expressed through the moment of inertia of the system and presented in the form

$$\Phi = \frac{1}{2}\sum_{1\le i\le n} m_i\left(\xi_i^2 + \eta_i^2 + \zeta_i^2\right)$$

Then taking into account (3.8), the previous equation can be rewritten in a very simple form as follows:

$$\ddot{\Phi} = 2E - U. \qquad (3.13)$$

This is the equation of dynamic equilibrium or Jacobi's virial equation describing both the dynamics of a system and its dynamic equilibrium using integral (volumetric) characteristics Φ and U or T.

Let us derive now another form of Jacobi's virial equation where the translational moment of the center of mass of the system is separated and all the characteristics depend only on the relative distance between the mass points of the system. For this purpose the Lagrangian identity can be used:

$$\left(\sum_{1\le i\le n} a_i^2\right)\left(\sum_{1\le i\le n} b_i^2\right) = \left(\sum_{1\le i\le n} a_i b_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n}\left(a_i b_j - b_i a_j\right)^2 \qquad (3.14)$$

r where ai and bi may acquire any values and *n* is any positive number.

Let us now put $a_i = \sqrt{m_i}$, and b_i equal to $\sqrt{m_i}\,\xi_i$, $\sqrt{m_i}\,\eta_i$ and $\sqrt{m_i}\,\zeta_i$ respectively.

Then three identities can be obtained from (3.7):

$$\left(\sum_{1\le i\le n} m_i\right)\left(\sum_{1\le i\le n} m_i\xi_i^2\right) = \left(\sum_{1\le i\le n} m_i\xi_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left(\xi_j - \xi_i\right)^2,$$

$$\left(\sum_{1\le i\le n} m_i\right)\left(\sum_{1\le i\le n} m_i\eta_i^2\right) = \left(\sum_{1\le i\le n} m_i\eta_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left(\eta_j - \eta_i\right)^2,$$

$$\left(\sum_{1\le i\le n} m_i\right)\left(\sum_{1\le i\le n} m_i\zeta_i^2\right) = \left(\sum_{1\le i\le n} m_i\zeta_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left(\zeta_j - \zeta_i\right)^2.$$

In summing up one finds

$$2m\Phi = \left(\sum_{1\le i\le n} m_i\xi_i\right)^2 + \left(\sum_{1\le i\le n} m_i\eta_i\right)^2 + \left(\sum_{1\le i\le n} m_i\zeta_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\Delta_j^2.$$

Using now Equation (3.13), the last equality can be rewritten in the form

$$2m\Phi = \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\Delta_j^2 + \left(a_1 t + b_1\right)^2 + \left(a_2 t + b_2\right)^2 + \left(a_3 t + b_3\right)^2 \qquad (3.15)$$

where

$$m = \sum_{1\le i\le n} m_i$$

is the total mass of the system.

Let us put

$$\Phi_0 = \frac{1}{4m}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\Delta_j^2.$$

The value Φ0 does not depend on the choice of the co-ordinate system and coincides with the value of the Jacobi function in the barycentric co-ordinate system. Moreover, from Equation (3.15) it follows that

$$\ddot{\Phi} = \ddot{\Phi}_0 + \frac{a_1^2 + a_2^2 + a_3^2}{m}.$$

Excluding the value Φ from Jacobi's equation (3.13) with the help of the last equality, the same equation can be obtained in the barycentric co-ordinate system:

$$\ddot{\Phi}_0 = 2E_0 - U, \quad (3.16)$$

Where E0 = T0 + U0 is the total energy of the system in the barycentric co-ordinate system equal to

$$E_0 = E - \frac{a_1^2 + a_2^2 + a_3^2}{2m}.$$

We can now show that the value of E0 does not depend on the choice of the co-ordinate system. For this purpose we can again use the Lagrangian identity (3.14). In this case

$a_i = \sqrt{m_i}$, and $b_i = \sqrt{m_i}\,\dot{\xi}_i$, $\sqrt{m_i}\,\dot{\eta}_i$ and $\sqrt{m_i}\,\dot{\zeta}_i$. Then the following three identities can be justified:

$$\left(\sum_{1\le i\le n} m_i\right)\left(\sum_{1\le i\le n} m_i\dot{\xi}_i^2\right) = \left(\sum_{1\le i\le n} m_i\dot{\xi}_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left(\dot{\xi}_j - \dot{\xi}_i\right)^2,$$

$$\left(\sum_{1\le i\le n} m_i\right)\left(\sum_{1\le i\le n} m_i\dot{\eta}_i^2\right) = \left(\sum_{1\le i\le n} m_i\dot{\eta}_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left(\dot{\eta}_j - \dot{\eta}_i\right)^2,$$

$$\left(\sum_{1\le i\le n} m_i\right)\left(\sum_{1\le i\le n} m_i\dot{\zeta}_i^2\right) = \left(\sum_{1\le i\le n} m_i\dot{\zeta}_i\right)^2 + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left(\dot{\zeta}_j - \dot{\zeta}_i\right)^2.$$

After summing and using (3.16) one obtains

$$2nT = \left(a_1^2 + a_2^2 + a_3^2\right) + \frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left[\left(\dot{\xi}_i - \dot{\xi}_j\right)^2 + \left(\dot{\eta}_i - \dot{\eta}_j\right)^2 + \left(\dot{\zeta}_i - \dot{\zeta}_j\right)^2\right]$$

$$T = \frac{\left(a_1^2 + a_2^2 + a_3^2\right)}{2m} + \frac{1}{2m}\left\{\frac{1}{2}\sum_{1\le i\le n}\sum_{1\le j\le n} m_i m_j\left[\left(\dot{\xi}_i - \dot{\xi}_j\right)^2 + \left(\dot{\eta}_i - \dot{\eta}_j\right)^2 + \left(\dot{\zeta}_i - \dot{\zeta}_j\right)^2\right]\right\} \qquad (3.17)$$

Here the second term on the right-hand side of Equation (3.17) coincides with the expression for the kinetic energy T_o of a system.

Substituting (3.17) into an expression for E_0, one obtains

$$E_0 = T_0 + U = \frac{1}{2m}\sum_{1\le i<j\le n} m_i m_j\left[\left(\dot{\xi}_i - \dot{\xi}_j\right)^2 + \left(\dot{\eta}_i - \dot{\eta}_j\right)^2 + \left(\dot{\zeta}_i - \dot{\zeta}_j\right)^2\right] - G\sum_{1\le i<j\le n}\frac{m_i m_j}{\Delta_j} \qquad (3.18)$$

Thus, the total energy of the system E0 depends only on the distance between the points of the system and on the velocity changes of these distances. But Jacobi's equation (3.16) appears to be invariant with respect to the choice of the co-ordinate system.

We can show now that the requirement of homogeneity of the potential energy function for deriving Jacobi's virial equation is not always obligatory. For this purpose we consider two examples.

Derivation of a Generalized Jacobi's Virial Equation for Dissipative Systems

Let us derive Jacobi's virial equation for a non-conservative system. We consider a system of n material points, the motion of which is determined by the force of their mutual gravitation interaction and the friction force. It is well known that the friction force always appears in the course of evolution of any natural system.

It is also known that there is no universal law describing the friction force. The only general statement is that the friction force acts in the direction opposite to the vector of velocity of a considered mass point.

Consider as an example the simplest law of Newtonian friction when its force is proportional to the velocity of motion of the mass:

$$\Xi_f = -km_i\dot{\xi}_i,$$
$$H_f = -km_i\dot{\eta}_i, \qquad (3.19)$$
$$Z_f = -km_i\dot{\zeta}_i.$$

Where ξ_i η_i , ζ_i are the components of the radius-vector of the velocity of the i-th mass point in the barycentric co-ordinate system; k is a constant independent of i; k>0.

Sometimes the friction force is independent of the velocity of the mass point. There are also some other laws describing the friction force.

We derive the equation of dynamical equilibrium for a system of *n* material points using the equations of motion (3.4) and taking into account the friction force expressed by Equation (3.19):

$$m_i\ddot{\xi}_i = -\frac{\partial U}{\partial \xi_i} - km_i\dot{\xi}_i,$$
$$m_i\ddot{\eta}_i = -\frac{\partial U}{\partial \eta_i} - km_i\dot{\eta}_i, \qquad (3.20)$$
$$m_i\ddot{\zeta}_i = -\frac{\partial U}{\partial \zeta_i} - km_i\dot{\zeta}_i,$$

where the value of the system's potential energy is determined by Equation (3.2).

Multiplying every of Equation (3.20) by ξ_i, η_i and ζ_i, respectively, and summing through all i, one obtains

$$\sum_{1\le i\le n} m_i\left(\xi_i\ddot{\xi}_i + \eta_i\ddot{\eta}_i + \zeta_i\ddot{\zeta}_i\right) = -\sum_{1\le i\le n}\left(\frac{\partial U}{\partial \xi_i}\xi_i + \frac{\partial U}{\partial \eta_i}\eta_i + \frac{\partial U}{\partial \zeta_i}\zeta_i\right) -$$
$$-\sum_{1\le i\le n} km_i\left(\xi_i\dot{\xi}_i + \eta_i\dot{\eta}_i + \zeta_i\dot{\zeta}_i\right)$$

Transforming the right- and left-hand sides of Equation (3.21) in the same way as in deriving Equation (3.13), one obtains

$$\ddot{\Phi} - 2T = U - k\dot{\Phi} \qquad (3.22)$$
$$\ddot{\Phi} = 2E - U - k\dot{\Phi}$$

Let us show that the total energy E of the system is a monotonically decreasing function of time. For this purpose we multiply each of the equation (3.20) by the vectors ξ_i η_i , ζ_i respectively, and sum over all from 1 to *n*, which results in

$$\sum_{1\le i\le n} m_i\left(\ddot{\xi}_i\dot{\xi}_i + \ddot{\eta}_i\dot{\eta}_i + \ddot{\zeta}_i\dot{\zeta}_i\right) = -\sum_{1\le i\le n}\left(\frac{\partial U}{\partial \xi_i}\dot{\xi}_i + \frac{\partial U}{\partial \eta_i}\dot{\eta}_i + \frac{\partial U}{\partial \zeta_i}\dot{\zeta}_i\right) -$$
$$-k\sum_{1\le i\le n} m_i\left(\dot{\xi}_i^2 + \dot{\eta}_i^2 + \dot{\zeta}_i^2\right)$$

The last expression can be rewritten in the form

$$\frac{d}{dt}(T) = -\frac{d}{dt}(U) - 2kT$$

or

$$dE = -2kTdt \,. \; (3.23)$$

Since the kinetic energy T of the system is always greater than zero, dE ≤ 0, i.e. the total energy of a gravitating system is a monotonically decreasing function of time. Thus the expression for the total energy E (t) of the system can be written as

$$E(t) = E_0 - 2k\int_{t_0}^{t} T(t)dt = E_0\left[1 + q(t)\right],$$

where q(t) is a monotonically increasing function of time.

Finally, the equation of dynamical equilibrium for a non-conservative system takes the form

$$\ddot{\Phi} = 2E_0\left[1 + q(t)\right] - U - k\dot{\Phi}\,. \qquad (3.24)$$

The second example where the requirement of homogeneity of the potential energy function for deriving Jacobi's virial equation is not obligatory is as follows. We derive Jacobi's virial equation for a system whose mass points interact mutually in accordance with Newton's law and move without friction in a spherical homogenous cloud whose density ρo is constant in time. Let, also, the geometric center of the cloud coincide with the center of mass of the considered system. The equations of motion for such a system can be written in the form:

$$m_i\frac{d^2\xi_i}{dt^2} = -\frac{4}{3}\pi G\rho_0 m_i\xi_i - \frac{\partial U}{\partial \xi_i},$$
$$m_i\frac{d^2\eta_i}{dt^2} = -\frac{4}{3}\pi G\rho_0 m_i\eta_i - \frac{\partial U}{\partial \eta_i}, \qquad (3.25)$$
$$m_i\frac{d^2\zeta_i}{dt^2} = -\frac{4}{3}\pi G\rho_0 m_i\zeta_i - \frac{\partial U}{\partial \zeta_i},$$

where i =1, 2, …, n.

It is obvious that the above system of equations possesses the ten first integrals of motion and that Jacobi's virial equation, written in the form

$$\frac{d^2\Phi}{dt^2} = 2E - U - \frac{8}{3}\pi G\rho_0\Phi. \qquad (3.26)$$

is valid for it.

The equation in the form (3.26) was first obtained by Duboshin et al. (1971). Equations (3.24) and (3.26) can be written in a more general form:

$$\ddot{\Phi} = 2E - U + X\left(t, \Phi, \dot{\Phi}\right). \qquad (3.27)$$

Where x (t ϋ ô) is a given function of time t, the Jacobi function Φ and first derivative Φ. Moreover, we can call Equation (3.27) a generalized equation of dynamical equilibrium.

The examples considered above justify the statement that for conditions of homogeneity of the potential energy function, required for the derivation of Jacobi's virial equation, is not always necessary. This condition is required for description of dynamics of conservative systems but not for dissipative systems or for systems in which motion is restricted by some other conditions.

Derivation of Jacobi's Virial Equation from Eulerian Equations

We now derive Jacobi's virial equation by transforming of the hydrodynamic or continuum model of a physical system. As is well known, the hydrodynamic approach to solving problems of dynamics is based on the system of differential equations of motion supplement, in the simplest case, by the equations of state and continuity, and by the appropriate assumptions concerning boundary conditions and perturbations affecting the system.

In this section, we understand by the term 'system' some given mass M of ideal gas localized in space by a finite volume V and restricted by a closed surface S. Let the gas in the system move by the forces of mutual gravitational interaction and of baric gradient. In addition, we accept the pressure within the volume to be isotropic and equal to zero on the surface S bordering the volume V. Then for a system in some Cartesian inertial co-ordinate system ξ, η, ζ, the Eulerian equations can be written in the form

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial}{\partial \xi}u+\rho v\frac{\partial}{\partial \eta}u+\rho w\frac{\partial}{\partial \zeta}u=-\frac{\partial p}{\partial \xi}+\rho\frac{\partial U_G}{\partial \xi},$$
$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial}{\partial \xi}v+\rho v\frac{\partial}{\partial \eta}v+\rho w\frac{\partial}{\partial \zeta}v=-\frac{\partial p}{\partial \eta}+\rho\frac{\partial U_G}{\partial \eta}, \qquad (3.28)$$
$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial}{\partial \xi}w+\rho v\frac{\partial}{\partial \eta}w+\rho w\frac{\partial}{\partial \zeta}w=-\frac{\partial p}{\partial \zeta}+\rho\frac{\partial U_G}{\partial \zeta},$$

Where ρ(ξ, η, ζ, t) is the gas density; u, v, w are components of the velocity vector (ξ, η, ζ, t) in a given point of space; p(ξ, η, ζ, t) is the gas pressure; UG is Newton's potential in a given point of space.

The value UG is given by

$$U_G=G\int_{(V)}\frac{\rho(x,y,z,t)}{\Delta}dxdydz, \qquad (3.29)$$

Where G is the gravity constant; $\Ddot{A}=\sqrt{(x-\hat{\imath})^2+(y-ç)^2+(z-æ)^2}$ is the distance between system points.

The potential energy of the gravitational interaction of material points of the system is linked to the Newtonian potential (3.29) by the relation

$$U=-\frac{1}{2}\int_{(V)}U_G\rho(\xi,\eta,\zeta,t)d\xi d\eta d\zeta.$$

To supplement the system of equations of motion we write the equation of continuity:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \xi}(\rho u)+\frac{\partial}{\partial \eta}(\rho v)+\frac{\partial}{\partial \zeta}(\rho w)=0 \qquad (3.30)$$

and the equation of state

$$p=f(\rho) \qquad (3.31)$$

Assuming at the same time that the processes occurring in the system are barotropic.

Let us obtain the ten classical integrals for the system whose motion is described by Equation (3.28).

We derive the integrals of the motion of the center of mass by integrating each of the equations (3.28) with respect to all the volume filled by the system. Integrating the first equation, we obtain

$$\int_{(V)}\rho\frac{du}{dt}d\xi d\eta d\zeta+\int_{(V)}\rho\left(u\frac{du}{d\xi}+v\frac{du}{d\eta}+w\frac{du}{d\zeta}\right)d\xi d\eta d\zeta=$$
$$=-\int_{(V)}\frac{dp}{d\xi}d\xi d\eta d\zeta+G\int_{(V)}\rho(\xi,\eta,\zeta,t)\left[\int_{(V)}\rho(x,y,z,t)\frac{x-\xi}{\Delta^3}dxdydz\right]d\xi d\eta d\zeta. \qquad (3.32)$$

The second term in the right-hand side of Equation (3.32) disappears because of the symmetry of the integral expression with respect to x and ξ. In accordance with the Gauss-Ostrogradsky theorem the first term in the right-hand side of Equation (3.32) terns to zero. In fact

$$\int_{(V)}\frac{dp}{d\xi}d\xi d\eta d\zeta=\int_{(S)}p\,d\eta d\zeta=0 \qquad (3.33)$$

As pressure p on the border of the considered system is equal to zero owing to the absence of outer effects.

Bearing in mind the possibility of passing to a Lagrangian co-ordinate system, and taking into account the law of the conservation of mass $dm=\rho dV=\rho_0 dV_0=dm_0$, we get

$$\int_{(V)}\rho\frac{du}{dt}d\xi\xi d\eta d+\int_{(V)}\rho\left(u\frac{du}{d\xi}+v\frac{du}{d\eta}+w\frac{du}{d\zeta}\right)d\xi\xi d\eta d=$$
$$=\int_{(V)}\rho\frac{du}{dt}dV=\int_{(V_0)}\rho_0\frac{du}{dt}dV_0=\frac{d}{dt}\int_{(V_0)}u\rho_0 dV_0=\frac{d}{dt}\int_{(V)}\rho u dV,$$

Where V_0 and ρo are the volume and the density in the initial moment of time t_0.

Finally, Equation (3.32) can be rewritten as

$$\frac{d}{dt}\int_{(V)}\rho u dV=0 \qquad (3.34)$$

Integrating (3.34) with respect to time and writing analogous expressions for two other equations of the system (3.28), we obtain the first three integrals of motion:

$$\int_{(V)}\rho u dV=a_1,$$
$$\int_{(V)}\rho v dV=a_2, \qquad (3.35)$$
$$\int_{(V)}\rho w dV=a_3.$$

Equation (3.42) represents the law of conservation of the system moments. Integration constants a1, a2, a3 can be obtained from the initial conditions.

We consider the first equation of the system (3.42) using again the law of conservation of mass. Then it is obvious that

$$\int_{(V)} u\rho dV = \int_{(V)} \frac{d\xi}{dt}\rho dV = \int_{(V_0)} \frac{d\xi}{dt}\rho_0 dV_0 = \frac{d}{dt}\int_{(V_0)} \xi\rho_0 dV_0 = \frac{d}{dt}\int_{(V)} \xi\rho dV = a_1. \quad (3.36)$$

Analogous expressions can be written for the two other equations (3.42). Integrating them with respect to time, we obtain integrals of motion of the center of mass of the system in the form

$$\int_{(V)} \xi\rho\, dV = a_1 t + b_1,$$

$$\int_{(V)} \eta\rho\, dV = a_2 t + b_2, \quad (3.37)$$

$$\int_{(V)} \zeta\rho\, dV = a_3 t + b_3.$$

We now derive three integrals of the moment of momentum of motion. For this purpose we multiply the second of Equation (3.28) by –ζ, the third by η, and then sum and integrate the resulting expressions with respect to volume V occupied by the system. We obtain

$$\int_{(V)} \rho\left(\eta\frac{dw}{dt} - \zeta\frac{dv}{dt}\right)dV = -\int_{(V)}\left(\eta\frac{\partial p}{\partial\zeta} - \zeta\frac{\partial p}{\partial\eta}\right)dV + \int_{(V)}\rho\left(\eta\frac{\partial U_G}{\partial\zeta} - \zeta\frac{\partial U_G}{\partial\eta}\right)dV. \quad (3.38)$$

Analogously, multiplying the first of Equation (3.28) by ζ, the third by –ξ, then summing and integrating with respect to volume V, we obtain

$$\int_{(V)} \rho\left(\zeta\frac{du}{dt} - \xi\frac{dw}{dt}\right)dV = -\int_{(V)}\left(\zeta\frac{\partial p}{\partial\xi} - \xi\frac{\partial p}{\partial\zeta}\right)dV + \int_{(V)}\rho\left(\zeta\frac{\partial U_G}{\partial\xi} - \xi\frac{\partial U_G}{\partial\zeta}\right)dV. \quad (3.39)$$

Multiplying the second of Equation (3.28) by ξ, the first by –η, and summing and integrating as above, and the third equality can be written

$$\int_{(V)} \rho\left(\xi\frac{dv}{dt} - \eta\frac{du}{dt}\right)dV = -\int_{(V)}\left(\xi\frac{\partial p}{\partial\eta} - \eta\frac{\partial p}{\partial\xi}\right)dV + \int_{(V)}\rho\left(\xi\frac{\partial U_G}{\partial\eta} - \eta\frac{\partial U_G}{\partial\xi}\right)dV. \quad (3.40)$$

We write the second integral in the right-hand side of Equation (3.38) in the form

$$\int_{(V)} \rho\left(\eta\frac{dw}{dt} - \zeta\frac{dv}{dt}\right)dV = G\int_{(V)} \rho(\xi,\eta,\zeta,t)\eta d\xi d\eta d\zeta \int_{(V)} \rho(x,y,z,t)\frac{z-\zeta}{\Delta^3}dx, dy, dz -$$

$$-G\int_{(V)} \rho(\xi,\eta,\zeta,t)\zeta d\xi d\eta d\zeta \int_{(V)} \rho(x,y,z,t)\frac{y-\zeta}{\Delta^3}dx, dy, dz.$$

The integral is equal to zero owing to the asymmetry expressed by the integral expressions with respect to z, ζ and y, η. Because the pressure at the border of the domain S is equal to zero, the first term in the right-hand side of Equation (3.38) is also equal to zero. Actually,

$$\int_{(V)}\left(\eta\frac{\partial p}{\partial\zeta} - \zeta\frac{\partial p}{\partial\eta}\right)dV = \int_{(V)}\left[\frac{d}{d\eta}(\xi p) - \frac{d}{d\xi}(\eta p)\right]dV = \int_{(V)}\left[\xi p d\xi d\zeta - \eta p d\eta d\zeta\right] = 0.$$

Taking into account the law of mass conservation, the left-hand side of Equation (3.38) in the Lagrange co-ordinate system can be rewritten as

$$\int_{(V)} \rho\left(\eta\frac{dw}{dt} - \zeta\frac{dv}{dt}\right)dV = \int_{(V)} p\frac{d}{dt}(\eta w - \zeta v)dV = \frac{d}{dt}\int_{(V)} p(\eta w - \zeta v)dV = 0. \quad (3.41)$$

Integrating this equation with respect to time, the first of the three integrals is obtained:

$$\int_{(V)} p(\eta w - \zeta v)dV = C_1.$$

The other two integrals can be obtained analogously. Thus the system of integrals of the moment of momentum has the form

$$\int_{(V)} p(\eta w - \zeta v)dV = C_1,$$

$$\int_{(V)} p(\zeta u - \xi w)dV = C_2, \quad (3.42)$$

$$\int_{(V)} p(\xi v - \eta u)dV = C_3.$$

To derive the tenth integral of motion representing the law of energy conservation, we multiply each of the system of equations (3.28) by u, v, and w accordingly, and then sum and integrate the equality obtained with respect to the system volume

$$\int_{(V)} \rho\left(\frac{du}{dt}u + \frac{dv}{dt}v + \frac{dw}{dt}w\right)dV = -\int_{(V)}\left(\frac{\partial p}{\partial\xi}u + \frac{\partial p}{\partial\eta}v + \frac{dp}{d\zeta}w\right)dV +$$

$$+\int_{(V)} \rho(\xi,\eta,\zeta,t)\left(\frac{\partial U_G}{\partial\xi}u + -\frac{\partial U_G}{\partial\eta}v + \frac{\partial U_G}{\partial\zeta}w\right)dV. \quad (3.43)$$

Applying the law of mass conservation for an elementary volume, it can easily be seen that the left-hand side of Equation (3.43) expresses the change of the velocity of kinetic energy of the system:

$$\int_{(V)} \rho\left(\frac{du}{dt}u + \frac{dv}{dt}v + \frac{dw}{dt}w\right)dV = \frac{d}{dt}\left[\frac{1}{2}\int_{(V)}\left(u^2 + v + w^2\right)dV\right] = \frac{d}{dt}(T).$$

The first integral in the right-hand side of Equation (3.43) can be transferred into

$$-\int_{(V)}\left(\frac{\partial p}{\partial\xi}u + \frac{\partial p}{\partial\eta}v + \frac{dp}{d\zeta}w\right)dV = 3\frac{d}{dt}\int_{(V)} p dV$$

and gives the change of velocity of the internal energy of the system.

The second integral in the right-hand side of the same equation expresses the velocity of the potential energy change:

$$\int_{(V)} \rho(\xi,\eta,\zeta,t)d\xi\xi d\eta d\left(\frac{\partial U_G}{\partial \xi}\frac{d\xi}{dt} + -\frac{\partial U_G}{\partial \eta}\frac{d\eta}{dt} + \frac{\partial U_G}{\partial \zeta}\frac{d\zeta}{dt}\right) =$$

$$= \frac{d}{dt}\left[-\frac{1}{2}\int_{(V)} \rho(\xi,\eta,\zeta,t)d\xi\xi d\eta dU_G\right] = -\frac{d}{dt}(U).$$

Finally, the law of energy conservation can be written in the form

$$T + U + W = E = const \qquad (3.44)$$

Where W is the internal energy of the system.

We now derive Jacobi's virial equation for a system described by Equations (3.28) and (3.31). For this purpose we multiply each of Equation (3.28) by ξ, η and ζ respectively, summing and integrating the resulting expressions with respect to the volume of the system:

$$\int_{(V)} \rho\left(\frac{du}{dt}\xi + \frac{dv}{dt}\eta + \frac{dw}{dt}\zeta\right)dV = -\int_{(V)}\left(\frac{\partial p}{\partial \xi}\xi + \frac{\partial p}{\partial \eta}\eta + \frac{dp}{d\zeta}\zeta\right)dV +$$

$$+ \int_{(V)} \rho\left(\frac{\partial U_G}{\partial \xi}\xi + -\frac{\partial U_G}{\partial \eta}\eta + \frac{\partial U_G}{\partial \zeta}\zeta\right)dV. \qquad (3.45)$$

Using the obtained identities considered in the previous section, we have

$$\frac{du}{dt}\xi = \frac{1}{2}\frac{d^2}{dt^2}(\xi^2) - u^2,$$

$$\frac{dv}{dt}\eta = \frac{1}{2}\frac{d^2}{dt^2}(\eta^2) - v^2,$$

$$\frac{dw}{dt}\zeta = \frac{1}{2}\frac{d^2}{dt^2}(\zeta^2) - w^2.$$

Taking into account the law of conservation of mass for elementary volume, we transform the left-hand side of Equation (3.45) as follows:

$$\int_{(V)} \rho\left(\frac{du}{dt}\xi + \frac{dv}{dt}\eta + \frac{dw}{dt}\zeta\right)dV = \frac{1}{2}\int_{(V)} \rho\frac{d^2}{dt^2}(\xi^2 + \eta^2 + \zeta^2)dV -$$

$$-\int_{(V)} \rho(u^2 + v^2 + w^2)dV = \ddot{\Phi} - 2T, \qquad (3.46)$$

where

$$\Phi = \frac{1}{2}\int_{(V)} \rho(\xi^2 + \eta^2 + \zeta^2)dV$$

is the Jacobi function and

$$-\int_{(V)}\left(\frac{\partial p}{\partial \xi}\xi + \frac{\partial p}{\partial \eta}\eta + \frac{dp}{d\zeta}\zeta\right)dV = -\int_{(V)}\left[\frac{\partial}{\partial \xi}(p\xi) + \frac{\partial}{\partial \eta}(p\eta) + \frac{d}{d\zeta}(p\zeta)\right]dV +$$

$$+ 3\int_{(V)} pdV = 3\int_{(V)} pdV. \quad (3.47)$$

is the kinetic energy of the system.

We now transform the first integral in the right-hand side of Equation (3.45). Using the Gauss-Ostrogradsky theorem and the equality with zero pressure at the border of the system, we can write

$$-\int_{(V)}\left(\frac{\partial p}{\partial \xi}\xi + \frac{\partial p}{\partial \eta}\eta + \frac{dp}{d\zeta}\zeta\right)dV = -\int_{(V)}\left[\frac{\partial}{\partial \xi}(p\xi) + \frac{\partial}{\partial \eta}(p\eta) + \frac{d}{d\zeta}(p\zeta)\right]dV +$$

$$+ 3\int_{(V)} pdV = 3\int_{(V)} pdV. \quad (3.47)$$

The obtained equation expresses the doubled internal energy of the system.

The second integral in the right-hand side of Equation (3.45) is equal to the potential energy of the gravitational interaction of mass particles within the system

$$\int_{(V)} \rho\left(\frac{\partial U_G}{\partial \xi}\xi + -\frac{\partial U_G}{\partial \eta}\eta + \frac{\partial U_G}{\partial \zeta}\zeta\right)dV = U. \qquad (3.48)$$

Substituting Equations (3.46), (3.47) and (3.48) into (3.45), Jacobi's virial equation is obtained in the form

$$\ddot{\Phi} - 2T = 3\int_{(V)} pdV + U. \qquad (3.49)$$

Taking into account the law of conservation of energy (3.44), we rewrite Equation (3.49) in a form which will be used farther:

$$\ddot{\Phi} = 2E - U, \qquad (3.50)$$

where E = T + U + W is the total energy of the system.

Centrifugal Effects as the Mechanism of the Solar System Bodies Creation from a Common Gaseous Cloud

The Conditions for a Body Separation and Orbiting

It is known that all the Solar System bodies (the planets, their satellites, comets and meteoric bodies) are identical in their substantial and chemical content and in this respect they are of common origin. But the search of a unified mechanism of the body creation has encountered on irresistible difficulty in their dynamics. The point is that the planets having only ~0.015% of the system mass possess 98% of the orbital angular momentum. At the same time ~99.85% of the Sun's mass produce no more than 2% of the angular momentum, which is accepted to be a conservative parameter? Also, the specific (for unit of the mass) angular momentum of the planets is increased together with the distance from the Sun. As it was discussed in the previous chapters, the above results follow from calculation model based on hydrostatic equilibrium state of the system, where the body motion results from the outer forces. It was shown that the hydrostatic equilibrium for celestial body dynamics appeared to be not correct physical conception.

We analyze the evolutionary problem of the Solar System based on fundamentals of the Jacobi dynamics, where the body motion initiates by the inner forces' action. Here, the energy loss in the form of radiation is accepted as the physical basis of the body evolution. And the centrifugal effect of elementary particles collision and scattering appears to be the mechanism of the energy generation and its redistribution. It is clear from observation that all celestial bodies are self-gravitating systems.

It is shown next that creation of a new body occurs within the parental cloud because of its separation in density on shells by the Archimedes low, when the outer shell reaches the state of weightlessness. Here, the orbital moment of momentum of a created secondary body represents the total kinetic moment of the parental body's cloud owing to the energy conservation law:

$$Q = \sum_i p_i r_i = \sum_i m_i \acute{i}_i r_i = \sum_i m_i \dot{r}_i r_i = \frac{d}{d}\left(\sum_i \frac{m_i r_i^2}{2}\right) = \frac{1}{2}\dot{I}_p \quad (4.1)$$

$$\frac{dQ}{dt} = \frac{1}{2}\ddot{I}_p = \sum_i \dot{r}_i \cdot p_i + \sum_i \dot{p}_i \cdot r_i = 2T + U. \quad (4.2)$$

Where Q is the moment of momentum of the parental cloud; p is moment of a particle; r is the radius; Ip is the polar moment of inertia of the cloud.

It means that the orbital moment of momentum of each planet represents the kinetic momentum of the ptotosun at the time of planet separation and orbiting. Kinetic moment of a body is equal to the sum of the rotational and oscillating moments, and the kinetic energy is equal to the sum of the rotational and oscillating energy, which follows from the energy conservation law. At the same time, the planet's orbital moment of momentum is formed by the total potential energy of the protosun. But the planet's angular moment of the axial rotation is formed by the tangential component of the own planet's potential energy (see below Equations (4.9) and (4.10)). Here the energy of axial rotation compiles small portion of the oscillating energy. As it was noted in Sec. 2.4, kinetic energy of the planets Earth, Mars, Jupiter, Saturn, Uranus, and Neptune compiles 10^{-3}–10^{-2}, and of the Mercury, Venus, the Moon and the Sun is about 10^{-4} from the total kinetic energy of each body.. For the bodies with uniform mass density distribution kinetic energy of rotation is equal to zero.

The interaction (collision and scattering) of mass particles is accompanied by continuous redistribution of the body's mass density. According to the Roche's tidal dynamics, redistribution of the mass density leads to the shell separation. It will be shown later that when the density of the upper shell reaches less than two third with respect to the underlying shells, then the upper shell becomes weightless (i.e., it loses weight). From physical point of view, it means that the own force field of the upper shell is in dynamical equilibrium with the parental force field. In this case, if the density of the upper shell has non-uniform density distribution, then by the difference in the potentials of the force field, the shell is converted into the secondary body. If the upper shell has uniform mass density distribution, then the shell forms a ring around equatorial plane of the parental body. In general case, the upper weightless shell is decayed into fragments with different amounts of mass. The comets were formed from the solar shell, the satellites and meteorites were created from the planet's shells. During evolution of a non-uniform gaseous body, it undergoes of the axial and equatorial oblations by an outer force field of the central parental body. This can be observed by inclination of the planet's and satellite's orbital plane slope relative to the parental equatorial plane. The polar outer force field pressure appears to be higher of equatorial. As a result, the outer polar force field values appeared to be higher than the equatorial. Because of this, the polar matter of the upper shell is continuously removed to the equatorial plane. This is why the created bodies are formed mainly in the equatorial plane and form equatorial disk.

So the orbital motion of separated secondary body is defined by the outer force field at the surface of the parental body. The value of this field at the body's surface is a fundamental parameter, which is determined by the body's law of the energy conservation. That is why the orbital velocity of a newly created body is equal to its parental first cosmic velocity. The direction of the orbital motion is determined by the Lenz's law. In this connection, it worth to note that from point of view of the solar system creation problem, the attempts to find explanation of the observed distribution of the moment of momentum between the axial rotation of the Sun and the planets' orbital motion are not rightful. This is because the planets' orbital velocity demonstrates parental relationship between the planets and the Sun by proving its identity with the first cosmic velocity and the law of the energy conservation.

Thus, it follows from the above scenario that the induced by the matter interaction outer force field of the Sun is responsible for the orbital motion of the planets. Analogously planets' force field is orbiting their satellites. Doing so, each body with high accuracy records the value of the parent's potential energy at the moment of orbiting. As to the shell's axial rotation, then its potential energy is determined by the value of its tangential component. The normal and tangential components of body's potential energy comprise the total potential energy, which is a conservative parameter.

It was shown earlier in [15], that in the framework of Jacobi dynamics solution of the Kepler's problem is given by equations:

$$\sqrt{\Phi} = \frac{B}{A}\left[1 - \varepsilon\cos(\lambda - \psi)\right], \quad (4.3)$$

$$t = \frac{4B}{(2A)^{3/2}}\left[\lambda - \varepsilon\sin(\lambda - \psi)\right] \quad (4.4)$$

$$\omega = \frac{2\pi}{T} = \frac{(2A)^{3/2}}{4B} = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{4}{3}\pi G\rho_0}, \quad (4.5)$$

where ε and ψ are the integration constants depending on the initial values of Jacobi's function Φ and its first derivative at the time moment t_0 (the time here is an independent variable); T is the period of virial oscillations; ω is the oscillation; λ is the auxiliary an independent variable; $A = A_0 = 1/2E > 0$, $B = B_0 = U\sqrt{\Phi_0}$ for radial oscillations; $A = Ar = 1/2E > 0$, $B = Br = U\sqrt{\Phi_r}$ for rotation of the body.

The product of the oscillation frequency ω of the outer force field and R of the body gives the value of the first cosmic velocity of an artificial satellite, that is, the velocity with which the satellite undertakes gravity attraction (the pressure induced by the outer force field). In order to undertake the attraction, satellite uses its own inner energy of the reactive engine. In this way the satellite reaches the first cosmic velocity and becomes weightlessness, i.e. the own outer force field reaches equilibrium with the planet's outer force field. After that the engine is switched off and its motion continues by the energy of outer force field. In order to be separated from the parental body, its outer shell must reach the state of the weightlessness, that is, its own force field reaches dynamical equilibrium with the parental force field. The secondary body, created from the outer shell, being completely in non-weighty state and in dynamical equilibrium with the parental outer force field, moves farther by that force field with the first cosmic velocity.

The Structure of Potential and Kinetic Energies of a Non uniform Body

In fact, all the celestial bodies of the Solar System, including the Sun, are non-uniform creatures. They have a shell structure and the shells themselves are also non-uniform components of a body. It was shown in Section 2.2 that according to the artificial satellite data all the measured gravitational moments of the Earth, including tesseral ones, have significant values. In geophysics this fact is interpreted as a deviation of the Earth from the hydrostatic equilibrium and attendance of the tangential forces which are continuously developed inside the body. From the viewpoint of the planet's dynamical equilibrium, the fact of the measured zonal and tesseral gravitational moments is a direct evidence of permanent development of the normal and tangential volumetric forces which are the components of the inner gravitational force field. In order to identify the above effects the inner force field of the body should be accordingly separated.

WE found that the force function and the polar moment of a non-uniform self-gravitating sphere can be expanded with respect to their components related to the uniform mean density mass and its non-uniformities. In accordance with the superposition principle these components are responsible for the normal and tangential dynamical effects of a non-uniform body. Such a separation of the potential energy and polar moment of inertia through their dimensionless form-factors α and β was done by Garcia Lambas et al. (1985) with our interpretation [11]. Taking into account that the observed satellite irregularities are caused by the non-uniform distribution of the mass density, an auxiliary function relative to the radial density distribution was introduced for the separation:

$$\Psi(s)=\int_0^s \frac{(\rho_r-\rho_0)}{\rho_0}x^2dx \qquad (4.6)$$

Where s = r/R is the ratio of the running radius to the radius of the sphere R; ρ_o is the mean density of the sphere of radius r; ρ_r is the radial density; x is the running coordinate; the value $(\rho_r-\rho_o)$ satisfies $\int_0^R(\rho_r-\rho_0)r^2dr$ = 0 and the function ψ(1) = 0.

The function ψ(s) expresses a radial change in the mass density of the non-uniform sphere relative to its mean value at the distance r/R. Now we can write expressions for the force function and the moment of inertia by using the structural form-factors α and β :

$$U=\alpha\frac{GM^2}{R}=4\pi G\int_0^R r\rho(r)m(r)dr\,, \qquad (4.7)$$

$$I=\beta^2MR^2=4\pi\int_0^R r^4\rho(r)dr\,. \qquad (4.8)$$

By (4.6) we can do the corresponding change of variables. As a result, the expressions for the potential energy U and polar moment of inertia I are found in the form of their components composed of their uniform and non-uniform constituents [11]:

$$U=4\pi G\int_0^R r\rho(r)m(r)dr=\alpha\frac{GM^2}{R}=\left[\frac{3}{5}+3\int_0^1\psi x dx+\frac{9}{2}\int_0^1\left(\frac{\psi}{x}\right)^2dx\right]\frac{GM^2}{R}, \qquad (4.9)$$

$$I=\beta^2MR^2=\left[\frac{3}{5}-6\int_0^1\psi x dx\right]MR^2. \qquad (4.10)$$

It is known that the moment of inertia multiplied by the square of the frequency ω of the oscillation-rotational motion of the mass is the kinetic energy of the body. Then Equation (4.10) can be rewritten as

$$K=I\omega^2=\beta^2MR^2\omega^2=\left[\frac{3}{5}-6\int_0^1\psi x dx\right]MR^2\omega^2. \qquad (4.11)$$

Let us clarify the physical meaning of the terms in expressions (4.9) and (4.11) of the potential and kinetic energies.

The first terms in (4.9) and (4.11), numerically equal to 3/5, represent α_0 and β_0^2 being the structural coefficients of the uniform sphere with radius r, the density of which is equal to its mean value. The ratio of the potential and kinetic energies of such a sphere corresponds to the condition of the body's dynamical equilibrium when its kinetic energy is realized in the form of oscillations.

The second terms of the expressions can be rewritten in the form

$$3\int_0^1\psi x dx\equiv3\int_0^1\left(\frac{\psi}{x}\right)x^2dx\,, \qquad (4.12)$$

$$-6\int_0^1\psi x dx\equiv-6\int_0^1\left(\frac{\psi}{x}\right)x^2dx\,. \qquad (4.13)$$

One can see here that the additive parts of the potential and kinetic energies of the interacting masses of the non-uniformities of each sphere shell with the uniform sphere having a radius r of the sphere shell there are written. Note that the structural coefficient β of the kinetic energy is twice as high as the potential energy and has the minus sign. It is known from physics that interaction of mass particles, uniform and non-uniform with respect to density is accompanied by their elastic and inelastic scattering of energy and appearance of a tangential component in their trajectories of motion. In this particular case the second terms in Equations (4.9) and (4.11) express the tangential (torque) component of the potential and kinetic energy of the body. Moreover, the rotational component of the kinetic energy is twice as much as the potential one.

The third term of Equation (4.9) can be rewritten as

$$\frac{9}{2}\int_0^1\left(\frac{\psi}{x}\right)^2dx\equiv\frac{9}{2}\int_0^1\left(\frac{\psi}{x^2}\right)^2x^2dx. \qquad (4.14)$$

Here, there is another additive part of the potential energy of the interacting non-uniformities. It is the non-equilibrated part of the potential energy which does not have an appropriate part of the reactive kinetic energy and represents a dissipative component. Dissipative energy represents the electromagnetic energy that is emitted by the body and it determines the body's evolutionary effects. This energy forms the electromagnetic field of the body.

Non uniformity of the density in this case and later is determined as difference between the density of the given spherical layer and mean value of density of the sphere with radius of the spherical layer.

Thus, by expansion of the expression of the potential energy and the polar moment of inertia we obtained the components of both forms of energy which are responsible for oscillation and rotation of the non-uniform body. Applying the above results we can write separate conditions of the dynamical equilibrium for each form of the motion and separate virial equations of the dynamical equilibrium of their motion.

Conclusions

The discovered nature of the planets and satellites orbital motion with the first cosmic velocity of their protoparents prove the correctness of the introduced fundamentals for dynamics of the natural systems based on dynamical equilibrium. The conditions of hydrostatical equilibrium used earlier for such problem solution should be considered as incorrect approach. Just this is the cause why the nature of the Newton's gravity force is actively discussed up to now. Solution of this problem appeared to be ordinary. Physical meaning of gravitation of a self-gravitating system and its rotation follows from the centrifugal effect of energy of its interacted elementary particles. The force of gravitation is the first derivative in time from the inner energy of the interacted elementary matter particles of the system. The energy itself, being the measure of that interaction, in general case, is the second derivative in time from the moment of inertia of the system. In the case of uniform system the energy equal to the first derivative in time from the moment of inertia. That is why the Einstein's equivalency principle is correct only for the uniform in density systems.

The only change of the force as the measure of interaction by the energy brings together the nature of the gravity and electromagnetic interactions. And if one takes into account the inner and outer force fields, then both interactions become equivalent in their capacity.

It was found that the process of the planets and satellites creation by separation from the parental bodies is coupled with conditions of the Universe expansion. Just in these conditions the hierarchic subsystems are appeared. The inner energy of the parental bodies releases only in the expansion conditions. In reality the process of a subsystem creation is only a part of more general process of decay of a system by elementary particles accompanied by release the bonded energy. The bullet points of dynamical effects of this process are self-gravitation ('weightness') and weightlessness. The process of the Universe attraction appears to be the next stage of its evolution. If the law of the energy conservation is held then this stage must come. It starts with creation of mass particles including electrons, nuclei of atoms and their isotopes by synthesis of the elementary weightlessness scalar particles. The bullet point of the dynamic effect of this process should be the simultaneous collision of π elementary particles with creation of a mass particle. Synthesis of the scalar elementary particles will be accompanied by absorption of energy from the force field for binding the particles. The hydrostatic pressure is also takes part in the process of attraction of the oscillating system in the framework of the Archimedes law.

In our opinion, the dynamics of the microcosm is a very interesting field for application of Jacobi dynamics. This book takes only the first step in this direction. It is shown that Jacibi dynamics is also applicable for the solution of this type of problem. An attractive idea is to use the dynamic approach for studying the physics of molecules, atoms and nuclei as dissipative systems, which might lead to discovery of many interesting effects [16-19].

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