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Research Article

Photons with Scalar Fields Should Exist

Shuang-ren Zhao*

Mutualenergy.org London ontario, Canada

***Corresponding Author**
Shuang-ren Zhao, Mutualenergy.org London ontario, Canada.

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Abstract

The author believes that scalar photons should exist. Scalar photons are photons corresponding to scalar fields ϕ. This type of photon is different from the photons we usually understand, as they correspond to the vector potential $\boldsymbol{A}$.This paper explores the theory of mutual energy flow within the framework of the Helmholtz equation, proposing a novel interpretation of particle-like energy transfer through the interaction of retarded and advanced waves. Traditional electromagnetic and quantum theories describe energy flow via self-energy flow (e.g., Poynting vector or probability current density), which fails to localize energy transfer. In contrast, mutual energy flow arises from the synchronized interaction between a retarded wave emitted by a source and an advanced wave emitted by a sink, forming a localized energy channel. We demonstrate that the Helmholtz equation must be modified - by introducing source-sink dynamics, defining a corrected magnetic field, and halving the mutual energy flow intensity - to derive a consistent energy conservation law. The resulting mutual energy flow density exhibits particle-like behavior: it is generated at the source, propagates directionally, and annihilates at the sink, with zero leakage outside this path. Crucially, self-energy flow is shown to represent reactive power, incapable of energy transfer, while mutual energy flow alone accounts for active energy propagation. This approach bridges electromagnetic theory and quantum mechanics, aligning with action-reaction principles and transactional interpretations. The findings suggest that particles can be modeled as mutual energy flows, resolving wave-particle duality by unifying retarded and advanced waves into a single physical entity. This work challenges classical field theories and offers a pathway to revise Maxwell's and Schrödinger's equations for particle-like localization.

Keywords: Poynting Theorem, Reciprocity Theorem, Huygens' Principle, Mutual Energy Flow, Helmholtz Equation, Wave Equation, Klein-Gordon Equation, Schrdinger Equation, Green's Function, Magnetic Field, Absorber Theory, Transactional Interpretation of Quantum Mechanics

1. Introduction

The theory of action and reaction at a distance has developed in two directions. First, from the physical perspective: the action-at-a-distance theory proposed by Schwarzschild and others after 1903; Dirac's self-action theory in the 1930s; Wheeler and Feynman's absorber theory from 1945 to 1948; Cramer's transactional interpretation of quantum mechanics around 1980; and Stephenson's advanced wave theory [1-7]. Second, in electromagnetic theory: Lorentz's reciprocity theorem proposed in 1896; Rumsey's action and reaction theory in 1954; Welch's time-domain reciprocity theorem in 1960; the author's concept of "mutual energy" and the "mutual energy theorem" proposed in 1987; de Hoop's cross-correlation reciprocity theorem in 1987; and Petrusenko's second Lorentz reciprocity theorem in 2009 [8-15].

These two directions are closely related, yet have long developed independently. The author follows the path of electromagnetic theory but emphasizes "mutual energy" rather than "reciprocity". unlike most others. Calling a theorem one of mutual energy implies it is a physical theorem; calling it reciprocal suggests a mathematical perspective. When the author proposed the mutual energy theorem in 1987, it was widely criticized for equating reciprocity with mutual energy. Thus, the author has long intended to clarify this issue. An initial attempt was made to prove the mutual energy theorem from the complex Poynting theorem, but it failed. This effort was delayed until the author revisited the topic and in 2017 succeeded in proving Welch's time-domain reciprocity theorem from the Poynting theorem, and then, through Fourier transform, derived the mutual energy theorem. This reinforced the author's belief in the mutual energy concept, leading to the proposal of mutual energy flow and the mutual energy flow theorem [16].

Moreover, the Welch reciprocity theorem implies that for it to be considered an energy theorem, advanced waves must physically exist. This led the author to study theories of advanced waves, including the absorber theory, Cramer's transactional interpretation, and Stephenson's advanced wave theory - further supporting the belief in the physical reality of advanced waves and the energy-theorem nature of mutual energy and mutual energy flow.

Later, the author realized that if mutual energy flow transmits energy, then self-energy flow must not - otherwise, it would imply the existence of two types of photons: mutual and self-energy flow photons, which would then have to be reconciled into one, a near-impossible task. Moreover, allowing both to transmit energy would exceed the correct total energy. Thus, the concept of reverse collapse of self-energy flow was introduced. The mutual energy flow, shaped like a tadpole - thin at both ends and thick in the middle - resembles the conceptual image of a photon. Hence, the author began to interpret particles as mutual energy flows [16].

It was then found that mutual energy flow is twice the self-energy flow since it includes two terms, $\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1$, whereas self-energy has only one: $\boldsymbol{E}_1 \times \boldsymbol{H}_1$ or $\boldsymbol{E}_2 \times \boldsymbol{H}_2$. To replace self-energy flow with mutual energy flow, normalization is needed to compress mutual energy flow to half [17].

Further, in calculating mutual energy flow in a dual-wire transformer, it was found that self-energy flow is reactive power. Thus, self-energy flow need not collapse. This led to the idea that electromagnetic radiation should also be reactive power. This implies a 90-degree phase shift between electric and magnetic fields, not in phase as in Maxwell's theory. Consequently, self-energy flow need not reverse collapse - but it also means that Maxwell's electromagnetic theory needs revision. The issue lies in the conditional validity of defining the magnetic field as the curl of the vector potential ($\boldsymbol{B} = \nabla \times \boldsymbol{A}$). This only holds in quasi-static conditions, not for radiative electromagnetic fields, which require corrections in the far field. Assuming:

$$\boldsymbol{B}_{Maxwell}^{(r)} = \nabla \times \boldsymbol{A}^{(r)}, \quad \boldsymbol{B}_{Maxwell}^{(a)} = \nabla \times \boldsymbol{A}^{(a)}, \tag{1}$$

where $\boldsymbol{A}^{(r)}$ and $\boldsymbol{A}^{(a)}$ are retarded and advanced vector potentials respectively, and the subscript "Maxwell" indicates quantities derived from Maxwell's equations, which may be incorrect. For instance, $\boldsymbol{B}_{Maxwell}^{(r)}$ may not be the true magnetic field. Assuming current has time dependence $\boldsymbol{J} = \boldsymbol{J}_0 \exp(j\omega t)$, then:

$$\boldsymbol{B}_n^{(r)} = \boldsymbol{B}_{Maxwell\ n}^{(r)}, \quad \boldsymbol{B}_n^{(a)} = \boldsymbol{B}_{Maxwell\ n}^{(a)}, \tag{2}$$

$$\boldsymbol{B}_f^{(r)} = -j\boldsymbol{B}_{Maxwell\ f}^{(r)}, \quad \boldsymbol{B}_f^{(a)} = j\boldsymbol{B}_{Maxwell\ f}^{(a)}. \tag{3}$$

Here $\boldsymbol{B}^{(r)}$ and $\boldsymbol{B}^{(a)}$ are the correct magnetic fields, with subscripts $\boldsymbol{n}$ and f denoting near and far fields. This shows that far-field components of magnetic fields from Maxwell's theory must be phase-shifted by 90 degrees to yield the correct fields [18].

It's worth noting that both directions of action-reaction theory conflict with Maxwell's ether and field theory. According to Maxwell, the transmitting antenna gives energy to the electromagnetic field/wave, which then transfers part of it to the receiving antenna, with the remainder continuing infinitely - even out of the universe. Transmission is independent of the receiver. However, action-reaction theory holds that the energy emitted by the transmitting antenna only affects the receiving antenna. If there is only one receiver in space, the energy acts solely on it. The receiver provides a reaction (advanced wave) when acted upon by the source (retarded wave), causing recoil of the transmitter. If no receiver exists, energy cannot be emitted. This requires that electromagnetic waves are reactive power, unable to escape the universe. In contrast, Maxwell's in-phase $\boldsymbol{E}$ and $\boldsymbol{H}$ fields imply active power emission - clearly an error in Maxwell's theory.

This paper applies the results of electromagnetic theory comprehensively to the Helmholtz equation, identifying corresponding mutual energy flow and particles, deepening the understanding of the wave-to-particle transition, and the wave-particle duality problem. The author believes that mutual energy flow formed from waves is the particle. The author's theory is similar to Cramer's transactional interpretation of quantum mechanics, both emphasizing the retarded wave from source and advanced wave from sink as physical realities. However, the author places more emphasis on mutual energy flow [19,20].

The author speculates that in nature, there may indeed exist a photon corresponding to a scalar potential ϕ. This type of photon is different from the photons we usually see, as they correspond to the vector potential $\boldsymbol{A}$. The author argues this from the Helmholtz equation.

1.1 Helmholtz Equation

The standard form of the Helmholtz equation is:

$$\nabla^2 u + k^2 u = 0. \tag{4}$$

where ∇^2 is the Laplace operator, and $u = u(r)$ is the unknown function, which can be a scalar or vector field; k is the wave number, usually a real or complex number representing spatial frequency. Our discussion also applies to the wave equation:

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial}{\partial t^2} u = 0. \tag{5}$$

Considering $u = u_0 \exp(-i\omega t)$, $\frac{\partial^2}{\partial t^2} = (-i\omega)^2 = -\omega^2$, we get:

$$\nabla^2 u + \frac{\omega^2}{c^2} u = 0. \tag{6}$$

with $k = \frac{\omega}{c}$. This method also applies to the Klein-Gordon equation.

1.2 Poynting-like Theorem

The Helmholtz equation is source-free. Adding a source term:

$$\nabla^2 u + k^2 u = \tau \tag{7}$$

Taking the complex conjugate:

$$\nabla^2 u^* + k^2 u^* = \tau^* \tag{8}$$

Multiplying (7) by u*:

$$\nabla^2 u u^* + k^2 u u^* = \tau u^* \tag{9}$$

Multiplying (8) by u:

$$\nabla^2 u^* u + k^2 u^* u = \tau^* u \tag{10}$$

Subtracting (9) from (10):

$$\nabla^2 u u^* - \nabla^2 u^* u = \tau u^* - \tau^* u \tag{11}$$

Consider:

$$\nabla \cdot (\nabla u u^* - \nabla u^* u)$$

$$= \nabla \cdot \nabla u u^* + \nabla u \cdot \nabla u^* - \nabla \cdot \nabla u^* u - \nabla u^* \cdot \nabla u$$

$$= \nabla^2 u u^* - \nabla^2 u^* u \tag{12}$$

Therefore:

$$\nabla \cdot (\nabla u u^* - \nabla u^* u) = \tau u^* - \tau^* u \tag{13}$$

Multiply both sides by $\frac{1}{2i}$:

$$\nabla \cdot \frac{1}{2i}(\nabla u u^* - \nabla u^* u) = \frac{1}{2i}(\tau u^* - \tau^* u) \tag{14}$$

Or:

$$\nabla \cdot \frac{1}{2}\left(\frac{1}{i}\nabla u u^* + \left(\frac{1}{i}\nabla u\right)^* u\right) = \frac{1}{2}\left(\frac{1}{i}\tau u^* + \left(\frac{1}{i}\tau\right)^* u\right) \tag{15}$$

The Helmholtz equation lacks magnetic fields, but in electromagnetic theory, we obtain particles by correcting the far-field magnetic field [18]. Hence, we introduce a magnetic field $\boldsymbol{h}_H$ to the Helmholtz equation and define its "Faraday's law" as follows:

$$\nabla u \equiv i\boldsymbol{h}_H \tag{16}$$

The above equation is analogous to Faraday's law in electromagnetic theory:

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} = -j\omega\boldsymbol{A} = i\omega\boldsymbol{A} \tag{17}$$

Here, $\boldsymbol{h}_H$ denotes the magnetic field in the Helmholtz equation. The subscript H indicates that this magnetic field is defined by the Helmholtz equation, but it is not yet the correct magnetic field. Therefore,

$$\boldsymbol{h}_H = \frac{1}{i}\nabla u \tag{18}$$

Consider

$$\frac{1}{i}\tau = -\rho \tag{19}$$

Thus, equation (15) becomes

$$\nabla \cdot \frac{1}{2}(\boldsymbol{h}_H u^* + (\boldsymbol{h}_H)^* u) = -\frac{1}{2}(\rho u^* + \rho^* u) \tag{20}$$

$$\mathrm{Re}(\nabla \cdot (\boldsymbol{h}_H u^*)) = -\mathrm{Re}(\rho u^*) \tag{21}$$

Bearing in mind the operator Re, the above becomes:

$$\nabla \cdot (\boldsymbol{h}_H u^*) = -\rho u^* \tag{22}$$

This equation is analogous to the complex Poynting theorem.

1.3 Reciprocity Theorem

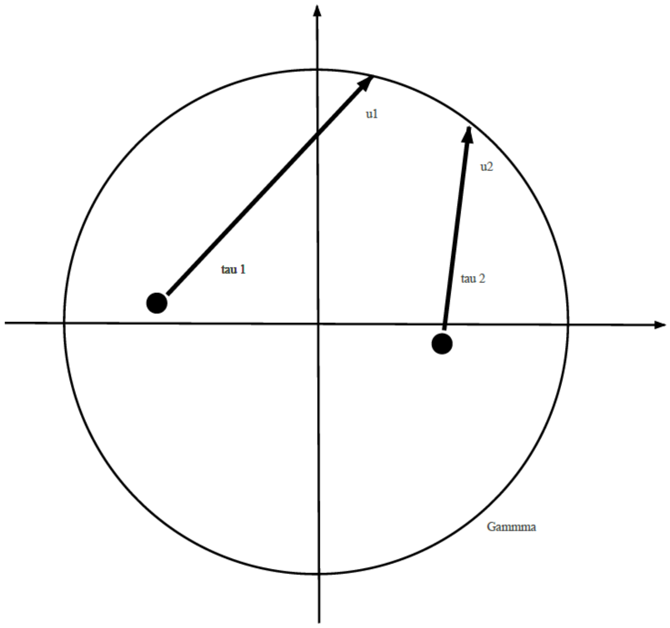

Figure 1: Within Boundary Γ are Two Sources τ_1, τ_2, where τ_1 is a Radiator Generating a Retarded Wave u_1, and τ_2 is a Sink Generating an Advanced Wave

Similarly, we have

$$\nabla \cdot (\boldsymbol{h}_{1H} u_1^*) = -\rho_1 u_1^*, \quad \nabla \cdot (\boldsymbol{h}_{2H} u_2^*) = -\rho_2 u_2^* \tag{23}$$

The above equations correspond to the Poynting theorem. Considering the principle of superposition:

$$\boldsymbol{h}_H = \boldsymbol{h}_{1H} + \boldsymbol{h}_{2H}, \quad u = u_1 + u_2, \quad \rho = \rho_1 + \rho_2 \tag{24}$$

We assume ρ_1 is a radiator generating a retarded wave and ρ_2 is a sink generating an advanced wave. See Figure 1. Assuming both retarded and advanced waves are of the same field and can be superposed, equation (22) becomes:

$$\nabla \cdot ((\boldsymbol{h}_{1H} + \boldsymbol{h}_{2H})(u_1^* + u_2^*)) = -(\rho_1 + \rho_2)(u_1^* + u_2^*) \tag{25}$$

Or

$$\nabla \cdot (\boldsymbol{h}_{1H} u_1^*) + \nabla \cdot (\boldsymbol{h}_{2H} u_2^*) + \nabla \cdot (\boldsymbol{h}_{1H} u_2^*) + \nabla \cdot (\boldsymbol{h}_{2H} u_1^*)$$

$$= -(\rho_1 u_1^* + \rho_2 u_2^* + \rho_1 u_2^* + \rho_2 u_1^*) \tag{26}$$

Subtracting the two equations in (23) from the above, we get:

$$\nabla \cdot (\boldsymbol{h}_{1H} u_2^* + \boldsymbol{h}_{2H} u_1^*) = -(\rho_1 u_2^* + \rho_2 u_1^*) \tag{27}$$

If we consider taking the real part, the equation can be rewritten as:

$$\nabla \cdot (\boldsymbol{h}_{1H} u_2^* + \boldsymbol{h}_{2H}^* u_1) = -(\rho_1 u_2^* + \rho_2^* u_1) \tag{28}$$

Taking volume integral:

$$-\oiint_\Gamma (\boldsymbol{h}_{1H} u_2^* + \boldsymbol{h}_{2H}^* u_1) \cdot \hat{n}\, d\Gamma = \int_V (\rho_1 u_2^* + \rho_2^* u_1)\, dV \tag{29}$$

This equation represents the mutual energy theorem (or reciprocity theorem) of the Helmholtz equation.

1.4 No-Radiation-Leakage Theorem

Consider the mutual energy flow $\oiint_{\Gamma}(\boldsymbol{h}_{1H}u_2^* + \boldsymbol{h}_{2H}^*u_1)\cdot\hat{n}\,d\Gamma$ not leaking out of the universe. When the radius of the surface $R = \infty$:

$$\oiint_{\Gamma}(\boldsymbol{h}_{1H}u_2^* + \boldsymbol{h}_{2H}^*u_1)\cdot\hat{n}\,d\Gamma = 0 \tag{30}$$

This implies that mutual energy does not leak out of the universe, because u_1 , u_2 are respectively a retarded and an advanced wave - one arrives at the surface Γ in the past, the other in the future - thus they do not exist simultaneously on Γ. Therefore the surface integral is zero, and:

$$\int_V (\rho_1 u_2^* + \rho_2^* u_1)\,dV = 0 \tag{31}$$

Or

$$-\int_{V_1}(\rho_1 u_2^*)\,dV = \int_{V_2}(\rho_2^* u_1)\,dV \tag{32}$$

This is also known as the mutual energy theorem of the Helmholtz equation.

1.5 Mutual Energy Flow Theorem

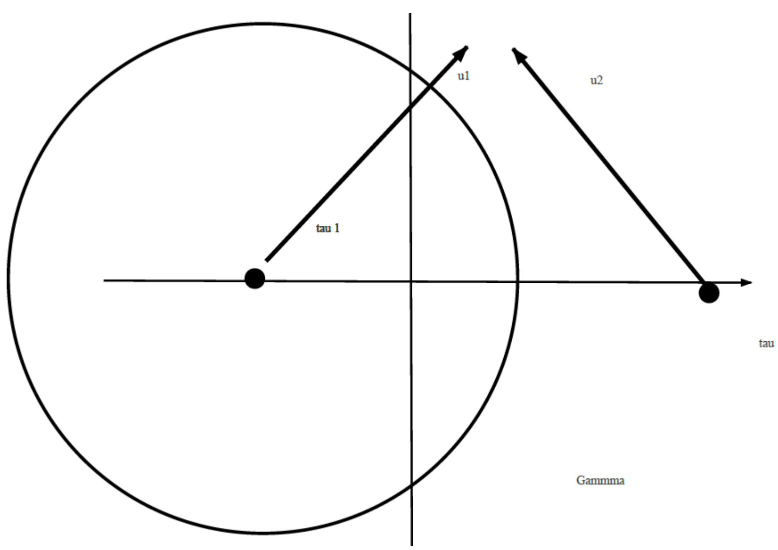

Figure 2: There is a Source τ_1 Inside the Boundary Γ. τ_1 is a Radiation Source Generating the Retarded Wave u_1, and τ_2 is a Sink Generating the Advanced Wave. τ_2 is Located Outside the Surface Γ.

In equation (29), by moving ρ_2 outside the region V, as shown in Figure 2, we obtain:

$$\oiint_{\Gamma_1}(\boldsymbol{h}_{1H}u_2^* + \boldsymbol{h}_{2H}^*u_1)\cdot\hat{n}d\Gamma = -\int_{V_1}(\rho_1 u_2^*)dV \tag{33}$$

From equations (33) and (32):

$$-\int_{V_1}(\rho_1 u_2^*)dV = \oiint_{\Gamma_1}(\boldsymbol{h}_{1H}u_2^* + \boldsymbol{h}_{2H}^*u_1)\cdot\hat{n}d\Gamma = \int_{V_2}(\rho_2^* u_1)dV \tag{34}$$

This is the mutual energy flow theorem. Although the surface Γ encloses the source ρ_1 during the derivation, in fact, the surface only needs to separate the sources ρ_1 and ρ_2. It can be a closed surface enclosing ρ_1, a closed surface enclosing ρ_2, or an infinite plane between ρ_1 and ρ_2. See Fiure 3.

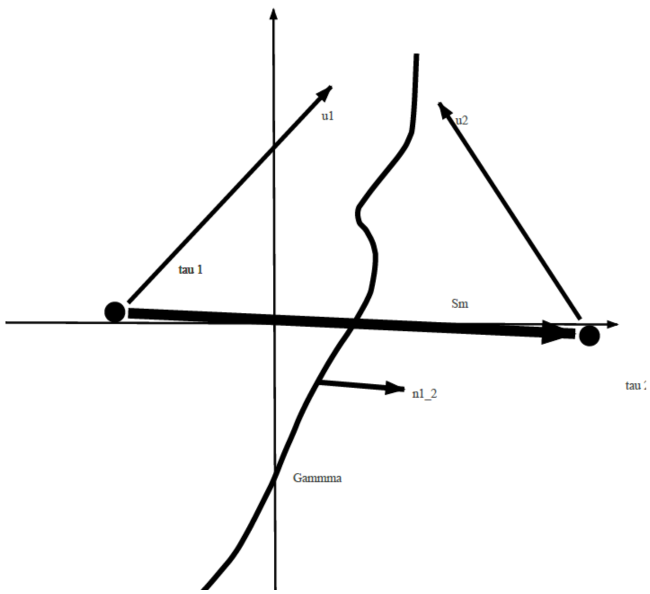

Figure 3: The Surface Γ Seperates the Source τ_1 and Sink τ_2 .

1.6 Huygens' Principle

Let

$$\rho_2 = \delta(\boldsymbol{x} - \boldsymbol{x}') \tag{35}$$

Then we obtain:

$$u_1(\boldsymbol{x}) = -\int_V (\rho_1 u_2^*) dV = \oiint_\Gamma (\boldsymbol{h}_{1H} u_2^* + \boldsymbol{h}_{2H}^* u_1) \cdot \hat{n} d\Gamma \tag{36}$$

This is Huygens' Principle. Originally, $u_1(\boldsymbol{x})$ was calculated from the radiation source ρ_1, but it can now be obtained through the surface integral $\oiint_\Gamma (\boldsymbol{h}_{1H} u_2^* + \boldsymbol{h}_{2H}^* u_1) \cdot \hat{n} d\Gamma$. In electromagnetic theory, although the derivation of Huygens' principle involves the mutual energy flow theorem, the author first obtained Huygens' principle, and nearly 30 years later proposed the concept of mutual energy flow and its theorem [12,16].

1.7 Law of Energy Conservation

There are still issues with the mutual energy flow theorem in equation (34). First, to replace self-energy flow with mutual energy flow as the measure of energy transfer, the mutual energy flow must be halved. Thus:

$$-\int_{V_1} (\rho_1 u_2^*) dV = \frac{1}{2} \oiint_\Gamma (\boldsymbol{h}_{1H} u_2^* + \boldsymbol{h}_{2H}^* u_1) \cdot \hat{n} d\Gamma = \int_{V_2} (\rho_2^* u_1) dV \tag{37}$$

Next, replacing the mutual energy flow density

$$\boldsymbol{S}_{Hm} = \frac{1}{2} (\boldsymbol{h}_{1H} u_2^* + \boldsymbol{h}_{2H}^* u_1) \tag{38}$$

with

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{h}_1 u_2^* + \boldsymbol{h}_2^* u_1) \tag{39}$$

and using

$$\boldsymbol{S}_1 = u_1 h_1^*, \quad \boldsymbol{S}_2 = u_2 h_2^* \tag{40}$$

instead of

$$\boldsymbol{S}_{H1} = u_1 h_{H1}^*, \quad \boldsymbol{S}_{H2} = u_2 h_{H2}^* \tag{41}$$

After this replacement, the self-energy flow $\boldsymbol{S}_1$ becomes reactive power, meaning self-energy flow does not transmit energy. The mutual energy flow theorem then becomes a mutual energy flow law or law of energy conservation:

$$-\int_V (\rho_1 u_2^*) dV = \frac{1}{2}\oiint_\Gamma (\boldsymbol{h}_1 u_2^* + \boldsymbol{h}_2^* u_1) \cdot \hat{n} d\Gamma = \int_V (\rho_2^* u_1) dV \tag{42}$$

We assume that both $\boldsymbol{h}_1$, $\boldsymbol{h}_2$ are far-field components. According to the correction formula in equation (3) considering the time factor $\rho \sim \exp(-i\omega t)$, the corrected magnetic fields are:

$$\boldsymbol{h}^{(r)} = i\boldsymbol{h}_H, \quad \boldsymbol{h}^{(a)} = -i\boldsymbol{h}_H \tag{43}$$

1.8 Quasi-Static Equation

It is worth mentioning that the mutual energy law in equation (42) is not derived from the Helmholtz equation (7), but from the so-called quasi-static Helmholtz equation. The quasi-static Helmholtz equation is obtained by removing the so-called "displacement current" term:

$$\nabla^2 u = \tau \tag{44}$$

This is essentially the Poisson equation. Our previous derivations still hold for the Poisson equation. Correspondingly, for the Poisson equation, the subscript H in the magnetic field can be omitted:

$$\boldsymbol{h}_H \leftarrow \boldsymbol{h} \tag{45}$$

This indicates replacing $\boldsymbol{h}_H$ with $\boldsymbol{h}$. Therefore, we can derive equation (42) from the Poisson equation (or the quasi-static Helmholtz equation). We then generalize this mutual energy flow theorem from the quasi-static condition to the case involving radiation. Our treatment of the Helmholtz equation is entirely consistent with our approach to the Schröodinger equation and Maxwell's equations. For further clarification, refer to [18].

2 Reasons for Correcting the Magnetic Field

The Helmholtz equation with a source is given by

$$\nabla^2 u + k^2 u = \tau \tag{46}$$

Previously, we applied

$$\frac{1}{i}\tau = -\rho \tag{47}$$

Therefore,

$$\tau = -i\rho \quad (48)$$

The Helmholtz equation becomes

$$\nabla^2 u + k^2 u = -i\rho \quad (49)$$

Its solution is

$$u = \int_V (i\rho)\frac{1}{r}\exp(ikx)dV \quad (50)$$

Previously, we defined Faraday's law within the Helmholtz framework as (16),

$$\boldsymbol{h}_H = \frac{1}{i}\nabla u \quad (51)$$

Hence,

$$\boldsymbol{h}_H = \frac{1}{i}\nabla u = \frac{1}{i}\nabla \int_V (i\rho)\frac{1}{r}\exp(ikx)dV$$

$$= \int_V \rho\nabla\left(\frac{1}{r}\exp(ikx)\right)dV$$

$$= \int_V \rho\left(-\frac{\boldsymbol{r}}{r^3} + ik\hat{r}\frac{1}{r}\right)\exp(ikx)dV$$

$$= -\int_V \rho\left(\frac{\boldsymbol{r}}{r^3}\right)\exp(ikx)dV + ik\int_V \rho\left(\frac{1}{r}\hat{r}\right)\exp(ikx)dV \quad (52)$$

The magnetic field $\boldsymbol{h}_H = \frac{1}{i}\nabla u$ is a physical quantity. Under quasi-static conditions (i.e., when $k = 0$), the equation becomes

$$\nabla^2 u = \tau = -i\rho \quad (53)$$

$$u = \int_V \frac{i\rho}{r}dV \quad (54)$$

$$\nabla u = \int_V \nabla\left(\frac{1}{r}\right)i\rho dV = -\int_V \frac{\boldsymbol{r}}{r^3}i\rho dV \quad (55)$$

$$\boldsymbol{h}_S = \frac{1}{i}\nabla u = -\int_V \frac{\boldsymbol{r}}{r^3}\rho dV \quad (56)$$

The author believes that the magnetic field derived from the Helmholtz equation under quasi-static conditions is the correct magnetic field, denoted as $\boldsymbol{h}$. Therefore, the above equation implies

$$\boldsymbol{h} = -\int_V \rho\left(\frac{\boldsymbol{r}}{r^3}\right)dV \quad (57)$$

We regard this $\boldsymbol{h}$ as the original definition of the magnetic field and the correct one. Therefore, the radiation field should reduce to the quasi-static field in the limit:

$$\lim_{kr\to 0}\boldsymbol{h}_H^{(r)}=\boldsymbol{h} \tag{58}$$

The superscript (r) denotes the retarded field, corresponding to the phase factor exp(ikx). The above equation requires that the radiative magnetic field must degenerate to the quasi-static magnetic field. However,

$$\lim_{kr\to 0}\boldsymbol{h}_H^{(r)}=\lim_{kr\to 0}\left(-\int_V\rho\left(\frac{\boldsymbol{r}}{r^3}\right)\exp(ikx)dV+ik\int_V\rho\left(\frac{1}{r}\hat{r}\right)\exp(ikx)dV\right)$$

$$=-\int_V\rho\frac{\boldsymbol{r}}{r^3}dV+ik\int_V\rho\frac{\boldsymbol{r}}{r^2}dV \tag{59}$$

Therefore,

$$\lim_{kr\to 0}\boldsymbol{h}_H^{(r)}\neq\boldsymbol{h} \tag{60}$$

The key issue is that the second term in (59) has a different phase from the first term, which is quite strange! The author believes that

$$\lim_{kr\to 0}\boldsymbol{h}^{(r)}=-\int_V\rho\left(\frac{\boldsymbol{r}}{r^3}\right)dV-k\int_V\rho\frac{\boldsymbol{r}}{r^2}dV \tag{61}$$

is the correct expression for the magnetic field. In this expression, both the far-field and near-field have the same phase factor. To achieve this, we must have

$$\boldsymbol{h}_n^{(r)}=\boldsymbol{h}_{Hn}^{(r)},\quad \boldsymbol{h}_f^{(r)}=i\boldsymbol{h}_{Hf}^{(r)} \tag{62}$$

Thus,

$$\boldsymbol{h}_n^{(r)}=\boldsymbol{h}_{Hn}^{(r)}=-\int_V\rho\frac{\boldsymbol{r}}{r^3}\exp(-ikx)dV \tag{63}$$

$$\boldsymbol{h}_f^{(r)}=i\boldsymbol{h}_{Hf}^{(r)}=i\left(ik\int_V\rho\frac{\boldsymbol{r}}{r^2}\exp(ikx)dV\right)=-k\int_V\rho\frac{\boldsymbol{r}}{r^2}\exp(ikx)dV \tag{64}$$

This ensures that both $\boldsymbol{h}_n^{(r)}$ and $\boldsymbol{h}_f^{(r)}$ share the same phase. Similarly,

$$\boldsymbol{h}_n^{(a)}=\boldsymbol{h}_{Hn}^{(a)},\quad \boldsymbol{h}_f^{(a)}=-i\boldsymbol{h}_{Hf}^{(a)} \tag{65}$$

The superscript (a) denotes the advanced field, corresponding to the phase factor exp(-ikx). If the reader is confused by this part of the author's procedure, please refer to the author's similar treatment of electromagnetic field theory [18].

3. Derivation of Mutual Energy Flow Theorem Using Green's Function Method

The derivation in Section 1.2 above essentially uses the real part of a complex quantity. That is, we obtained only the real part of Equation (42), not the full equation. Below we derive it again using Green's function method to fully recover Equation (42), including its imaginary part.

Rewrite the Helmholtz equations (7) as

$$\nabla^2 u_1 u_2^* + k^2 u_1 u_2^* = \tau_1 u_2^* \tag{66}$$

$$\nabla^2 u_2^* u_1 + k^2 u_2^* u_1 = \tau_2^* u_1 \tag{67}$$

Subtracting the two equations:

$$\nabla^2 u_1 u_2^* - \nabla^2 u_2^* u_1 = \tau_1 u_2^* - \tau_2^* u_1 \tag{68}$$

Consider

$$\nabla \cdot (\nabla u_1 u_2^* - \nabla u_2^* u_1)$$
$$= \nabla \cdot \nabla u_1 u_2^* + \nabla u_1 \cdot \nabla u_2^* - \nabla \cdot \nabla u_2^* u_1 - \nabla u_2^* \cdot \nabla u_1$$
$$= \nabla^2 u_1 u_2^* - \nabla^2 u_2^* u_1 \tag{69}$$

Equation (68) can then be rewritten as

$$\nabla \cdot (\nabla u_1 u_2^* - \nabla u_2^* u_1) = \tau_1 u_2^* - \tau_2^* u_1 \tag{70}$$

Define the magnetic fields:

$$\nabla u_1 = i\boldsymbol{h}_{H1}, \quad \nabla u_2 = i\boldsymbol{h}_{H2} \tag{71}$$

Then (70) becomes

$$\nabla \cdot (i\boldsymbol{h}_{H1} u_2^* - (i\boldsymbol{h}_{H2})^* u_1) = \tau_1 u_2^* - \tau_2^* u_1 \tag{72}$$

Or

$$\frac{1}{i}\nabla \cdot (i\boldsymbol{h}_{H1} u_2^* - (i\boldsymbol{h}_{H2})^* u_1) = \frac{1}{i}(\tau_1 u_2^* - \tau_2^* u_1) \tag{73}$$

Or

$$\nabla \cdot (\boldsymbol{h}_{H1} u_2^* + (\boldsymbol{h}_{H2})^* u_1) = -\left[\left(-\frac{1}{i}\tau_1\right) u_2^* + \left(-\frac{1}{i}\tau_2\right)^* u_1\right] \tag{74}$$

Considering

$$-\frac{1}{i}\tau_1 = i\tau_1 = \rho_1, \quad -\frac{1}{i}\tau_2 = i\tau_2 = \rho_2 \tag{75}$$

We obtain

$$\nabla \cdot (\boldsymbol{h}_{H1} u_2^* + (\nabla\boldsymbol{h}_{H2})^* u_1) = -(\rho_1 u_2^* + \rho_2^* u_1) \tag{76}$$

From this, we derive the mutual energy theorem:

$$\oiint_{\Gamma}(\boldsymbol{h}_{H1}u_2^* + (\boldsymbol{h}_{H2})^* u_1) \cdot \hat{n} d\Gamma = -\int_V (\rho_1 u_2^* + \rho_2^* u_1) dV \tag{77}$$

Consider that the mutual energy flow does not overflow the universe. When the radius of the surface Γ is $R = \infty$, we have

$$\oiint_{\Gamma}(\boldsymbol{h}_{H1}u_2^* + (\boldsymbol{h}_{H2})^* u_1) \cdot \hat{n} d\Gamma = 0 \tag{78}$$

From this, it follows that

$$\int_V (\rho_1 u_2^* + \rho_2^* u_1) dV = 0 \tag{79}$$

Or,

$$-\int_{V_1} (\rho_1 u_2^*) dV = \int_{V_2} (\rho_2^* u_1) dV \tag{80}$$

Now consider moving ρ_2 out of the region V. Equation (77) becomes

$$\oiint_{\Gamma_1}(\boldsymbol{h}_{H1}u_2^* + (\boldsymbol{h}_{H2})^* u_1) \cdot \hat{n} d\Gamma = -\int_{V_1} (\rho_1 u_2^*) dV \tag{81}$$

Combining equations (81) and (80), we get

$$-\int_{V_1} (\rho_1 u_2^*) dV = \oiint_{\Gamma_1}(\boldsymbol{h}_{H1}u_2^* + (\boldsymbol{h}_{H2})^* u_1) \cdot \hat{n} d\Gamma = \int_{V_2} (u_1 \rho_2^*) dV \tag{82}$$

The above is the mutual energy flow theorem. However, this expression is still not correct; we must compress the mutual energy flow to its half value. That is,

$$-\int_{V_1} (\rho_1 u_2^*) dV = \oiint_{\Gamma} \frac{1}{2}(\boldsymbol{h}_{H1}u_2^* + (\boldsymbol{h}_{H2})^* u_1) \cdot \hat{n} d\Gamma = \int_{V_2} (u_1 \rho_2^*) dV \tag{83}$$

Even this expression is incorrect. The mutual energy flow

$$\boldsymbol{S}_{Hm} = \frac{1}{2}(\boldsymbol{h}_{H1}u_2^* + \boldsymbol{h}_{H2}{}^* u_1) \tag{84}$$

should be replaced by

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{h}_1 u_2^* + \boldsymbol{h}_2{}^* u_1) \tag{85}$$

Thus, the correct mutual energy flow theorem is

$$-\int_V (\rho_1 u_2^*) dV = \oiint_{\Gamma} \frac{1}{2}(\boldsymbol{h}_1 u_2^* + \boldsymbol{h}_2{}^* u_1) \cdot \hat{n} d\Gamma = \int_V (u_1 \rho_2^*) dV \tag{86}$$

Previously, we obtained (42). During the derivation of the mutual energy flow theorem, the real part was taken. Therefore, it is only valid for the real part, while both the real and imaginary parts of the above expression are correct. Considering that self-energy flow corresponds to reactive power, the above expression has become the law of energy conservation or law of mutual energy flow.

4. Calculating Mutual Energy Flow Density Using Plane Waves i.e., the Particle's Energy Density

In this section, we calculate the mutual energy flow density. We aim to prove that our definition of mutual energy flow exhibits particle-like properties - that is, the mutual energy flow is generated at the source and annihilated at the sink. The density of the mutual energy flow is given by

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{h}_1 u_2^* + \boldsymbol{h}_2{}^* u_1) \tag{87}$$

We know that

$$\boldsymbol{h}_{H1} = \frac{1}{i}\nabla u_1 \tag{88}$$

And,

$$\boldsymbol{h}^{(r)} = i\boldsymbol{h}_H, \quad \boldsymbol{h}^{(a)} = -i\boldsymbol{h}_H \tag{89}$$

The superscript (r) indicates retarded, and (a) indicates advanced. The author assumes that the source randomly emits retarded waves u_1, and the sink randomly emits advanced waves u_2. Initially, u_1 and u_2 are spherical waves emitted in all directions. However, when the retarded wave u_1 reaches the sink ρ_2 and ρ_2 happens to emit an advanced wave, this advanced wave synchronizes with the retarded wave. After synchronization, the advanced wave becomes the waveguide of the retarded wave and vice versa. Ultimately, both the retarded and advanced waves become quasi-plane waves propagating from the source to the sink. Between the source and sink, u_1 is a retarded wave, and u_2 is an advanced wave. At the source and sink, retarded and advanced waves flip. Thus, u_1, h_1 are advanced waves when $x < 0$ and retarded when $x \geq 0$,

$$\boldsymbol{h}_1 = \begin{cases} -i\boldsymbol{h}_{H1} & x < 0 \\ i\boldsymbol{h}_{H1} & 0 \leq x \end{cases} = \begin{cases} -i\frac{1}{i}\nabla u_1 & x < 0 \\ i\frac{1}{i}\nabla u_1 & 0 \leq x \end{cases} = \nabla u_1 \begin{cases} -1 & x < 0 \\ 1 & 0 \leq x \end{cases} \tag{90}$$

u_2, $\boldsymbol{h}_2$ are advanced waves when $x \leq L$ and retarded when $x > L$,

$$\boldsymbol{h}_2 = \begin{cases} -i\boldsymbol{h}_{H2} & x \leq L \\ i\boldsymbol{h}_{H2} & L < x \end{cases} = \begin{cases} -i\frac{1}{i}\nabla u_2 & x \leq L \\ i\frac{1}{i}\nabla u_2 & L < x \end{cases} = \nabla u_2 \begin{cases} -1 & x \leq L \\ 1 & L < x \end{cases} \tag{91}$$

Note that choosing the left side of the source and sink as advanced waves and the right side as retarded waves is consistent with the author's choice in electromagnetic theory [18]. Thus,

$$\boldsymbol{S}_m = \frac{1}{2}(\boldsymbol{h}_1 u_2^* + \boldsymbol{h}_2{}^* u_1) \tag{92}$$

$$= \frac{1}{2}\begin{cases} -\nabla u_1 u_2^* - \nabla u_2{}^* u_1 & x < 0 \\ \nabla u_1 u_2^* - \nabla u_2{}^* u_1 & 0 \leq x \leq L \\ \nabla u_1 u_2^* + \nabla u_2{}^* u_1 & L < x \end{cases} \tag{93}$$

Assume

$$u_1 = \exp(ikx) \tag{94}$$

This is a simple plane wave. In the region $0 \leq x \leq L$,

$$\boldsymbol{h}_1 = -i\boldsymbol{h}_{H1} = -i\left(\frac{1}{i}\nabla u_1\right) = \nabla u_1 = ik\hat{x}\exp(ikx) \tag{95}$$

u_2, $\boldsymbol{h}_2$ should be synchronized with u_1, $\boldsymbol{h}_1$. For the first term of equation (92) to be real, u_2 and $\boldsymbol{h}_1$ must have the same phase. So,

$$u_2 \sim ik\exp(ikx) \tag{96}$$

To match dimensions with u_1, take

$$u_2 = i\exp(ikx) \tag{97}$$

Substitute (94) and (96) into (93) to get
Substituting equations (94, 96) into (93) yields

$$\boldsymbol{S}_m = \frac{1}{2}\begin{cases} -\nabla\exp(ikx)(i\exp(ikx))^* - \nabla(i\exp(ikx))^*\exp(ikx) & x < 0 \\ \nabla\exp(ikx)(i\exp(ikx))^* - \nabla(i\exp(ikx))^*\exp(ikx) & 0 \le x \le L \\ \nabla\exp(ikx)(i\exp(ikx))^* + \nabla(i\exp(ikx))^*\exp(ikx) & L < x \end{cases}$$

$$= \frac{1}{2}\hat{x}\begin{cases} -ik\exp(ikx)(i\exp(ikx))^* + ik(i\exp(ikx))^*\exp(ikx) & x < 0 \\ ik\exp(ikx)(i\exp(ikx))^* + ik(i\exp(ikx))^*\exp(ikx) & 0 \le x \le L \\ ik\exp(ikx)(i\exp(ikx))^* - ik(i\exp(ikx))^*\exp(ikx) & L < x \end{cases}$$

$$= \frac{1}{2}k\hat{x}\begin{cases} -i(i)^* + i(i)^* & x < 0 \\ i(i)^* + i(i)^* & 0 \le x \le L \\ i(i)^* - i(i)^* & L < x \end{cases} = k\hat{x}\begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le L \\ 0 & L < x \end{cases} \tag{98}$$

Therefore, $\boldsymbol{S}_m$ is generated at $x = 0$ and annihilated at $x = L$. $\boldsymbol{S}_m$ can be used to describe a particle! In the region $0 \le x \le L$, the momentum of the particle can be expressed as $\hbar\boldsymbol{S}_m = \hbar k\hat{x} = \boldsymbol{p}$. Moreover,

$$\boldsymbol{S}_1 = u_1\boldsymbol{h}_1^* = u_1\nabla u_1^*\begin{cases} -1 & x < 0 \\ 1 & 0 \le x \end{cases}$$

$$= \exp(ikx)(-ik\hat{x})\exp(ikx)^*\begin{cases} -1 & x < 0 \\ 1 & 0 \le x \end{cases}$$

$$= (-ik\hat{x})\begin{cases} -1 & x < 0 \\ 1 & 0 \le x \end{cases} \tag{99}$$

Since the above expression is imaginary, the self-energy flow $\boldsymbol{S}_1$ is imaginary, so $\boldsymbol{S}_1$ represents reactive power and does not transfer energy. Similarly, $\boldsymbol{S}_2$ is also imaginary and represents reactive power, meaning it also does not transfer energy. If the self-energy flow does not transfer energy and all energy is transferred by the mutual energy flow, then the mutual energy flow theorem becomes the law of energy conservation - and a localized one at that. The mutual energy flow density is generated at the source and annihilated at the sink, thus it can be regarded as a particle. Therefore, equation (98) also represents the energy density of a particle. Of course, saying it is the particle energy density is still somewhat imprecise, for example, a particle must decay radially, such as

$$\boldsymbol{S}_m = f(R)k\hat{x}\begin{cases} 0 & x < 0 \\ 1 & 0 \le x \le L \\ 0 & L < x \end{cases} \tag{100}$$

where

$$f(R)=\frac{1}{\pi a^2}\begin{cases}1 & R\le a\\ 0 & R>a\end{cases},\quad R=\sqrt{x^2+y^2} \tag{101}$$

a is beam width. So the mutual energy flow is

$$Q_m=\hbar\oiint_{\Gamma}\boldsymbol{S}_m\cdot\hat{n}d\Gamma$$

$$=\int_0^a\int_0^{2\pi}f(R)(\hbar k\hat{x}\begin{cases}0 & x<0\\ 1 & 0\le x\le L\\ 0 & L<x\end{cases})RdR$$

$$=\hbar k\hat{x}\begin{cases}0 & x<0\\ 1 & 0\le x\le L\\ 0 & L<x\end{cases}=\boldsymbol{p}\begin{cases}0 & x<0\\ 1 & 0\le x\le L\\ 0 & L<x\end{cases} \tag{102}$$

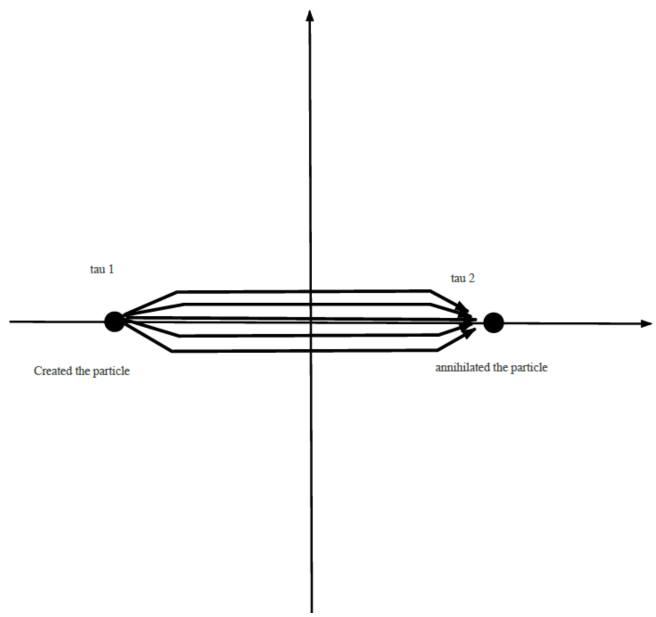

Figure 4: The Mutual Energy Flow is Generated at the Emitter τ_1 and Annihilated at the Sink τ_2. The Mutual Energy Flow is Essentially the Particle Itself

In this way, the mutual energy flow Q_m corresponds to the particle's momentum p in the region $0\le x\le L$ and is zero outside this region. Thus, Q_m can be regarded as the particle itself. See Figure 4.

5. Conclusion

This paper starts from the Helmholtz equation and demonstrates that the energy flow of a particle should be described by mutual energy flow, rather than self-energy flow. The so-called self-energy flow corresponds to the Poynting vector in electromagnetic field theory and to the probability current density in quantum theory. The self-energy flow consists of waves generated by a single source. In contrast, mutual energy flow must be composed of waves from two different sources: one being a radiation source generating retarded waves, and the other a sink generating advanced waves. The mutual energy flow is formed by the combination of both retarded and advanced waves. However, particles cannot be directly constructed from the Helmholtz equation alone. It is necessary first to modify the Helmholtz equation by adding a source and a sink, introducing a magnetic field, and then correcting the magnetic field. Additionally, the intensity of the mutual energy flow must be compressed to half of its original value. These modifications imply that mutual energy flow is not derived directly from the Helmholtz equation, but rather, the Helmholtz equation is used as an auxiliary equation. By utilizing the quasi-static form of the Helmholtz equation, mutual energy flow is obtained and then extended to radiation conditions, resulting in the mutual energy flow law, or the energy conservation law. The fields in the mutual energy flow thus derived already deviate from the solutions of the Helmholtz equation. Only with such deviations can a proper description of particles be constructed.

In traditional electromagnetic theory and quantum theory, starting from Maxwell's equations or Schrödinger's equation never leads to the formation of particles. The reason is that, in order to obtain particles, one must deviate from the solutions of these equations. These equations all yield self-energy flow, whereas constructing the correct mutual energy flow requires deviations from them. It is necessary to correct the so-called "magnetic field". Only after such corrections can the correct particle energy flow be constructed. These energy flows are mutual energy flows. The mutual energy flow density is precisely the energy flow density of a particle. In this way, particles are constructed using retarded and advanced waves.

The author speculates that the scalar potential ϕ corresponding to the electromagnetic field can also form a scalar photon or light particle.

This work establishes a unified theoretical framework connecting electromagnetic wave theory, classical field equations, and quantum mechanics through the concept of mutual energy flow. By rigorously analyzing the Helmholtz, wave, Klein-Gordon, and Schrödinger equations, we demonstrate that:
Energy Localization Mechanism: Traditional self-energy flow (Poynting vector/probability current) describes reactive power, while mutual energy flow forms particle-like energy channels via retarded-advanced wave synchronization.
Field Equation Revisions: A corrected magnetic field definition and halved mutual energy flux are required to derive energy-conserving solutions, resolving classical radiation paradoxes.
Wave-Particle Unification: Particles emerge as quantized mutual energy flows, bridging Maxwell's and Schrödinger's equations while aligning with absorber theory and transactional interpretation.
Key Implications: Challenges classical field theory's treatment of radiation reaction and proposes testable advanced wave signatures. Future work will focus on experimental validation and relativistic extensions.

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