## Research Article

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# Optimization of 3D Trajectory of the Vehicle with Dynamic Principle of Maintaining According to the Criterion of Minimum Average True Geometric Altitude 

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#### Abstract

The paper proposes to minimize the altitude of a low-flying vehicle near the waved sea surface due to the desire to lay a trajectory in smooth maneuvers mainly over the troughs of sea waves. The approach to the wave hollow is carried out by comparing the measurements of two radar altimeters and deviating the course by a certain amount in the direction of the altimeter, which showed higher altitude. The change in altitude is due to the property of self-stabilization of the vehicle and vertical maneuvers. The effectiveness of the proposed algorithm was evaluated by computer simulation of the movement of a light highly maneuverable vehicle under conditions of 6 sea state number. The simulation results showed the possibility of reducing the altitude from 3.0 to 2.35 meters and increasing the aerodynamic quality (lift-to-drag ratio) of the winged vehicle by $20 \%$. It was also found that the greatest efficiency of minimizing altitude is achieved with the reference heading of the vehicle perpendicular to the general direction of sea wave's propagation. The modeling did not take into account the influence of wind disturbances and accidental touches of the sea surface by the vehicle body, as well as the punching of the sea surface by air cushion. Consideration of these factors will be the direction of further research.


Keywords: Sea Waves, Optimization, Trajectory, Maneuvering, Ground Effect, True Geometric Altitude

## Introduction

Different types of transport use different physical principles to ensure their motion is in the required mode. Wheel based transport (cars, classic trains) is based on the use of wheels - one of the great inventions of mankind in the field of mechanics, which has existed for more than two thousand years. The design of displacement vessels is based on the laws of hydrostatics and hydrodynamics, the most significant of which are Pascal's law and Bernoulli's law. Air transport uses the laws of aerodynamics and very well-adapted physical principles implemented by theoretical and computational methods. Air cushion vessels (hovercraft) do not have direct contact with the underlying surface, but they are not air transport, as they are rigidly "tied" to the ground. The vehicles on using dynamic air cushion are less attached to the ground, amongst which a special place is occupied by ekranoplanes (WIG-craft).

Ekranoplanes use an interesting physical phenomenon - the ground effect, which consists in sharp increase in the aerodynamic
quality of the wing when it is lowered in flight to a height of less than $1 / 5$ of the wing chord. Naturally, when flying in the ground effect mode, a dynamic air cushion is formed between the wing and the surface, that can increase the lift and reduce drag, as a result of which the aerodynamic quality (lift/drag ratio) under ideal conditions of a flat underlying surface can approximately double, and with a real not quite even surface - approximately $20-40 \%$. Improving aerodynamic quality can easily be counted as an increase in payload or range of flight. It is important that ekranoplane can fly over any physical surface - water, land without trees and buildings, ice, snow, swamp.

The surface of water can be disturbed to a certain extent, depending on the size of the ekranoplane. The largest ekranoplanes built in Russia have had dimensions (wing span and length) of the order of 100 m and chord of the wing of about 30 m . It is clear that the altitude of its flight should not exceed 6 m , if we count the altitude in relation to the average undisturbed sea level. When flying at this altitude, even taking into account permissible errors of altitude stabilization, the permissible amplitude of sea waves can be $3-4 \mathrm{~m}$, which corresponds to sea waves of 6-7 points. This provides a very
good seaworthiness and the possibility of ocean applications. Unfortunately, small ekranoplanes do not possess such seaworthiness. When comparing with the displacement vessels.

The undoubted advantage of eranoplanes is their high speed of the order of 0.6 M (i.e., commensurate with aviation). The disadvantage of ekranoplane is the restriction on weather conditions, i.e. on the scale of sea waves, especially during take-off and landing.

Note that the problem of limiting the unevenness of the underlying surface is acute not only for ekranoplanes, but also for wheeled (automobile) transport. It is known that a poor highway or off-road force the driver to maneuver in a horizontal plane, avoiding obstacles (pits and potholes). In principle, the same problem of maneuvering when avoiding the largest obstacles should be considered for ekranoplane. For drivers or automotive drones, detouring high obstacles or deep holes is a long-mastered easily automated procedure that allows saving money on car suspension repairs and does not take into account some small extension of the path due to the replacement of a straight line with a slightly winding one. Consideration of a similar problem for the ekranoplane is also absolutely necessary, and the beginning of such a study is described later in this article.

The physical formulation of the problem is formulated as follows. The dependence of the aerodynamic quality of the ekranoplane on the geometric altitude of the flight above the waved sea is given. There is a mathematical model of ekranoplane dynamics. Mathematical model of sea waves as random anisotropic stationary field, is given. It is known that the current geometric height of two separated points of the hull is measured with precision positional altimeters, which allows to control the shape of the wave surface under the winged craft. It is required to:

- propose an algorithm for a small change in the ekranoplane's course in an attempt to approach the hollow of a large wave and move away from its crest, and as a result - reduce the average geometric altitude of the flight;
- assess the possibility and feasibility of light vertical and lateral maneuvers of ekranoplane of light vertical;
- offer the ekranoplane maneuver in order to enhance the effect of the ground effect and improve aerodynamic quality;
- evaluate the feasibility of introducing light maneuvers of an ekranoplane with the aim of reducing the average geometric altitude for large and small ekranoplanes.

Optimization of the 3D trajectory is relevant not only for ekranoplanes, but also for other types of vehicles with dynamic principle of support, for example, air boats and hovercraft. Minimizing the average altitude of the movement increases the likelihood of the vehicle body touching the crests of the sea waves. Rare, slight touches are permissible since the body of the vehicle must withstand them. Also, wave crests are partially squeezed out by an air cushion under the vehicle's body.

Driving at low altitude can give low-flying vehicles advantages in solving some problems. For example, when using the ground ef-
fect, reducing the altitude increases the aerodynamic quality of the vehicle, thereby increasing its carrying capacity. The disadvantage of driving at extremely low altitude is the high risk of collision with other objects or uneven ground surfaces. The body of all types of marine low- flying vehicles is capable of withstanding the "point" of water contact at high speed.

It was shown in $[5,6]$ that, using smooth lateral maneuvering, it is possible to accurately plot a trajectory over the troughs of low-frequency component of sea waves. The high-frequency component is slightly squeezed by air cushion, reducing the need for accounting. The ordinate spectrum of sea waves, which is under the trajectory of the vehicle, tending to move over the troughs of sea waves, is lower-frequency than the spectrum of the ordinates of sea waves under the vehicle, moving rectilinearly. Due to which, as a result of lateral maneuvering, the region of intersection of the ordinate spectrum of sea waves and the frequency spectrum of vertical maneuvering of the aircraft, increases. The increase in the intersection of the spectrum means that the low-flying vehicle can more accurately go around the ordinate of sea waves, trying to maintain a given safe distance to it. This article proposes a method for optimizing 3D trajectory of the vehicle with the dynamic principle of maintaining, according to the criterion of minimum, the average true geometric altitude. And also studies the effectiveness of reducing the altitude only due to vertical maneuvering and at the same time vertical and horizontal maneuvering. In the study of the effectiveness of reducing the average true geometric altitude of the vehicle due to only vertical maneuvering, sea waves were modeled by using shaping filter according to the known wave spectrum. In studying the efficiency of reducing the altitude due to both vertical and horizontal maneuvering, a three-dimensional irregular model of sea waves by V. Pearson was taken, which is by far the most accurate description of a waved sea surface.

The task of minimizing the absolute altitude can be represented by the task of stabilizing a given true geometric altitude $h$ of the vehicle relative to the ordinate of sea waves, mainly using its natural property of self-stabilization due to air cushion. Also, to increase the maneuverability of the vehicle and the quality of stabilization, it is allowed to use the elevator and flaps.

Up to date, the altitude of low-flying vehicles near the sea surface may be decreased only due to changes in their design and mode of flight. The novelty of this work lies in the proposal to minimize the altitude due to the desire to lay a path mainly over the troughs of sea waves. The features of low-altitude movement near the sea surface, which must be taken into account when laying the trajectory, are described in [7-13]. An analysis of the geometric characteristics of sea waves and the maneuvering characteristics of some modern small and medium low-flying vehicles showed the relevance of a deeper study of the possibility of striving to lay their trajectory over the low-periodic components of sea waves. Of particular importance for solving these problems is the self-stabilization of the ekranoplane. The principles of constructing the measuring equipment that allows with sufficient accuracy to evaluate the param-
eters of the vehicle movement under conditions of sea waves are described in $[14,15,12]$. In the simplest case, one can measure the altitude with a two-channel altimeter that integrates the readings of location altimeters and vertical accelerometers. Features, properties and methods of modeling sea waves are described in the book [10]. It is shown here that sea waves have quite powerful low-frequency components.

For laying the trajectory mainly over the hollows of the waves, it is necessary to perform almost continuous maneuvers, causing overload of the vehicle. If there are people on board the vehicle, it is necessary to use anti-loading suits. Ways to combat overload and their effect on the human body are described in [16-19, 13]. During a long flight in the wave envelope mode, the crew of the vehicle should be in anti-loading suits.

## Vertical maneuvering

It is known that the movement of the vehicle with the dynamic principle of maintenance can be described separately as longitudinal and as lateral [5]. Let us consider separately the longitudinal movement of the vehicle. To begin with, we assume that the vehicle moves with a constant track angle and horizontal velocity.

Using MATLAB-Simulink medium, the motion of a low-flying vehicle under conditions of irregular sea waves was simulated. As a result of the simulation, the trajectories were determined in the mode of minimizing altitude at any intensity of sea waves. After that, the average decrease in the absolute altitude was estimated due to the tendency to vertically bend around the ordinate of sea waves. The modeling did not take into account the influence of wind. At the initial stage of the study, this assumption is acceptable, since the purpose of the article is not to evaluate the effectiveness of minimizing the absolute altitude of a low-flying vehicle under real conditions of movement, but to show the possibility of significantly reducing the absolute altitude by going around the ordinates of sea waves mainly using the natural property of self-stabilization of a low-flying vehicle. Taking into account the influence of wind disturbances is a further area of research.

Sea waves were simulated using a fractional rational approximation of the spectrum of irregular sea waves. A polynomial with coefficients in the denominator up to the third order was taken as the transfer function of the low-flying vehicles. This model is acceptable because nonlinear effects in the formation of wind sea waves begin to act only with strong stormy waves, the score of which exceeds 6 . The transport vehicles considered in this article are not used in such a strong stormy sea. Some researchers consider nonlinear effects to be more significant even with a lower degree of excitement, but practically do not describe in detail the physics and mathematics of these effects, which complicates the modeling. Their results are based on statistical processing of experimental
data, which cannot be comprehensive and reflect the current properties of a general non-stationary stochastic field.

The reaction of low-flying vehicles, striving to maintain a constant true geometric altitude, to a change in the ordinate of sea waves has a certain delay. It is possible to reduce the delay by starting to perform vertical maneuvers in advance, evaluating the values of the wave ordinate at a certain distance $r$ in front of the vehicle. For this, the radiation pattern of the radio altimeter must be directed at a certain angle. The distance $r$ depends on the speed of the vehicles relative to the sea surface. In radiolocation, these issues were resolved back in the 60 s when trying to organize the stabilization of a ship by the principle of combined control by error and disturbance. Indignation - the incident wave was not just predicted for a dozen seconds, but was directly measured by radar with a beam tilted forward. Naturally, such a radar device was well stabilized to avoid the influence of pitching on the readings of the indicated incident wave sensor. The stabilizer is based on a combination of inertial and position sensors.

Let us assume that for a low-flying vehicle moving above sea disturbance of certain intensity, the average reaction delay of the vehicle within a given time interval is known. Knowing the ground speed of the vehicle, it is possible to determine the distance $r$, by means of which the sea surface ordinate should be estimated to minimize the control error created by the vehicle reaction delay. The sum of the sea surface ordinate ahead on course through the distance $r$ and the true given geometric altitude hgiv is called the desired absolute altitude of the vehicle through a period of time equal to the vehicle reaction delay. The vector direction from the vehicle's current absolute altitude to the desired absolute altitude by means of the distance $r$ is called the desired motion inclination. In order for the low-altitude vehicle to maintain a true given geometric altitude, it is proposed to continuously deflect the elevator by an angle that provides the desired path inclination. The proposed method of the true geometric altitude stabilization is not optimal by the criterion of the minimum control error since it is possible to increase the vertical maneuverability of the low-altitude vehicle by setting a greater elevator angle and gradually reducing it by the completion of the maneuver. This method improvement is a further challenge.

To assess the potential of the altitude minimization using the proposed method, modeling and simulation were performed in Simulink environment. The developed model can be presented in the form of four large units: simulation of sea disturbance, simulation of the measuring and control system of the low-altitude vehicle (LAV), simulation of the LAV dynamic motion, evaluation of the altitude minimization effectiveness. An enlarged block diagram of the simulation system for the motion of the low-altitude vehicle seeking to round the low-frequency components of the sea disturbance, is shown in Figure 1.

Figure 1: Generalized block diagram of the simulation system for the motion of the low-altitude vehicle seeking to round the low-frequency components of the sea disturbance using vertical maneuvering.


The sea disturbance simulation unit is a white noise $\mathrm{w}(\mathrm{t})$ generator that transmits a signal to the shaping filter. The output of the shaping filter gives the predicted ordinate value of sea disturbance $\xi(t)$. The specified parameter of this unit is the three percent supportability value $h_{3} \%$ of sea disturbance. This value is used to calculate the transfer function parameters of the shaping filter $\mathrm{H}_{\mathrm{v}}(\mathrm{s})$. The estimated value of the sea disturbance ordinate $\xi(t)$ is transmitted to the simulation unit input of the measuring and control system, in which the elevator angle $\delta_{a}(t)$ is calculated and then transmitted to the LAV motion simulation unit input. Motion simulation unit LAV generates the time dependence of the absolute altitude $\mathrm{habs}(\mathrm{t})$ of the LAV seeking to round sea disturbance and determines to what extent the average value of the vehicle absolute altitude has decreased. A more detailed block diagram of the low-altitude vehicle motion simulation system is shown in Figure 3. Parameters evaluation units for the shaping filter and the LAV dynamic simulation unit designate here as parameter calculation units PCU1 and PCU2, respectively, and M is an array consisting of elements,

$$
M_{Z 0}^{\omega_{z}}, M_{Z 0}^{\alpha}, M_{Z 0}^{\delta_{0}}, Y_{0}^{\alpha}
$$

Figure 2: Block diagram of the low-altitude vehicle motion simulation system.


The wave ordinate dispersion $D_{r}$, the correlation function parameter $\beta$ and the attenuation coefficient $\alpha$ are calculated. For arisen wind disturbance, the ratio $\alpha=0,21 \beta$ is fulfilled. All these parameters are necessary to determine the spectrum of sea disturbance, from which the transfer function of the shaping filter is determined by factorization on complex-conjugate multipliers. The wave ordinate dispersion $D_{\mathrm{r}}$ is related to the sea disturbance altitude of three percent probability $h_{3} \%$ by the formula $\operatorname{Dr}=0,0358 h_{3}^{2} \%$

Sea disturbance is described by exponential and rational spectra. Only rational spectra can be used for modeling of the sea surface
using the shaping filter. They are slightly shifted relative to the exponential ones into the low-frequency region. The following spectrum was used in the work [10].

$$
\begin{equation*}
S_{h}(\Omega)=\frac{4 D_{r} \alpha \sigma^{2}}{\sigma^{4}+2\left(\alpha^{2}-\beta^{2}\right) \sigma^{2}+\left(\alpha^{2}+\beta^{2}\right)^{2}} \tag{1}
\end{equation*}
$$

with $\Omega$ as the sea disturbance spatial frequency. The frequency of spectrum maximum $\Omega_{m}$ is determined by the ratio $\Omega_{m}=1,42 / \sqrt{h_{3 \%}}$ and almost coincides with $\beta$, since $\Omega_{m}=\sqrt{\alpha^{2}+\beta^{2}}=\beta \sqrt{1+(\alpha / \beta)^{2}}=1,02 \beta$. The shaping filter for a signal with such spectrum has the transfer function.

$$
\begin{equation*}
H(s)=\frac{2 \sqrt{\alpha D_{r} s}}{s^{2}+2 \alpha s+\left(\alpha^{2}+\beta^{2}\right)} \tag{2}
\end{equation*}
$$

Next, the difference between the desired absolute altitude by means of distance $r$ and its current absolute altitude is calculated. Since the desired absolute altitude is the sum of the estimated wave altitude and the given value of the true geometric altitude, the formula takes the following form: $\Delta h_{a b s}(t)=\xi(t)+h_{g i v}-h_{a b s}(t)$. After that, the formula $\theta=a \tan 2\left(\Delta h_{a b s}, r\right)$ determines the desired path inclination. The ratio of $\Delta h_{a b s}(t)$ to $r$ is the tangent of the desired path inclination. Elevator angle $\delta_{a}$ is calculated on the basis of (3) subject to calculated $\theta$. it is assumed that the $\delta$ a linearly depends on the $\Delta \theta$. The stabilization principle of the true geometric altitude of the vehicle due to vertical manoeuvring is explained in Figure 2.

The function $\operatorname{atan} 2(x, y)$ is similar to the arctangent of $x / y$ except that it has no discontinuities and is suitable for calculating the angle between the opposite leg of x and the adjacent leg of $y$. The function name is borrowed from the corresponding function in the programming languages $\mathrm{C}++$ and MATLAB.

Figure 3: Stabilization principle of the true geometric altitude of the vehicle due to vertical manoeuvring.


Further, the path inclination is determined by the transfer function (3) of path inclination changing relative to the control action. After that, the absolute altitude at the next moment of time is calculated by the known velocity modulus and the path inclination.

The imaged equations of the aircraft longitudinal motion have the form
$S^{2} \theta(S)+a_{1} s \theta(s)+a_{2} \alpha(s)=-a_{3} \delta_{a}(s)=a_{4} M_{B Z}(S)-S \alpha(S)+a_{5} \alpha(s)+S \theta(S)=a_{6} F-$ ${ }_{B Y}(S)$

Let's exclude the variable $\alpha(s)$ from the equation and get the path inclination angle transfer function on the elevator angle

$$
\begin{equation*}
H_{\theta}^{\delta_{a}}(s)=\frac{a_{3} a_{5}}{s\left[s^{2}+\left(a_{1}-a_{5}\right) s+\left(a_{2}-a_{1} a_{5}\right)\right]} \tag{3}
\end{equation*}
$$

$a_{5}=Y_{0}{ }^{\alpha} / m V_{0}, a 4$ as specified coefficient, $\alpha(\mathrm{s})$ as the angle of attack Laplace transformation, $M_{B Z}(s)$ - disturbance torque, $F_{B Y}(s)$ - disturbance force.

Here $I_{z}$ is vehicle inertia relative to the axis $\mathrm{OZ}, M_{\mathrm{ZO}}^{\omega_{\mathrm{z}}}$ - a variation of pitching moment due to pitching $\mathrm{M}_{\mathrm{zo}}^{\alpha}$ - static longitudinal stability, $M_{z O}^{\delta_{B}}$-control torque appearing in case of the elevator deflection of the horizontal tail, $Y_{0}{ }^{\alpha}$ - vehicle lift force in case of the undisturbed motion, $m$ - vehicle mass, $V 0$ - vehicle speed.

Reference to the formula (3) shows that it is necessary to know the vehicle inertial characteristics for obtaining $\mathrm{H}_{\theta^{B}}^{\left.\delta^{( }\right)}$.

During the entire simulation time, the obtained values of the sea disturbance ordinate, and the absolute altitude of the low-altitude vehicle are stored in the memory. Upon completion of the simulation, the reduction value of the absolute altitude average value is calculated by using the proposed method. Also, the efficiency evaluation unit can use other criteria for evaluating the method effectiveness, for example, increasing the aerodynamic quality of the vehicle using ground effect.

Two-dimensional images of sea disturbance were obtained by simulation in Simulink.

For the 4-point sea disturbance $h_{3 \%}=2 \mathrm{~m}$, whence it follows that Dr=0.143 $\mathrm{m}^{2}, \beta=1.004 \mathrm{~s}^{-1}, \alpha=0.210 \mathrm{~s}^{-1}$.

$$
\begin{equation*}
H(s)=\frac{b_{0} s}{s^{2}+a_{1} s+a_{2}} \tag{4}
\end{equation*}
$$

For the 6-point sea disturbance $h_{3 \%}=6 \mathrm{~m}$, whence it follows that $D_{r}=1.289 m_{2^{\prime}}, \beta=0.5683 \mathrm{~s}^{-1}, \alpha=0.1193 \mathrm{~s}^{-1}$, with $b_{0}=0.7843 \mathrm{~s}^{-1} ; a_{1}=$ $0.2386 \mathrm{~s}-1 ; a_{2}=0.3372 \mathrm{~s}^{-2}$.

The transfer function (3) coefficients took the following values: $a_{1}=$ $1.154 \mathrm{~s}^{-1} ; a_{2}=1.40 \mathrm{~s}^{-2} ; a_{3}=0.427 \mathrm{~s}^{-2} ; a_{5}=0.415 \mathrm{~s}^{-1}$. These values were determined based on the average inertial characteristics of small and medium sized low-altitude vehicles, taken from open sources. Some characteristics are given in $[7,12,13,20]$. A fragment of the optimized trajectory during the flight under 6-point sea disturbance is shown in Figure 4.

Figure 4: Fragment of the vehicle motion path (dotted line) and 6 -point sea disturbance (solid line).


In rectilinear motion, the true average geometric altitude is 3 m ; when moving in the altitude minimization mode with zero reaction delay, the average true geometric altitude was equal to 2.35 m .

Since in solving the problem of the true geometric altitude stabilization, the vehicle is forced to perform intensive manoeuvres almost continuously, it is important to assess the overload $n y=V \theta^{\prime} / g$. The acceleration response transfer function for the path inclination changing has the following easy form

$$
\begin{equation*}
H_{n_{y}}^{\delta_{a}}(s)=-\frac{V s}{g} H_{\theta}^{\delta_{a}}(s) \tag{5}
\end{equation*}
$$

A fragment of the time dependence of vehicle overload is shown in Figure 5.

Figure 5: Fragment of time dependence of vehicle overload


In the first 300 seconds, the maximum overload modulus has achieved 0.15 g at 130th second of flight. Throughout the flight, the overload did not exceed 0.16 g . It is indicated that with weak turbulence, aircraft passengers experience an overload of $0.8-1.2 \mathrm{~g}$, and under storm, overload exceeds the value of 2 g . The most unpleasant and annoying is the effect on any person of low-frequency overloads with a frequency of $0.1-0.5 \mathrm{~Hz}$ given in [21]. But due to the low magnitude, the overload resulting from vertical maneuvering to minimize altitude the average geometrical altitude of flight is acceptable. In addition, it is possible to safeguard the crew and passengers from overloads using anti-overload protection, the method of operation of which is described in details in $[7,11,15,16,19,21]$.

Horizontal maneuvering
After studying the effectiveness of minimizing the altitude by maneuvering in the vertical plane, let us separately consider maneuvering in the horizontal plane, evaluating the effectiveness of the tendency to lay the trajectory over the troughs of sea waves so that the ordinate of sea waves under the fuselage of the vehicle is as small as possible. In parallel with the laying of the trajectory mainly over the hollows of the sea waves, let us reduce the altitude of the vehicle due to vertical maneuvering as in the previous section of the article.
For horizontal maneuvering, the vehicle heading $\gamma$ is determined by the direction of the smallest gradient of height of the sea waves in the sector $\left[\gamma_{\text {min }}, \gamma_{\text {max }}\right]$.

$$
\begin{equation*}
\gamma(t)=\min \nabla \xi(t, x, y) \text {, at } \gamma_{\min }(t)<\gamma(t)<\gamma_{\min }(t)(6) \tag{6}
\end{equation*}
$$

where $\xi(\mathrm{x}, \mathrm{y})$ is height of the sea surface at the point with coordinates ( $\mathrm{x}, \mathrm{y}$ ).

The sector [ymin, $\gamma \max$ ] depends on velocity, distance to the endpoint, the vehicle's path angle, wave height and has a center directed to the endpoint. As the vehicle approaches the endpoint, the sector $\left[\gamma_{\text {min }}, \gamma_{\max }\right]$ narrows to ensure that the vehicles arrive at the desired point.

To determine the minimum gradient of the underlying surface in a limited sector, at least two point altimeters must be used. When the difference in measured altitudes is greater than a predetermined threshold, the vehicle's course changes by a fixed angle in the direction of the meter that showed the maximum true altitude, since a large height means proximity to the hollow. For example, we can place two altimeters located in the left and right halves of the wing; if the altitude is exceeded, the heading of the vehicle increases by a fixed value towards the right altimeter, and decreases towards left altimeter. A diagram illustrates the algorithm for calculating the deviation of the rudder depending on the readings of the left and right radio altimeters, is shown in Figure 6.

Figure 6: Diagram showing the dependence of the rudder angle on the readings of the left and right radio altimeters


$$
\operatorname{sector}\left(\gamma, \gamma_{\min }, \gamma_{\max }\right)=\left\{\begin{array}{l}
\gamma=\gamma_{\max }, \quad \text { if } \gamma>\gamma_{\max }  \tag{7}\\
\gamma=\gamma, \quad \text { if } \gamma_{\min }<\gamma<\gamma_{\max } \\
\gamma=\gamma_{\min }, \quad \text { if } \gamma<\gamma_{\min }
\end{array}\right.
$$

The rudder deflection angle $\Delta \varphi$ is calculated by the following algorithm:

1. Find the difference in the readings of radio altimeters $\Delta \mathrm{h}$
2. Compare the difference modulus $|\Delta h|$ with a threshold $\mathrm{h}^{*}$ to determine a significant difference between the measured altitudes
3. Determine the desired direction of change in the course of $\Delta \gamma$, which increases the likelihood of a vehicle entering the sea wave hollow (if the altitude is greater under the left altimeter hl , then the rotation is carried out by -D grad, if the height is greater under the right altimeter hr , then the rotation is carried out by D grad)
4. To carry out the correction $\Delta \gamma$ taking into account the boundaries of the sector of permitted traffic. If $\Delta \gamma$ goes beyond the
boundaries, then take $\Delta \gamma$ so that the corrected direction of movement $\gamma+\Delta \gamma$ coincides with the nearest boundary of the sector of permitted movement
5. Using the given coefficient $K$ and $\Delta \gamma$, determine the angle of deviation of the rudder $\varphi$.

The transfer function of the dependence of the steering angle $\gamma$ of the vehicle on the angle of rotation of the rudder $\delta \mathrm{d}$ has the following form

$$
\begin{equation*}
W_{\gamma}^{\delta_{d}}=\frac{k}{s\left[T^{2} s^{2}+2 \zeta T s+1\right]} \tag{8}
\end{equation*}
$$

where $k$ is the scale factor, $T$ is the time constant of the vehicle, $\zeta$ is the damping coefficient.

Sea waves were modeled according to the following formula [8]

$$
\begin{equation*}
\xi(t, x, y)=\sum_{i=1}^{n} \sum_{j=1}^{n} r_{i y} \cos \left(\Omega_{i} t-k_{i} x_{0} \cos \chi_{j}-k_{i} y_{0} \sin \chi_{j}+\varepsilon_{y}\right) \tag{9}
\end{equation*}
$$

where $r_{i j}$ is the amplitude of the wave with the $i$-th frequency and the $j$-th direction of propagation; $n$ is the number of harmonics with different frequencies; $m$ is the number of harmonic waves with different directions of propagation; $\Omega_{\mathrm{i}}$ is the circular frequen$\mathrm{cy} ; k_{\mathrm{i}}$ is the frequency of the form; $\chi_{j}$ is the direction of propagation of the elementary wave relative to the direction of the wind and the axis $0 x_{0} ; \varepsilon_{i j}$ - uniformly distributed initial phase in the range $[0,2 \pi)$.

$$
r_{i j}=\sqrt{2 \pi^{-1} S_{y}\left(\Omega_{i}, \chi\right) \Delta \Omega \Delta \chi}
$$

$\mathrm{S}(\Omega, \chi)$ is a function of the distribution of sea waves power in the frequencies and directions of propagation, $\Delta \Omega$ is discrete in the circular frequency, functions $S(\Omega, \chi) ; \Delta \chi$ is discrete in the direction of propagation of the elementary wave, $\mathrm{ki}=\Omega_{i}^{2} / g$.

A fragment of optimized three-dimensional trajectory of the vehicle with dynamic principle of support is shown in Figure 7. Points $A$ and $B$ denote the start and endpoints of the path fragment.

Figure 7: A fragment of optimized 3D trajectory of the vehicle with a dynamic principle of maintenance


With 6-point sea waves, due to the desire to lay the trajectory of the vehicle mainly over the troughs of sea waves with the help of lateral maneuvering, it was possible to reduce the ordinate of the sea surface under the hull by $6-14 \%$, depending on the direction of motion. The greatest minimization of ordinate is achieved with the reference track angle of the vehicle perpendicular to the direction of wave propagation. When optimizing the 3D trajectory due to maneuvering simultaneously in horizontal and vertical planes, the average altitude of the vehicle was reduced from 3 m to 2.17 m , which is 0.18 m more than only during vertical maneuvering. The simulation results show that this can increase the aerodynamic quality of the vehicle with dynamic principle of maintenance by almost $20 \%$. To assess the increase in aerodynamic quality, the formula was used [8].

$$
\begin{equation*}
K / K_{\infty}=1+b / 25 h \text { at } b \geq 0.03 \tag{10}
\end{equation*}
$$

where $b$ is the wing chord, $m$.
Due to poor knowledge of the influence of disturbing influences on a maneuvering low-flying vehicle near the sea surface with a dynamic principle of support, they were not taken into account in
the simulation. Evaluation of the effectiveness of 3D path optimization taking into account disturbing influences is a further area of our research.

## Conclusions

The article proposes a method for optimizing the 3D trajectory of the vehicle with dynamic principle of maintaining, according to the criterion, the minimum average true geometric altitude. Minimization of altitude is carried out by the desire to plot a trajectory in the direction of the hollows of sea waves and the desire to withstand the true reference geometric altitude of the vehicle. The simulation results have showed the ability to reduce the altitude of the vehicle from 3.0 to 2.35 m when maneuvering only in the vertical plane, and to 2.18 m when maneuvering simultaneously in both vertical and horizontal planes. In the latter case, a decrease in altitude increases the average aerodynamic quality of the vehicle by almost $20 \%$. The next step of the exploration is to study the effect on the optimization of disturbing influences, such as wind, hull touches of the sea surface as a result of maneuvering, etc.

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