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On the Efficacy of Average Weightings Predicting Joint Choices in Bilateral Negotiations with Attribute Tradeoffs

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Abstract

Previous research suggests negotiators demonstrate a predisposition toward weight averaging or the averaging of attribute values. One outcome is that weight averaging generates a maximizing joint choice using a Multi-Attribute Utility of joint preference or 'utility' over known attribute values. Thus, attribute weightings are a fixed variable in the MAU model and determine maximum joint preference over a joint negotiation set with uncorrelated attribute values. We test an alternative form in a simulated experiment predicting joint choice when there are strict trade-offs in the negotiation set and negotiators average their intrinsic weightings paramorphic to a multi -attribute model. We show that joint weights by averaging, or negotiation must be equal (a 50/50 weighing of the two attributes) to reflect maximum joint preference or utility. Consistent with prior research, results find that a relative 50/50 weighing of the two attributes is dependent both on the symmetry or asymmetry of negotiators' preference functions for the two attributes and possible cognitive conflict inherent to negotiators intrinsic individual weightings. For instance, negotiators' individual weightings (.2,.8) vs (.8,.2) that result in average weights of (.5,.5) involve less cognitive conflict than negotiating weightings of (.7,.3) vs (.1,.9) to achieve joint weights of (.5,.5). The paper concludes by summarizing the primary results of the simulation related to extant research and presents possible modeling for negotiations over three attributes of which at least two attributes are strictly tradeoffs.

Keywords: Neurodidactics, Neuroeducation, Ecological Intervention, Pedagogical Practices, Teaching

Introduction

Research into marketing negotiations has an extensive history in family, sales, distribution, and product decisions [1-3]. The research presented here represents these kinds of bilateral negotiations but over *attributes with tradeoffs* expressed by each negotiator's preferences for attributes X and Y written as ν(x-),ν(y). Although such a mapping of attribute values to utility or preference is unique to the individual, research tells us that an overall joint evaluation is also a function of the shape of joint preferences [4]. Specifically, we present a model whereby the members of a negotiating dyad place inverse salience on the attributes X or Y and that each party concedes to the preferences of their opponent's more salient attribute. For instance, if negotiator 1 values X≻Y and negotiator 2 values Y≻X, each concedes to the other's more salient preference, ν(y) by negotiator 1 and ν(x) by negotiator 2. Negotiators also hold either positive linear or nonlinear (concave) preference functions in their more salient attribute thus creating four preference frontiers given by f[ν(x-),ν(y)]. Mathematically, we show the maximizing joint choice is where the pair negotiates equal weightings for the two attributes paramorphic to a Weighted Linear Model (WLM) or a Multi-Attribute Utility (MAU) model. Historically, research using the WLM or MAU show that partners demonstrate a predisposition toward weight averaging or the averaging of attribute values [5, 6]. The question arises: How efficient is averaging in predicting joint choice when there are trade-offs of attributes X and Y? We test this research question by a simulated experiment designed to predict joint choice using two solution concepts based on mathematical optimality (e.g., equal weighting) or a revision in joint preferences (e.g., unequal weighting). However, we also argue how this is an issue of 'egg vs. chicken.' In other words, does a negotiated choice simply reflect joint attribute weightings as scaling constants per Keeney and Raiffa, or do negotiators employ a cognitive weighting scheme to then decide on a joint choice? [7].

Background

Using some elementary modeling of individual (one person) decision making [7, 8]: (1) A value function in the natural scales of two attributes is $f(x, y) = x + \lambda y$ and (2) in preference space, $v(x, y) = v_x + \lambda v_y$, where $v_x = f(x)$ and $v_y = f(y)$ are value or preference functions on the interval $(0 \le v(\cdot) \le 1.0)$. The basic 'short-hand' model used in this paper is $p_j = w_x(v_x) + w_y(v_y)$; where w_x and w_y are joint subjective weightings, cognitively interpreted as the salience (importance) a negotiating pair assigns to attributes *X* and *X*. Negotiators then find their maximum joint preference (or 'utility), p_j, by applying their joint weights to preferences v_x and v_y.

This two-attribute negotiation describes a tradeoff between preference values ν_x and ν_y under certainty, implying negotiators use cardinal or interpersonal comparisons [7, 9]. The additive model is feasible if (1) preferences (ν_x) for X and preferences (ν_x) for Y are independent and (2) a difference in preference between two values of X is invariant to values of Y. Negotiators find their maximum joint preference (or 'utility), p_j, by applying their joint weights to preferences ν_x and ν_y. Under the independence assumption for values assigned to options in the choice set, this is a 'baseline' model of multiattribute choice where "• •weights capture the subjective importance of the attributes and values [of resolution] depend on attribute specific utility functions" [10]. Similarly, in axiomatic decision theory the baseline model is a "preference aggregation rule" derived from a multiplicative model of group preference [11]. Bhatia et al. point out that such multiattribute models are more common to psychology and business than economics where the emphasis has been on decision models of risky and intertemporal choice.

Joint Weights for Preferences $v(x)$, $v(y)$

Averaging versus adding strategies is a common topic in the historical literature on multi- attribute decision making in psychology and marketing [12, 13]. For example, Acikgoz in a study of employment offers, found that subjects were more prone to averaging attribute values when evaluating a single offer but more prone to adding attribute values when there were multiple offers [14]. In a two-attribute environment, averaging is the equivalent of assigning equal weights to each attribute, so that in normalized form, $w_x = (1 - w_y) = .5$ Alternatively, if attribute weights are unequal, researchers then deem a decision maker (DM) to be using an additive (weighted) process. Moreover, experimental research on two-party decisions show that negotiators can effectively recognize and/or learn intrinsic, subjective weightings (see also a summary of empirical studies in Dhami and Olsson) and that their joint weightings reflect an averaging strategy short of capitulation to the other party [15-18]. When two negotiators have opposite preference orderings for attributes as in $X \succ Y$ and $Y \succ X$, a negotiation determining joint weights would thus be: w_x vs $(1 - w_y)$ or w_y vs $(1 - w_x)$; where w_x is the weight assigned by the negotiator whose more important attribute is X, and w_y is the weight given by the negotiator whose more important attribute is Y. This is a logical negotiation given negotiators preferential differences for X or Y as outlined in section 1 above. For instance, assume the two negotiators have respectively subjective weights of (.6, .4) and (.2, .8) for (X, Y). Their joint weightings by averaging are then (.4, .6) and their joint preference (or utility) for option j is $p_j =$
$.4\nu_x + .6\nu_y$.

Choice Set Over Preferences $v(x)$, $v(y)$

We define a choice set of options that correspond to the efficient frontier or Pareto set (P) for preferences ν_x and ν_y by measuring preferences of each party only for their most important attribute. This generates a joint distribution of corresponding preferences for the two attributes (X, Y) by each negotiator conceding to the preference values of their opponent's more important attribute so that $\nu_x = f(x)$ are the X preferences of one negotiator and $\nu_y = f(y)$ are the Y preferences of the other negotiator [19]. This is essentially a single stage log rolling procedure for two issues (attributes) that describe as a negotiated policy resulting in a single utility model [20].

The parameters of such preference frontiers are difficult to determine, but we do know they are concave downward if $\nu_x = f(x)$ and $\nu_y = f(y)$ are positive linear or positive concave. Figures 1 through 4 in section 3.3 below, show four such preference frontiers for store options rated by Consumer Reports on the attributes price and selection. Each frontier represents negotiators holding respective linear (L) or nonlinear (N) preferences for price and selection. Appendix A presents derivations for the four preference frontiers examined in this paper when $\nu_y = f(\nu_x)$ is unknown

Maximizing Joint Preference

For the simulated experiment subsequently described in section 3, we define an optimal joint choice where that choice maximizes $p_j = w_x(\nu_x) + w_y(\nu_y)$ over options on the efficient frontier. We present contrasting perspectives on how two negotiators might maximize their joint preference or satisfaction. The first stipulates joint maximization occurs where the optimal choice has slope $\lambda = -1$ at preference values (ν_x, ν_y), indicating an equal tradeoff of preferences for the specific attribute values (x, y). The second perspective stipulates that maximization of p_j arises from an a' priori set of negotiated or average weightings. In other words, the joint weights w_x and w_y are constants in maximizing p_j. Curry and Menasco mathematically made this argument using a symmetric frontier (e.g., the positive quadrant, quarter-circle), such that for any set of fixed joint weightings the maximization of p_j yields values of (x, y) that lie on the quarter circle [5]. A distinction here is that this maximization is not equivalent to a maximization based on equal tradeoffs unless weightings are (.5, .5). The experimental focus of this paper is on the former under the assumption negotiators wish an equal tradeoff between preferences for X and Y.

Appendix A derives the value, $m=-w/(1 - w)$, for a Pareto frontier given by $y=a+b_1 x-b_2 x^2$ by specifying the multi-attribute function:

$$p_j = w \cdot x + (1 - w) \cdot y \quad (1)$$

where the generic weight, w, is the joint weight for attribute X. Maximizing and solving for x when $m = -1$ yields the point (x, y) on P where $\lambda = -1$. In other words, a choice option (if available) is optimal at (x, y) and joint weights must be $w_x = w_y = .5$. This procedure is the same as maximizing $y = f(x)$ without consideration of joint weightings. However, as noted above, preference frontiers are difficult to identify mathematically for $P \equiv \nu_y = f(\nu_x)$; thus, preventing a straightforward maximization. By substituting $\nu_x = f(x)$ and $\nu_y = f(y)$ into (1) above:

$$p_j = w[\nu_x = f(x)] + (1 - w)[(\nu_y = f(y)] \quad (2)$$

where $y = a + b_1x - b_2x^2$ in this case. Maximizing eq. (2) and solving for (x, y) when $m = -1$ yields the values of ν_x and ν_y using the known value functions $\nu_x = f(x)$ and $\nu_y = f(y)$. This is the maximum joint preference for attributes (X, Y) requiring joint weights, $w_x = w_y = .5$.[1]

Alternatively, maximizing p_j given fixed weights could generate a point outside the positive quadrant. For example, with an asymmetric frontier as in Appendix A, if joint weights are $w_x = .7$ and $w_y = .3$, then $m = -2.3$ and solving for x in attribute or in linear preference space gives x = 62.5 and y = −15.2. This solution does not exist within the limits of the Pareto frontier in the positive quadrant, 10 < (x, y) > 50. As a practical matter, the joint weights (.7, .3) would predict a local maximization where (ν_x, ν_y) maximizes p_j over j discrete choices in S. We show such predictions in the experimental sections using a simple logit model. For example using Figures 1 through 4 as a template, let S be the subset of discrete choices $S = \{(\nu_x, \nu_y)^{(1)}, (\nu_x, \nu_y)^{(2)}, \cdots, (\nu_x, \nu_y)^{(J)}\}$ along the efficient frontier ($S \in P$). An idyllically rational negotiating pair would jointly choose the option where (ν_x, ν_y) most closely approximates $\lambda = -1$. This is largely an impossible task for even sophisticated negotiators short of sketching the frontier and then making their 'best guess.' Paramorphic to equation 2 above, negotiators instead select the option with preference values (ν_x, ν_y) that maximize p_j; thereby, approximating an equal tradeoff between unit preferences.

Preference Concessions

The *compromise effect* is the tendency of individuals or groups to choose a midpoint or intermediate option. Several studies in marketing of individual consumer choice report the effect, and it is also prevalent from studies in household negotiations [1, 21-24].

As noted above, an optimizing compromise requires an equal joint weighting of attributes or preferences, but it is also true that any joint choice imposes a joint weighting for that choice. A plausible alternative is that negotiators agree on a jointly preferred option by simply acknowledging the others' preferences for their preferred attribute as in section 2.2. Thus, outcomes of negotiators' concessions of preferences might be quite different than for predictions by attribute weightings. We describe a simple gain and loss model based on interpersonal utility comparisons and captured by Prospect Theory to account for preference concessions [25-27]. Letting X be the preferred attribute for negotiator 1 and Y the preferred attribute for negotiator 2, we define equal gains (*g*) or losses (*l*) in each negotiator's preferred attribute as

$$g = (\nu_x - \nu_x^{min}) = (\nu_y - \nu_y^{min}) \text{ and} \quad (3)$$

$$l = (\nu_x^{max} - \nu_x) = (\nu_y^{max} - \nu_y). \quad (4)$$

Minimum attainable preferences $(\nu^{min}_x, \nu^{min}_y)$ or maximally attainable preferences $(\nu^{max}_x, \nu^{max}_y)$ are endpoints on the continuous frontier, P. Negotiators jointly select a choice such that they each gain or lose an equal amount of 'utility' g or l in their more important attribute X or Y. Practically, a discrete choice from $S \in P$ with negotiated preferences (ν_x, ν_y) approximates an equal gain (g) or equal loss(l). Appendix B presents a derivation of equations (3) and (4) above for $\nu_x = f(x)$ and $\nu_y = f(y)$.

A Simulated Experiment

The scenario is a household choice of store from which to purchase electronic products (audio, video, security, etc.) based on evaluations by Consumer Reports (CR) for prices and product selection of eighteen regional or national stores [28]. The CR store ratings reflect both a physical and online presence. Any two household members such as spouses or parent and child would constitute a bilateral negotiation. The choice set consists of seven nondominated stores amongst the eighteen evaluated by Consumer Reports.

We construct four experimental conditions analogous to the interpersonal conflict (IPC) paradigm based on the average importance (weights) household negotiators assign to each of the store attributes. Each condition simulates how strongly a negotiator weights their more important attribute based on their bias toward that attribute. The bias is either strong or weak and assumed exogenous to the negotiation. We then use average weightings within each of the four cognitive conflict conditions to predict optimal and concession store choices (section 3.6) over four preference frontiers constructed in sections 3.2 and 3.3. Section 3.7 presents two basic research questions: (1) How well does weight averaging predict Pareto choices versus concession choices under the four conflict schemes? and (2) How does the shape of the preference frontier influence these predictions?

Sample Weights

We simulated how strongly a negotiator subjectively weights their more important attribute based on their bias toward that attribute. The bias is either strong or weak and assumed exogenous to the negotiation.2 We used an on-line algorithm (Mersenne Twister) to randomly sample from four populations of size n = 50 (N = 200). Sample values from the four populations varied on the interval, $0 \leq x \leq .5$, so that an observation's calculated weight is $w = x + .5$. Two samples (**P1, S1**) skew to the left (negative), and two (P2, S2) skews to the right (positive). Table 1 shows the statistical results for the four samples. **P1** and **S1** represent populations that statistically show greater importance for price (**P1**) or selection (**S1**); alternatively, **P2** and **S2** represent populations displaying lesser importance for price and selection. Sample sizes of 50 would be reasonable in any experiment of this sort based on the effective costs of sampling and conducting experiments.

Regardless of whether individuals hold exogenous weightings or context-induced weightings, we would expect variation in sampling as in any realistic research [29]. We express the 'robustness' of skewness in each sample as the percent of weightings occurring relative to a cutoff of w = .7. Eighty percent of both **P1** and **S1** sample weights are robust at $w_i \geq .7$, while 76% and 68% are robust at $w_i \leq .7$ for **P2** and **S2**, respectively.

Table 1: Statistics for Four Distributions of Weights Skewed Negative or Positive

	P1	S1	P2	S2
N	50	50	50	50
Mean	.852	.852	.639	.667
Median	.910	.890	.590	.650
Std. dev.	.146	.141	.137	.152
Skewness	-.731	-.921	1.141	.624

(P1, S1): price and selection negatively skewed
(P2, S2): price and selection positively skewed.

Average Weightings

Using the 50 observations from each of the four samples reported in Table 1, we created four experimental conditions based on the statistical skewness of weights for price and selection: (*L*) sample price and selection weights skewed left or negative (**P1** and **S1**); (*R*) sample price and selection weights skewed right or positive (**P2** and **S2**); (*LR*) price-weights skewed left, and selection-weights skewed right (**P1** and **S2**); and (*RL*) price-weights skewed right, and selection- weights skewed left (**P2** and **S1**). We calculated the average weight for price (w_p) in each experimental condition as the baseline dependent measure by pairing the respective weights for price and selection in their numerical sampling orders (i = 1,. . . ,50). Thus, we created 50 faux negotiations for each experimental condition (*L, R, LR and RL*). The averaging computations are $[\boldsymbol{P1} + (1 - \boldsymbol{S1})] \div 2$ for *L*; $\boldsymbol{P2} + (1 - \boldsymbol{S2})$ for *R*; $\boldsymbol{P1} + (1 - \boldsymbol{S2})$ for *LR*; and $\boldsymbol{P2} + (1 - \boldsymbol{S1})$ for *RL*. The average joint weights (rounded) for price under each skewness condition are $\overline{w}_p$ =.50 for *L*, $\overline{w}_p$ = .49 for *R*, $\overline{w}_p$ = .59 for *LR* and $\overline{w}_p$ = .39 for *RL*.

From footnote 1, $m = -\frac{w}{1-w}$ values of w (in this case, w_p is also the slope lambda (λ) of the joint indifference curves given Thus, $\lambda = -\frac{w_P}{1-w_P}$ should reflect values near 1.0 for average weights under conditions *L* and *R*, greater than 1.0 under *LR*, and less than 1.0 under *RL*. The median lambdas confirm these expected tradeoffs over the four skewed weight distributions:

L	R	LR	RL
1.00	.95	1.58	.60

A dyad's tradeoff favors neither price nor selection under *L* and *R*, while favoring price under *LR* and selection under *RL*.

Cognitive Conflict

In the context of literature on the impact of cognitive conflict on decision making, Table 2 abstracts two typologies of cognitive conflict derived from K. R. Hammond's Interpersonal Conflict (IPC) Paradigm with *L* and *R* as examples of *symmetric* conflict and *LR* and *R*L as examples of *asymmetric* conflict [30]. Previous experimental research using the IPC has shown symmetric conflict to be a more efficient predictor of 'desirable' outcomes than asymmetric conflict [15, 16, 31]. In our case, a desirable outcome is an optimal Pareto joint choice predicted by joint weights of (.5, .5) under the hypothesis $E(\tilde{w}_p) = E(\tilde{w}_S)$. Both L and R satisfy this expectation, whereas *LR* and *RL* are examples of asymmetric conflict given $E(\tilde{w}_p) > E(\tilde{w}_S)$ for LR and $E(\tilde{w}_S) > E(\tilde{w}_p)$ for *RL*.The expected weights shown in Table 2 are the mean weightings for price (**P1** or **P2**) and selection (**S1** or **S2**) from Table 1. Thus, each member of a hypothetical dyad assigns the most weight to their more important attribute. For example, in condition L (**P1** vs **S1**) given in Table 2, a negotiator favoring price has normative weights, (.85,.15) respectively for price and selection; while a negotiator favoring selection has respective normative weights of (.15, .85). Condition *R* (**P2** vs **S2**) is a less extreme form of symmetric conflict based on the skewness in the distribution of negotiators weightings of their more important attribute (**P2** = .64, **S2** = .67). In both conditions, averaging yields joint weights, $w_{price} \approx w_{selection}$ = .5. Averaging under conditions L and R results in equal weights required to reach an optimal choice per section 2.3. Conditions *LR* (**P1** vs **P2**) and *RL* (**P2** vs **S1**) result in expected average weights more favorable to price in *LR* but more favorable to selection in *RL*, and thus, cognitive conflict is asymmetric. Table 2 shows these comparisons as Expected vs Optimal weightings.

Table 2: Four Conditions of Cognitive Conflict Based on Expected Weightings for Price and Selection

Condition	Symmetric	Asymmetric	Expected Average vs Optimal
L: P1 vs S1	(.85, .15) vs (.15, .85)		(.50, .50) vs (.50, .50)
R: P2 vs S2	(.64, .36) vs (.33, .67)		(.49, .51) vs (.50, .50)
LR: P1 vs S2		(.85, .15) vs (.33, .67)	(.59, .41) vs (.50, .50)
RL: P2 vs S1		(.64, .36) vs (.15, .85)	(.40, .60) vs (.50, .50)

Store Frontier

Using CR's five-point rating scale (1-5) for price (P) and service (S), we constructed a Pareto frontier of seven stores shown in Table 3. Converting to 50-point scales with 10 being the lowest score and 50 the highest, the seven nondominated stores yield a negotiation frontier given by $S = 50 + .2P - .02P^2$ ($R^2 = .996$). Higher ratings mean lower prices or better selection for a given store. Table 3 shows the store brand and their respective converted CR ratings. Our choice of a continuous frontier supports general conclusions not possible with a discrete choice set, particularly as manufacturers attempt to improve products by altering product attributes and/or introducing new products to fill gaps along the frontier. Many products evaluated by Consumer Reports tend to exhibit this frontier 'convexity.' In fact, likely less than one percent of products evaluated by Consumer Reports are not concave to the origin (Menasco, 2017, pp. 24-26).

Table 3: Consumer Reports ratings of Seven Pareto stores

Store	Price	Selection
Apple Store	20	46
Independent	26	42
Best Buy	30	38
Staples	35	32
Walmart	37	30
Target	40	26
Costco	45	18

Preference Frontiers

We created four preference frontiers by assigning linear or nonlinear preferences to price or selection as the more important attribute to each negotiator. The functional forms are $\nu^L(P) = -.25 + .025P$ or $\nu^L(S) = -.25 + .025(S)$ for linear preferences and $\nu^N(P) = 1.808 - 5.714P^{-.5}$ or $\nu^N(S) = 1.808 - 5.714S^{-.5}$ for nonlinear preferences. Per convention, zero preference was set to the minimum, transformed CR scale value, $P = S = 10$. Combining preferences yields the four preference frontiers and estimated feasible settlement space shown in

figures 1, 2, 3, and 4. Figure 1 (LL) presents a frontier where both parties have linear preferences in their more important attribute. Figure 2 (NL) shows a nonlinear preference for price and a linear preference for selection in the more important attribute of each party. Figure 3 (LN) shows the inverse with a linear preference for price and a nonlinear preference for selection. Figure 4 (NN) represents nonlinear preferences in both attributes.

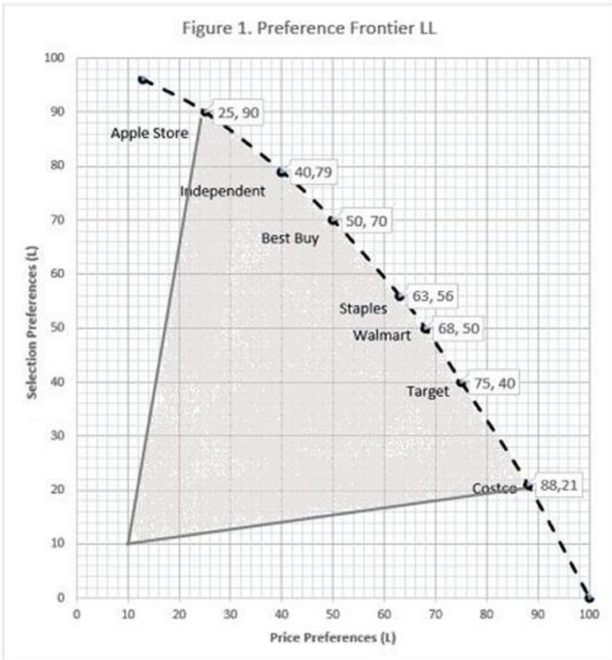


Figure 1: Preference Frontier LL

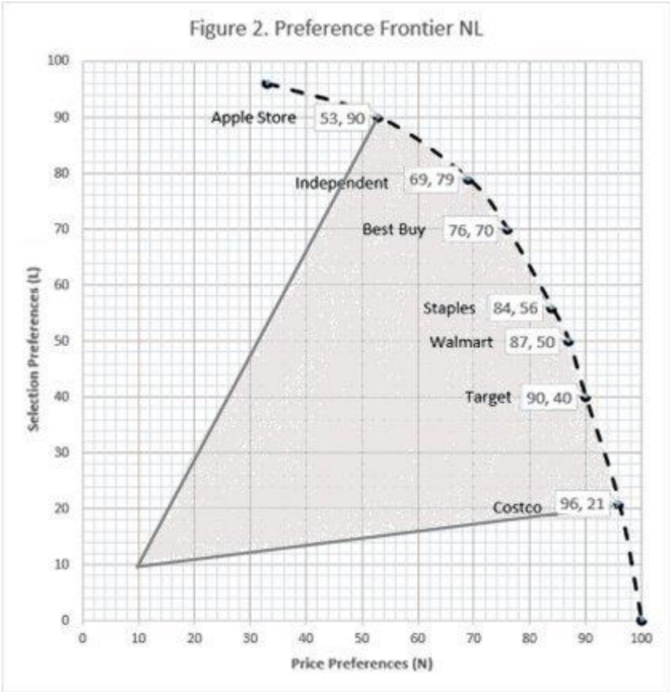


Figure 2: Preference Frontier NL

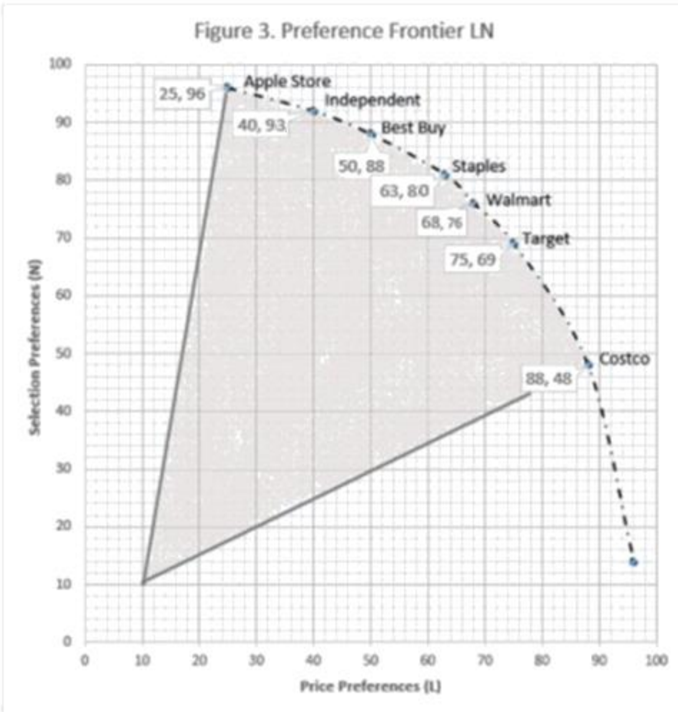


Figure 3: Preference Frontier LN

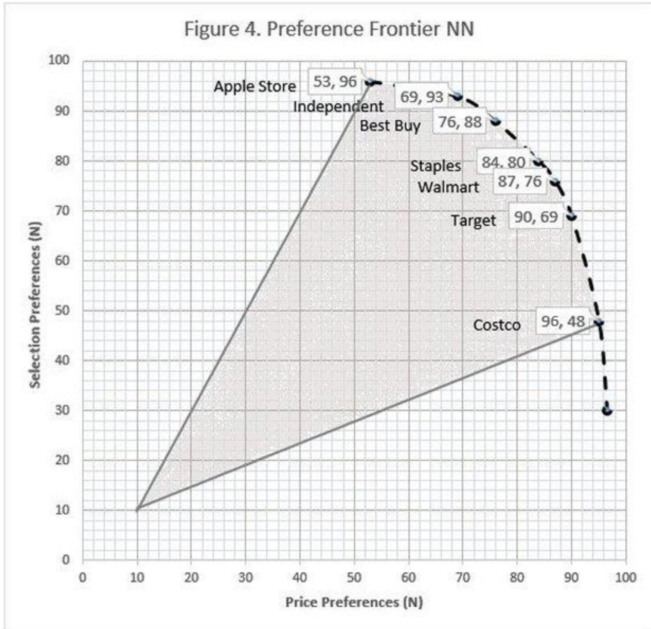


Figure 4: Preference Frontier NN

Optimal (Maximizing) Choices

From section 2.3, we know the maximizing or optimal choice in two attributes must reflect a 50/50 allocation of attribute weightings or in this case, $w_p = w_s = .5$, yielding the optimal two- attribute preference (v_p, v_s) when $m = -1.0$. Let S be the set of eight discrete store choices, $S \in P$ in figures 1, 2, 3 and 4. Assuming normative rationality, negotiators would jointly choose the store option where (v_p, v_s) most closely approximates $\lambda = -1$. These choices are Best Buy (50, 70) in Figure1 (LL), Independent (69, 79) in Figure 2 (NL), Target (75, 69) in Figure 3 (LN), and Target (90, 69) in Figure 4 (NN). Comparatively, the actual (mathematical) optimums are (50, 70) for LL, (69, 79) for NL, (74, 70) for LN and (89, 71) for NN.

Concession Choices

We are also interested in how well weight averaging predicts concessions. Accordingly, we estimated the quadratic forms of each preference frontier to calculate equal gains and losses using the end-point preferences for Apple Store and Costco. Each preference frontier should theoretically yield $\rho = -1$. As such, the R^2s were (1.000, .998, .993, .974) for LL, NL, LN and NN, respectively. Using Euclidian proximity (d), the following stores are spatially the closest to preferences satisfying $g \ or \ l$: For LL, Staples (63, 56) is spatially closest to both the normative gain and loss solutions at (62, 58) and (59, 61). For NL, Staples (84,56) is closest to a gain (85, 53) and Best Buy(76, 70) closest to a loss (76, 71). For LN, Staples (63, 80) is closest to (63, 82) for a gain and Walmart (68, 76) is closest to a loss (66, 75). Last, NN mirrors LL with Staples (84, 80) closest both to the gain at (84, 79) and the loss at (83, 82). Note, that all concession choices are the same for each frontier depicted in figures 1 through 4 and are the three mid-point store preferences on each frontier. A choice of one of these stores based on preferences would constitute a 'compromise effect' discussed in section 2.4.

Predictions of Joint Choice

We classified store choices into one of three Pareto categories applicable to each of the four frontiers: (1) left-Pareto (Apple, Independent), (2) compromise or mid-Pareto (Best Buy, Staples, Walmart), and (3) right-Pareto (Target, Costco). Table 4 shows optimal and concession choices by category for the four preference frontiers LL, NL, LN and NN. Note that concession choices occur under category 2 for all four preference frontiers, and when preferences for price and selection are symmetric as with LL and NN, Staples is both the equal gain and equal loss choice. When preferences are asymmetric, an equal loss choice favors the party with a linear preference, e.g., Best Buy under NL in Figure 2 and Walmart under LN in Figure 3. Optimal choices also reflect symmetry or asymmetry of preferences except for NN. Under NL, a linear preference for selection favors that party with the choice of Independent (Figure 2); while for LN, a linear preference for price favors that party with the optimal choice of Target (Figure 3). It is useful to consider the table as a template for weight averaging under the four skewness conditions. That is, predicted choices by weight averaging, nominally, should be consistent over type of preference frontier. Section 4.2 presents results of predicted choices that are contradictory to this assumption.

Table 4: Normative Store Choices Categorized Over Frontier

Category	1	2	3
LL		BB(o)	
		ST (g-l)	
NL	IN(o)	ST(g)	
		BB(l)	
LN		ST(g)	TG(o)
		WM(l)	
NN		ST(g-l)	TG(o)

BB-Best Buy, ST-Staples, IN-Independent, WM-Walmart, TG-Target; (o)-optimum, (g-l)-gain, loss

The principal dependent measure is the frequency of category choice. We did this by first calculating the probability of individual store choice ($j = 1,\cdot\cdot\cdot, 7$) for each negotiation pair ($n = 1,\cdot\cdot\cdot,50$) under *L, R, LR and RL* over each preference frontier (LL, NL, LN and NN). We used a logit formulation, $prob_{(nj)} = \frac{\exp p_{nj}}{\sum_{j=1}^{7} \exp p_{nj}}$; where a pair's overall store preference is $p_{nj} = w_P(v_P) + w_S(v_S)$, and w_P, w_S are the pair's average weighting for price and selection. We then calculated a probability of category choice for each negotiation pair by summing the probabilities of choice for each store within each category. The category with the largest probability then becomes the 'joint choice' for that pair.

Anticipated Category Choices

(1) From section 3.1.2, conditions L and R are cognitively symmetric because under both conditions $E(\tilde{w}_P) = E(\tilde{w}_S)$; thus, we expect an overall higher frequency of category choices containing the optimal choice. Whereas conditions LR and RL are cognitively asymmetric; overall, right-Pareto choices (category 3) should dominate for LR given $E(\tilde{w}_P) > E(\tilde{w}_S)$, and left Pareto choices (category 1) should dominate for RL given $E(\tilde{w}_P) < E(\tilde{w}_S)$. (2) Specifically, symmetric conflict should result in a greater frequency of category 2 choices under LL, a greater frequency of category 1 choices under LN, and a greater frequency of category 3 choices under

both LN and NN (see Table 4). Alternatively, for asymmetric conflict, category choices should not vary over type of preference frontier. Taken together, a preponderance of concession (category 2) choices should occur only for symmetric conflict under LL (Table 4).

Results

The basic design is a one-way analysis with the four frontiers (LL, NL, LN and NN) as repeated measures under conditions L, R, LR, and RL as shown in Tables 5 and 6. In each table, there are four independent contingency evaluations. By combining L and R (symmetric conflict) and LR and RL (asymmetric conflict), Table 5 aggregates frequencies of category choice shown in Table 6 to the correctly predicted category versus the other two categories. In Table 5, all contingencies but LN are significant, simply indicating there is no statistical association between category choice and type of conflict under LN. A three-way contingency statistic is not possible because cognitive conflict and preference frontier are not independent factors as result of applying common weightings (in the logit) to each preference frontier. From section 4.1, we assume weightings to be cognitively stable, internally held values applied by the decision maker over different decision contexts. Alternatively, a sampling of weights for each frontier would instead be consistent with research showing that a decision maker might use different weightings in different contexts [29].

Table 5: Frequency of Predicted and Other Category Choices Over Preference Frontier and Conflict

Conflict	Symmetric	Asymmetric	Total
LL			
Predicted Other	49 51	67 33	116 84
NL			
Predicted Other	72 28	46 54	118 82
LN			
Predicted Other	31 69	37 63	68 132
NN			
Predicted Other	1 99	34 66	35 165
Total			
Predicted Other	153 247	184 216	337 463

$$p(\chi^2_{LL}) = .01,\ p(\chi^2_{NL}) = .000,\ p(\chi^2_{LN})\ n.s.,\ p(\chi^2_{NN}) = .000$$

Table 6: Frequency of Category Choices Over Preference Frontier and Skewness of Weightings

Skew	L	R	LR	RL	Total
LL					
1	19	17	6	38	80
2	23	26	15	11	75
3	8	7	29	1	45
NL					
1	34	38	18	46	136
2	16	12	32	4	64
3	0	0	0	0	0
LN					
1	0	0	0	4	4
2	32	37	17	39	125
3	18	13	33	7	71
NN					
1	12	11	2	28	53
2	37	39	42	22	140
3	1	0	6	0	7
Total	200	200	200	200	800

Main Effects of Skewness (Conflict)

From section 3.7 both conditions L and R (symmetric conflict) should generate a high frequency of optimal category choices, while conditions LR and RL (asymmetric conflict) should yield a high frequency of category 1 or category 3 choices. Of the 200 predictions in each symmetric skewness condition (L and R) given in Table 6, L yields n = 76 and R yields n = 77 respective optimal category choices as shown in Table 5, for an equivalent percentage of 38% of total predictions (n = 153 of 400 logit predictions). Per Table 6, of the remaining 247 predictions under symmetric conflict in Table 5, seventy-four are non-optimal, category 1 or 3 choices and 173 are non-optimal, concession (category 2) choices. These results suggest symmetric conflict is statistically, overall, a relatively poor predictor of optimal choice.

Under asymmetric conflict in Table 5, LR category 3 choices are n = 68 of 200 choices (34%) and RL category 1 choices are n = 116 of 200 choices (58%). Overall, asymmetric conflict correctly predicts 46% of the 400 choices (Table 5). Thus, asymmetric conflict is only marginally better than symmetric conflict at predicting joint choices. In sum, neither symmetric nor asymmetric conflict are reliable predictors of category choice based on weight averaging.

Interaction Between Main Effects and Replications

The marginal totals in Table 6 are the empirical equivalent of Table 4. Beginning with LL, marginal frequencies do not support a majority of normative category 2 choices whether optimal or concession. Nonetheless, weight averaging yields a majority of optimal (category 2) choices for both L (n = 23) and R (n = 26). Taken together, symmetric conflict *(L+R)* accounts for 65% of

75 marginal choices. On the other hand, as expected, condition LR contributes heavily to the occurrence of category 3 choices (n = 29 of 45 marginal choices or 64%); while *RL* is a lesser majority contributor to expected category 1 choices (n = 38 of 80 marginal choices or 48%).

Consistent with Table 4, NL marginals show a preponderance of optimal category 1 choices (n = 136 of 200 marginal choices or 65.3%). Consistent with predictions by weight averaging, symmetric conflict *(L+R)* contributes n = 72 and asymmetric *(RL)* conflict contributes n = 46 choices, respectively; for a total of 118 correctly predicted choices. Note, there are zero category 3 marginal choices for *LR* which is an unexpected result.

Inconsistent with Table 4, LN marginals do not show a majority of optimal category 3 choices. Instead, there is a preponderance of category 2 (concession) choices (n = 125 or 63%) with n = 69 of those choices due to symmetric conflict *(L+R)* and n = 56 due to asymmetric conflict *(LR+RL).* The *LR* condition, otherwise, correctly predicts a majority of category 3 choices (n = 33), but that *RL* incorrectly predicts a majority of category 2 rather than category 1 choices (n = 39 of 50 predictions).

Inconsistent with Table 4, NN marginals show only n = 7 category 3 choices. Symmetric conflict yields n = 99 of 100 choices that are either category 1 or category 2 instead of predicted category 3 choices. *LR* should predict a majority of category 3 choices, but only six such choices occur, while *RL* correctly predicts only 53% of marginal category 1 choices (n = 28 of 53 predictions). Thus, we must conclude that weight averaging is a weak predictor of normative category choice over nonlinear preferences.

Summary of Results

Behavioral research that 'trains' individuals to learn a particular weighting scheme is quite different than sampling individuals who are participants in a bilateral negotiation [16]. Precision will be much less in the latter case, largely because of the distribution of joint weightings based on skewness. Overall, we thus find type of cognitive conflict to be a weak predictor of normative category choice with asymmetric conflict a slightly better predictor (46%) than symmetric conflict (38%). However, symmetric conflict *(L+R)* marginally predicts more optimal category choices (n = 153) versus asymmetric conflict (n = 136 for *LR+RL*). These counts are from Table 6 corresponding to optimal category choices given in Table 4. Table 6 also shows symmetric conflict to be a better predictor of concession or category 2 choices. Excluding the LL preference frontier because the optimal choice is also category 2, symmetric conflict yields n = 173 of 300 predictions or 58% concession choices. Comparatively, only 46% of asymmetric choices are category 2 (n = 182 of 400 predictions).

As Table 6 shows, skewness clearly yields choices dependent on shape of the preference frontier. If our logit predictions fairly mimic a negotiating pair's probability of category choice, it is certainly the case that joint choice will vary by different preference structures, particularly when individuals hold stable internal attribute weightings. Table 7 presents the range of the joint weighting on price required to classify each of the three choice categories presented in Table 6. Table 7 shows reasonable consistency of required weight ranges over both skewness conditions and replications.3 But notably, the combination of linear and nonlinear preferences have a large impact on the allocation of weightings required to classify category choice. For example, the two asymmetric frontiers require nearly the same weight ranges for category 1 and 2 choices under **NL** but category 2 and 3 choices under **LN**, while the symmetric frontiers **LL** and **NN** similarly allocate required weightings over category choice independent of skewness. However, across levels of skewness *(L, R, LR and RL),* Table 7 shows a nearly nonexistent required range for the price weight under **NN** for category 3 choices (containing the optimal, maximizing choice of Target). As noted in the previous section, when average weights are (. 5, .5), two nonlinear preferences do not accurately predict the maximizing category.

Table 7: Empirical Ranges of Joint Weight on Price Required by Category Choice

Skew	L	R	LR	RL
LL				
1	.32~.47	.30~.46	.35~.46	.26~.46
2	.48~.58	.48~.58	.48~.58	.47~.58
3	.60~.73	.60~.70	.60~.74	.69~.69
NL				
1	.32~.55	.30~.54	.35~.54	.26~.54
2	.56~.73	.56~.70	.56~.74	.56~.69
3	---	---	---	---
LN				
1	---	---	---	.26~.28
2	.32~.53	.30~.53	.35~.52	.29~.50
3	.54~.73	.54~.70	.54~.74	54~.69
NN				
1	.32~.41	.30~.39	.35~.40	.26~.39
2	.42~.70	.40~.70	.42~.71	.40~.69
3	73~.73	---	.72~.74	---

~ defines the inclusive interval for w_p.

Discussion

The perspective taken in this paper on attribute weightings is cognitive; that is subjective weights captured by a multi-attribute model are statistically separable from attribute scores or attribute preferences, and therefore, represent cognitive importance to the individual. For instance, Curry and Menasco present early experimental evidence for this effect and replicated in later research [23, 32]. A question about the efficacy of weight averaging thus has legitimate antecedents in prior research. As noted in section 2.3, two different viewpoints on averaging are possible: (1) when averaging yields equal weights, i.e., (.5. , .5), the resultant attribute scores or preferences maximize joint satisfaction along the efficient frontier. (2) Alternatively, for any set of negotiated (or average) weights dependent on the normalized sum of the two parties' individual weight vectors, only the attribute scores or preferences required by those weightings maximize joint satisfaction [17]. Any other set of scores will not maximize joint satisfaction or utility. For the research presented in this paper, we examined the former perspective, because it preserves the original form of the efficient frontier from which only one option maximizes joint satisfaction; thereby, necessitating that a negotiating dyad exhibit an equal tradeoff of attribute or preference values.

Key Findings

The simulated experiment described in this paper is a characteristic marketing example of a negotiation in which joint choice [settlements] "… can capitalize on differences between disputants' relative weights." [33]. Based on the Interpersonal Conflict Paradigm, we modeled negotiators assessments of joint weights that hypothetically yield a mutually satisfactory joint choice [16]. Our results show that a simple averaging strategy of attribute weights largely ignores the maximizing choice much less a heuristic choice based on concessions of joint preference. For cognitive conflict that is symmetric, enough variation in individual weightings probably occurs to deny a large proportion of negotiators arriving at joint equal weights by averaging, while asymmetric weightings will never produce 50/50 relative joint weightings by averaging.

Second, Consistent with Munpower and Darling and Munpower and Rohrbaugh, our results show that negotiators' preference functions determine the distribution of efficient choices along the frontier which in turn dictates required weightings for category choices [20, 33]. When two parties share linear and nonlinear preferences in their more important attribute, category 2 choices across skewness conditions require a higher range of joint weights on price under **NL**, while **LN** requires a similarly higher range for category 3 choices (Table 7). This 'shifting' effect is not as pronounced when negotiators hold symmetric preferences as in the frontiers **LL** and **NN** (although the **NN** frontier is a mediocre predictor of category 3 choices).

Limitations and Extensions

Negotiating weights over two attribute preferences is a limiting case because most negotiations involve multiple issues; nevertheless, one can imagine disputants negotiating joint weightings as with negotiation support systems (NSS) that utilize a joint multi-attribute utility model (MAU). Thus, using an NSS, negotiators' find an optimizing joint preference by identifying their intrinsic, individual attribute weightings then negotiate the set of joint weightings. This is equivalent to maximizing for a fixed set of weights determined by the average weight for attributes, X_1, X_2,• • •, X_n [17]. In practice, it is not likely that many negotiations over n attributes would involve more than two strict tradeoffs. For example, an examination of the technical specifications of brands of refrigerator/freezer combinations (n = 43) evaluated by Consumer Reports show this to be the case: Both refrigerator and freezer size in cubic ft. are negatively correlated with energy efficiency, while refrigerator and freezer size are positively correlated (likely reflecting consumer preference for total size) [28]. Imagine a negotiation between Marketing and Product Development teams for a new market entry. (1) They could

negotiate weights for the three attributes and maximize over potential product profiles by using a facsimile objective function as with a negotiation support system (NSS). (2) Alternatively, they could investigate tradeoffs over an efficient frontier by negotiating or averaging their attribute weights for energy efficiency and total cubic ft. size (refrigerator plus freezer). For a proposed entry along the frontier, the sides would then negotiate relative refrigerator and freezer size. This is an abstract example but does illustrate the potential use of partial tradeoffs for a variety of practical negotiations. Further research could also clarify the prevalence of other forms normative frontiers describing attribute tradeoffs besides the simple quadratic used in this paper. For example, Menasco shows that 3-space frontiers such as hyper-linear, spherical, and ellipsoidal can model partial tradeoffs such as the CR refrigerator example above [19]. Appendix C presents an example of three tradeoffs for a spherical frontier (P) given by the octant $(x_1 - x_0)$, $(y_1 - y_0)$, $(z_1 - z_0)$.

Footnotes

1. Note that the ratio $m = -\frac{w}{1-w}$, is the slope of the joint indifference curves tangent to the negotiation set in the two-attribute or two-preference space as function of attribute values (x, y) or their corresponding preference values (v_x, v_y). Given w_x in the numerator and $w_y = 1 - w_x$ in the denominator, m is a measure of the dyad's joint tradeoffs relative to the X attribute. Thus, a dyad's tradeoff favors X when joint $w_x > .5$ and favors Y when joint $w_y > .5$.

2. We assume weights are known by compositional (e.g., self-explication) or decompositional (e.g., general linear model) methods (see Lewis and Shakun, and Johnson, et al., for eclectic, respective examples) [34, 35]. In either case, subject weightings for price and selection would need to be determined over a representative data set, that in this case, could be CR's ratings on price and selection for all 18 stores evaluated–the so called 'consideration set' of stores from which the Pareto frontier defines the 'choice set' of stores ultimately evaluated. Using the choice set is an inappropriate sample because of possible contextual effects leading to model misspecification [21]. In this case, the negatively correlated attributes price and selection could induce parties "· picking an alternative not because it has the best overall evaluation, but because it is the best on the most important attribute" [35]. Weight averaging somewhat overcomes this objection even if each negotiator uses a noncom-

pensatory rule implying respective weightings of (1, 0) and (0, 1), because the joint preference would then be $p_j = .5(price) + .5(selection)$. It is noteworthy to point out that preference models from equations (1) and (2) are deterministic but that any decompositional procedure to recover weightings is stochastic. For example, we could have the statistical model, $p_{ij} = w_P(v_P) + w_S(v_S) + u$; where p_{ij} is individual i's estimated overall preference for store j and (v_P, v_S) are respective preferences for attribute values price and selection as derived in section 2. As shown in any elementary econometrics text, coefficients estimated from a sample exhibiting extreme negative correlations between independent variables are likely to lead to serious estimation errors [36]. Thus, we presume the sampled weights reported in Table 1 are from a larger, orthogonal sample, e.g., the consideration (or feasible) set of 18 stores evaluated by CR. The correlation between price and selection for the 18 stores is $r = .003$.

3. Required ranges of the joint weight for price were empirically determined based on category classification described in section 3.6. Although theoretically, each preference frontier is continuous, Table 7 would not appear much different in terms of required weight ranges even if calculating logit probabilities over a continuous distribution were feasible. This is because each faux negotiating pair uses the same joint weights to calculate overall store preferences on each discrete frontier with a fixed category classification for each frontier [36].

Appendix A

We show solutions for the value m for two Pareto examples that are either mathematically asymmetric or symmetric in the positive quadrant of two-attribute space. Both frontiers are based on ratings of price and selection by Consumer Reports (per text) for stores offering a range of electronic products. CR ratings are rescaled from ($1 \geq R \leq 5$) to ($10 \geq R \leq 50$)

A. Derivation of m for an asymmetric frontier: For the Pareto frontier given by the quadratic $y = a + b_1x - b_2x^2$, the general solution for $m = -w/(1 - w)$ is $= b_1 + b_22x$ (b_2 negative) by maximizing $w \cdot x + (1 - w) \cdot y$, where w is the joint weight assigned to attribute X. For the attribute frontier, $S = 50 + .2P - .02P^2$, we derive m for the four preference frontiers:

1. When *linear preference* $v(P) = -.25 + .025P$ and *linear preference* $v(S) = -.25 + .025S$: The attribute scores ($P = 30$, $S = 38$) satisfy $m = -1$ in both attribute and preference spaces with $v_P = .50$ and $v_S = .70$ by maximizing $f = w(-.25 + .025P) + (1 - w)(-.25 + .025S)$, $S = 50 + .2P - .02P^2$.
2. When *nonlinear preference* $v(P) = 1.808 - 5.714P^{-.5}$ and linear preference $v(S) = -.25 + .025S$: The attribute scores ($P = 26.2$, $S = 41.5$) satisfy $m = -1$ for $v_P = .69$ and $v_S = .79$, by maximizing $f = w(1.808 - 5.714P^{-.5}) + (1 - w)(-.25 + .025S)$. $m = (.005 - .001P)/2.857P^{-1.5}$.
3. When *linear preference* $v(P) = -.25 + .025P$ and *nonlinear preference* $v(S) = 1.808 - 5.714S^{-.5}$: The attribute scores ($P = 39.4$, $S = 26.8$) satisfy $m = -1$ for $v_P = .74$ and $v_S = .70$, by maximizing $f = w(-.25 + .025P) + (1 - w)(1.808 - 5.714S^{-.5})$. $m = [(.5714 - .1035P)(50 + .2P - .02P^2)^{-1.5}] \div .025$.
4. When *nonlinear* $v(P)$ and $v(S)$ *preferences*: The attribute scores ($P = 39.1$, $S = 27.2$) satisfy $m = -1$ for $v_P = .89$ and $v_S = .71$. $m = [(50 + .2P - .02P2)^{-1.5}(.2 - .02P)] \div P^{-1.5}$

B. Derivation of m for a symmetric frontier: Assuming the quarter circle $y = \sqrt{r^2 - (x - h)^2} + l$ where $r^2 = (x - h)^2 + (y - l)^2$, the general solution for $m = -w/(1 - w)$ is $-x + h/\sqrt{r^2 - (x - h)^2}$ by maximizing $w \cdot x + (1 - w) \cdot (\sqrt{r^2 - (x - h)^2} + l)$. Using the CR rating scale for Price and Selection [$10 \geq (P, S) \leq 50$], the quarter-circle frontier is $S = \sqrt{40^2 - (P - 10)^2} + 10$. Solutions for m given combinations of linear and nonlinear preferences are:

1. When *Linear* $v(P) = v(S)$ preference: $m = (-P + 10)/\sqrt{40^2 - (P - 10)^2}$ by maximizing $w \cdot P + (1 - w) \cdot S$, $m = -1$ when ($P = S = 38.3$) or ($v_P = v_S = .707$).
2. When *Nonlinear* $v(P)$ and *linear* $v(S)$ *preferences*: $m = (-.025P + .25)[40^2 - (P - 10)^{-1}] \div 2.857P^{-1.5}$ by maximizing $f = w \cdot (1.808 - 5.714P^{-.5}) + (1 - w) \cdot [-.25 + .025(\sqrt{(40^2 - (P - 10)^2} + 10)]$, $m = -1$ when ($P = 32$, $S = 43.4$) or ($v_P = .798$, $v_S = .835$).
3. *Linear* $v(P)$ and *nonlinear* $v(S)$ preferences: Since the attribute frontier is symmetric, $m = -1$ is the inverse of 2. above when ($P = 43.4$, $S = 32$) or ($v_P = .835$, $v_S = .798$)

Appendix B

We derive concessions in negotiations over preferences $v(X)$ and $v(Y)$. In general, $g \triangleq (v_x - v_x^{min}) = (v_y - v_y^{min})$ and $l \triangleq (v_x^{max} - v_x) = (v_y^{max} - v_y)$. Also, $g = l$ if $v_x^{min} = v_y^{min}$ and $v_x^{max} = v_y^{max}$. (1) If $v_y = f(v_x)$ is known, the solution for g and l is straightforward by calculating v_y. (2) Otherwise, we substitute the preference functions, $v_x = f(x)$ and $v_y = f(y)$.

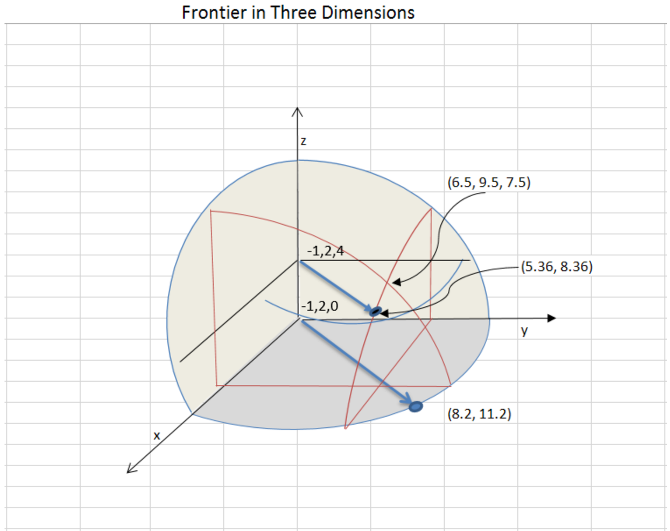
1. Substituting $v_y = f(v_x)$ in both g and l and solving, yields the joint preferences (v_x, v_y).
2. Substituting $v_x = f(x)$ and $v_y = g(y = f(x))$ in both g and l, yields the joint preferences (v_x, v_y) by solving for x and converting, $v_x = f(x)$, $v_y = f(y)$.

Appendix C

Derivation of attribute values for a three-attribute space: The spherical frontier *(P)*.

The midpoint on P is {$\pm.577(x_1 - x_0)$, $\pm.577(y_1 - y_0)$, $\pm.577(z_1 - z_0)$, because the unit sphere, $1.0 = x^2 + y^2 + z^2$ and solving the partial derivatives yields $x = y = z = \pm.577$ as the midpoint (in the appropriate octant). Now let ($x_0 = -1$, $y_0 = 2$, $z_0 = 0$) and ($x_1 = 12$, $y_1 = 15$, $z^1 = 13$). Using |. 577| as a scalar; $x = .577(12 + 1) = 7.5$, $y = .577(15 - 2) = 7.5$, and $z = .577(13 - 0) = 7.5$. The midpoint on P is then $x_o = 6.5$, $y_o = 9.5$ and $z_o = 7.5$; *e. g.*, $(6.5 + 1) = (9.5 - 2) = (7.5 - 0) = 7.5$. Thus, the squared radius is $13^2 = 3(7.5^2) = 169$.

The point (6.5, 9.5, 7.5) is the intersection of the x and y planes shown below and is the midpoint solution in Cartesian coordinates. Yet, this point is not feasible if X, Y and Z have tradeoffs. The figure shows a midpoint (compromise) at ($x = 8.2$, $y = 11.2$) when the origin, $z_0 = 0$. The other two midpoints are ($y = 11.2$, $z = 9.2$, $x = 0$) and ($x = 8.2$, $z = 9.2$, $y = 0$) by solving $y = f(x)$, $y = f(z)$, $z = f(x)$. In other words, if three attributes exhibit pairwise tradeoffs, one attribute must take a minimum value or some other separately negotiated value as in the above example with relative refrigerator and freezer size. Consider the case when a renegotiation of z_0 is appropriate: We illustrate using $z_0 = 4$. The new origin is (−1, 2, 4) and a compromise is the midpoint ($x = 5.36$, $y = 8.36$) illustrated in the figure. The new compromise is reasonable, because cross-sections of the hemisphere in (X, Y) are circles perpendicular to Z. Thus, the new frontier is a quarter-circle in the quadrant given by the radius, $(z_1 - z_0) = 13 - 4 = 9$ and the joint choice has scores ($x = 5.36$, $y = 8.36$, $z = 4.0$). In lieu of a strict compromise of attribute scores, if negotiators can identify intrinsic weights for X and Y, then average or negotiated joint weightings would yield maximum (x, y) scores over the quarter circle frontier for $z_0 = 4$.



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