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Short Communication

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On the Doppler Spectrum for the Signal of a Space Borne SAR Operated Over the Ocean

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Abstract
An analytical formula for the Doppler spectrum is obtained and, on its basis, a method is proposed to increase the efficiency of existing algorithms for determining the Doppler centroid.

At present, the generally accepted method for remote measurement of current velocity on the ocean surface is to determine the Doppler centroid, i.e., the position of the maximum of the backscattered signal Doppler spectrum (see, for example, [1] and the list of references there). However, if you look at the only realization of the Doppler spectrum it immediately becomes clear that finding the centroid is not a very simple task. Obviously, when observing from space, due to high speed of SAR carrier it is not possible to accumulate a sufficient number of realizations in order to smooth out random fluctuations in statistical estimates of the spectrum. Nevertheless, knowledge of the Doppler spectrum around which its components oscillate is useful. Below, an analytical formula for the spectrum is obtained and, on its basis, a method is proposed to increase the efficiency of existing algorithms for determining the Doppler centroid.

We consider the issue indicated by the title in relation to microwave synthetic aperture radar (SAR). The operation of aperture synthesis is determined by the matching filtering formula

$$a_{SAR}(t) = \frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} dt' a(t')\exp\left[-i\frac{k}{R}V^2(t'-t)^2\right] \tag{1}$$

Here, $a(t')$ and $a_{SAR}(t)$ are the complex amplitudes (i.e., the Doppler parts) of the backscattered field and the SAR signal, respectively, and Δt is the integration time determining the nominal SAR resolution (the factor before the integral is introduced to preserve the dimension). Besides, $k = 2\pi/\lambda$, where λ is the radar wavelength, V and R are the SAR carrier speed and the slant range, respectively; the nominal azimuthal SAR resolution

$$\Delta_{SAR} = \frac{\lambda R}{2V\Delta t} \tag{2}$$

It is assumed that the SAR moves in parallel the x-axis, and the size of resolution cell Δ_y along the ground range y is small compared to the characteristic wavelength of the large ocean wave.

We used the following expression for $a(t')$ (see [2] for details):

$$a(t) = \frac{2k^2}{\sqrt{\pi}} e^{2ikR} \int_{\Delta\vec{r}} d\vec{r}' m(\vec{r}',t)\, \xi(\vec{r}',t) \exp\{2ik[(y'-y)\sin\theta_0 - \zeta(\vec{r}',t)\cos\theta_0 + \frac{1}{2R}(x'-Vt)^2]\} \tag{3}$$

Here, $\xi(\vec{r}', t)$ is a statistically homogeneous ("standard") small-scale ripple with statistical characteristics constant along the profile of a large wave, $\zeta(\vec{r}', t)$ are surface elevations due to large waves, a factor $m(\vec{r}', t)$ takes into account the amplitude modulation of the scattered field; integration is within the physical resolution cell. The amplitude $a\,(x, t)$ is normalized so that the intensity averaged over the realizations of small-scale ripples is equal to the normalized radar cross section σ_0.

We introduce the factor

$$\Phi(t - t') = \exp\left[-4\left(\frac{t - t'}{\Delta t}\right)^2\right] \tag{4}$$

to the integrand of (1) in order to extend integration to infinite limits, and then let's create a correlation function

$$R(\tau) = \langle a_{SAR}(t) a_{SAR}^*(t + \tau)\rangle$$

After a rather long series of transformations similar to those performed when obtaining the SAR signal intensity formula (see [2]), we obtain:

$$a_{SAR}(t) a_{SAR}^*(t + \tau) = \frac{\pi}{2}\Delta_y \exp\left(-\frac{\pi^2}{8}\frac{V^2}{\Delta_{SAR}^2}\tau^2\right)\int dx\, \sigma_0 \exp\left[-\frac{\pi^2}{2\Delta_{SAR}^2}D^2 - i\frac{2kV}{R}D\tau - i2kv_{rad}\tau\right] \tag{5}$$

$$D = [V(t + \tau/2) - x] - \frac{R}{V}v_{rad}$$

where $v_{rad} = \bar{v}_{rad} + v_{orb.rad}$ is the radial component of velocity on the ocean surface, which consists of an average (regular) part $\bar{v}_{rad}$ and a random one caused by orbital movements of small ripples.

To get the correlation function, we have to average the right part of (5) over $vrad$. Obviously, the averaging operation is insensitive to the position of the straight line $V\,(t + \tau\,/\,2) - x$ with respect to the x-axis; only its slope is important here. Therefore, we put $V\,(t + \tau\,/\,2) = 0$ and assume $D = -x - \frac{R}{V}v_{rad}$, then for the correlation function we can write down:

$$R(\tau) = \frac{1}{2}\bar{\sigma}_0\Delta_y\frac{\Delta_{SAR}}{\sigma_{v.rad}}\frac{V}{R}\exp\left(-\frac{\bar{v}_{rad}^2}{2\sigma_{rad}^2}\right)\exp\left(-\frac{\pi^2}{8}\frac{V^2}{\Delta_{SAR}^2}\tau^2\right)\times$$

$$\int dx \exp\left[-\frac{\pi^2}{2\Delta_{SAR}^2}x^2 + i\frac{2kV}{R}x\tau\right]\langle exp\left[-\frac{\pi^2}{2\Delta_{SAR}^2}\left(\frac{R}{V}\right)^2 v_{rad}^2 - \frac{\pi^2}{\Delta_{SAR}^2}\frac{R}{V}xv_{rad}\right]\rangle \tag{6}$$

Here, the angle brackets mean the averaging over v_{rad}, which we assume is distributed normally with parameters $\bar{v}_{rad,}$ and $\sigma_{v.rad}$. In addition, we assume that there is no significant statistical relationship between σ_0 and the values in angle brackets.

It can be shown that if the some random value u has the normal distribution with parameters $\bar{u}$ and σ_u then

$$\langle exp(-au^2 - bu)\rangle = \frac{exp\left(-\frac{\bar{u}^2}{2\sigma_u^2}\frac{2a\sigma_u^2}{2a\sigma_u^2 + 1}\right)}{\sqrt{2a\sigma_u^2 + 1}} exp\left(\frac{b^2\sigma_u^2/2 - b\bar{u}}{2a\sigma_u^2 + 1}\right) \tag{7}$$

Here, according to (6),

$$a = \frac{\pi^2}{2\Delta_{SAR}^2}\left(\frac{R}{V}\right)^2 \quad \text{and} \quad b = \frac{\pi^2}{\Delta_{SAR}^2}\frac{R}{V}x \tag{8}$$

In the case of space borne SAR, condition $2a\sigma_{v.rad}^2 \gg 1$ is usually satisfied. That's why

$$\frac{exp\left(-\frac{\bar{u}^2}{2\sigma_u^2}\frac{2a\sigma_u^2}{2a\sigma_u^2+1}\right)}{\sqrt{2a\sigma_u^2+1}} \cong \frac{\Delta_{SAR}}{\pi\sigma_{v.rad}}\frac{V}{R}\exp\left(-\frac{\bar{v}_{rad}^2}{2\sigma_{v.rad}^2}\right), \tag{9}$$

$$\frac{b^2\sigma_u^2/2 - b\bar{u}}{2a\sigma_u^2+1} = \frac{b^2}{4a\sigma_u^2(1+1/2a\sigma_u^2)} - \frac{b\bar{u}}{2a\sigma_u^2(1+1/2a\sigma_u^2)} \cong \frac{b^2}{4a}\left(1-\frac{1}{2a\sigma_u^2}\right) - \frac{b\bar{u}}{2a\sigma_u^2}\left(1-\frac{1}{2a\sigma_u^2}\right) =$$

$$\frac{\pi^2}{2\Delta_{SAR}^2}x^2 - \frac{1}{2\sigma_{v.rad}^2}\left(\frac{V}{R}\right)^2 x^2 - \frac{V}{R}\frac{\bar{v}_{rad}}{\sigma_{v.rad}^2}x + \Delta_{SAR}^2\left(\frac{V}{R}\right)^3\frac{\bar{v}_{rad}}{\sigma_{v.rad}^4}x$$

It is easy to notice that the last term of the sum above is usually two orders of magnitude smaller than the penultimate one. Taking this into account, we write:

$$R(\tau) = \frac{1}{2}\bar{\sigma}_0\Delta_y\frac{\Delta_{SAR}}{\sigma_{v.rad}}\frac{V}{R}\exp\left(-\frac{\bar{v}_{rad}^2}{2\sigma_{v.rad}^2}\right)\exp\left(-\frac{\pi^2}{8}\frac{V^2}{\Delta_{SAR}^2}\tau^2\right)\times \tag{10}$$

$$\int dx\ \exp\left[-\frac{1}{2\sigma_{v.rad}^2}\left(\frac{V}{R}\right)^2 x^2 - \frac{V}{R}\left(\frac{\bar{v}_{rad}}{\sigma_{v.rad}^2} - i2k\tau\right)x\right]$$

This integral is taken, therefore

$$R(\tau) = \frac{1}{\sqrt{2\pi}}\bar{\sigma}_0\Delta_y\Delta_{SAR}\exp\left[\left(-\frac{\pi^2}{8}\frac{V^2}{\Delta_{SAR}^2} + 2k^2\sigma_{v.rad}^2\right)\tau^2 - i2k\bar{v}_{rad}\tau\right] \tag{11}$$

Obviously, in the case of spaceborne SAR

$$\frac{\pi^2}{8}\frac{V^2}{\Delta_{SAR}^2} \gg 2k^2\ \sigma_{v.rad}^2 \tag{12}$$

and, therefore

$$R(\tau) = \frac{1}{\sqrt{2\pi}}\bar{\sigma}_0\Delta_y\Delta_{SAR}\exp\left(-\frac{\pi^2}{8}\frac{V^2}{\Delta_{SAR}^2}\tau^2 - i2k\bar{v}_{rad}\tau\right) \tag{13}$$

From here it follows that the spectrum has the shape-factor

$$\hat{S}(\omega) = \exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2V^2}(\omega - 2k\bar{v}_{rad})^2\right] \tag{14}$$

As can be seen, the spectrum has a Gaussian shape with the maximum at $2k\bar{v}_{rad}$ and decreases to a level of 0.5 at the frequency

$$f_{0.5}[Hz] = \frac{1}{2\sqrt{2}}\frac{V}{\Delta_{SAR}}(-\ln 0.5)^{1/2} \tag{15}$$

Hence, the Doppler spectrum width at level 0.5 for a space SAR can be estimated as

$$\Delta S_{0.5} = 2f_{0.5} \approx 0.6\frac{V}{\Delta_{SAR}} \sim 1KHz \tag{16}$$

Here it is appropriate to ask the question: does the width of the Doppler spectrum really not depend on the state of the ocean surface? In fact, there is such dependence, but in the case of spaceborne SAR it is weak due to relation (12).

Let us turn to the article [3], were, in particular, the results of the numerical modelling of the Doppler spectrum are given (see Figure.13 there). On can see that the averaged spectrum has the Gaussian shape, the width $\Delta S_{0.5}$ is about $1KHz$, and the curves that envelop the random values of the spectrum estimation components from above and below (if they were drawn) would repeat (each with its coefficient $1 \pm \alpha$) the averaging curve. We introduce the confidence interval and present its upper and lower boundaries:

$$\hat{S}_{\pm}(\omega) = (1 \pm \alpha)\exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega - 2k\bar{v}_{rad})^2\right] \tag{17}$$

In addition, from the mentioned Figure.13 of [3], we can conclude that the value $\alpha \cong 0.1$ provides a probability close to 100% of finding the estimate within the confidence interval. In the general case, the value of α is set based on the specific requirement for the probability parameter of the confidence interval.

Now let's turn to our Figure.1 and pay attention to the horizontal line $\hat{S}(\omega) = 1$ drawn through the Doppler centroid point; denote the two points of intersection of this line with the upper boundary of the confidence interval ω_1 and ω_2. Note that with the probability specified for the confidence interval, the relation $\hat{S}(\omega) < 1$ is valid for all frequencies outside the interval $\omega_1 < \omega < \omega_2$. Therefore, with the same probability we can exclude frequency regions outside this interval from the Doppler centroid search area and thereby narrow the search area. The right boundary of the specified interval can be found using the equation

$$(1 + \alpha)exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega_2 - 2k\bar{v}_{rad})^2\right] = 1 \tag{18}$$

and the left one symmetrically about the point $\omega = 2k\bar{v}_{rad}$.

It should be noted here that the true position of the centroid point is, strictly speaking, unknown to us. However, existing algorithms (see [3] and references there) give centroid uncertainty (standard deviation and bias) within 10 Hz, while the narrowed (i.e., effective) search bandwidth, as shown below, is about 200 Hz. Obviously, in this case, some blurring of the boundaries of the effective search area is insignificant.

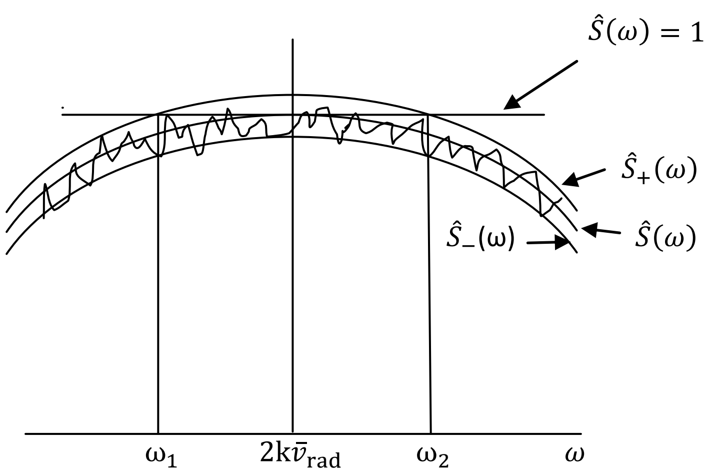

Figure 1: Section of random realization of spectral estimate (the scheme).

Having assumed

$$exp\left[-\frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega_2 - 2k\bar{v}_{rad})^2\right] \cong 1 - \frac{2\Delta_{SAR}^2}{\pi^2 V^2}(\omega_2 - 2k\bar{v}_{rad})^2 \tag{19}$$

then at $\alpha \ll 1$ we obtain the effective bandwidth

$$\Delta f_{eff}[Hz] \cong \sqrt{\alpha/2}\,\frac{V}{\Delta_{SAR}} \quad (20)$$

and at $\alpha = 0.1$ we obtain $\Delta f_{eff} \cong 200\ Hz$ along with the fact that the width of the Doppler spectrum at level 0.5 is about 1 KHz.

Thus, we propose that when finding the Doppler centroid, first use one of the existing algorithms, and then, having obtained the first approximation centroid, apply the same algorithm to a narrowed search area.

References

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