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Non-Violations in Bell's Inequality

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Abstract

Intuitively, violations of Bell's inequality make no sense because one cannot violate a mathematical relationship unless it is wrong, or has been misused. However, the rotating polarizer experiment has been argued to provide such violations, and it is asserted this requires non-locality. It is shown here that at least one form of the inequality is violated because the results violate a condition used in deriving the inequality. The problem arises from mixed frames of reference being utilized, and if an external reference frame is employed, such as a polarized source, the inequality should be complied with. This could be verified by experiment. It is also important to resolve this as entanglement is to be potentially involved equipment such as quantum computing. Correct understanding may assist in such design.

According to Wikipedia, quantum entanglement is actively being researched or applications such as communications, quantum computation and quantum radar. Quantum entanglement means that properties of well-separated particles have correlated properties, thus if a pair of photons are entangled, if you measure a particular property of one, you immediately know the value of the property of the other, even if the pair are photons going in opposite directions and hence can never communicate. The Copenhagen Interpretation of quantum mechanics asserts the value of the first variable was determined at measurement. If so, since the second has to have its value instantly conferred, it is stated that the combined state of the entangled pair is non-local. There is an alternative. If the properties of the particles were conferred on them when the entanglement was created, then there is no mystery that the correlation can be shown to be measured faster than any possible communication could have occurred; the properties were always determined before anyone measured anything. However, this is generally ruled out because we observe violations of Bell's Inequality.

It is generally held that the rotating polarizer experiment, exemplified first by Aspect *et al*.(1981), and later by Weihs *et al.* (2) demonstrate violations of Bell's Inequality (1982) [1,2,3]. A violation of a mathematical relationship usually either means the relationship is wrong, or it has been misused. The above experiments have a concern that arises because the procedure to ensure only entangled photons are counted makes the first detector the frame of reference. The joint detection depends on the $\sin^2$ of the difference in the angle between the detector filters (after adding $\pi/2$).

The question then is, if a common frame of reference were to be employed for all terms in the inequality, are there violations? That can be experimentally tested. However, to ensure the common frame of reference, all terms must be referenced individually, even if they are measured jointly, and we need a version of the inequality that permits this.

Bell's Inequality requires tests that either pass (+) or fail (-) at three conditions. Let the probabilities of such a result be given by capitals. Because a test must either pass or fail, but not both,

$$A = (A+) + (A-) = 1 \quad (1)$$

and similarly for B and C. Therefore

$$(A+)(B-) = (A+)(B-)[(C+) + (C-)] \quad (2)$$

because the bracketed term equals 1, the sum of the passes and fails at C. Similarly

$$(B+)(C-) = [(A+) + (A-)] (B+)(C-) \quad (3)$$

By adding and expanding,

$$(A+)(B-) + (B+)(C-) = (A+)(C-)[(B+)+(B-)] + (A+)(B-)(C+) + (A-)(B+)(C-) \quad (4)$$

Since the bracketed term equals 1 and the last two terms are positive numbers, or at least zero, we have

$$(A+)(B-) + (B+)(C-) \geq (A+)(C-) \quad (5)$$

Consider a wave of amplitude a with polarization at an angle of θ to the y axis generated by one photon. The projected amplitude on the y axis is $a\cos\theta$, and on the x axis, $a\sin\theta$. Accordingly, the way the probabilities P are projected onto the two axes are

$$P_y + P_x = a^2\cos^2\theta + a^2\sin^2\theta = a^2 = A = 1 \quad (6)$$

Thus probability, or energy, is properly conserved. If the polarization is on the axis of the detector's polarization, then A(+) = 1, and no signal is found on the x axis because $\sin^2 0 = 0$. [In practice, no equipment runs perfectly so there were minor discrepancies. For the rest of this paper, I assume all equipment was running perfectly. The ideal result is, of course, the basis of equation (1)] This was experimentally complied with in the Aspect experiments, and demonstrates that the results also comply with the Malus Law.

The Aspect rotations were, for detector A = 0°, B = 22.5°, C = 45°. Following Bell the joint probabilities are given by $\sin^2(\delta\Theta)$ where $\delta\Theta$ is the difference in rotations, required because the second photons are only counted if they are entangled with the first and hence have the same polarization plane, from which we get for the photons counted that were countable for that detector combination,

$$0.146 + 0.146 \geq 0.5 \qquad (7)$$

Which, as Bell noted, is clearly not true. This is the basis of the violation. (The usual approach is to use the CHSH inequality, with which Weihs et al. clearly demonstrate the $\sin^2\theta$ relationship [3]. That uses joint detections, and while the results are true, it hides the problem.)

The problem lies in the use of mixed frames of reference. Each of the detectors read the same because the source is rotationally invariant, but not all the photons recorded with one are entangled with the other. To separate these out, the second detector, say, counts only photons that arrive within a given time of the first one. That makes photons at the first detector to have a probability of 1 because they have already been detected (and thus eliminates the need for the $\cos^2$ term in the Malus law) but it also means there is not a constant external frame of reference. The inequality requires three different conditions. Thus Bell demonstrated the inequality through washing socks at three different temperatures. You could always tell which condition a sock was washed at by inserting a thermometer. Because the source is rotationally invariant it is impossible to tell whether a measurement is an A, a B or a C from the equipment. One could argue that the laboratory walls offer a frame of reference. That might be true but no measurement refers to or is affected by the laboratory walls.

The net consequence is that as the reference frame changes between determinations, so do the values of the terms. As a first example, if B(+) = 1, then B(-) has to equal 0 or the condition (equation 1) in the derivation does not work. From rotational invariance A(+) = B(+) = C(+) = 1, so all minus terms equal 0, and (4) now becomes 0 + 0 = 0 + 0 +0. The B(+) and B(-) terms used in (7) to arrive at the Bell figures are in different reference frames, which leads to B = 1.146. That is a violation of probability, and of the condition used to derive the inequality.

Let us try as a thought experiment a constant frame of reference, namely a polarized source. Let the initial orientation of the detector filters align with the polarization of the wave. Assume we can have sequences with a common number of photons. Now A(+)B(-) = 0.146 and A(+)(C-) = 0.5, the $\sin^2\theta$ relationship as above, but now B(+) does not equal 1 because not all the polarized photons can be detected by the B+ polarizer, but also far more are detected by the C detector. Now B(+)C(-) = 0.427 (the amplitudes are cos 22.5 and -sin 45) and Bell's inequality is obeyed since 0.146 + 0.427 > 0.5 (6)

Thus the violations do not arise from any equipment failure or loophole, but rather the data put into the inequality violates the derivation of the inequality. This arises because the second photon is in the frame of reference of the first, which leads to only employing the $\sin^2$ relationship as the $\cos^2$ term is 1. That leads to two frames of reference in one relationship, wherein (C-) takes two different values at two appearances, yet the derivation requires it to have one value. The problem with the CHSH inequality is that by counting joint detections or non-detections, the issue of reference frame is buried.

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