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New Law or Boundary Condition of Electromagnetic Wave Theory: Radiation Shall Not Overflow The Universe

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Abstract
Introducing a new boundary condition at infinity for electromagnetic theory.The theory of mutual energy flow using electromagnetic fields.The electromagnetic waves of the dual planar antenna, including self energy flow and mutual energy, have been successfully calculated. Among them, self energy flow is reactive power. Mutual energy flow is active power, which is generated from the source and annihilated at the sink.According to Maxwell's electromagnetic theory, or classical electromagnetic theory, posits that a changing current on an antenna induces an electromagnetic wave that can propagate in space. When this wave reaches a receiving antenna, it transfers electromagnetic energy and momentum, allowing the reception of signals. However, the author challenges this standard description, aligning with Wheeler Feynman's absorber theory. According to this theory, the radiation from a transmitting antenna is influenced not only by its own changing current but also by the current changes of the remote receiving antenna, the absorber's charge, and environmental materials.The author supports Wheeler Feynman's perspective and introduces a new electromagnetic theory. In this theory, all transmitting and receiving antennas, as well as radiation and absorber materials, are assumed to be near the origin. The key assumption is that no material on an infinitely large sphere can absorb electromagnetic waves, preventing the transmission of electromagnetic energy to infinity. This idea is integrated into Maxwell's electromagnetic theory by adding a boundary condition, contradicting the theory's original Sliver-Muller radiation condition, which requires a good absorber material in the far field. The author proposes relaxing Maxwell's equations, specifically the mutual energy principle equivalent to Maxwell's equation, to incorporate the new boundary condition that radiation should not overflow the universe. This relaxation results in a novel electromagnetic theory. By introducing this additional boundary condition, the author aims to provide a more accessible description of the entire electromagnetic theory. While the author initially presented their theory from an energy conservation standpoint, emphasizing its adherence to the law of conservation of energy, adding the new boundary condition may offer readers a more easily acceptable perspective.

Keywords: Poynting Theorem, Sliver-Muller, Boundary Conditions, Maxwell Equation, Quasi static, Magnetic Quasi-Static, Transactional Interpretation, Absorber theory, Welch, Reciprocity theorem, Conservation of Energy, Energy Flow, Photon

1. Introduction

This author put forward the mutual energy theorem in 1987 [1-3]. Later, this author found that the theorems similar to the mutual energy theorem are all called reciprocity theorems. Among them is the time-domain reciprocity theorem proposed by Welch in 1960 [4]. Rumsey proposed a new reciprocity theorem in 1963 [5]. The correlation reciprocity theorem proposed by de Hoop [6]. Since 2015, this author has found this problem, that is, the positioning of the same electromagnetic field formula is different. Others call it the reciprocity theorem, while this author calls it the mutual energy theorem. The reciprocity theorem is equivalent to some kind of Green's function, which is positioned as a mathematical formula to help solve Maxwell's equation, not a physical theorem. The mutual energy theorem is an energy theorem, so it is also a physical theorem. After finding this problem, this author began to further study this problem. He hopes to find out whether this formula is the energy theorem or the reciprocity theorem. Through research, it is found that this theorem is not only the energy theorem, but also the energy conservation law. It is not only the energy conservation law, but also there is an energy flow theorem. Therefore, this energy conservation law is actually a localized energy conservation law. It is a strong law of conservation of energy.

Therefore, this author further proposed the mutual energy flow theorem and interpreted quantum mechanics with mutual energy flow [7]. This interpretation is close to the transactional interpretation of quantum mechanics proposed by Cramer [8, 9]. Both believe that the advanced wave is an objective physical reality. The proposal of transactional interpretation is based on Wheeler Feynman's absorber theory [10, 11]. The absorber theory is put forward based on the principle of a-action-at-distance [12-14]. This author agrees with many viewpoints of the absorber theory, that is, the absorber material does not passively receive electromagnetic waves, but receives electromagnetic waves by radiating advanced waves.

On the basis of absorber theory and transactional interpretation, this author further improved his own theory on electromagnetic mutual energy and mutual energy flow, put forward the Huygens principle based on the mutual energy flow theorem [15, 16] the theory of macroscopic electromagnetic waves composed of photons [17] The wave-particle duality theory of mutual energy flow corresponding to Schrodinger equation [15]. This author further studies and proposes a new class of Green functions to solve the electromagnetic problem [18]. This author also extends this theory to electromagnetic stress flow and Newton's Third Law at a long distance [19]. And some other applications [20, 21].

This paper attempts to re-establish the electromagnetic theory of mutual energy proposed by this author. This time, this author proposes a new axiom that electromagnetic waves do not overflow the universe. This can also be regarded as a new electromagnetic boundary condition. According to Maxwell's electromagnetic theory, if an antenna has a change in current, the antenna will radiate electromagnetic waves. If we build a sphere with an infinite radius around the antenna, the energy flow density or Poynting vector of the electromagnetic wave will not be zero for the area of the whole sphere. This author believes that electromagnetic waves should not overflow the universe. There is a problem with Maxwell's electromagnetic theory. Correction required. After correction, The average of the Poynting vector corresponding to the energy flow on the sphere with infinite radius must be zero.

Of course, this author does not want to deny Maxwell's electromagnetic theory. This author finds that Maxwell's electromagnetic theory actually implies a condition, that is, an absorber material that can completely absorb electromagnetic waves is uniformly arranged on a sphere with an infinite radius. This is achieved by the Sliver-Muller radiation condition. But for the real situation, the boundary of the universe is not always full of absorber materials. Especially for photons, they are always absorbed by an absorber charge. Photons are absorbed at one point when they radiate. Photons have nothing to do with whether the cosmic boundary is full of absorber materials. The new electromagnetic field theory established by this author is based on the condition that there is no absorber on the spherical surface with infinite radius. Of course, this boundary condition is not consistent with the Sliver-Muller radiation condition. Instead, offer a new boundary condition that the radiation does not overflow of the universe. The boundary condition of Maxwell's electromagnetic theory can only take the Sliver-Muller radiation condition. Maxwell's electromagnetic theory conflicts with the boundary condition that radiation does not overflow universe proposed by author. In order to solve this conflict, this author adds a relaxation process to Maxwell's electromagnetic theory. Through this relaxation process, the electromagnetic equation releases another degree of freedom, which just allow to add the boundary condition that radiation does not overflow the universe.

In order to explain to the readers that this author is justified in doing so, this author first explains that Maxwell's electromagnetic theory and magnetic quasi-static electromagnetic theory are completely different electromagnetic theories, so different symbols should be used to represent this electromagnetic field. That is, the lower case letters e and h are used to represent the radiated electromagnetic field satisfying Maxwell's equation. Use ***E, H*** to represent quasi-static electromagnetic field or magnetic quasi-static electromagnetic field. The electromagnetic field theory proposed by this author has made a relaxation process for Maxwell's electromagnetic field theory, and added the boundary condition that radiation does not overflow the universe, so that the symbols of ***E*** and ***H*** electromagnetic fields can be restored. This is because the electromagnetic field theory proposed by this author is closer to the quasi-static electromagnetic field, or magnetic quasi-static electromagnetic field.

Finally, this paper compares three kinds of electromagnetic fields, quasi-static electromagnetic field, Maxwell's radiation electromagnetic field, and the radiation electromagnetic field proposed by this author. It shows that although the electromagnetic field proposed by this author is also a radiation electromagnetic field, it is closer to the quasi-static electromagnetic field, and even meets the same basic laws. Therefore, the electromagnetic field proposed by this author should be restored to use E and H.

Although the electromagnetic field proposed by this author has many better properties than Maxwell's electromagnetic field, especially this new theory is suitable for explaining photons, photons are mutual energy flow (electromagnetic wave energy flow to be accurate), and explaining the wave particle duality problem. But at present, this author's electromagnetic theory can not completely solve its electromagnetic field like Maxwell's electromagnetic theory. Fortunately, this author's electromagnetic field theory can draw on the calculation results of Maxwell's electromagnetic theory. At this time, however, Maxwell's equation does not appear as a physical law, but as an auxiliary mathematical tool. The laws of physics are still a new set of electromagnetic equations proposed by this author.

Contribution of this article:

1. A new electromagnetic field axiom or boundary condition has been proposed, which states that radiation cannot overflow the universe.
2. A relaxation process was proposed, which involves relaxing Poynting's theorem and incorporating new boundary conditions to transition from Maxwell's electromagnetic theory to this author's electromagnetic theory.
3. A new method for calculating inductance has been proposed, in which the size of the device is large, so both retarded and advanced effects must be considered.

2. Radiation Must not Spill Out of the Universe

2.1. Far Field Radiation Conditions of Maxwell's Electromagnetic Theory

According to Maxwell's electromagnetic theory, the following Sliver-Muller radiation boundary conditions are available for radiated electromagnetic fields [25],

$$\lim_{r\to\infty} r|\boldsymbol{E}| = q_1 \tag{1}$$

$$\lim_{r\to\infty} r|\boldsymbol{H}| = q_2 \tag{2}$$

$$\lim_{r\to\infty} r(\boldsymbol{E} - \eta\boldsymbol{H} \times \hat{\mathrm{r}}) = 0 \tag{3}$$

$$\lim_{r\to\infty} r(\boldsymbol{H} - \frac{1}{\eta}\hat{\mathrm{r}} \times \boldsymbol{H}) = 0 \tag{4}$$

According to this boundary condition, the electromagnetic wave emitted from the source points in all directions. And the electric field and magnetic field are both

$$\lim_{r\to\infty} |\boldsymbol{E}| \sim \frac{1}{r} \tag{5}$$

$$\lim_{r\to\infty} |\boldsymbol{H}| \sim \frac{1}{r} \tag{6}$$

"~" means proportional.

In the far field, the electric field and magnetic field are in phase. We know that the microwave antenna should be placed in the microwave anechoic chamber when measuring the pattern or other parameters of the microwave antenna. Very good electromagnetic wave absorbing materials must be placed on the wall of the microwave anechoic chamber. Only in this way can good measurement results be obtained. The wall with absorbing material is placed to make the electromagnetic wave radiated by the antenna meet the boundary conditions of the Sliver-Muller as much as possible. When we solve the same electromagnetic problem with Helmholtz equation, there are similar radiation conditions Sommerfeld radiation conditions. This condition, like the boundary condition of the Sliver-Muller, requires that the boundary of the universe be filled with absorbing materials, as shown in the figure 0.

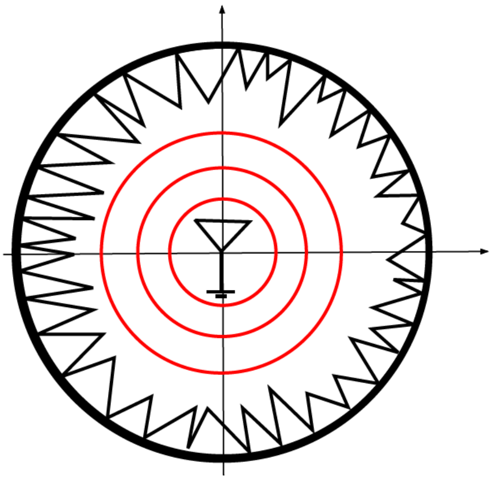

Figure 0: The Microwave Antenna Needs to Be Placed in The Microwave Anechoic Chamber For Measurement

2.2. New Boundary Conditions

We know that nothing can overflow the universe, here this author says that radiation cannot overflow the universe, that means,

$$\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot d\Gamma = 0. \tag{7}$$

Where Γ is an arbitrary surface surrounding all current elements. $\oint_{\Gamma}$ is closed surface integral. $\boldsymbol{E}$ is the radiated electric field, and $\boldsymbol{H}$ is the radiated magnetic field. A special case is that Γ is a sphere with radius R, and the radius R of the sphere tends to infinity,

$$\lim_{R\to\infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot d\Gamma = 0. \tag{8}$$

The above formula is obviously valid for magnetic quasi-static electromagnetic fields, or quasi-static electromagnetic fields, because at this time,

$$\lim_{R\to\infty} \boldsymbol{E} \sim \frac{1}{R^2}, \tag{9}$$

$$\lim_{R\to\infty} \boldsymbol{H} \sim \frac{1}{R^2}, \tag{10}$$

$$\lim_{R\to\infty} \boldsymbol{E} \times \boldsymbol{H} \sim \frac{1}{R^4}, \tag{11}$$

$$\lim_{R\to\infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot d\Gamma \sim \lim_{R\to\infty} \frac{1}{R^4} R^2 = \lim_{R\to\infty} \frac{1}{R^2} = 0. \tag{12}$$

"~"means proportional. We know that according to Maxwell's electromagnetic theory,

$$\lim_{R\to\infty} \boldsymbol{E} \sim \frac{1}{R} \tag{13}$$

$$\lim_{R\to\infty} \boldsymbol{H} \sim \frac{1}{R} \tag{14}$$

therefore

$$\lim_{R\to\infty} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E} \times \boldsymbol{H}) \cdot d\Gamma \sim \lim_{R\to\infty} \frac{1}{R^2} R^2 = \lim_{R\to\infty} 1 \neq 0 \tag{15}$$

Therefore, according to Maxwell's electromagnetic theory, the energy flow of electromagnetic radiation field is bound to overflow the universe. When the radiation satisfies the Sliver-Muller boundary condition, the above equation also exists. This means that electromagnetic radiation can flow out of the universe. However, this author believes that this is a misunderstanding of Maxwell's electromagnetic theory. Electromagnetic field radiation can not overflow the universe. this author puts forward a new law of electromagnetic field and adds the rule that radiation does not overflow the universe to the original classical electromagnetic theory as an axiom. We know that Maxwell's electromagnetic theory has just been solved. Now there is a new constraint. The problem will definitely be " overreached" . Overdetermination will lead to unsolved problems. However, we can relax the original computing system appropriately, so that Maxwell's classical electromagnetic theory can obtain a new degree of freedom through relaxation. In this way, our new axiom can be added. The new axiom can also be regarded as a new boundary condition.

It is worth mentioning that the corresponding boundary condition of Maxwell's electromagnetic radiation theory is the Sliver-Muller radiation boundary condition. This boundary condition actually means that the boundary of the universe is filled with materials that absorb electromagnetic waves. This is not always the case for boundary conditions in practice. Especially for photons of high-frequency radio waves, photons are always absorbed by an absorber charge, regardless of whether the cosmic boundary is full of absorber materials. Photons are only related to two current elements, one is the source of light, that is, the light source of photons, and the other is the sink, that is, the object that absorbs photons. Photons will be generated from the source and annihilated at the sink. The absorbing material on the boundary does not participate in the electromagnetic wave process of photons. Therefore, the Sliver-Muller condition is not suitable for electromagnetic systems containing a small number of photons.

In addition, if the light source or transmitting antenna is not placed in the microwave anechoic chamber, but in space, the boundary of the universe in space is still considered to be translucent, so this does not conform to Maxwell's electromagnetic theory and the Sliver-Muller boundary condition. Electromagnetic energy always overflows the universe this is not reasonable.

2.3. Mutual Energy Flow and Self Energy Flow Shall not Overflow The Universe

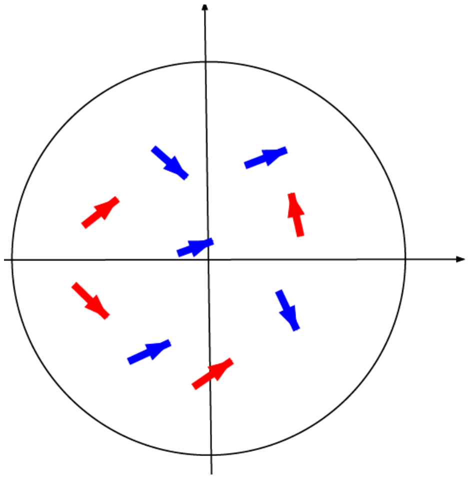

Figure 1: There are *N* Current Sources in The Space. The Red one Is The Radiation Source, Which Radiates The Retarded Wave, And The Blue one is The Sink, Which Radiates The Advanced Wave

We assume that there are *N* current elements $\boldsymbol{J}_i$, i=1,⋯N. See 1. The red arrow represents the source and the blue arrow represents the sink. The radiation source outputs electromagnetic energy, and the sink absorbs electromagnetic energy. Retarded wave radiated by the source, advanced wave radiated by the sink.

$$\boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{J}_i \tag{16}$$

The superposition principle tells us

$$\boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{E}_i, \cdots \boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{H}_i \tag{17}$$

Substitute (16, 17) into (7) to get,

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot d\Gamma = 0 \tag{18}$$

The above formula can be divided into two formulas

$$\sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot d\Gamma = 0 \tag{19}$$

$$\sum_{i=1}^{N} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot d\Gamma = 0 \tag{20}$$

The two formulas above add up to (18). (19) can be rewritten as

$$\sum_{i=1}^{N} \sum_{j=1}^{j<i} \int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot d\Gamma = 0 \tag{21}$$

We define,

$$\boldsymbol{S}_m = \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot d\Gamma \tag{22}$$

is mutual energy flow between $\boldsymbol{J}_i$ and $\boldsymbol{J}_j$. Formula (21) means that the mutual energy flow does not overflow the universe. Here Γ is a sphere with an infinite radius. Define,

$$\boldsymbol{S}_i = \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot d\Gamma \tag{23}$$

$\boldsymbol{S}_i$ is the self energy flow corresponding to current element $\boldsymbol{J}_i$. (20) means that the self energy flow cannot overflow the universe. Therefore, the boundary condition that radiation does not overflow the universe has now become two. Mutual energy flow does not overflow the universe and self energy flow does not overflow the universe.

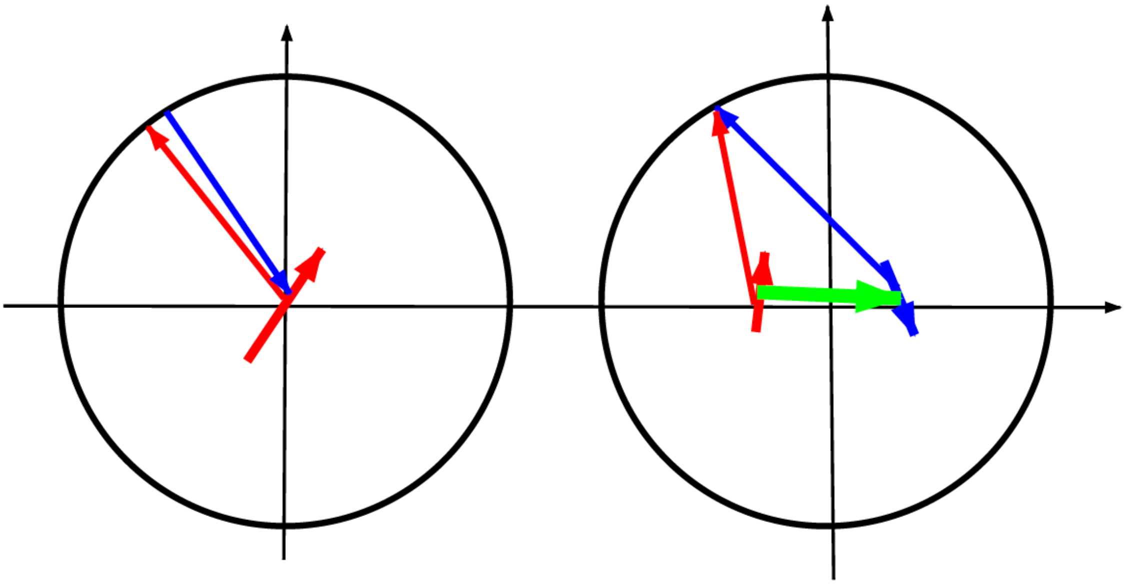

Figure 2: Assume that the Current on Circuit C_1,C_2 are I_1, I_2.

The figure 2 shows two cases. On the left is the self energy flow. If the self energy flow has a outward (red) radiation energy flow, there must be a time reversal energy flow, that is, a blue energy flow pointing to the current element. Therefore, although there is an outward radiating energy flow, this energy flow is offset by the inward energy flow. Maxwell's electromagnetic theory does not support electromagnetic waves with time reversal. But if the electromagnetic wave is reactive power, it will transfer energy in the positive direction and negative direction in one period, and the average energy transferred is zero. This view of point will be discussed in detail later in this article. On the right side of the figure 2 is mutual energy flow. In the case of mutual energy flow, there are at least two current elements. The red one is the source, which radiates the retarded wave, and the blue one is the sink, which generates the advanced wave. Green is the mutual energy flow from the source to the sink. The mutual energy flow will be further explained later in this paper. The radiation source includes the primary coil of the transformer, the transmitting antenna, and the radiator charge. The sink includes a secondary coil of the transformer, a receiving antenna, and an absorber charge. The retarded wave and the advanced wave will not reach the remote boundary at the same time, because the retarded wave will reach the surface Γ at some time in the future, and the advanced wave will reach the surface Γ at some time in the past, so the retarded wave and the advanced wave will not reach the surface Γ at the same time, so the mutual energy flow must be zero on the surface Γ. That is, (21) must be true.

The reader may ask, what if both current elements are radiation sources (or sink)? In this case, we can combine two sources into one source, and the situation on the left of the figure 2 has been met.

3. Neumann's Law of Electromagnetic Induction

Next, we will derive the electromagnetic field theory of quasi-static or magnetic quasi-static. Starting from Neumann's electromagnetic field theory. From Neumann's law of electromagnetic induction, we derive magnetic quasi-static, quasi-static electromagnetic theory. Then see how Maxwell developed his own theory of radiated electromagnetic fields from quasi-static electromagnetic fields. Here we say development, not derivation, because it is impossible to derive Maxwell's radiation electromagnetic field theory from the quasi-static electromagnetic field theory. The transition from quasi-static electromagnetic field to radiated electromagnetic field can only be achieved through genius's guess. The following is Faraday's law of electromagnetic induction proposed by Neumann. Quasi-static electromagnetic field is usually obtained from this as a starting point. The electromagnetic induction electromotive force is,

$$\mathcal{E}_{2,1} = -\frac{d}{dt}\frac{\mu_0}{4\pi}\oint_{C_2}\oint_{C_1}\frac{I_1 d\boldsymbol{l}_1\cdot d\boldsymbol{l}_2}{r} \tag{24}$$

$$r = \sqrt{||\boldsymbol{x}_2 - \boldsymbol{x}_1||}$$

The two circuits are shown in 3,

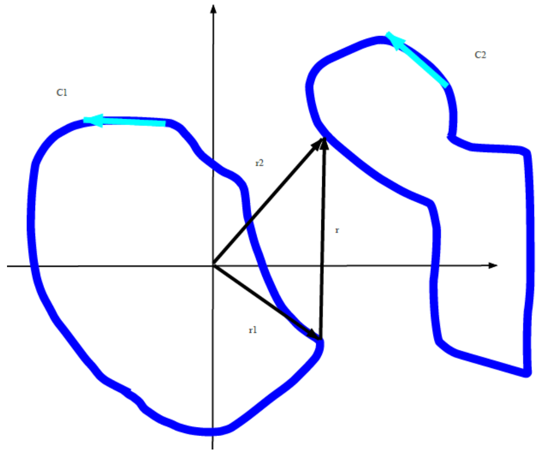

Figure 3: Two Circuits C_1,C_2 Assume that the Current on Circuit 1 is I_1, The Current on Circuit 2 is I_2

Define magnetic vector,

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi} \oint_{C_1} \frac{I_1 d\boldsymbol{l}_1}{r} \tag{25}$$

The induced electromotive force (24) can be written as,

$$\mathcal{E}_{2,1} = -\frac{d}{dt} \oint_{C_2} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{26}$$

Consider the transformation from line current to body current

$$\oint_{C_1} \cdots I_1 d\boldsymbol{l}_1 \rightarrow \iiint_V \cdots \boldsymbol{J}_1 dV \tag{27}$$

The vector potential can be rewritten as,

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi} \oint_{C_1} \frac{I_1 d\boldsymbol{l}_1}{r} \rightarrow \frac{\mu_0}{4\pi} \iiint_{V_1} \frac{\boldsymbol{J}_1}{r} dV \tag{28}$$

The definition of considering electromotive force is,

$$\mathcal{E}_{2,1} \equiv \oint_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_2 \tag{29}$$

Where $\boldsymbol{E}_1$ is the induced electric field generatedthe by the current $\boldsymbol{J}_1$. (24) can be rewritten as,

$$\oint_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_2 = -\frac{d}{dt} \oint_{C_2} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{30}$$

Assumption C_2 is It's a closed surface. The order of the differential and integral can be exchanged,

$$\oint_{C_2} \boldsymbol{E}_1 \cdot d\boldsymbol{l}_2 = -\oint_{C_2} \frac{\partial}{\partial t} \boldsymbol{A}_1 \cdot d\boldsymbol{l}_2 \tag{31}$$

Or,

$$\oint_{C_2} (\boldsymbol{E}_1 + \frac{\partial}{\partial t} \boldsymbol{A}_1) \cdot d\boldsymbol{l}_2 = 0 \tag{32}$$

Consider,

$$\oint_{C_2} (\nabla \psi_1) \cdot d\boldsymbol{l}_2 = 0 \tag{33}$$

ψ_1 is a arbitory function, hence there is,

$$\boldsymbol{E}_1 + \frac{\partial}{\partial t}\boldsymbol{A}_1 = \nabla\psi_1 \tag{34}$$

Assume,

$$\psi_1 = -\phi_1 \tag{35}$$

There is,

$$\boldsymbol{E}_1 = -\nabla\phi_1 - \frac{\partial}{\partial t}\boldsymbol{A}_1 \tag{36}$$

Define,

$$\boldsymbol{E}_1^{(I)} \equiv -\frac{\partial}{\partial t}\boldsymbol{A}_1 \tag{37}$$

as an induced electric field. The superscript "(I)" means induction. Define

$$\boldsymbol{E}_1^{(C)} \equiv -\nabla\phi_1 \tag{38}$$

as Coulomb electrostatic field. The superscript " (C)" means Coulomb. Define the magnetic field,

$$\boldsymbol{B}_1 \equiv \nabla \times \boldsymbol{A}_1 \tag{39}$$

as magnetic field. Thus

$$\nabla \cdot \boldsymbol{B}_1 = 0 \tag{40}$$

$$\nabla \times \boldsymbol{E}_1 = \nabla \times (-\nabla\phi_1 - \frac{\partial}{\partial t}\boldsymbol{A}_1) = (-\nabla \times \frac{\partial}{\partial t}\boldsymbol{A}_1) = (-\frac{\partial}{\partial t}\nabla \times \boldsymbol{A}_1) \tag{41}$$

The above equation considers the mathematical formula,

$$\nabla \times \nabla\phi_1 = 0 \tag{42}$$

Considering (39), we obtain,

$$\nabla \times \boldsymbol{E}_1 = -\frac{\partial}{\partial t}\boldsymbol{B}_1 \tag{43}$$

Find the divergence of the formula (38),

$$\nabla \cdot \boldsymbol{E}_1^{(C)} = -\nabla \cdot \nabla\phi_1 = -\nabla^2\phi_1 \tag{44}$$

According to the electrostatic electromagnetic theory ϕ_1 meet Poisson equation,

$$\nabla^2\phi_1 = -\frac{\rho_1}{\epsilon_0} \tag{45}$$

Thus,

$$\nabla \cdot (\epsilon_0 \boldsymbol{E}_1^{(C)}) = \rho_1 \tag{46}$$

Consider,

$$\boldsymbol{A}_1 = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}_1}{r} dV \tag{47}$$

Take two curls to get,

$$\nabla \times \nabla \times \boldsymbol{A}_1 = \nabla(\nabla \cdot \boldsymbol{A}_1) - \nabla^2\boldsymbol{A}_1 \tag{48}$$

Consider,

$$\nabla \cdot \boldsymbol{A}_1 = \frac{\mu_0}{4\pi} \nabla \cdot \iiint_V \frac{\boldsymbol{J}_1}{r} dV$$

$$= \frac{\mu_0}{4\pi} \iiint_V \nabla \frac{1}{r} \cdot \boldsymbol{J}_1 dV$$

$$= -\frac{\mu_0}{4\pi} \iiint_V \nabla' \frac{1}{r} \cdot \boldsymbol{J}_1 dV \tag{49}$$

Among them, the following is considerred,

$$\nabla \frac{1}{r} = -\nabla' \frac{1}{r} \tag{50}$$

$$\iiint_V \nabla' \cdot (\frac{1}{r} \boldsymbol{J}_1) dV = \iiint_V (\nabla' \frac{1}{r} \cdot \boldsymbol{J}_1) dV + \iiint_V (\frac{1}{r} \nabla' \cdot \boldsymbol{J}_1) dV \tag{51}$$

The right side of the above equation,

$$\iiint_V \nabla' \cdot (\frac{1}{r} \boldsymbol{J}_1) dV = \oint_\Gamma (\frac{1}{r} \boldsymbol{J}_1) \cdot \hat{n} d\Gamma = 0$$

It is considered above that Γ is taken outside ***J***, so the above formula is 0. So we have,

$$-\iiint_V (\nabla' \frac{1}{r} \cdot \boldsymbol{J}_1) dV = \iiint_V (\frac{1}{r} \nabla' \cdot \boldsymbol{J}_1) dV \tag{52}$$

Hence,

$$\nabla \cdot \boldsymbol{A}_1 = -\frac{\mu_0}{4\pi} \iiint_V \nabla' \frac{1}{r} \cdot \boldsymbol{J}_1 dV = \frac{\mu_0}{4\pi} \iiint_V (\frac{1}{r} \nabla' \cdot \boldsymbol{J}_1) dV \tag{53}$$

Considering the continuity equation of current,

$$\nabla' \cdot \boldsymbol{J}_1 = -\frac{\partial}{\partial t} \rho_1 \tag{54}$$

We get

$$\nabla \cdot \boldsymbol{A}_1 = -\frac{\mu_0}{4\pi} \iiint_V (\frac{1}{r} \frac{\partial}{\partial t} \rho_1) dV \tag{55}$$

Or,

$$\nabla \cdot \boldsymbol{A}_1 = -\frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \iiint_V (\frac{1}{r} \rho_1) dV \tag{56}$$

Or,

$$\nabla \cdot \boldsymbol{A}_1 = -\mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi_1 \tag{57}$$

The above formula is the Lorenz gauge. Where it is considerred the definition,

$$\phi_1 \equiv \frac{1}{4\pi\epsilon_0} \iiint_V (\frac{1}{r} \rho_1) dV \tag{58}$$

Further consider,

$$\nabla^2 \boldsymbol{A}_1 = \nabla^2 \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}_1}{r} dV = \mu_0 \iiint_V \nabla^2 (\frac{1}{4\pi r}) \boldsymbol{J}_1 dV$$

$$= \mu_0 \iiint_V (-\delta(\boldsymbol{x} - \boldsymbol{x}')) \boldsymbol{J}_1(\boldsymbol{x}') dV \tag{59}$$

It has taken into account that,

$$\nabla^2 (\frac{1}{4\pi r}) = -\delta(\boldsymbol{x} - \boldsymbol{x}') \tag{60}$$

Hence,

$$\nabla^2 \boldsymbol{A}_1 = -\mu_0 \boldsymbol{J}_1 \tag{61}$$

Since,

$$\nabla \times \nabla \times \boldsymbol{A}_1 = \nabla(\nabla \cdot \boldsymbol{A}_1) - \nabla^2 \boldsymbol{A}_1 \tag{62}$$

Consider (57, 61),

$$\nabla \times \nabla \times \boldsymbol{A}_1 = \nabla(-\mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi_1) + \mu_0 \boldsymbol{J}_1 \tag{63}$$

Or,

$$\nabla \times \boldsymbol{B}_1 = \nabla(-\mu_0 \epsilon_0 \frac{\partial}{\partial t} \phi_1) + \mu_0 \boldsymbol{J}_1 \tag{64}$$

Or,

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \frac{\partial}{\partial t}(-\epsilon_0 \nabla \phi_1) \tag{65}$$

Or,

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \frac{\partial}{\partial t} \epsilon_0 \boldsymbol{E}_1^{(C)} \tag{66}$$

Where,

$$\boldsymbol{E}_1^{(C)} = -\nabla \phi_1 \tag{67}$$

3.1. Quasi Static Electromagnetic Field

From this we get the Gauss law (46),

$$\nabla \cdot \boldsymbol{E}_1^{(C)} = \frac{\rho_1}{\epsilon_0} \tag{68}$$

Faraday's law (43),

$$\nabla \times \boldsymbol{E}_1 = -\frac{\partial}{\partial t} \boldsymbol{B}_1 \tag{69}$$

Magnetic Gauss law (40),

$$\nabla \cdot \boldsymbol{B}_1 = 0 \tag{70}$$

Ampere's circuital law (66)

$$\nabla \times \boldsymbol{H}_1 = \boldsymbol{J}_1 + \frac{\partial}{\partial t}(\epsilon_0 \boldsymbol{E}_1^{(C)}) \tag{71}$$

In fact, we can also get,

$$\nabla \cdot \boldsymbol{E}_2^{(C)} = \frac{\rho_2}{\epsilon_0} \tag{72}$$

Faraday's law

$$\nabla \times \boldsymbol{E}_2 = -\frac{\partial}{\partial t} \boldsymbol{B}_2 \tag{73}$$

Magnetic Gauss law

$$\nabla \cdot \boldsymbol{B}_2 = 0 \tag{74}$$

Ampere's circuital law

$$\nabla \times \boldsymbol{H}_2 = \boldsymbol{J}_2 + \frac{\partial}{\partial t}(\epsilon_0 \boldsymbol{E}_2^{(C)}) \tag{75}$$

subscript 1 and subscript 2 can be removed or kept in mind. In this way, we can write as,

$$\nabla \cdot \boldsymbol{E}^{(C)} = \frac{\rho}{\epsilon_0} \tag{76}$$

Faraday's law,

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{77}$$

Magnetic Gauss law,

$$\nabla \cdot \boldsymbol{B} = 0 \tag{78}$$

Ampere's circuital law,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}(\epsilon_0 \boldsymbol{E}^{(C)}) \tag{79}$$

The above equation is the electromagnetic field equation under quasi-static condition. Although the above four formulas have no subscript, we should note that they actually have subscript 1 or subscript 2 in the derivation process. These subscripts are omitted. In fact, these subscrits always exist. If subscript 1 is the primary coil, subscript 2 represents the secondary coil. The above four formulas are the equations that the primary coil and the secondary coil meet respectively. These equations are very similar to Maxwell's equations, but they are not Maxwell's equations, but quasi-static equations of electromagnetic field. The quasi-static equation is suitable for the circuit system with capacitors and inductors.

3.2. Magnetic Quasistatic Equation

In the magnetic quasi-static case, the above formula can be simplified as,

$$\nabla \cdot \boldsymbol{E}^{(C)} = \frac{\rho}{\epsilon_0} \tag{80}$$

Faraday's Law:

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{81}$$

Gauss law of magnetic field:

$$\nabla \cdot \boldsymbol{B} = 0 \tag{82}$$

Ampere circuital law:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \tag{83}$$

Above is the equation of magnetic quasi-static electromagnetic field. Magnetic quasi-static is suitable for circuit systems without capacitors and only with inductors and resistance.

3.3. Maxwell's Equation

Above are the electric and magnetic field equations under quasi-static and magnetic quasi-static conditions. From the electromagnetic equation under two static conditions to Maxwell's equation is not a derivation process, but a guessing process. We know that the earliest definition of electric field is,

$$\boldsymbol{E} = \boldsymbol{E}^{(C)} = -\nabla\phi \tag{84}$$

With Faraday's Law, Maxwell defined a new electric field

$$\boldsymbol{E} = \boldsymbol{E}^{(C)} + \boldsymbol{E}^{(I)} \tag{85}$$

Among them,

$$\boldsymbol{E}^{(I)} = -\frac{\partial}{\partial t}\boldsymbol{A} \tag{86}$$

$E^{(I)}$ is an induced electric field. So as far as the definition of electric field is concerned,

$$\boldsymbol{E}^{(C)} \rightarrow \boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{87}$$

We have transited from the Coulomb electric field to the total electric field. For this reason, we made the transformation (87) as above (76, 79), and obtain,

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{88}$$

Faraday's law remains unchanged

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{89}$$

Magnetic Gauss law remains unchanged

$$\nabla \cdot \boldsymbol{B} = 0 \tag{90}$$

We obtain Maxwell-Ampere's circuital law,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{E} \tag{91}$$

Note that in the above equation

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \tag{92}$$

In this way, a new set of equations (88-91) is obtained after transformation. This new set of equations is Maxwell's equation. According to this new set of equations, the theory of electromagnetic radiation is included. In classical electromagnetic theory, the new set of equations and the electromagnetic field obtained under quasi-static conditions are usually regarded as electromagnetic fields, so the same symbol is used. In fact, this has caused confusion. No matter whether they are electromagnetic fields or not, they use completely different forms of mathematical equations, so the solutions are completely different in mathematics. In order to distinguish them, this author represents Maxwell's radiated electromagnetic field with lowercase letters, so (88-91) is rewritten as,

Gauss law for electric field,

$$\nabla \cdot \boldsymbol{e} = \frac{\rho}{\epsilon_0} \tag{93}$$

Faraday's law,

$$\nabla \times \boldsymbol{e} = -\frac{\partial}{\partial t}\boldsymbol{b} \tag{94}$$

Magnetic Gauss law,

$$\nabla \cdot \boldsymbol{b} = 0 \tag{95}$$

Maxwell-Ampere's circuital law,

$$\nabla \times \boldsymbol{h} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{e} \tag{96}$$

This author uses capital letters

$$\boldsymbol{E}, \boldsymbol{H} \tag{97}$$

to represents quasistatic and magnetic quasistatic electromagnetic fields. Use lowercase letters

$$\boldsymbol{e}, \boldsymbol{h} \tag{98}$$

to represent the radiated electromagnetic field that satisfies Maxwell's equation. In the following chapter, this author will further explain that these two electromagnetic fields are very different in nature. Indeed, they should not be confused.

4. Poynting's Theorem and Energy

4.1. For Quasistatic Electromagnetic Fields

Considering (88-91),

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \nabla \times \boldsymbol{E} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{H}$$

$$= (-\frac{\partial}{\partial t}\boldsymbol{B}) \cdot \boldsymbol{H} - \boldsymbol{E} \cdot (\boldsymbol{J} + \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}^{(C)}) \tag{99}$$

Hence,

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = -\boldsymbol{E} \cdot \boldsymbol{J} - \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} - \boldsymbol{E} \cdot \epsilon_0 \frac{\partial}{\partial t}\boldsymbol{E}^{(C)} \tag{100}$$

Poynting's theorem is,

$$\oint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = -\iiint (\boldsymbol{E} \cdot \boldsymbol{J} + \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B} + \boldsymbol{E} \cdot \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}^{(C)}) dV \tag{101}$$

The magnetic field energy term is,

$$\iiint (\boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B}) dV = \frac{\partial}{\partial t} \iiint (\frac{1}{2}\boldsymbol{H} \cdot \boldsymbol{B}) dV \tag{102}$$

Therefore, the energy of the magnetic field is

$$U_M = \iiint (\frac{1}{2}\boldsymbol{H} \cdot \boldsymbol{B}) dV \tag{103}$$

The subscript " M" indicates the magnetic field, and the electric field energy is,

$$\iiint (\boldsymbol{E} \cdot \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}^{(C)}) dV = \iiint (\boldsymbol{E}^{(I)} + \boldsymbol{E}^{(C)}) \cdot \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}^{(C)}) dV$$

$$= \iiint (\boldsymbol{E}^{(I)} \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}^{(C)}) dV + \frac{\partial}{\partial t} \iiint (\frac{1}{2}\boldsymbol{E}^{(C)} \cdot \epsilon_0 \boldsymbol{E}^{(C)}) dV \tag{104}$$

The superscript " I" indicates inductive, " C" indicates coulomb electric field, and the energy of coulomb electric field is

$$U_E = \iiint (\frac{1}{2}\boldsymbol{E}^{(C)} \cdot \epsilon_0 \boldsymbol{E}^{(C)}) dV \tag{105}$$

For hybrid term,

$$\iiint (\boldsymbol{E}^{(I)} \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}^{(C)}) dV \tag{106}$$

This term may not generate energy. There are two possibilities, which are not energy but orthogonal in mathematics,

$$\iiint (\boldsymbol{E}^{(I)} \frac{\partial}{\partial t}\epsilon_0 \boldsymbol{E}^{(C)}) dV = 0. \tag{107}$$

Another possibility is that although the above integral is not zero, in physics, the above quantity does not constitute energy. Another possibility is that the above quantities do constitute an energy, a mixed energy. However, although this author did not understand this point, it is not the focus of this article. We went on down.

4.2. For Magnetic Quasi-Static Electromagnetic Fields

Poynting's theorem is,

$$\oint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = -\iiint (\boldsymbol{E} \cdot \boldsymbol{J} + \boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B}) dV \tag{108}$$

Under magnetic quasi-static conditions, there are only magnetic field energy items,

$$\iiint (\boldsymbol{H} \cdot \frac{\partial}{\partial t}\boldsymbol{B}) dV = \frac{\partial}{\partial t} \iiint \frac{1}{2}(\boldsymbol{H} \cdot \boldsymbol{B}) dV$$

So the magnetic field energy is,

$$U_m = \iiint \frac{1}{2}(\boldsymbol{H} \cdot \boldsymbol{B}) dV$$

Note the induced electric field,

$$\boldsymbol{E}^{(I)} = \frac{\partial}{\partial t}\boldsymbol{A}$$

Although ∂/∂t A is not 0, no energy is generated under magnetic quasi-static conditions. This is very important. The induced electric field does not generate electric field energy under the magnetic quasi-static condition. If you don't understand this point, please refer to another paper of this author [20].

4.3. Radiation Electromagnetic Field of Maxwell

$$\nabla \cdot (\boldsymbol{e} \times \boldsymbol{h}) = \nabla \times \boldsymbol{e} \cdot \boldsymbol{h} - \boldsymbol{e} \cdot \nabla \times \boldsymbol{h} \tag{109}$$

$$= (-\frac{\partial}{\partial t}\boldsymbol{b}) \cdot \boldsymbol{b} - \boldsymbol{e} \cdot (\boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{d}) \tag{110}$$

$$\nabla \cdot (\boldsymbol{e} \times \boldsymbol{h}) = -\boldsymbol{e} \cdot \boldsymbol{J} - \boldsymbol{h} \cdot \frac{\partial}{\partial t}\boldsymbol{b} - \boldsymbol{e} \cdot \frac{\partial}{\partial t}\boldsymbol{d} \tag{111}$$

Poynting's theorem is,

$$\oint_{\Gamma} (\boldsymbol{e} \times \boldsymbol{h}) \cdot \hat{n} d\Gamma = -\iiint (\boldsymbol{e} \cdot \boldsymbol{J} + \boldsymbol{h} \cdot \frac{\partial}{\partial t}\boldsymbol{b} + \boldsymbol{e} \cdot \frac{\partial}{\partial t}\boldsymbol{d}) dV \tag{112}$$

The magnetic field items are,

$$\iiint (\boldsymbol{h} \cdot \frac{\partial}{\partial t}\boldsymbol{b}) dV = \frac{\partial}{\partial t}\iiint (\frac{1}{2}\boldsymbol{h} \cdot \boldsymbol{b}) dV \tag{113}$$

So the magnetic field energy is,

$$U_m = \iiint (\frac{1}{2}\boldsymbol{h} \cdot \boldsymbol{b}) dV \tag{114}$$

The electric field term is,

$$\iiint (\boldsymbol{e} \cdot \frac{\partial}{\partial t}\boldsymbol{d}) dV = \frac{\partial}{\partial t}\iiint (\frac{1}{2}\boldsymbol{e} \cdot \boldsymbol{d}) dV \tag{115}$$

$$U_e = \iiint (\frac{1}{2}\boldsymbol{e} \cdot \boldsymbol{d}) dV = \iiint (\frac{1}{2}(\boldsymbol{e}^{(I)} + \boldsymbol{e}^{(C)}) \cdot \epsilon_0(\boldsymbol{e}^{(I)} + \boldsymbol{e}^{(C)})) dV$$

$$= \frac{1}{2}\epsilon_0 \iiint (e^{(I)} \cdot \boldsymbol{e}^{(I)} + 2\boldsymbol{e}^{(C)} \cdot \boldsymbol{e}^{(I)} + \boldsymbol{e}^{(C)} \cdot \boldsymbol{e}^{(C)}) dV \tag{116}$$

In the case of Maxwell radiation electromagnetic field, the energy of electric field is more than that of induction electric field,

$$\frac{1}{2}\epsilon_0 \iiint (e^{(I)} \cdot \boldsymbol{e}^{(I)}) dV = \frac{1}{2}\epsilon_0 \iiint (\frac{\partial}{\partial t}\boldsymbol{A} \cdot \frac{\partial}{\partial t}\boldsymbol{A}) dV \tag{117}$$

The above equation is the energy of the induced electric field, which is stored in the electromagnetic wave. Another energy flow,

$$\frac{1}{2}\epsilon_0 \iiint (\boldsymbol{e}^{(C)} \cdot \boldsymbol{e}^{(C)}) dV \tag{118}$$

It is stored in a capacitor. As for mixed energy,

$$\epsilon_0 \iiint (\boldsymbol{e}^{(C)} \cdot \boldsymbol{e}^{(I)}) dV \tag{119}$$

It may or may not be zero. This author is not clear about this part of energy.

4.4. Energy of Induced Electric Field

Earlier we found that for Maxwell's radiation electromagnetic field theory,

$$\frac{1}{2}\epsilon_0 \iiint (\boldsymbol{e}^{(I)}\cdot\boldsymbol{e}^{(I)})dV = \frac{1}{2}\epsilon_0 \iiint (\frac{\partial}{\partial t}\boldsymbol{A}\cdot\frac{\partial}{\partial t}\boldsymbol{A})dV \tag{120}$$

It is the energy of induced electric field, but it can also be calculated for quasi-static electric field,

$$\frac{1}{2}\epsilon_0 \iiint (\boldsymbol{E}^{(I)}\cdot\boldsymbol{E}^{(I)})dV = \frac{1}{2}\epsilon_0 \iiint (\frac{\partial}{\partial t}\boldsymbol{A}\cdot\frac{\partial}{\partial t}\boldsymbol{A})dV \tag{121}$$

Although the above equation (121) is not zero in calculation, its energy does not appear in the corresponding Poynting's law (101, 108), so it does not participate in the calculation of energy. But (120) appears in Poynting's theorem (112) corresponding to Maxwell's equation. This means

$$\boldsymbol{e}, \boldsymbol{h} \tag{122}$$

and

$$\boldsymbol{E}, \boldsymbol{H} \tag{123}$$

They are really two different kinds of electromagnetic fields! They have different energy expressions. Therefore, it is completely correct for this author to distinguish them with two different symbols. In the classical electromagnetic field theory, it is often considered that the Maxwell radiation electromagnetic field equation is an " accurate" electromagnetic field equation, and the magnetic quasi-static electromagnetic field equation is an approximate electromagnetic field equation. Therefore, the energy of the induced electric field appearing in the Maxwell electromagnetic theory is also the energy of the magnetic quasi-static electromagnetic field. This leads to the energy paradox of the induced electric field, and this author specifically discusses this problem [20].

5. Whether can Degenerate into Quasistatic Field

For the solution of Maxwell equation, the magnetic vector potential and scalar potential can be written in the form of retarded potential,

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV \tag{124}$$

$$\phi = \frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV \tag{125}$$

$\boldsymbol{A}$ and ϕ have a time factor $\exp(j\omega t)$. This time factor is omitted in the above formula. The induced electric field can be written as,

$$\boldsymbol{e}^{(I)} = -\frac{\partial}{\partial t}\boldsymbol{A} = -j\omega\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV \tag{126}$$

The electrostatic Coulomb electric field can be written as,

$$\begin{aligned}\boldsymbol{e}^{(C)} &= -\nabla\phi = -\nabla\frac{1}{4\pi\epsilon_0}\iiint_V \frac{\rho}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV\\ &= -\frac{1}{4\pi\epsilon_0}\iiint_V \nabla(\frac{1}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}))\rho dV\\ &= -\frac{1}{4\pi\epsilon_0}\iiint_V (\nabla(\frac{1}{r})\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})) + \frac{1}{r}\nabla\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}))\rho dV\\ &= -\frac{1}{4\pi\epsilon_0}\iiint_V (-\frac{\boldsymbol{r}}{r^3}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})) + \frac{-j\boldsymbol{k}}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}))\rho dV\end{aligned} \tag{127}$$

The magnetic field is

$$\begin{aligned}\boldsymbol{b} &= \nabla\times\boldsymbol{A} = \nabla\times\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV\\ &= \frac{\mu_0}{4\pi}\iiint_V \nabla(\frac{1}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}))\times\boldsymbol{J}dV\\ &= \frac{\mu_0}{4\pi}\iiint_V (-\frac{\boldsymbol{r}}{r^3}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}) + \frac{-j\boldsymbol{k}}{r}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}))\times\boldsymbol{J}dV\end{aligned} \tag{128}$$

Consider a spatial scale *l, l* is the scale of this electromagnetic system, and suppose lambda is the wavelength of electromagnetic wave. consider,

$$l \ll \lambda \tag{129}$$

Or,

$$\frac{2\pi l}{\lambda} \ll 1 \tag{130}$$

Consider $k = \frac{2\pi}{\lambda}$, with

$$kl \ll 1 \tag{131}$$

This shows that there are,

$$|\boldsymbol{k} \cdot \boldsymbol{r}| \ll 1 \tag{132}$$

Or,

$$\boldsymbol{k} \cdot \boldsymbol{r} \to 0 \tag{133}$$

Or,

$$\exp(-j\boldsymbol{k} \cdot \boldsymbol{r}) \to 1 \tag{134}$$

Therefore, in this case (126-128),

$$\boldsymbol{e}^{(I)} = -j\omega \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} \exp(-j\boldsymbol{k} \cdot \boldsymbol{r}) dV \to -j\omega \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} dV = \boldsymbol{E}^{(I)} \tag{135}$$

$$\begin{aligned}\boldsymbol{e}^{(C)} &= -\frac{1}{4\pi\epsilon_0} \iiint_V (-\frac{\boldsymbol{r}}{r^3}\rho) + \frac{-j\boldsymbol{k}}{r}\rho)\exp(-j\boldsymbol{k} \cdot \boldsymbol{r}) dV \\ &\to -\frac{1}{4\pi\epsilon_0} \iiint_V (-\frac{\boldsymbol{r}}{r^3}\rho) + \frac{-j\boldsymbol{k}}{r}\rho) dV \\ &= \boldsymbol{E}^{(C)} + \frac{1}{4\pi\epsilon_0} \iiint_V \frac{j\boldsymbol{k}}{r}\rho dV \\ &\neq \boldsymbol{E}^{(C)}\end{aligned} \tag{136}$$

For magnetic fields,

$$\begin{aligned}\boldsymbol{b} &= \frac{\mu_0}{4\pi} \iiint_V (-\frac{\boldsymbol{r}}{r^3} \times \boldsymbol{J} + \frac{-j\boldsymbol{k}}{r} \times \boldsymbol{J})\exp(-j\boldsymbol{k} \cdot \boldsymbol{r}) dV \\ &\to \frac{\mu_0}{4\pi} \iiint_V (-\frac{\boldsymbol{r}}{r^3} \times \boldsymbol{J} + \frac{-j\boldsymbol{k}}{r} \times \boldsymbol{J}) dV \\ &= \boldsymbol{B} + \frac{\mu_0}{4\pi} \iiint_V \frac{-j\boldsymbol{k}}{r} \times \boldsymbol{J}) dV \\ &\neq \boldsymbol{B}\end{aligned} \tag{137}$$

It can be seen from the above formula that the induced electromagnetic field $\boldsymbol{e}^{(I)}$ of Maxwell's electromagnetic theory degenerates into a quasi-static induced electric field $\boldsymbol{E}^{(I)}$ when the wavelength is large. Maxwell's Coulomb electric field $\boldsymbol{e}^{(C)}$ cannot degenerate into a quasi-static electric field $\boldsymbol{E}^{(C)}$. Maxwell's radiated magnetic field ***b*** cannot be degenerated into a quasi-static magnetic field ***B***. Because Maxwell radiation electromagnetic field can not degenerate into quasi-static electromagnetic field. Maxwell's electromagnetic fields ***e, b*** and quasi-static electromagnetic fields ***E, B*** are indeed not the same electromagnetic fields. It is entirely correct for this author to distinguish them by different symbols. It is wrong to confuse these two kinds of electromagnetic fields in the text books of classical electromagnetic field theory.

6. Test Through the Law of Conservation of Energy

We should test Maxwell radiated electromagnetic fields and quasi static electromagnetic fields with the law of conservation of energy. Consider that there are N current elements in figure 4, some of which are sources and some are sinks. The red arrow in the figure shows the electromagnetic energy released from the radiation source. The blue ones are sinks, which receive electromagnetic energy. Suppose that N is huge, it has included all current elements of the whole universe.

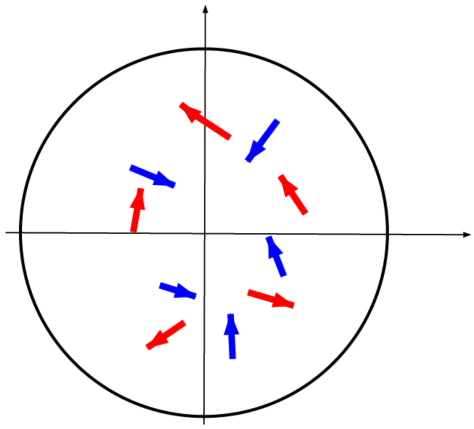

Figure 4: There are *N* Current Elements in the Space. The Red One Is The Source, And The Blue One is the Sink

We know that the energy conservation law of electromagnetic field is

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty}dt\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dV=0 \tag{138}$$

The above formula is the law of conservation of electromagnetic energy. This law of conservation of energy is self explanatory. Its establishment should be wordless. Because we can believe that there are only *N* charges in space. Of course *N* is big. If the energy one of the current elements $\boldsymbol{J}_j$ is lost, other charges will increase the same energy sooner or later, so the total energy will not change. Therefore, there is the above formula. The above law of conservation of energy can be proved from the quasi-static equation. However, it is impossible to prove the above formula by the radiating electromagnetic field from Maxwell. This shows that the electromagnetic theory represented by Maxwell's equation is different from the quasi-static electromagnetic theory. Let's prove it.

6.1. Prove The Law of Conservation of Energy From the Magnetic Quasi-Static Poynting Theorem

Poynting's law of magnetic quasi-static electromagnetic field can be derived from (81,83),

$$\oint_\Gamma(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma=-\iiint(\boldsymbol{J}\cdot\boldsymbol{E}+\boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B})dV \tag{139}$$

Considering the superposition principle (16, 17),

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\oint_\Gamma(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma=-\sum_{i=1}^{N}\sum_{j=1}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{E}_j+\boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_j)dV \tag{140}$$

Split into two formulas

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\oint_\Gamma(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma=-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{E}_j+\boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_j)dV \tag{141}$$

$$\sum_{j=1}^{N}\oint_\Gamma(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma=-\sum_{i=1}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{E}_i+\boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_i)dV \tag{142}$$

The sum of the above two formulas is (140). By integrating the above two equations with time,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty}dt\oint_\Gamma(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma=-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty}dt\iiint(\boldsymbol{J}_i\cdot\boldsymbol{E}_j+\frac{\partial}{\partial t}\boldsymbol{B}_j)dV \tag{143}$$

$$\sum_{j=1}^{N}\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i)\cdot \hat{n} d\Gamma = -\sum_{i=1}^{N}\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i + \boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_i) dV \quad (144)$$

To prove the law of conservation of energy of formula (138), we need to prove the following five formulas,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j)\cdot \hat{n} d\Gamma = 0 \quad (145)$$

$$\sum_{j=1}^{N}\int_{t=-\infty}^{\infty} dt \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i)\cdot \hat{n} d\Gamma = 0 \quad (146)$$

$$\sum_{i=1}^{N}\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i) dV = 0 \quad (147)$$

$$\sum_{i=1}^{N}\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_i) dV = 0 \quad (148)$$

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_j) dV = 0 \quad (149)$$

Proof (145), assuming $r \to \infty$,

$$\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j)\cdot \hat{n} d\Gamma \sim \frac{1}{r^4} r^2 = \frac{1}{r^2} \to 0 \quad (150)$$

(145) proof finished.
Proof (146). We know that for magnetic quasi-static electromagnetic fields, suppose $r \to \infty$,

$$\boldsymbol{E}_i \sim \frac{1}{r^2} \quad (151)$$

$$\boldsymbol{B}_i \sim \frac{1}{r^2} \quad (152)$$

$$\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i)\cdot \hat{n} d\Gamma \sim \frac{1}{r^4} r^2 = \frac{1}{r^2} \to 0 \quad (153)$$

(146) proof finished.
Prove (148),

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_i) dV = \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t}\iiint (\frac{1}{2}\boldsymbol{H}_i \cdot \boldsymbol{B}_i) dV$$

$$= \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} U = U(\infty) - U(-\infty)$$

$$= 0 - 0 = 0 \quad (154)$$

where,

$$U = \iiint (\frac{1}{2}\boldsymbol{H}_i \cdot \boldsymbol{B}_i) dV$$

$U(\infty)$ The process we discussed has ended, so $U(\infty)=0$. $U(-\infty)$ The process we discussed has not yet started, so $U(-\infty)=0$. The proof of (148) is finished.
Prove (149)

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_j) dV$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{j<i}\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{H}_i \cdot \frac{\partial}{\partial t}\boldsymbol{B}_j + \boldsymbol{H}_j \cdot \frac{\partial}{\partial t}\boldsymbol{B}_i) dV$$

$$= \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t}\sum_{i=1}^{N}\sum_{j=1}^{j<i}\iiint (\boldsymbol{H}_i \cdot \boldsymbol{B}_j) dV$$

$$= \int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} U$$

$$= U(\infty) - U(-\infty) = 0 \quad (155)$$

Where,

$$U = \sum_{i=1}^{N} \sum_{j=1}^{j<i} \iiint (\boldsymbol{H}_i \cdot \boldsymbol{B}_j) dV \quad (156)$$

The reasons for $U(\infty)$ and $U(-\infty)$ are the same as those above, both of which are 0. The proof of (149) is finished.

The following is the proof of (147),

$$\sum_{i=1}^{N} \int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i) dV$$

$$= \sum_{i=1}^{N} \int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i) dV$$

$$= -\sum_{i=1}^{N} \frac{\mu_0}{4\pi} \int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \frac{\partial}{\partial t} \iiint \frac{\boldsymbol{J}_i}{r} dV) dV$$

$$= -\sum_{i=1}^{N} \frac{\mu_0}{4\pi} \int_{t=-\infty}^{\infty} dt \iiint \iiint \frac{1}{r} \boldsymbol{J}_i \cdot \frac{\partial}{\partial t} \boldsymbol{J}_i dV' dV$$

$$= -\int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} \sum_{i=1}^{N} \frac{\mu_0}{4\pi} \iiint \iiint \frac{1}{2} \frac{1}{r} \boldsymbol{J}_i \cdot \boldsymbol{J}_i dV' dV$$

$$= -\int_{t=-\infty}^{\infty} dt \frac{\partial}{\partial t} U$$

$$= -[U(\infty) - U(-\infty)]$$

$$= 0 \quad (157)$$

Where the following is considerred,

$$\boldsymbol{E} = -\frac{\partial}{\partial t} \boldsymbol{A} = -\frac{\partial}{\partial t} \frac{\mu_0}{4\pi} \iiint \frac{\boldsymbol{J}}{r} dV \quad (158)$$

$$U = \sum_{i=1}^{N} \frac{\mu_0}{4\pi} \iiint \iiint \frac{1}{2} \frac{1}{r} \boldsymbol{J}_i \cdot \boldsymbol{J}_i dV' dV \quad (159)$$

The $U(\infty)$ process has ended, so there is, $U(\infty)=0$ The $U(-\infty)$ process has not started yet, so there is a formula $U(-\infty)=0$. The proof of (147) is finished.

Consider (145-149) in formula (143 and 144) and we get (138). Therefore, the law of energy conservation (138) of magnetic quasi-static electromagnetic field is satisfied.

6.2. Failed to Prove the Law of Conservation of Energy From Poynting Theorem of Maxwell Radiation Electromagnetic Field

Use a similar approach. Poynting's law of Maxwell radiated electromagnetic field is,

$$\oint_{\Gamma} (\boldsymbol{e} \times \boldsymbol{h}) \cdot \hat{n} d\Gamma = -\iiint (\boldsymbol{J} \cdot \boldsymbol{e} + \boldsymbol{e} \cdot \frac{\partial}{\partial t} \boldsymbol{d} + \boldsymbol{h} \cdot \frac{\partial}{\partial t} \boldsymbol{b}) dV \quad (160)$$

Considering the superposition principle,

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_j) \cdot \hat{n} d\Gamma = -\sum_{i=1}^{N} \sum_{j=1}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{e}_j + \boldsymbol{e}_i \cdot \frac{\partial}{\partial t} \boldsymbol{d}_j + \boldsymbol{h}_i \cdot \frac{\partial}{\partial t} \boldsymbol{b}_j) dV \quad (161)$$

The above formula can be divided into two formulas. After considering the time integration,

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_j) \cdot \hat{n} d\Gamma = -\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{e}_j + \boldsymbol{e}_i \cdot \boldsymbol{d}_j + \boldsymbol{h}_i \cdot \frac{\partial}{\partial t} \boldsymbol{b}_j) dV \tag{162}$$

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma = -\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{e}_i + \boldsymbol{e}_i \cdot \frac{\partial}{\partial t} \boldsymbol{d}_i + \boldsymbol{h}_i \cdot \boldsymbol{b}_i) dV \tag{163}$$

To prove the law of conservation of energy, we need to prove that,

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma = 0 \tag{164}$$

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{e}_i) = 0 \tag{165}$$

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \iiint (\boldsymbol{e}_i \cdot \frac{\partial}{\partial t} \boldsymbol{d}_i + \boldsymbol{h}_i \cdot \frac{\partial}{\partial t} \boldsymbol{b}_i) dV = 0 \tag{166}$$

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_j) \cdot \hat{n} d\Gamma = 0 \tag{167}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \iiint (\boldsymbol{e}_i \cdot \frac{\partial}{\partial t} \boldsymbol{d}_j + \boldsymbol{h}_i \cdot \frac{\partial}{\partial t} \boldsymbol{b}_j) dV = 0 \tag{168}$$

But we know that the far-field characteristics of the radiated electromagnetic field satisfying Maxwell's equation are,

$$\boldsymbol{e}_i \sim \frac{1}{r} \tag{169}$$

$$\boldsymbol{b}_i \sim \frac{1}{r} \tag{170}$$

$$\oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma \rightarrow \frac{r^2}{r^2} \neq 0 \tag{171}$$

Therefore

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma \neq 0 \tag{172}$$

We know that for Maxwell's radiated electromagnetic field, there is always a Poynting energy flow that is not zero. We know from the calculation of any antenna that the above formula is not zero. In fact, the Sliver-Muller radiation condition also tells us this. We will not continue to verify other formulas. As long as (164) is not zero, we cannot prove the law of conservation of energy (138). But we should write this formula in lowercase letters,

$$\sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{e}_i \cdot \boldsymbol{J}_j) dV = 0 \tag{173}$$

The above proof shows that the above law of conservation of energy does not hold true for Maxwell's radiated electromagnetic field! It is worth mentioning that the above formula is still valid, but it can only be used as an energy theorem, not a law of conservation of energy. To prove the law of conservation of energy, it must be proved that (164-168) are satisfied. But if the above equation is proved to be the energy theorem, we only need to prove (167-168). This is because if we want to proved that the above equation is an energy theorem, we can get (162) by subtracting (163) from (161). Since (161) is the Poynting energy theorem of N current elements, (163) is the sum of N Poynting theorems of the ith, and is also the energy theorem, their difference formula (162) is also the energy theorem. If (167-168) is satisfied, the formula (173) is obtained. Therefore (173) can be established as an energy theorem. So we proved it is a energy theorem.

The energy theorem is still much weaker than the law of conservation of energy. When we say it is the law of conservation of energy, it must contain all energy, but when $\oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot n\, d\Gamma \neq 0$, there are other energy items. $\sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{e}_i \cdot \boldsymbol{J}_j) dV$ does not contain all energy, so (173) is not a law of conservation of energy. It is only an energy theorem in Maxwell's theoretical system. This is why this author called it the mutual energy theorem in his 1987 paper [1, 2, 3].

We should understand that (138 and 173) are energy conservation laws. In Maxwell's electromagnetic theory, (173) is not energy conservation law, which proves that Maxwell's electromagnetic theory is missing.

6.3. History of The Law of Conservation of Energy

It is worth mentioning that the understanding of formulas (138,173) has a history of development. Let (N=2) in (138) get,

$$\sum_{i=1}^{2} \sum_{j=1,j\neq i}^{2} \int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV = 0 \tag{174}$$

Or

$$-\int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV = \int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV \tag{175}$$

The above is Welch's time-domain reciprocity theorem (1960) [4]. Welch's reciprocity theorem is a special case (τ=0) of th de Hoop's reciprocity theorem (at the end of 1987) [6],

$$-\int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t+\tau)) dV = \int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t+\tau)) dV \tag{176}$$

the Fourier transform of de Hoop reciprocity theorem is,

$$-\iiint_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV = \int_{t=-\infty}^{\infty} dt \iiint_V (\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1) dV \tag{177}$$

The above formula is Rumsey's new reciprocity theorem (1963), which is also the mutual energy theorem proposed by this author in early of 1987 [1-3, 5]. The above formula can be obtained from the Lorentz reciprocity theorem (1900-1930) [24, 22].

$$\iiint_V (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV = \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2) dV \tag{178}$$

Through conjugate transformation,

$$\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J} \rightarrow \boldsymbol{E}^*, -\boldsymbol{H}^*, -J^* \tag{179}$$

Conjugate transformation is a transformation preserving Maxwell's equation [23]. That is, if Maxwell equation is satisfied before transformation, Maxwell equation is still satisfied after transformation.

So this theorem was first discovered by Welch in 1960 as the reciprocity theorem. But in 1987, this author first redefined it as the energy theorem. It was not until 2017 that this author discovered that this mutual energy theorem is actually the law of conservation of energy [7]. That is to say, the formula (138, 173) is actually the law of conservation of energy. However, this law of conservation of energy cannot be proved from Maxwell's equation. This author thinks that Maxwell's classical electromagnetic theory is problematic on this point. In the past, this author always hoped to persuade readers to accept this author's point of view through the law of conservation of energy. Maxwell's electromagnetic theory has loopholes or bugs, but few readers agree with it. Now, this author continues to persuade readers in another way.

7. Revision of Maxwell's Electromagnetic Theory

This author introduces a new electromagnetic field law that radiation does not overflow the universe. This law can be regarded as a new axiom or a new boundary condition. However, Maxwell's electromagnetic radiation theory only allows for the Sliver-Muller radiation condition. Therefore, we need to properly revise Maxwell's electromagnetic theory.

7.1. Relaxation Process of Maxwell's Equation

We can see that Maxwell's radiation electromagnetic field and quasi-static electromagnetic field or magnetic quasi-static electromagnetic field are not really same electromagnetic fields, so can we properly modify Maxwell's electromagnetic theory to make it a new electromagnetic field, which is close to the original magnetic quasi-static electromagnetic field and quasi-static electromagnetic field, but is there the electromagnetic wave radiation property of Maxwell radiation electromagnetic field? In fact, it is possible. Poynting's law is,

$$\oint_\Gamma (\boldsymbol{e} \times \boldsymbol{h}) \cdot \hat{n} d\Gamma = -\iiint \left(\boldsymbol{J} \cdot \boldsymbol{e} + \boldsymbol{e} \cdot \frac{\partial}{\partial t} \boldsymbol{d} + \boldsymbol{h} \cdot \frac{\partial}{\partial t} \boldsymbol{b}\right) dV \tag{180}$$

Consider the superposition principle,

$$\boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{J}_i, \qquad \boldsymbol{e} = \sum_{i=1}^{N} \boldsymbol{e}_i, \qquad \boldsymbol{h} = \sum_{i=1}^{N} \boldsymbol{h}_i, \tag{181}$$

We have,

$$\sum_{i=1}^{N}\sum_{i=1}^{N}\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_j)\cdot\hat{n}d\Gamma=-\iiint(\boldsymbol{J}_i\cdot\boldsymbol{e}_j+\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_j+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_j)dV \tag{182}$$

Disassemble it into,

$$\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_j)\cdot\hat{n}d\Gamma=-\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{e}_j+\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_j+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_j)dV \tag{183}$$

$$\sum_{i=1}^{N}\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma=-\sum_{i=1}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{e}_i+\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_i+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_i)dV \tag{184}$$

Time integration for the above

$$\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_j)\cdot\hat{n}d\Gamma=-\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{e}_j+\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_j+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_j)dV \tag{185}$$

$$\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma=-\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{e}_i+\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_i+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_i)dV \tag{186}$$

Consider,

$$\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}\iiint(\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_j+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_j)dV$$

$$=\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\sum_{i=1}^{j<i}\iiint(\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_j+\boldsymbol{e}_j\cdot\frac{\partial}{\partial t}\boldsymbol{d}_i+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_j+\boldsymbol{h}_j\cdot\frac{\partial}{\partial t}\boldsymbol{b}_i)dV$$

$$=\int_{t=-\infty}^{\infty}dt\frac{\partial}{\partial t}\sum_{i=1}^{N}\sum_{i=1}^{j<i}\iiint(\boldsymbol{e}_i\cdot\boldsymbol{d}_j+\boldsymbol{h}_i\cdot\boldsymbol{b}_j)dV$$

$$=\int_{t=-\infty}^{\infty}dt\frac{\partial}{\partial t}U=U(\infty)-U(-\infty)=0 \tag{187}$$

Where,

$$U=\sum_{i=1}^{N}\sum_{i=1}^{j<i}\iiint(\boldsymbol{e}_i\cdot\boldsymbol{d}_j+\boldsymbol{h}_i\cdot\boldsymbol{b}_j)dV \tag{188}$$

The $U(\infty)$ process has ended, and all $U(\infty)=0$. $U(-\infty)$ process has not started, $U(-\infty)=0$.

$$\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\iiint(\boldsymbol{e}_i\cdot\frac{\partial}{\partial t}\boldsymbol{d}_i+\boldsymbol{h}_i\cdot\frac{\partial}{\partial t}\boldsymbol{b}_i)dV$$

$$=\int_{t=-\infty}^{\infty}dt\frac{\partial}{\partial t}\sum_{i=1}^{N}\iiint\frac{1}{2}(\boldsymbol{e}_i\cdot\boldsymbol{d}_i+\boldsymbol{h}_i\cdot\boldsymbol{b}_i)dV$$

$$\int_{t=-\infty}^{\infty}dt\frac{\partial}{\partial t}U=U(\infty)-U(-\infty)=0 \tag{189}$$

Where,

$$U=\sum_{i=1}^{N}\iiint\frac{1}{2}(\boldsymbol{e}_i\cdot\boldsymbol{d}_i+\boldsymbol{h}_i\cdot\boldsymbol{b}_i)dV \tag{190}$$

The $U(\infty)$ process has ended, and all $U(\infty)=0$. $U(-\infty)$ process has not started, $U(-\infty)=0$. Thus there is,

$$\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_j)\cdot\hat{n}d\Gamma=-\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\sum_{i=1,j\neq i}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{e}_j)dV \tag{191}$$

$$\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma=-\int_{t=-\infty}^{\infty}dt\sum_{i=1}^{N}\iiint(\boldsymbol{J}_i\cdot\boldsymbol{e}_i)dV \tag{192}$$

The above two equations are actually more relaxed than (183,184) because of the time integration. This relaxation allows us to replace ***e, h*** with ***E, H*** In this way we obtain,

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{i=1, j\neq i}^{N} \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma = -\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{i=1, j\neq i}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_j) dV \qquad (193)$$

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = -\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i) dV \qquad (194)$$

This author believes that the above equation is already a " real" equation of the radiation electromagnetic field, rather than Maxwell's equation of the radiation electromagnetic field. (193) is called the relaxed mutual energy principle. The corresponding (183) is the principle of mutual energy. We can also call the formula (194) the relaxed self energy principle. (184) is the principle of self energy.

7.2. Law of Conservation of Energy

Now we can add the electromagnetic field law that this author put forward earlier to the above equation (193 and 194). i.e., the radiation does not overflow the universe.

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \qquad (195)$$

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{i=1, j\neq i}^{N} \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma$$

$$= \int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{i=1}^{j<i} \oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma = 0 \qquad (196)$$

Where (195) means that the self energy flow cannot overflow the universe, and (196) means that the mutual energy flow cannot overflow the universe. If (195, 196) is substituted into (193, 194),

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \sum_{i=1, j\neq i}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_j) dV = 0 \qquad (197)$$

and

$$\int_{t=-\infty}^{\infty} dt \sum_{i=1}^{N} \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i) dV = 0 \qquad (198)$$

The sum sign of the above equation can be omitted,

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i) dV = 0 \qquad (199)$$

(197) is the law of conservation of energy, and (199) is the law of self energy and non radiation. Neither of these two laws can be derived from the radiation electromagnetic theory of Maxwell's equation. These two equations are obtained by first relaxing Maxwell's equation, and then adding this author's law that radiation does not overflow the universe.

It is worth mentioning that this relaxation process is very important. This relaxation process allows a redundant degree of freedom to be released, so that we can add the boundary condition that radiation does not overflow the universe. If a new boundary condition is added to the original Maxwell equation, it will definitely make the problem overdetermined, and there will be no solution.

7.3. Mutual Energy Flow Theorem or Energy Flow Theorem

It can be proved that the following mutual energy flow theorem,

$$-\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{e}_j) dV = \int_{t=-\infty}^{\infty} dt (\xi_i, \xi_j) = \int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_j \cdot \boldsymbol{e}_i) dV \qquad (200)$$

Where,

$$(\xi_i, \xi_j) \equiv \oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_j + \boldsymbol{e}_j \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma$$

Similarly, the relaxed mutual energy principle (193) can prove that the following energy flow law theorem,

$$-\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_j)dV = \int_{t=-\infty}^{\infty} dt(\xi_i, \xi_j) = \int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_j \cdot \boldsymbol{E}_i)dV \qquad (201)$$

Where,

$$(\xi_i, \xi_j) \equiv \oint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_j + \boldsymbol{E}_j \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma \qquad (202)$$

is the electromagnetic energy flow. Corresponding to Maxwell equation, since the self energy flow is not zero, (200) can only be an energy flow theorem. The corresponding formula (201) is an energy flow law because the self energy flow does not transfer energy. The formula (200) is used together with the law of conservation of energy (197). The above mutual energy flow theorem and energy flow law will not be proved here. This theorem has been proved many times in other papers of this author, such as [7]. Here we only emphasize that under the radiation electromagnetic field condition of Maxwell's equation, this formula is the mutual energy flow theorem. In this author's theoretical system, because Maxwell's theory has been modified, that is, the relaxation process, and after the relaxation, the law that radiation does not overflow the universe has been introduced, so we get the law of energy flow. With this energy flow law, the formula (197) is not only the law of energy conservation, but also the localized law of energy conservation. This law of conservation of energy has reached another level!

8.Verify the Principle of Self Energy

8.1. Verification Under Magnetic Quasi-Static Conditions

Earlier we got (199)

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{E}_i)dV = 0 \qquad (203)$$

If in the system of Maxwell's equations radiating electromagnetic fields, we know that,

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{e}_i)dV \neq 0 \qquad (204)$$

Then we still need to verify (203). In order to facilitate our work, we first omitted the corner mark i,

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J} \cdot \boldsymbol{E})dV = 0 \qquad (205)$$

In the frequency domain, the formula can be rewritten as,

$$\Re\left(\iiint (\boldsymbol{J}^* \cdot \boldsymbol{E})dV\right) = 0 \qquad (206)$$

$\Re$ is the real part. The superscript "*" indicates complex conjugation. If under magnetic quasi-static conditions,

$$\boldsymbol{E} = -\frac{d}{dt}\boldsymbol{A} = -j\omega\boldsymbol{A} = -j\omega\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV$$

Hence,

$$\iiint (\boldsymbol{J}^* \cdot \boldsymbol{E})dV = \iiint \left(\boldsymbol{J}^* \cdot \left(-j\omega\frac{\mu_0}{4\pi}\iiint_V \frac{\boldsymbol{J}}{r}dV'\right)\right)dV$$

$$= -j\omega\frac{\mu_0}{4\pi}\iiint_V \iiint_V \frac{\boldsymbol{J}^*(\boldsymbol{x})\cdot\boldsymbol{J}(\boldsymbol{x}')}{r}dV'dV$$

$$= -j\omega L I^* I \qquad (207)$$

Where,

$$L = \frac{1}{II^*}\frac{\mu_0}{4\pi}\iiint_V \iiint_V \frac{\boldsymbol{J}^*(\boldsymbol{x})\cdot\boldsymbol{J}(\boldsymbol{x}')}{r}dV'dV \qquad (208)$$

is the inductance of the system, which is a real number. Therefore, $\iiint (\boldsymbol{J}^*\cdot\boldsymbol{E})dV$ is a pure imaginary number. Therefore, there is (206) and there is (203). Proof is finished.

8.2. For Maxwell Radiated Electromagnetic Fields

Now let's take a look at the situation under Maxwell's electromagnetic theory. In Maxwell's electromagnetic theory, the vector potential becomes the retarded potential. Therefore,

$$\boldsymbol{e} = -j\omega \boldsymbol{A}^{(r)} = -j\omega \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV \tag{209}$$

The superscript "(r)" means retardation. $\boldsymbol{A}^{(r)}$ is the retarded potential.

$$\iiint (\boldsymbol{J}^* \cdot \boldsymbol{e})dV = \iiint (\boldsymbol{J}^* \cdot (-j\omega \frac{\mu_0}{4\pi} \iiint_V \frac{\boldsymbol{J}}{r} \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV'))dV$$

$$= -j\omega \frac{\mu_0}{4\pi} \iiint_V \iiint_V \frac{\boldsymbol{J}^*(\boldsymbol{x})\cdot\boldsymbol{J}(\boldsymbol{x}\prime)}{r} \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV'dV$$

$$= -j\omega L I^* I \tag{210}$$

Where,

$$L = \frac{1}{II^*}\frac{\mu_0}{4\pi} \iiint_V \iiint_V \frac{\boldsymbol{J}^*(\boldsymbol{x})\cdot\boldsymbol{J}(\boldsymbol{x}\prime)}{r} \exp(-j\boldsymbol{k}\cdot\boldsymbol{r})dV'dV \tag{211}$$

We will find that with the addition of the retardation factor exp($-j\boldsymbol{k}\cdot\boldsymbol{r}$), the system inductance is no longer a pure real number, so we have,

$$\Re(\iiint (\boldsymbol{J}^* \cdot \boldsymbol{e})dV) \neq 0 \tag{212}$$

Hence,

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J}_i \cdot \boldsymbol{e}_i)dV \neq 0 \tag{213}$$

The formula (204) has been proved. The self energy principle in Maxwell's electromagnetic theory is not satisfied.

8.3. This Author's Electromagnetic Theory

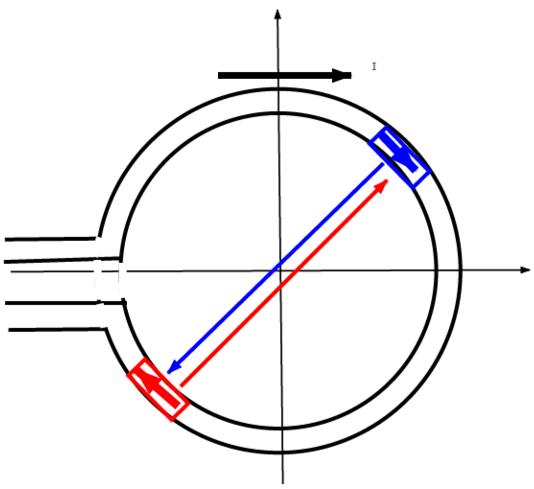

Figure 5: An Inductive Coil. There Are any two Current Elements Above

How to ensure the following formula in this author's radiation electromagnetic theory is true?

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J} \cdot \boldsymbol{E})dV = 0 \tag{214}$$

This author thinks that it is wrong to consider only the retarded wave in Maxwell's electromagnetic theory. In fact, the advanced wave always exists with the retarded wave. We know that Neumann's electromagnetic formula is,

$$\boldsymbol{E}_{2,1} = -\frac{d}{dt}\frac{\mu_0}{4\pi}\int_{C_2}\int_{C_1}\frac{I_1 d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r}$$

$$= -j\omega\frac{\mu_0}{4\pi}\int_{C_2}\int_{C_1}\frac{I_1 d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r}$$

$$= -j\omega L I_1 \quad (215)$$

Where,

$$L = \frac{\mu_0}{4\pi}\int_{C_2}\int_{C_1}\frac{d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2}{r} \quad (216)$$

Divide the inductance into small segments,

$$L = \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{\mu_0}{4\pi}\frac{\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}}{r}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{j<i}\frac{\mu_0}{4\pi}\frac{\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}+\Delta\boldsymbol{l}_{1j}\cdot\Delta\boldsymbol{l}_{2i}}{r} \quad (217)$$

Consider $\Delta\boldsymbol{l}_{1i}$ (red line segment in figure 5) give $\Delta\boldsymbol{l}_{2j}$ (blue line in figure 5) a retarded wave. In fact, $\Delta\boldsymbol{l}_{2j}$ will definitely give $\Delta\boldsymbol{l}_{1i}$ an advanced wave, so there must be

$$\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j} \to \Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}_{ij}) \quad (218)$$

$$\Delta\boldsymbol{l}_{1j}\cdot\Delta\boldsymbol{l}_{2i} \to \Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(+j\boldsymbol{k}\cdot\boldsymbol{r}_{ij}) \quad (219)$$

Of course, it may also be as follows:,

$$\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j} \to \Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(+j\boldsymbol{k}\cdot\boldsymbol{r}_{ij}) \quad (220)$$

$$\Delta\boldsymbol{l}_{1j}\cdot\Delta\boldsymbol{l}_{2i} \to \Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}_{ij}) \quad (221)$$

→ means the transition from the magnetic quasi-static electromagnetic system to this author's electromagnetic system. Hence there is,

$$L = \sum_{i=1}^{N}\sum_{j=1}^{j<i}\frac{\mu_0}{4\pi}\frac{\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}+\Delta\boldsymbol{l}_{1j}\cdot\Delta\boldsymbol{l}_{2i}}{r}$$

$$\to \sum_{i=1}^{N}\sum_{j=1}^{j<i}\frac{\mu_0}{4\pi}\frac{\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}_{ij})+\Delta\boldsymbol{l}_{1j}\cdot\Delta\boldsymbol{l}_{2i}\exp(+j\boldsymbol{k}\cdot\boldsymbol{r}_{ij})}{r}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{j<i}\frac{\mu_0}{4\pi}\frac{\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(+j\boldsymbol{k}\cdot\boldsymbol{r}_{ij})+\Delta\boldsymbol{l}_{1j}\cdot\Delta\boldsymbol{l}_{2i}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}_{ij})}{r}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{j<i}\frac{\mu_0}{4\pi}\frac{2\Re(\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}_{ij}))}{r}$$

$$= 2\Re\sum_{i=1}^{N}\sum_{j=1}^{j<i}\frac{\mu_0}{4\pi}\frac{(\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}_{ij}))}{r}$$

$$= \Re\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{\mu_0}{4\pi}\frac{(\Delta\boldsymbol{l}_{1i}\cdot\Delta\boldsymbol{l}_{2j}\exp(-j\boldsymbol{k}\cdot\boldsymbol{r}_{ij}))}{r} \quad (222)$$

It can be seen that the system inductance L is still a real number, and we still have it,

$$\iiint (\boldsymbol{J}^* \cdot \boldsymbol{E})dV \rightarrow \int_C \boldsymbol{E} \cdot d\boldsymbol{l}I^* = \mathcal{E}I = -j\omega LII^*$$

is a pure imaginary number, and hence,

$$\Re \iiint (\boldsymbol{J}^* \cdot \boldsymbol{E})dV = 0 \tag{223}$$

Thus we get (205). The proof is finished.
Although we have proved the above formula, some people may still be dissatisfied with our proof, so we provide a second proof.

$$\iiint (\boldsymbol{J}^* \cdot \boldsymbol{E})dV = \sum_{i=1}^{N} I_i^* \Delta \boldsymbol{l}_i \cdot \boldsymbol{E}(i) \tag{224}$$

$$\boldsymbol{E}(i) = \sum_{j=1, j\neq i}^{N} \boldsymbol{E}_j \tag{225}$$

$\boldsymbol{E}(i)$ is the electric field of current elements other than current element i. $\boldsymbol{E}_j$ Is current element $I_j \Delta \boldsymbol{l}_j$ Electric field of.

$$\iiint (\boldsymbol{J}^* \cdot \boldsymbol{E})dV = \sum_{i=1}^{N} I_i^* \Delta \boldsymbol{l}_i \cdot \sum_{j=1,j\neq i}^{N} \boldsymbol{E}_j = \sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} I_i^* \Delta \boldsymbol{l}_i \cdot \boldsymbol{E}_j \tag{226}$$

Considering the frequency domain formula of the law of conservation of energy (138),

$$\Re(\sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j^*)dV) = 0 \tag{227}$$

there is,

$$\Re \iiint (\boldsymbol{J}^* \cdot \boldsymbol{E})dV = \Re(\sum_{i=1}^{N} \sum_{j=1,j\neq i}^{N} I_i^* \Delta \boldsymbol{l}_i \cdot \boldsymbol{E}_j) = 0 \tag{228}$$

The formula (205) has been proved. In the formula (138), the current element is N separated current elements. In (226), the current elements are connected together, but they can still be regarded as N separate current elements. Therefore, the law of conservation of energy of formula (138) can be applied.

The formula (205) can be regarded as the principle of self energy, which has also been proved in this author's theoretical system.

In Maxwell's electromagnetic field theory system, the principle of self energy

$$\int_{t=-\infty}^{\infty} dt \iiint (\boldsymbol{J} \cdot \boldsymbol{e})dV = 0$$

is not true, which shows that Maxwell's electromagnetic theory is flawed.

9. Electromagnetic Wave of Plate Current of Maxwell's Theory

In this section, we solve the electromagnetic wave of infinite plane current according to Maxwell's classical electromagnetic theory. Many previous theories have been given, but readers may still not really understand the problem. Especially that mysterious relaxation process, is it really amazing that the original Maxwell electromagnetic theory can be relaxed, and just release a degree of freedom, and allow the addition of a new boundary condition? All this must be clearly shown to the reader in the following examples. The electromagnetic field of plate current, finite plate or infinite plate are good examples. Finally, this author will explain the mutual energy theory of electromagnetic field proposed by this author with double plate current.

9.1. Calculation of Retarded Wave According to Maxwell Radiation Electromagnetic Field

As shown in 6, the current is an infinite plate current. Retarded waves are generated on both sides of the plate current.

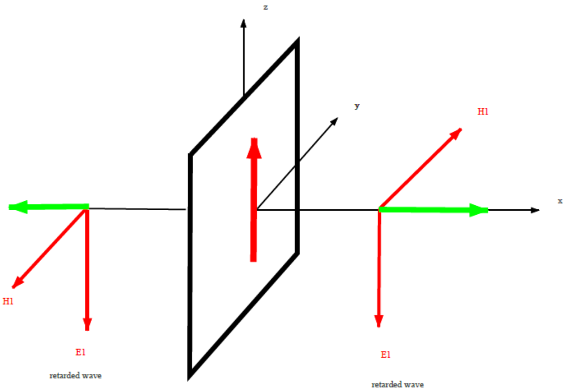

Figure 6: The Electromagnetic Retarded Wave of a Flat Plate Current

According to Maxwell's radiation electromagnetic theory, the current density is assumed to be,

$$\boldsymbol{J} = J_0\hat{z} \tag{229}$$

The time factor of current is exp($j\omega t$), which has been omitted. The magnetic field can follow Ampere's circuital law

$$\oint_C \boldsymbol{h} \cdot d\boldsymbol{l} = I \tag{230}$$

Obtain,

$$\boldsymbol{h}(x = 0^+) = \frac{J_0}{2}\hat{y} \tag{231}$$

The above equation $x = 0^+$ is on the right side of the current. Therefore, considering that the electromagnetic wave propagates in the x direction, the magnetic field of the electromagnetic wave should be,

$$\boldsymbol{h} = \frac{J_0}{2}\exp(-jkx)\hat{y} \tag{232}$$

Where $k = \omega\sqrt{(\epsilon_0 \mu_0)}$. The electric field can be calculated according to the differential equation of Ampere-Maxwell's circuital law, considering that the current J in the area where $x > 0$ is zero,

$$\nabla \times \boldsymbol{h} = \frac{\partial}{\partial t}\boldsymbol{d} = j\omega\epsilon_0\boldsymbol{e} \tag{233}$$

Or,

$$(-jk\hat{x}) \times \boldsymbol{h} = j\omega\epsilon_0\boldsymbol{e} \tag{234}$$

Or,

$$\boldsymbol{e} = \frac{1}{j\omega\epsilon_0}(-jk\hat{x}) \times \boldsymbol{h}$$

$$= \frac{1}{j\omega\epsilon_0}(-jk\hat{x}) \times \frac{J_0}{2}\exp(-jkx)\hat{y}$$

$$= \frac{k}{\omega\epsilon_0}\frac{J_0}{2}\exp(-jkx)(-\hat{z})$$

$$= \frac{\omega\sqrt{\mu_0\epsilon_0}}{\omega\epsilon_0}\frac{J_0}{2}\exp(-jkx)(-\hat{z})$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0}}\frac{J_0}{2}\exp(-jkx)(-\hat{z})$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{J_0}{2} \exp(-jkx)(-\hat{z})$$

$$= \frac{\eta_0 J_0}{2} \exp(-jkx)(-\hat{z}) \quad (235)$$

Where the following is considerred,

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad (236)$$

$$k = \omega\sqrt{\mu_0 \epsilon_0} \quad (237)$$

The Poynting vector to the right of the current is,

$$\boldsymbol{s}_r = \boldsymbol{e} \times \boldsymbol{h}^*$$

$$= (\frac{\eta_0 J_0}{2} \exp(-jkx)(-\hat{z})) \times (\frac{J_0}{2} \exp(-jkx)\hat{y})^*$$

$$= (\frac{\eta_0 J_0}{2})(\frac{J_0^*}{2})\hat{x}$$

$$= \frac{\eta_0 J_0 J_0^*}{4} \hat{x} \quad (238)$$

Similarly, we can calculate that on the left side of the current,

$$\boldsymbol{s}_l = \frac{\eta_0 J_0 J_0^*}{4} (-\hat{x}) \quad (239)$$

In addition,

$$\boldsymbol{e}(x = 0) \cdot \boldsymbol{J}^* = (\frac{\eta_0 J_0}{2} (-\hat{z})) \cdot (J_0 \hat{z})^*$$

$$= -\frac{\eta_0 J_0 {J_0}^*}{2} \quad (240)$$

So we verified,

$$\boldsymbol{s}_r \cdot \hat{x} + \boldsymbol{s}_l \cdot (-\hat{x}) = -\boldsymbol{e}(x = 0) \cdot \boldsymbol{J}^* \quad (241)$$

The equivalence of the above formula and Poynting's theorem,

$$\oint_\Gamma \boldsymbol{s} \cdot \hat{n} d\Gamma = -\iiint \boldsymbol{e} \cdot \boldsymbol{J}^* dV \quad (242)$$

Where,

$$\boldsymbol{s} = \boldsymbol{e} \times \boldsymbol{h}^* \quad (243)$$

Therefore, we can obtain the electromagnetic radiation of the infinite plane current according to Maxwell's electromagnetic theory.

9.2. Advanced Wave

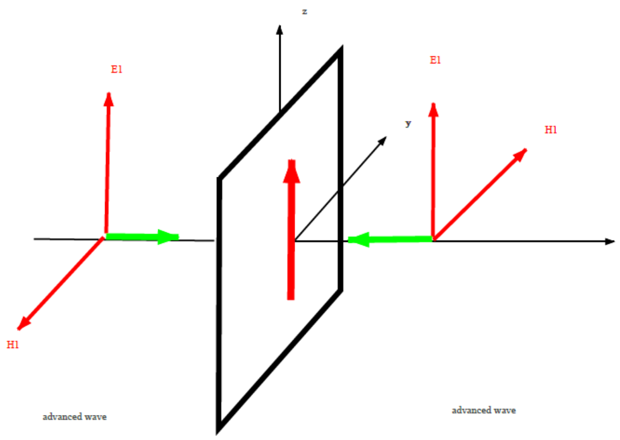

Figure 7: The Electromagnetic Advanced Wave of a Flat Plate Current

First, consider the right side of the current. If it is the advanced wave, the magnetic field, it is obtained in the same way as the previous method,

$$\boldsymbol{h} = \frac{J_0}{2}\hat{y} \tag{244}$$

However, because it is an advanced wave, there should be a factor exp(+*jkx*), so it is,

$$\boldsymbol{h} = \frac{J_0}{2}\exp(+jkx)\hat{y} \tag{245}$$

The electric field is obtained from the Ampere's circuital law of maxwell's equation as before,

$$\nabla \times \boldsymbol{h} = \frac{\partial}{\partial t}\boldsymbol{d} = j\omega\epsilon_0\boldsymbol{e} \tag{246}$$

Or,

$$(+jk\hat{x}) \times \boldsymbol{h} = j\omega\epsilon_0\boldsymbol{e} \tag{247}$$

Or,

$$\boldsymbol{e} = \frac{1}{j\omega\epsilon_0}(+jk\hat{x}) \times \boldsymbol{h}$$

$$= \frac{1}{j\omega\epsilon_0}(+jk\hat{x}) \times \frac{J_0}{2}\exp(-jkx)\hat{y}$$

$$= \frac{k}{\omega\epsilon_0}\frac{J_0}{2}\exp(+jkx)(+\hat{z})$$

$$= \frac{\eta_0 J_0}{2}\exp(-jkx)(+\hat{z}) \tag{248}$$

The Poynting vector to the right of the current is,

$$\boldsymbol{s}_r = \boldsymbol{e} \times \boldsymbol{h}^*$$

$$= (\frac{\eta_0 J_0}{2}\exp(+jkx)(+\hat{z})) \times (\frac{J_0}{2}\exp(+jkx)\hat{y})^*$$

$$= -(\frac{\eta_0 J_0}{2})(\frac{J_0^*}{2})\hat{x}$$

$$= -\frac{\eta_0 J_0 J_0^*}{4}\hat{x} \tag{249}$$

The energy flow of the advanced wave is directed to the current plate. That is correct.
Similarly, we can calculate that it is on the left side of the current,

$$\boldsymbol{s}_l = \frac{\eta_0 J_0 J_0^*}{4}(\hat{x}) \tag{250}$$

In addition,

$$\boldsymbol{e}(x=0)\cdot\boldsymbol{J}^* = (\frac{\eta_0 J_0}{2}(+\hat{z}))\cdot(J_0\hat{z})^*$$

$$= +\frac{\eta_0 J_0 J_0{}^*}{2} \tag{251}$$

We can still have,

$$\boldsymbol{s}_r\cdot\hat{x} + \boldsymbol{s}_l\cdot(-\hat{x}) = -\frac{\eta_0 J_0 J_0^*}{4} - \frac{\eta_0 J_0 J_0^*}{4}$$

$$-\boldsymbol{e}(x=0)\cdot\boldsymbol{J}^* = -\frac{\eta_0 J_0 J_0{}^*}{2} \tag{252}$$

Poynting's theorem is verified,

$$\boldsymbol{s}_r\cdot\hat{x} + \boldsymbol{s}_l\cdot(-\hat{x}) = -\boldsymbol{e}(x=0)\cdot\boldsymbol{J}^* \tag{253}$$

The above formula is,

$$\oint_\Gamma \boldsymbol{s}\cdot\hat{n}d\Gamma = -\iiint \boldsymbol{e}\cdot\boldsymbol{J}^* dV \tag{254}$$

Therefore, Poynting's theorem is also verified for the advanced wave.

9.3. The Problems

This author finds that according to Maxwell's electromagnetic theory, although Poynting's theorem is satisfied, on the right side of the current, the initial directions of the electric field of the retarded wave and the advanced wave are exactly opposite. According to Wheeler Feynman's absorber theory, any current generates half retarded wave and half advanced wave. If the current does produce half the retarded wave and half the advanced wave, the total electric field of the retarded wave and the advanced wave at x=0 is just zero, because the electric field of the retarded wave and the advanced wave is zero on the current surface. We know that near the current plate,

$$kx \to 0 \tag{255}$$

$$\exp(kx) \to 1 \tag{256}$$

At this time, the radiated electromagnetic field should degenerate into a magnetic quasi-static electromagnetic field. According to our experience, the magnetic quasi-static electric field near the current plate is not zero. This shows that Maxwell's electromagnetic theory cannot support Wheeler Feynman's absorber theory. This is why many people think that the Wheeler Feynman absorber theory is correct, but the Wheeler Feynman absorber theory still cannot be recognized by most people. Because Maxwell's electromagnetic theory cannot support it!

This author supports Wheeler Feynman's point of view and thinks that Wheeler Feynman's point of view is correct. If Maxwell's electromagnetic field theory cannot support Wheeler Feynman's point of view, it should be that Maxwell's electromagnetic field theory has a problem.

10. Electromagnetic Wave Of Plate Current According To Author's Theory

To solve the electromagnetic wave of infinite plane current according to this author's electromagnetic theory, in this chapter, the reader should pay attention to how this author adds the mysterious relaxation process to the system theory and how to add new boundary conditions. In addition, we should also pay attention to the difference between this author's electromagnetic theory and Maxwell's electromagnetic theory in the previous chapter.

10.1. Retarded Wave

First, consider the right side of the plane current. For this problem, the magnetic field is calculated as above in section 9,

$$\boldsymbol{H} = \boldsymbol{h} = \frac{J_0}{2}\exp(-jkx)\hat{y} \tag{257}$$

The factor exp(-*jkx*) above represents the retarded wave, and the electric field is calculated according to the principle of self energy

$$\Re(\oint (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{x} = 0 \tag{258}$$

Or,

$$\Re(\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{x} = 0 \tag{259}$$

The above equation requires the phase difference of $\boldsymbol{E}$ and $\boldsymbol{H}$ to have ±90 degrees. We know that the direction of the electric field should be perpendicular to the magnetic field, and there should be,

$$||\boldsymbol{E}|| = ||\boldsymbol{e}|| \tag{260}$$

That is to say, the electric field of this author's theory should be equal to that of Maxwell's electromagnetic theory. Therefore, according to this author's electromagnetic theory,

$$\boldsymbol{E} \sim (\pm j)\eta_0 \hat{x} \times \boldsymbol{H} \tag{261}$$

$$= (\pm j)\frac{\eta_0 J_0}{2}\exp(-jkx)(-\hat{z}) \tag{262}$$

There are still signs to be determined in the above formula. This sign can be determined according to the electric field of the magnetic quasi-static electromagnetic field. Because this author's electromagnetic field method should be able to degenerate into a magnetic quasi-static electromagnetic field. Therefore, it can be considered that,

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} = -j\omega \iint \frac{J\hat{z}}{r} \sim j(-\hat{z}) \tag{263}$$

In the formula, ~ means proportional, and this symbol is phase preserving but not value preserving. Comparing the above equation (263) with (262), we can see that,

$$\boldsymbol{E} = j\frac{\eta_0 J_0}{2}\exp(-jkx)(-\hat{z}) \tag{264}$$

We can calculate Poynting vector

$$\boldsymbol{S}_r = \boldsymbol{E} \times \boldsymbol{H}^*$$

$$= j\frac{\eta_0 J_0}{2}\exp(-jkx)(-\hat{z}) \times (\frac{J_0}{2}\exp(-jkx)\hat{y})^*$$

$$= j\frac{\eta_0 J_0 J_0^*}{4}\hat{x} \tag{265}$$

The subscript " r" indicates that it is on the right side of the current,

$$\Re(\boldsymbol{S}_r \cdot \hat{x}) = \Re(j\frac{\eta_0 J_0 J_0^*}{4}) = 0 \tag{266}$$

It can be seen from the above that according to this author's electromagnetic field theory, Poynting vector is a pure imaginary number, that is, it is reactive power. This author calls this wave of reactive power the reactive power wave. Reactive power waves propagate energy in the forward and in the opposite directions at the same cycle. The reactive power wave has two quarter cycles that propagate power in the forward and two quarter cycles that propagate power backward. Therefore, the average forward propagation power is zero. This kind of wave does not lose electromagnetic energy in the process of propagation. All radiated electromagnetic wave energy flow will eventually return to the transmitting antenna or light source.

10.2. Advanced Wave

First, consider the right side of the plane current. For this problem, the magnetic field is calculated as above 9, but the advanced wave factor exp(jkx) must be considered,

$$\boldsymbol{H} = \boldsymbol{h} = \frac{J_0}{2}\exp(jkx)\hat{y} \tag{267}$$

The electric field is calculated according to the principle of self energy,

$$\Re(\oint (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{x} = 0 \tag{268}$$

$$\Re(\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \hat{x} = 0 \tag{269}$$

The above equation requires the phase difference of $\boldsymbol{E}$ and $\boldsymbol{H}$ to have ±90 degrees. We know that the direction of the electric field should be perpendicular to the magnetic field, and,

$$||\boldsymbol{E}|| = ||\boldsymbol{e}|| \tag{270}$$

That is to say, the electric field of this author's theory should be equal to that of Maxwell's electromagnetic theory. So according to this author's electromagnetic theory,

$$\boldsymbol{E} \sim (\pm j)\eta_0 \hat{x} \times \boldsymbol{H}$$

$$= (\pm j)\frac{\eta_0 J_0}{2}\exp(+jkx)(-\hat{z}) \tag{271}$$

There are still signs to be determined in the above formula. This sign can be determined according to the electric field of the magnetic quasi-static electromagnetic field. Because this author's electromagnetic field method should be able to degenerate into a magnetic quasi-static electromagnetic field. Therefore, it can be considered that,

$$\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} = -j\omega \iint \frac{J\hat{z}}{r} \sim j(-\hat{z}) \tag{272}$$

In the formula, ~ means proportional, and this symbol is phase preserving but not value preserving. From this we know that,

$$\boldsymbol{E} = j\frac{\eta_0 J_0}{2}\exp(+jkx)(-\hat{z}) \tag{273}$$

We can calculate Poynting vector

$$\boldsymbol{S}_r = \boldsymbol{E} \times \boldsymbol{H}^*$$

$$= j\frac{\eta_0 J_0}{2}\exp(+jkx)(-\hat{z}) \times (\frac{J_0}{2}\exp(+jkx)\hat{y})^*$$

$$= j\frac{\eta_0 J_0 J_0^*}{4}\hat{x} \tag{274}$$

$$\Re(\boldsymbol{S}_r \cdot \hat{x}) = \Re(j\frac{\eta_0 J_0 J_0^*}{4}) = 0 \tag{275}$$

It can be seen from the above that according to this author's electromagnetic field theory, Poynting vector is a pure imaginary number, that is, it is reactive power. this author calls this wave of reactive power the reactive power wave. Reactive power waves propagate energy forward and in the opposite direction at the same cycle. Reactive power wave has two quarter cycles of forward propagation power and two quarter cycles of backward propagation power. Therefore, the average propagation power is zero. This wave does not lose electromagnetic energy.

10.3. Supporting Wheeler Feynman Absorber Theory

In this author's electromagnetic theory, the initial values of the electric field and magnetic field of the retarded wave and the advanced wave are identical. Therefore, the current can radiate the retarded wave and the advanced wave at the same time, that is, it radiates half of the retarded wave and half of the advanced wave. The electric fields of the retarded wave and the advanced wave are superimposed on the current boundary and do not offset. Therefore, this author's electromagnetic field theory provides support for Wheeler Feynman absorber theory. In addition, when x=0, that is, near the current plate, this author's electromagnetic theory can degenerate into a magnetic quasi-static electromagnetic field. Maxwell's radiation electromagnetic theory does not support Wheeler Feynman's absorber theory, nor can it degenerate into magnetic quasi-static electromagnetic field! For the degenerated magnetic quasi-static electromagnetic field, we can also consider a finite plate current. The electric field of a finite plate current does not tend to infinity under the magnetic quasistatic condition. In this case, we can see the significance of this author's electromagnetic theory.

10.4. Electromagnetic Wave Propagating to the Right

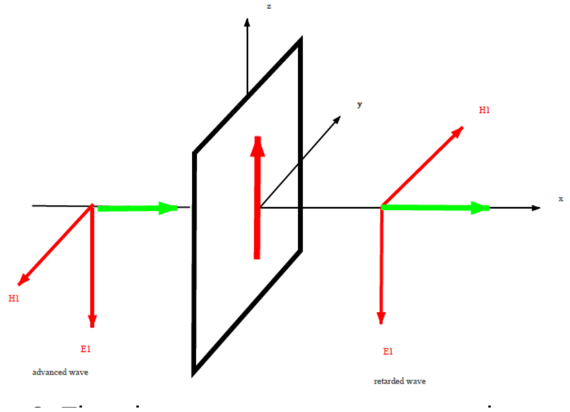

Figure 8: The Plate Current can Generate Electromagnetic Waves that Propagate to The Right

According to this author's electromagnetic field theory, any current element can radiate the retarded wave and the advanced wave at the same time, which is consistent with Wheeler's and Feynman's absorber theory [10, 11]. In absorber theory, current element can radiate half retarded wave and half advanced wave. In this author's electromagnetic field theory, because electromagnetic wave is reactive power wave, this wave does not lose energy. This wave is a self energy flow. In this author's electromagnetic theory, the self energy flow is reactive power, so the propagation does not lose energy. So this kind of wave may or may not have! This author will deal with mutual energy flow later in this example. In this author's electromagnetic theory, energy is transferred by mutual energy flow. The mutual energy flow is generated by the retarded wave from the source and the advanced wave from the sink. Therefore, if the retarded wave does not encounter the advanced wave after it is sent from the source, it is an invalid electromagnetic wave, because it does not lose energy anyway. It is the effective electromagnetic wave if it encounters the advanced wave.

This author assumes that the retarded wave on the right side of the current in our discussion is an effective electromagnetic wave, because in the next section, we will place a new current plate on the right side of this current plate. This new current plate is a sink, which is used to receive electromagnetic waves (and emit advanced waves).

This author assume that the advanced wave on the left side of the current plate is an effective electromagnetic wave. The advanced wave on the left side of the current plate has a space factor exp(-*jkx*). The retarded wave on the left side is invalid. Now let's calculate the electromagnetic field on the left side of the current plate. The magnetic field can be obtained from Ampere's circuital law,

$$\boldsymbol{H} = -\frac{J_0}{2}\exp(-jkx)\hat{y} \qquad (276)$$

The electric field is the same as the previous calculation

$$\boldsymbol{E} = j\frac{\eta_0 J_0}{2}\exp(-jkx)(-\hat{z}) \qquad (277)$$

Of course, considering various combinations, the plate current can generate retarded waves on both sides, advanced waves on both sides, waves propagating to the right on both sides, and electromagnetic waves propagating to the left on both sides. However, in the following example, we only need to consider the wave to the right on both sides. This means that the right side of the plate current is the retarded wave, and the left side of the plate current is the advanced wave.

11. Electromagnetic Wave of Double-Plate Current With Author's Theory

11.1. Field to the Right of the First Current

The first current plate has been calculated in front (267,273), but the electric field and magnetic field add a subscript 1, which is easy to distinguish. It is indicated by the subscript "r" to the right of the current.

$$\boldsymbol{H}_{1r} = \frac{J_{10}}{2}\exp(-jkx)\hat{y} \tag{278}$$

$$\boldsymbol{E}_{1r} = j\frac{\eta_0 J_{10}}{2}\exp(-jkx)(-\hat{z}) \tag{279}$$

The field on the left side of the current plate is in accordance with (276, 277)

$$\boldsymbol{H}_{1l} = -\frac{J_{10}}{2}\exp(-jkx)\hat{y} \tag{280}$$

$$\boldsymbol{E}_{1l} = j\frac{\eta_0 J_{10}}{2}\exp(-jkx)(-\hat{z}) \tag{281}$$

See figure 9. This author has previously calculated the electromagnetic wave generated by a single current plate according to this author's electromagnetic theory, and this electromagnetic wave propagates to the right. Of course, this current can also generate electromagnetic waves propagating to the left. However, the second current plate is on the right of the first one, as shown in the figure 9. This author assumes that the direction of electromagnetic energy flow is from the first current plate to the second current plate, so it is an electromagnetic wave propagating to the right. Therefore, we assume that both current plates generate electromagnetic waves propagating to the right. Then the electromagnetic wave propagating to the left is invalid. Refer to figure 9.

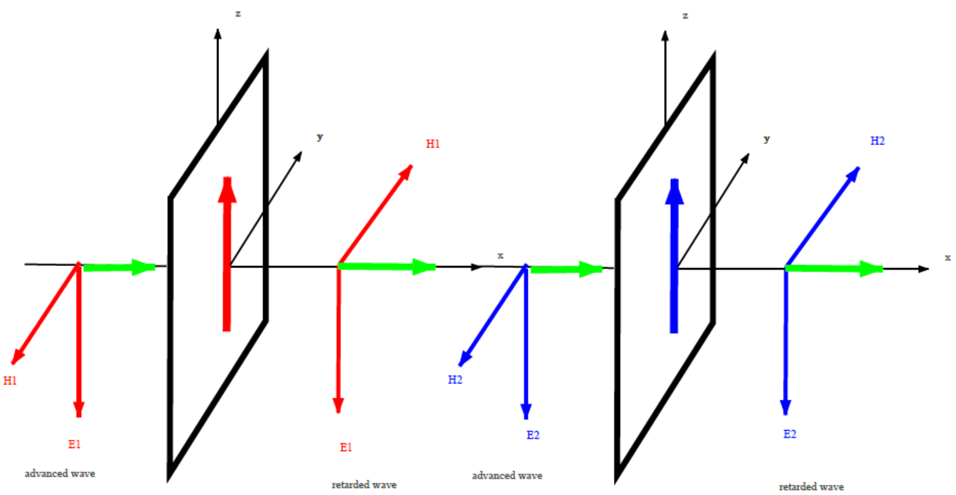

Figure 9: Double Plate Currents. Each Current Produces An Electromagnetic Wave That Travels To the Right

11.2. Electromagnetic Field of the Second Current Plate

We assume that

$$x = l \tag{282}$$

There is a second current plate at, and there is an electromagnetic field on the right of the current,

$$\boldsymbol{H}_{2r} = \boldsymbol{h} = \frac{J_{20}}{2}\exp(-jk(x-l))\hat{y} \tag{283}$$

$$\boldsymbol{E}_{2r} = j\frac{\eta_0 J_{20}}{2}\exp(-jk(x-l))(-\hat{z}) \tag{284}$$

On the left side of the current plate

$$\boldsymbol{H}_{2l} = -\frac{J_{20}}{2}\exp(-jk(x-l))\hat{y} \tag{285}$$

The electric field is the same as the previous calculation

$$\boldsymbol{E}_{2l} = j\frac{\eta_0 J_{20}}{2}\exp(-jk(x-l))(-\hat{z}) \tag{286}$$

11.3. Calculation of Mutual Energy Flow Between Two Plates

$$S_m = \boldsymbol{E}_{1r} \times \boldsymbol{H}_{2l}^* + \boldsymbol{E}_{2l}^* \times \boldsymbol{H}_{1r}$$

$$= (j\frac{\eta_0 J_{10}}{2}\exp(-jkx))(-\hat{z})) \times (-\frac{J_{20}}{2}\exp(-jk(x-l))\hat{y})^*$$

$$+(j\frac{\eta_0 J_{20}}{2}\exp(-jk(x-l))(-\hat{z}))^* \times (\frac{J_{10}}{2}\exp(-jkx)\hat{y})$$

$$= [(j\frac{\eta_0 J_{10}}{2}\exp(-jkx)))(-\frac{J_{20}}{2}\exp(-jk(x-L)))^*$$

$$+(j\frac{\eta_0 J_{20}}{2}\exp(-jk(x-L)))^*(\frac{J_{10}}{2}\exp(-jkx))]\hat{x}$$

$$= \frac{1}{2}\eta_0 J_{10} J_{20}^* j^* \exp(-jk(-l))^* \hat{x} \tag{287}$$

Now let's calculate the induced potential on the current of the second plate. At this time, we assume that the length of the current plate is l_{length}, width is l_{width}.

$$\mathcal{E}_{2,1} = \int \boldsymbol{E}_{1r} \cdot d\boldsymbol{l} = l_{length}\boldsymbol{E}_{1r}(x=l)\cdot\hat{z}$$

$$= l_{length}(j\frac{\eta_0 J_{10}}{2}\exp(-jkl))(-\hat{z}))\cdot\hat{z}$$

$$= -jl_{length}\frac{\eta_0 J_{10}}{2}\exp(-jkl) \tag{288}$$

The impedance of the second current plate is,

$$Z_2 = j\omega L_2 + R_2$$

The induced current of the second current plate is,

$$I_2 = \frac{\mathcal{E}_{2,1}}{Z_2} = \frac{\mathcal{E}_{2,1}}{j\omega L_2 + R_2} \to \frac{\mathcal{E}_{2,1}}{R_2} = -jl_{length}\frac{\eta_0 J_{10}}{2R_2}\exp(-jkl) \tag{289}$$

Assumption in the above formula,

$$R_2 \gg \omega L_2 \tag{290}$$

Hence,

$$j\omega L_2 + R_2 \to R_2 \tag{291}$$

l_{length} is the length of the current plate from top to bottom. This author previously assumed that the current plate is infinite, but to calculate the induced electromotive force, we have to assume the length of a current plate. The current density of the second current plate is,

$$J_{20} = \frac{I_2}{l_{width}} = -jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}}\exp(-jkl) \tag{292}$$

l_{width} is the width of the current plate. Calculate mutual energy flow,

$$S_m = \frac{1}{2}\eta_0 J_{10} J_{20}^* j^* \exp(-jk(-l))^* \hat{x}$$

$$= \frac{1}{2}\eta_0 J_{10}(-jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}}\exp(-jkl))^* j^* \exp(-jk(-l))^* \hat{x}$$

$$= \frac{1}{2}\eta_0 J_{10} l_{length}\frac{\eta_0 J_{10}^*}{2R_2 l_{width}}\hat{x}$$

$$= \frac{\eta_0^2 J_{10} J_{10}^* l_{length}}{4R_2 l_{width}}\hat{x} \tag{293}$$

It can be seen that the mutual energy flow points to the $\hat{x}$ direction.

$$S_m \sim \hat{x} \tag{294}$$

11.4. Mutual Energy Flow at $x < 0, x > l$

Consider the mutual energy flow as,

$$\boldsymbol{S}_m = \boldsymbol{S}_{12} + \boldsymbol{S}_{21}$$

$$= \boldsymbol{E}_{1r} \times \boldsymbol{H}_{2l}^* + \boldsymbol{E}_{2l}^* \times \boldsymbol{H}_{1r} \tag{295}$$

$$\boldsymbol{S}_{12} \triangleq \boldsymbol{E}_{1r} \times \boldsymbol{H}_{2l}^* \tag{296}$$

$$\boldsymbol{S}_{21} \triangleq \boldsymbol{E}_{2l}^* \times \boldsymbol{H}_{1r} \tag{297}$$

In the range of $0 < x < l$ $\boldsymbol{S}_{12}$ and $\boldsymbol{S}_{21}$ is superimposed. But when x<0, because the direction of $\boldsymbol{H}_{1l}$ suddenly changes (just opposite to the direction of $\boldsymbol{H}_{1r}$), $\boldsymbol{S}_{12}$ and $\boldsymbol{S}_{21}$ becomes offset. In $x > l$ the direction of magnetic field of $\boldsymbol{H}_{2r}$ changes (just opposite to the direction of $\boldsymbol{H}_{2l}$), so $\boldsymbol{S}_{12}$ and $\boldsymbol{S}_{21}$ becomes offset. So there is,

$$\boldsymbol{S}_m = 0, \quad x \notin [0, l] \tag{298}$$

Merge (293 and 298), so we have

$$\boldsymbol{S}_m = \frac{\eta_0^2 J_{10} J_{10}^* l_{length}}{4R_2 l_{width}}\hat{x}\begin{cases}0 & x < 0\\ 1 & 0 < x < l\\ 0 & x > l\end{cases} \tag{299}$$

11.5. Verify That the Energy Flow Law Meets

We need to verify the law of energy flow and energy conservation (201). Considering that in the frequency domain and $N = 2$,

$$-\iiint (\boldsymbol{J}_1 \cdot \boldsymbol{E}_2^*) dV = (\xi_1, \xi_2) = \iiint (\boldsymbol{J}_2^* \cdot \boldsymbol{E}_1) dV \tag{300}$$

Where,

$$(\xi_1, \xi_2) \equiv \oint_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{301}$$

The energy flow formula we need to verify corresponding to the plate current is,

$$-\boldsymbol{J}_1 \cdot \boldsymbol{E}_{2l}^* = \boldsymbol{S}_m \cdot \hat{x} = \boldsymbol{J}_2^* \cdot \boldsymbol{E}_{1r} \tag{302}$$

$$-\boldsymbol{J}_1 \cdot \boldsymbol{E}_{2l}^*|_{x=0} = -J_{10}\hat{z} \cdot (j\frac{\eta_0 J_{20}}{2}\exp(-jk(x-l))(-\hat{z}))^*$$

$$= J_{10}(j\frac{\eta_0 J_{20}}{2}\exp(-jk(x-l)))^*$$

$$= J_{10}(j\frac{\eta_0 J_{20}}{2}\exp(jkl))^*$$

$$= J_{10}(j\frac{\eta_0(-jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}}\exp(-jkl))}{2}\exp(jkl))^*$$

$$= J_{10}J_{10}^*\eta_0^2(\frac{l_{length}}{4R_2 l_{width}}) \tag{303}$$

$$\boldsymbol{J}_2^* \cdot \boldsymbol{E}_{1r}|_{x=l} = (-jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}}\exp(-jkl)\hat{z})^* \cdot (j\frac{\eta_0 J_{10}}{2}\exp(-jkl))(-\hat{z}))$$

$$= (jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}}\exp(-jkl))^*(j\frac{\eta_0 J_{10}}{2}\exp(-jkl)))$$

$$= (l_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}})^*(\frac{\eta_0 J_{10}}{2}))$$

$$= J_{10}J_{10}^*\eta_0^2\frac{l_{length}}{4R_2 l_{width}} \tag{304}$$

it is known that the formula (302) is verified, which shows that the formula of the energy flow law (300) is verified. Note that there is no set of differential equations similar to Maxwell's equation in this author's electromagnetic theory. To verify whether this solution is correct is to pass this energy flow law, which is also the law of energy conservation. Of course, we also require that,

$$\Re(\boldsymbol{S}_{1r} \cdot \hat{x}) = \Re(\boldsymbol{S}_{1l} \cdot \hat{x}) = 0 \tag{305}$$

$$\Re(\boldsymbol{S}_{2r} \cdot \hat{x}) = \Re(\boldsymbol{S}_{2l} \cdot \hat{x}) = 0 \tag{306}$$

where

$$\boldsymbol{S}_{1r} = \boldsymbol{E}_{1r} \times \boldsymbol{H}_{1r}^*$$

$$\boldsymbol{S}_{1l} = \boldsymbol{E}_{1l} \times \boldsymbol{H}_{1l}^*$$

$$\boldsymbol{S}_{2r} = \boldsymbol{E}_{2r} \times \boldsymbol{H}_{2r}^*$$

$$\boldsymbol{S}_{2l} = \boldsymbol{E}_{2l} \times \boldsymbol{H}_{2l}^*$$

These two formulas (305, 306) have been verified previously. As long as the law of energy flow, the law of conservation of energy 300, and the principle of self energy flow (305, 306) are satisfied, this solution satisfies this author's electromagnetic theory.

11.6. Comparison of Particle Models In Transactional Interpretation of Quantum Mechanics With Cramer

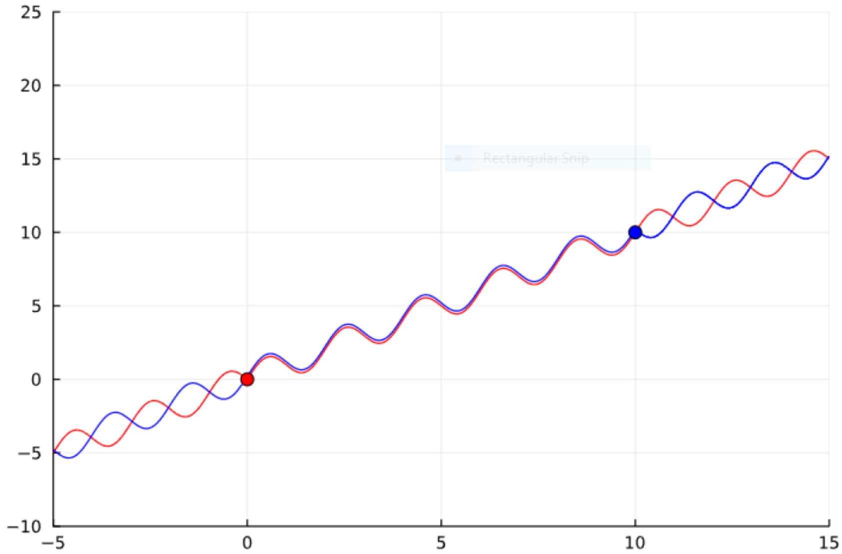

Figure 10: Cramer's Particle Model in the Transactional Interpretation of Quantum Mechanics

The surface mutual energy current of the above equation (299) is generated on the first current plate and annihilated on the second current plate. Keep the same value between the two current plates, and point from the first plate to the second plate. The mutual energy flow has the property of photons. This is the reason why this author interprets photons with mutual energy flow. The classical electromagnetic theory uses self energy current as electromagnetic wave, that is, Poynting vector $\boldsymbol{e}_1 \times \boldsymbol{h}_1$ or $\boldsymbol{e}_2 \times \boldsymbol{h}_2$ as the energy flow of electromagnetic wave, the energy flow will not annihilate since its generation and will continue to move. Therefore, self energy flow can not be used to describe photons.

Figure 10 describes the particle model in Cramer's transactional interpretation of quantum mechanical [8, 9]. In this model, there are source (shown by red dots) and sink (shown by blue dots). Cramer developed the current model of Wheeler and Feynman absorber theory. In Wheeler and Feynman absorber theory, any current element can emit half retarded wave and half advanced wave. In Cramer's model, the source and sink also emit half retarded wave and half advanced wave, but the retarded wave and advanced wave can be in different directions of the source and sink. For example, the retarded wave is sent to the right and the advanced wave is sent to the left. In addition, the starting phase of the retarded wave and the advanced wave in Cramer model can also be adjusted at will. Further, the source sends a retarded wave to the right, and the sink sends an advanced wave to the left. It is assumed that the retarded wave and the advanced wave have exactly the same phase between the source and the sink, so they are superposed. The retarded wave from the sink on the right side of the sink exactly keeps a 180 degree phase angle with the retarded wave from the source, so they cancel each other. On the left side of the source, the advanced wave from the source and the advanced wave from the sink maintain a phase difference of 180 degrees, so it is just offset. Cramer's particle model is very sophisticated. Its papers [8, 9] have been cited by thousands of people. However, this model is only a qualitative theory and a guess. There are still many problems. For example, the 180 degree phase difference mentioned above is very difficult to understand. Why is there a 180 degree phase difference? In addition, why does the current source send retarded wave to the right and advanced wave to the left, instead of sending retarded wave and advanced wave to the left and right at the same time? In the original model of Wheeler and Feynman, the retarded wave and advanced wave are also omnidirectional, rather than the directivity of Cramer.

The formula (299) is basically consistent with Cramer's quantum mechanical particle model, but the retarded wave and advanced wave are not superimposed between the source and sink. It is a mutual energy flow composed of retarded wave and advanced wave $\boldsymbol{S}_{12}$ and $\boldsymbol{S}_{21}$. Because the sign of one of the magnetic fields changes outside the range of the source and sink, the superposition becomes cancellation. This change in magnetic field completely explains the cause of 180 degrees. In addition, in this author's theory, the retarded wave from the source is a reactive power wave, which does not lose energy. Therefore, if it does not form a mutual energy flow with other advanced waves, it can be completely ignored. Or it is considered as invalid electromagnetic wave. We can assume that the retarded wave on the right side of the source is an effective wave and the advanced wave on the right side is an invalid wave. The retarded wave on the left side of the source is an invalid wave, and the advanced wave on the left side is an effective wave. In this way, the source can form a wave propagating to the right. Similarly, we can assume that the wave from the sink also propagates to the right. In this way, we also fully explain Cramer's further development of Wheeler and Feynman's absorber theory. In Wheeler Feynman's model, both the retarded wave and the advanced wave are omnidirectional radiation. In Cramer's model, the wave has directivity. Although the electromagnetic wave in our model is omnidirectional like that in Wheeler Feynman's model, it can also propagate to the right, so it is consistent with Cramer's model too.

11.7. The Interval l Between two Plates is Very Small

Let's assume that the distance between the two current plates is very close,

$$l \to 0 \tag{307}$$

$$kx \to 0 \tag{308}$$

$$\exp(-jkx) \to 1 \tag{309}$$

In this case, this author's electromagnetic field degenerates into a magnetic quasi-static electromagnetic field

$$\boldsymbol{H}_{1r} = \frac{J_{10}}{2}\hat{y} \tag{310}$$

$$\boldsymbol{E}_{1r} = j\frac{\eta_0 J_{10}}{2}(-\hat{z}) \tag{311}$$

On the left side of the current plate

$$\boldsymbol{H}_{1l} = -\frac{J_{10}}{2}\hat{y} \tag{312}$$

$$\boldsymbol{E}_{1l} = j\frac{\eta_0 J_{10}}{2}(-\hat{z}) \tag{313}$$

For the electromagnetic field of the second current plate

$$\boldsymbol{H}_{2r} = \frac{J_{20}}{2}\hat{y} \tag{314}$$

$$\boldsymbol{E}_{2r} = j\frac{\eta_0 J_{20}}{2}(-\hat{z}) \tag{315}$$

$$\boldsymbol{H}_{2l} = -\frac{J_{20}}{2}\hat{y} \tag{316}$$

$$\boldsymbol{E}_{2l} = j\frac{\eta_0 J_{20}}{2}(-\hat{z}) \tag{317}$$

Consider,

$$\boldsymbol{S}_m = \boldsymbol{E}_{1r} \times \boldsymbol{H}_{2l}^* + \boldsymbol{E}_{2l}^* \times \boldsymbol{H}_{1r}$$

$$= (j\frac{\eta_0 J_{10}}{2}(-\hat{z})) \times (-\frac{J_{20}}{2}\hat{y})^* + (j\frac{\eta_0 J_{20}}{2}(-\hat{z}))^* \times (\frac{J_{10}}{2}\hat{y})$$

$$= [(j\frac{\eta_0 J_{10}}{2})(-\frac{J_{20}}{2})^* + (j\frac{\eta_0 J_{20}}{2})^*(\frac{J_{10}}{2})]\hat{x}$$

$$= \frac{\eta_0 J_{10}}{4}[(j)(-J_{20})^* + (jJ_{20})^*]\hat{x} \tag{318}$$

Consider,

$$J_{20}=\frac{I_2}{l_{width}}=-jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}} \tag{319}$$

$$\boldsymbol{S}_m=\frac{\eta_0 J_{10}}{4}[(j)(-J_{20})^*+(jJ_{20})^*]\hat{x}$$

$$=\frac{\eta_0 J_{10}}{4}[(j)(-(-jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}}))^*+(j(-jl_{length}\frac{\eta_0 J_{10}}{2R_2 l_{width}}))^*]\hat{x}$$

$$=\frac{\eta_0^2 J_{10}J_{10}^* l_{length}}{8R_2 l_{width}}[(j)(j)^*+(j(-j))^*]\hat{x}$$

In this case, this author's electromagnetic fields are degenerated into magnetic quasi-static electromagnetic fields. The reader can verify that the calculation in the magnetic quasi-static case is completely similar to the above. The mutual energy flow finally has a factor similar to that above

$$[(j)(j)^*+(j(-j))^*]\hat{x}=2\hat{x} \tag{320}$$

$$\boldsymbol{S}_m=\frac{\eta_0^2 J_{10}J_{10}^* l_{length}}{4R_2 l_{width}}\hat{x} \tag{321}$$

Similary we can have,

$$\boldsymbol{S}_m=\frac{\eta_0^2 J_{10}J_{10}^* l_{length}}{4R_2 l_{width}}\hat{x}\begin{cases}0 & x<0\\ 1 & 0<x<l\\ 0 & x>l\end{cases} \tag{322}$$

Therefore, the radiation electromagnetic field defined by this author can indeed degenerate into a magnetic quasi-static electromagnetic field. However, in the magnetic quasi-static case, we cannot generally assume that the current plate is infinite. Then the electric field will tend to infinity. Therefore, we generally take a finite current plate.

11.8. Calculate the Energy Flow of This Author's Electromagnetic Theory With Maxwell's Electromagnetic Theory

According to Maxwell's electromagnetic field theory, we have,

$$\boldsymbol{h}_{1r}=\frac{J_{10}}{2}\exp(-jkx)\hat{y} \tag{323}$$

$$\boldsymbol{e}_{1r}=\frac{\eta_0 J_{10}}{2}\exp(-jkx)(-\hat{z}) \tag{324}$$

$$\boldsymbol{h}_{2l}=\frac{J_{20}}{2}\exp(+jk(l-x))(-\hat{y}) \tag{325}$$

$$\boldsymbol{e}_{2l}=\frac{\eta_0 J_{20}}{2}\exp(+jk(l-x))\hat{z} \tag{326}$$

We can calculate the mutual energy flow,

$$\boldsymbol{s}_m=\boldsymbol{e}_{1r}\times\boldsymbol{h}_{2l}^*+\boldsymbol{e}_{2l}^*\times\boldsymbol{h}_1$$

$$=(\frac{\eta_0 J_{10}}{2}\exp(-jkx)(-\hat{z}))\times(\frac{J_{20}}{2}\exp(+jk(l-x))(-\hat{y}))^*$$

$$+(\frac{\eta_0 J_{20}}{2}\exp(+jk(l-x))\hat{z})^*\times(\frac{J_{10}}{2}\exp(-jkx)\hat{y})$$

$$= \frac{\eta_0 J_{10}}{4} J_{20}^* \exp(+jkl)^* 2(-\hat{x})$$

$$= \frac{\eta_0 J_{10}}{2} J_{20}^* \exp(+jkl)^* (-\hat{x}) \quad (327)$$

Calculate the electromotive force,

$$\mathcal{E}_{2,1} = \int \boldsymbol{e}_{1r} \cdot d\boldsymbol{l} = l_{length} e_{1r}(x = l) \cdot \hat{z}$$

$$= l_{length} (\frac{\eta_0 J_{10}}{2} \exp(-jkl))(-\hat{z})) \cdot \hat{z}$$

$$= -l_{length} \frac{\eta_0 J_{10}}{2} \exp(-jkl) \quad (328)$$

The impedance of the second current plate is,

$$Z_2 = j\omega L_2 + R_2 \quad (329)$$

The induced current of the second current plate is,

$$I_2 = \frac{\mathcal{E}_{2,1}}{Z_2} = \frac{\mathcal{E}_{2,1}}{j\omega L_2 + R_2} \rightarrow \frac{\mathcal{E}_{2,1}}{R_2} = -l_{length} \frac{\eta_0 J_{10}}{2R_2} \exp(-jkl) \quad (330)$$

Assumption in the above formula,

$$R_2 \gg \omega L_2 \quad (331)$$

Hence,

$$j\omega L_2 + R_2 \rightarrow R_2 \quad (332)$$

l_{length} is the length of the current plate from top to bottom. We previously assumed that the current plate is infinite, but to calculate the induced electromotive force, we have to assume the length of a current plate. The current density of the second current plate is,

$$J_{20} = \frac{I_2}{l_{width}} = -l_{length} \frac{\eta_0 J_{10}}{2R_2 l_{width}} \exp(-jkl) \quad (333)$$

l_{width} is the width of the current plate.

$$\boldsymbol{s}_m = \frac{\eta_0 J_{10}}{2} J_{20}^* \exp(+jkl)^* (-\hat{x})$$

$$= \frac{\eta_0 J_{10}}{2} (-l_{length} \frac{\eta_0 J_{10}}{2R_2 l_{width}} \exp(-jkl))^* \exp(+jkl)^* (-\hat{x})$$

$$= \frac{\eta_0 J_{10}}{2} (-l_{length} \frac{\eta_0 J_{10}^*}{2R_2 l_{width}})(-\hat{x})$$

$$= \frac{\eta_0^2 J_{10} J_{10}^* l_{length}}{4R_2 l_{width}} \hat{x} \quad (334)$$

Therefore, we conclude that the interval of $0 \le x \le L$ has

$$\boldsymbol{S}_m = \boldsymbol{s}_m \quad (335)$$

Or,

$$\boldsymbol{E}_{1r} \times \boldsymbol{H}_{2l}^* + \boldsymbol{E}_{2l}^* \times \boldsymbol{H}_{1r} = \boldsymbol{e}_{1r} \times \boldsymbol{h}_{2l}^* + \boldsymbol{e}_{2l}^* \times \boldsymbol{h}_{1r} \quad (336)$$

In this way, we can directly use Maxwell's electromagnetic theory to calculate the mutual energy flow of this author's electromagnetic field. It is worth mentioning that we can only calculate the electromagnetic energy flow (mutual energy flow) in the interval of $0 \leq x \leq 1$. Using Maxwell's electromagnetic theory, we can't get the formula like (299). This is because according to Maxwell's electromagnetic theory, the advanced wave on the left side of the current changes not only the sign of the magnetic field, but also the sign of the electric field. See the field on the right of the current in figure 6 and the field on the left in figure 6. This shows that Maxwell's radiation electromagnetic field theory fail to support the photon model in Cramer's transactional interpretation of quantum mechanics.

11.9. A Problem of The Maxwell's Theory

We have known from Maxwell's theory that a single plate current can produce retarded wave radiation. It has been calculated in Chapter 9 that Poynting vectors on both sides of the plate current satisfy,

$$\boldsymbol{s}_r \cdot \hat{x} + \boldsymbol{s}_l \cdot (-\hat{x}) = -\boldsymbol{e}(x=0) \cdot \boldsymbol{J}^* \tag{337}$$

For the solution satisfying Maxwell's equation, this author said that it actually means that the solution satisfies the boundary radiation boundary condition, the Sliver-Muller radiation boundary condition. For our case of infinite plate current, of course, the direct Sliver-Muller boundary condition is incorrect, but there are always similar radiation boundary conditions. This boundary condition actually means that there is a very good absorption condition outside the plate current. The infinity is filled with good absorbing materials, which can absorb all the electromagnetic wave energy radiated from the current $\boldsymbol{J}$. In the previous section, we found that if we placed another current plate on the right side of the current plate under the above conditions, a mutual energy flow $\boldsymbol{s}_m$ was established between the first current plate and the second current plate, see the formula (334). This makes Maxwell's electromagnetic theory contradict his field theory (aether). Here, the field theory refers to Maxwell's theory that the change of current source can cause electromagnetic radiation. When the electromagnetic field radiates from the source, it moves independently from the source. That is, the movement has its own independent degree of freedom. When the electromagnetic wave moves to the receiving antenna, it transfers energy to the receiving antenna. Therefore, the electromagnetic wave should move away from the radiation source, so that when we add the second current plate to the system, it should not react on the original current source. The problem is that the addition of the second board will indeed affect the radiation of the first board! The total radiation energy is increased. This is contrary to the original assumption that there is the best absorbing material at infinity that can absorb all the waves emitted from the radiation source.

Of course, the reader can also argue that the second current plate radiates the advanced wave. In classical electromagnetic theory, there is no advanced wave. The classical electromagnetic theory denies the existence of advanced waves. Indeed, if Maxwell's theory is to conform to the aforementioned field theory or aether theory, the advanced wave must be removed from the theory. Otherwise, a mixing theory is formed. Here, mixing means that part of the energy flow is field theory, which is composed of Poynting vector $\boldsymbol{s}_r$. One part is action and reaction, and this part of energy flow consists of mutual energy flow $\boldsymbol{s}_m$. These two kinds of thoughts are opposite and cannot be allowed to mix together.

Therefore, in Maxwell's theory, it is better not to have advanced waves, but only retarded waves. Only the transmitting antenna is the source of the electromagnetic field. In Maxwell's electromagnetic theory, the sink should not be introduced, although Maxwell's equation allows the existence of advanced waves. Sink is also allowed. In this way, Maxwell's electromagnetic theory, as a complete theory of radiation electromagnetic field, is still good. The radiation electromagnetic field here refers to an environment where only radiation sources exist and perfect electromagnetic absorption materials are arranged in the distance. In this case, Maxwell's electromagnetic theory holds.

Of course, this situation does not include all situations. If there are conductors in the system, these conductors will consume electromagnetic energy. Maxwell's electromagnetic theory is not suitable for the existence of substances that consume electromagnetic wave energy. The substance that consumes electromagnetic energy, that is, the substance that absorbs electromagnetic waves, can only appear on the boundary of the infinite element, and must also be uniformly arranged on the remote boundary, which conforms to the Sliver-Muller boundary condition. But in reality, there is always conductor and resistance in the electromagnetic system, which will consume electromagnetic wave energy. Maxwell's electromagnetic radiation theory does not allow this kind of material to appear, which is obviously not a complete electromagnetic theory!

this author's electromagnetic theory has made a correction on Maxwell's electromagnetic theory. Because of this correction, the essence of this author's electromagnetic theory is the electromagnetic theory of action and reaction (action-at-a-distance). The radiation of the antenna itself, that is, Poynting vector, is reactive power. This wave can spread to any place in space, but does not consume electromagnetic energy. The energy transmitted by electromagnetic wave only occurs when the retarded wave (action) from the transmitting antenna happens to be synchronized with the advanced wave (reaction) from a receiving antenna. In this case, mutual energy flow is generated. Mutual energy flow is photons. This photon carries the energy flow composed of the retarded wave and the advanced wave from the transmitting antenna to the receiving antenna. This electromagnetic energy flow is generated on the transmitting antenna and annihilated on the receiving antenna, which perfectly explains the problem of wave particle duality, which is very magical.

12. Comparison of Three Electromagnetic Fields

Below we list the comparison of three electromagnetic theories in the table. The three cases are 1) quasi-static electromagnetic field theory, including magnetic quasi-static electromagnetic field theory. 2) Maxwell's radiation electromagnetic field theory refers to the electromagnetic theory that satisfies Maxwell's equation. Here, the Ampere circuital law of Maxwell's equation includes displacement current (93-96). In this paper, the electromagnetic field in Maxwell's electromagnetic theory is represented by the lowercase letters ***e*** and ***h***. 3) this author's electromagnetic field theory refers to the electromagnetic field that satisfies the electromagnetic field equation (195-201). In this paper, this author's electromagnetic field is represented by ***E*** and ***H***.

	Quasistatic field	Maxwell's theory	This author's theory
Energy conservation law	Y	N	Y
does not overflow the universe	Y	N	Y
Sliver-Muller boundary condition	N	Y	N
Degenerate into quasi-static field	Y	N	Y
Self energy principle	Y	N	Y
Calculation ease	Y	Y	N
Wheeler Feynman absorber theory		N	Y
Cramer's interpretation		N	Y
90 degree phase difference	Y	N	Y
action-at-a-distance	Y	N	Y
field theory (aether theory)	N	Y	N
Presence of depleting substances	Y	N	Y

The law of conservation of energy in the above table refers to (138,173). The boundary condition that radiation does not overflow the universe refers to (20,21). The Sliver-Muller radiation boundary condition refers to (1-4). Self energy principle refers to (203).

It can be seen from the above table that this author's electromagnetic radiation theory is very close to the quasi-static electromagnetic field theory. On contrary, Maxwell's electromagnetic theory is incompatible with the quasi-static electromagnetic field. This is also this author's suggestion not to use the same symbol for the radiated electromagnetic field derived from Maxwell's theory and the quasi-static electromagnetic field, which may cause misunderstanding. For this author's electromagnetic field theory, this author can only solve relatively simple cases, such as the case of infinite plate current. Or even for the electromagnetic field far away from the transmitting antenna. Although a set of equations proposed by this author can determine the electromagnetic field solution of this author in principle, there is still no very effective method to solve the complex situation. However, the method provided by Maxwell's electromagnetic theory can provide a reference. For radiation problems, it is often only necessary to know the far field. The far field of this author's electromagnetic theory can be obtained from Maxwell's electromagnetic theory, and then the 90 degree phase difference between electromagnetic field and magnetic field in this author's electromagnetic theory can be considered. Therefore, Maxwell's electromagnetic theory cannot be abandoned. There are some electromagnetic problems, such as solving the radiation pattern of the transmitting antenna. Such problems actually assume that there is a good absorbing material at the cosmic boundary, which means that the Sliver-Muller radiation boundary condition is valid. In this case, Maxwell's electromagnetic theory is still very effective. Therefore, this author's electromagnetic theory can be regarded as the electromagnetic theory under different boundary conditions, that is, the electromagnetic theory under the condition that radiation does not overflow the universe.

13. Conclusion

This author has proposed the mutual energy theorem of electromagnetic field since 1987. In 2017, he found that this theorem is actually the law of conservation of electromagnetic field energy, and proposed the mutual energy flow theorem and mutual energy principle. Based on these, a complete new set of radiation electromagnetic field theory is developed by this author, which is different from Maxwell's. In the past, this author mainly emphasized that compared with Maxwell's electromagnetic field theory, the electromagnetic field theory proposed by this author is an electromagnetic field theory that satisfies the law of conservation of energy. Although this is not wrong, it is not easy to persuade readers to accept this author's views. In order to further clarify this author's point of view and persuade readers to accept this author's point of view as much as possible, this author chooses another approach in this paper. This paper presents a new law that radiation does not overflow the universe. This law can also be used as a new boundary condition. This boundary condition is different from the Sliver Muller boundary condition satisfied by Maxwell equation. The Sliver-Muller boundary condition actually requires that the cosmic boundary be filled with absorber materials. this author makes no such assumption. It is assumed that there is nothing on the boundary of the universe. Thus Maxwell's equation must be in conflict with the new boundary conditions proposed by this author. This author has carried out a relaxation process for Maxwell's equation. Maxwell's equation cannot be relaxed. In fact, this author relaxes the mutual energy principle, which is equivalent to Maxwell's equation. Relaxed mutual energy principle can add the boundary condition that radiation does not overflow the universe. In this way, this author's electromagnetic theory can be fully deduced. As an example, the electromagnetic field of plate current is discussed. The electromagnetic field and energy flow of double plate current are also discussed. This author hopes that readers can see the rationality of the electromagnetic field theory proposed by this author from these examples. On the contrary, Maxwell's electromagnetic theory is flawed. In other words, Maxwell's electromagnetic theory is only suitable for the Sliver-Muller boundary condition. The electromagnetic field theory of this author is suitable for the boundary condition that radiation does not overflow the universe. In this way, this author's electromagnetic theory is completely contained in a new boundary condition. This author hopes that readers can more easily accept this author's new electromagnetic theory.

In this author's electromagnetic theory, there is an energy flow theorem from the source to the sink. This energy flow is also called mutual energy flow, which has the property of photons. The mutual energy flow is generated from the radiation source and annihilated at the sink. This is very close to the particle model in Cramer's transactional interpretation of quantum mechanics. It can be said that this author's electromagnetic theory supports Cramer's quantum mechanical model. It also supports Wheeler Feynman's absorber theory. This author's electromagnetic theory provides a good solution to the problem of wave particle duality.

In Maxwell's electromagnetic theory, the phase of electromagnetic wave is in phase. When we were in middle school, our teacher told us that electromagnetic waves in space are transmitted in the way that electric field is converted into magnetic field and magnetic field is converted into electric field. But according to Maxwell's electromagnetic field theory, the electric field and magnetic field are in phase, and the magnetic field also decreases when the electric field decreases, so the electric field cannot be converted into a magnetic field. Magnetic fields cannot be converted into electric fields. In this author's electromagnetic field theory, the electric field and magnetic field of electromagnetic wave keep 90 degree phase difference to overcome this contradiction. As the electric field and magnetic field have 90 degree phase difference, the magnetic field increases when the electric field decreases, and the electric field increases when the magnetic field decreases. In this way, the propagation of electromagnetic wave in space becomes more reasonable.

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