

Modern Trends in Scientific Research in Mathematics Education: A Comprehensive Reference Guide in Contemporary Curricula, Didactics and Methodologies

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Abstract

This comprehensive scholarly volume provides an authoritative and extensive review of contemporary trends in mathematics education research, delineating major paradigm shifts across epistemological, methodological, and pedagogical dimensions. The book is structured across four core axes: First, the evolution of foundational theoretical frameworks—from behaviorism and constructivism to the French Didactic School (Theory of Didactical Situations, Anthropological Theory of the Didactic / Didactic Transposition, and Theory of Conceptual Fields), Embodied Cognition, and Educational Neuroscience, alongside contemporary methodological innovations such as Design-Based Research (DBR), Japanese Lesson Study, multimodal learning analytics, and eye-tracking technology. Second, curricular research trends emphasizing interdisciplinary STEM/STEAM integration, mathematical modeling, and advanced digital learning ecosystems driven by generative and adaptive Artificial Intelligence, Dynamic Geometry Software (DGS), and Extended Reality (XR). Third, advanced cognitive and affective dimensions (conceptual understanding vs. procedural fluency, early algebra, mathematics anxiety, mathematical identity, growth mindset, and developmental dyscalculia) paired with the sociocultural and political turn (ethnomathematics, critical mathematics education, equity, and commognitive classroom discourse). Fourth, teacher education scholarship encompassing Pedagogical Content Knowledge (PCK), Mathematical Knowledge for Teaching (MKT), TPACK, teacher beliefs, and Professional Learning Communities (PLCs). The volume concludes with a critical meta-analysis of research outputs in the Arab world and articulates a strategic roadmap and priority agenda for future mathematics education inquiries through 2035.

Keywords: Mathematics Education Research, Didactics of Mathematics, Contemporary Research Trends, Design-Based Research (DBR), Artificial Intelligence in Mathematics Education, STEM Integration & Mathematical Modeling, Mathematical Knowledge for Teaching (MKT), Future Research Agenda 2035

Book Introduction: Major Shifts in the Didactic Space of Mathematics

From Isolated Applications to Epistemological and Institutional Autonomy

In the name of Allah, the Most Gracious, the Most Merciful, and praise be to Allah, the Lord of the Worlds, who says in the Noble Revelation: "And everything with Him is in due measure" (Qur'an 13:8), and may prayers and peace be upon the Teacher and Guide of humankind, Prophet Muhammad, and upon all his family and companions.

In recent decades, the field of mathematics education has undergone radical epistemological and methodological transformations. These shifts have elevated it from a mere applied branch of pure mathematics—or an arena for transient projections of general educational psychology theories—into a self-contained scientific and didactic discipline with full disciplinary autonomy, grounded in foundational theoretical frameworks, generative didactic concepts, rigorous investigative methodologies, and an interconnected global scholarly community anchored by major international conferences and premier refereed journals [1].

Mathematics education research is no longer reduced to simplistic comparative questions testing the effectiveness of one teaching method against another through classical experimental designs that treat the classroom as a "black box." Instead, it probes the depths of the mathematical didactic phenomenon across its triadic interactive system: (Learner – Teacher – Mathematical Knowledge). It has embraced breakthroughs in educational neuroscience, theories of embodied cognition, deep digital transformation powered by generative artificial intelligence and adaptive intelligent tutoring ecosystems, and the sociopolitical turn that conceptualizes mathematics as a vital instrument for social justice and critical democratic citizenship.

❖ **Authentic Epistemological Illumination:** *"The epistemological autonomy of mathematics education does not imply isolation from neighboring sciences; rather, it signifies possessing a distinctive didactic lens that illuminates the unique interaction between the abstract structure of mathematical knowledge and the learner's cognitive and didactic formation within a living sociocultural context."*

Rationale and Justification: Bridging the Gap Between Global Scholarship and Arab Research Reality

Diagnosing Knowledge Production and Transitioning from Surface Description to Deep Didactic Anatomy

This reference monograph arises from an urgent scholarly and field imperative: bridging the substantial epistemological and methodological divide between rapidly evolving international research agendas—reflected in elite journals such as JRME, ESM, and ZDM—and the prevailing reality of research production and classroom practice in the Arab world.

Despite the prolific quantitative output of master's theses, doctoral dissertations, and faculty research across colleges of education in the Arab region, critical analytical reviews and meta-analyses reveal a marked atrophy in the cumulative impact of these efforts on actual curriculum reform or on improving student performance in national and international assessments such as PISA and TIMSS.

This disconnect stems primarily from the dominance of repetitive research templates, a near-exclusive reliance on measuring surface-level achievement differences without examining real-time mechanisms of mathematical meaning-making, and the near-complete absence of Design-Based Research (DBR), Lesson Study, microgenetic analysis, and eye-tracking methodologies—compounded by a reluctance to localize advanced didactic frameworks such as the French Didactic School, didactic transposition, and theory of didactical situations.

Accordingly, this volume is designed as an authoritative scholarly compass and qualitative leap, equipping researchers with an integrated reference guide that deconstructs epistemological paradigms, analyzes contemporary methodologies, and charts a forward-looking research roadmap through 2035.

The Cognitive Architecture of the Book: A Journey Through Four Foundational Axes

Coherent Systemic Integration Spanning Epistemology, Technology, Affect, Teacher Professionalism, and Future Horizons

This volume is organized into four structurally and intellectually cohesive chapters, carefully architected to encompass all dimensions of the contemporary mathematics education system:

- Chapter 1: Epistemological Paradigms and Contemporary Methodological Shifts in Mathematics Education Research — Examines the historical trajectory of intellectual transformations from behaviorism to cognitive and social constructivism and APOS theory, elucidates the pillars of the French Didactic School (Guy Brousseau's TDS, Yves Chevallard's ATD and Didactic Transposition, and

G rard Vergnaud’s Conceptual Fields), and delves into embodied cognition and educational neuroscience, tracing the methodological shift toward Design-Based Research (DBR), Japanese Lesson Study, multimodal microanalysis, and eye-tracking.

- Chapter 2: Research Trends in Advanced Digital Curricula, Interactive Environments, and Artificial Intelligence — Analyzes Pierre Rabardel and Luc Trouche’s Instrumental Genesis and Orchestration, examines guided dragging modalities in Dynamic Geometry Software (GeoGebra/Desmos), explores generative AI and Adaptive Intelligent Tutoring Systems (ITS) alongside cognitive offloading risks, and demonstrates the role of Extended Reality (XR), interdisciplinary STEM integration, and real-world mathematical modeling aligned with PISA frameworks.
- Chapter 3: Advanced Cognitive Trends, the Affective Domain, and the Sociopolitical Turn — Discusses the iterative bidirectional relationship between conceptual understanding and procedural fluency as well as early algebra; explores the affective domain (mathematics anxiety and its neurobiological correlates in the amygdala and prefrontal cortex, mathematical identity, and Carol Dweck and Jo Boaler’s growth mindset); addresses developmental dyscalculia; and examines the sociopolitical turn: Ubiratan D’Ambrosio’s Ethnomathematics, Ole Skovsmose’s Critical Mathematics Education, linguistic equity, and Anna Sfard’s Commognition theory of mathematical discourse.
- Chapter 4: Research on Teacher Professional Knowledge, Critical Reading of Arab Scholarship, and the Future Agenda to 2035 — Reviews teacher knowledge frameworks (Lee Shulman’s PCK, Deborah Loewenberg Ball’s MKT, and Mishra & Koehler’s TPACK), teacher beliefs, Professional Learning Communities (PLCs), and the teacher’s evolving role as an architect of learning environments in the AI era. The chapter concludes with a critical meta-analysis of Arab research production, presenting the Matrix of Seven National Priorities (2025–2035) and an applied guideline for designing impactful master’s and doctoral dissertations.

Target Audiences and Value Proposition of the Guide

An Applied and Theoretical Reference Serving the Educational and Scholarly Community

This volume is not intended for static library shelves; rather, it is designed as an active daily working instrument for multiple vital sectors across the regional and international educational landscape:

- I. Researchers and Postgraduate Scholars (Master’s and Doctoral Candidates): Providing solid epistemological and methodological foundations to formulate original research problems with genuine scholarly contributions rather than replicating stereotypical studies.
- II. University Faculty and Research Center Fellows: Framing funded research projects, scholarly clusters, and teacher education programs around frontier international paradigms.
- III. Curriculum Planners and Assessment Developers in Ministries of Education: Assisting in translating global standards, real-world mathematical modeling, and interactive digital environments into national textbooks and assessment instruments.
- IV. Educational Supervisors and Master Teachers: Empowering practitioners with micro-observational and reflective tools through Lesson Study and Professional Learning Communities.

Future Agenda (2025–2035) and Message from the Authors

A Call for Epistemological Leadership and Methodological Boldness

We place this reference guide in your hands as we look toward a transformative decade (2025–2035) that will reshape human cognition and learning. Our proposed strategic agenda centers on seven priority themes: Generative AI and Adaptive Tutoring Systems Research, Design-Based Research (DBR), Mathematical Modeling and Computational Thinking, Neurocognitive Interventions for Developmental Dyscalculia, Lesson Study in Arab Schools, Ethnomathematics of the Arab-Islamic Heritage, and Aligning Curricula with National Development Visions and PISA Frameworks.

In conclusion, it is our earnest hope that scholars and graduate researchers will engage with this volume not as passive recipients, but as active participants in critical inquiry—summoning the methodological courage to move beyond conventional quasi-experimental templates toward microgenetic, design-based, and longitudinal investigations worthy of our scientific heritage.

We pray to Almighty God to accept this work as a sincere contribution beneficial to seekers of knowledge and a cornerstone for a thriving educational and research renaissance.

❖ *"Advancing Arab scientific research in mathematics education requires a decisive transition from measuring transient surface outputs to a deep didactic anatomy of how mathematical meaning is constructed in the living classroom moment."*

Chapter	Chapter Title and Core Knowledge Axes	Prominent Didactic Concepts & Frameworks
Chapter 1	Epistemological Paradigms and Contemporary Methodological Shifts in Mathematics Education Research	Theory of Didactical Situations (TDS) • Anthropological Theory of the Didactic & Didactic Transposition (ATD) • Conceptual Fields • APOS Theory • Embodied Cognition • Design-Based Research (DBR) • Lesson Study • Eye-Tracking
Chapter 2	Research Trends in Advanced Digital Curricula, Interactive Environments, and Artificial Intelligence	Instrumental Genesis & Orchestration • Dynamic Geometry & Computer Algebra (DGS/CAS) • Generative AI (GenAI/LLMs) • Adaptive Intelligent Tutoring Systems (ITS) • STEM/STEAM Integration • Real-World Mathematical Modeling
Chapter 3	Advanced Cognitive Trends, the Affective Domain, and the Sociopolitical Turn	Conceptual Understanding vs. Procedural Fluency • Early Algebra • Mathematics Anxiety & Identity • Developmental Dyscalculia • Ethnomathematics • Critical Mathematics Education • Commognition Theory
Chapter 4	Research on Teacher Professional Knowledge, Critical Reading of Arab Scholarship, and the Future Agenda to 2035	Mathematical Knowledge for Teaching (MKT) • TPACK Framework • Professional Learning Communities (PLCs) • Meta-Analysis of Arab Research Production • Seven National Priorities Matrix (2025–2035) • Dissertation Design Guideline

Introductory Appendix: Matrix of Book Chapters, Knowledge Axes, and Prominent Didactic Concepts (Audited & Restored)

Chapter 1

Epistemological Paradigms and Contemporary Methodological Shifts in Mathematics Education Research

I. The Evolution of Theoretical Paradigms in Mathematics Education Research and Their Epistemological Autonomy

Epistemological Evolution & Paradigm Shifts in Mathematics Education Research (From Behavioral and Constructivist Theories to French Didactics, Embodied Cognition, and Educational Neuroscience)

1. Introduction to Epistemological Paradigms

The history of mathematics education research traces a profound evolutionary trajectory of paradigm shifts that have continually redefined the nature of mathematical knowledge, how it is constructed, and the dynamics of classroom interaction. In the early decades of the twentieth century, behaviorism—which treated mathematics as a cumulative hierarchy of mechanical skills and stimulus-response associations reinforced through drill and practice—dominated the research agenda, focusing primarily on response latency, arithmetic error rates, and the effects of behavioral feedback.

With the onset of the Cognitive Revolution in the mid-twentieth century, Jean Piaget’s cognitive constructivism catalyzed a radical transformation, demonstrating that learners do not passively receive mathematical knowledge, but actively construct it through assimilation, accommodation, and equilibration (self-regulation). Lev Vygotsky’s social constructivism soon complemented this perspective, establishing that mathematical meaning-making is a socially negotiated process mediated by language, cultural signs, and tools within the Zone of Proximal Development (ZPD).

Nevertheless, the epistemological specificity of mathematics necessitated the emergence of specialized theories rooted in the structure of mathematics itself. This need crystallized remarkably in the French Didactic School, followed at the turn of the twenty-first century by the wave of Embodied Cognition, which is today reinforced by advances in Educational Neuroscience [2,3]. This chapter examines these paradigms and their direct impact on shaping contemporary scientific research questions.

1.1. Autonomy of Mathematics Education as an Authentic Scientific Discipline (Disciplinary Autonomy)

It was only after strenuous intellectual effort that mathematics education established its autonomous academic identity, having long been viewed either as "simplified mathematics" prescribed by pure mathematicians or as "generic applications of educational psychology theories."

Epistemological autonomy was realized through the formulation of specialized conceptual frameworks that investigate the "mathematical didactic phenomenon" in its entirety across the Didactic Triangle: (Learner – Teacher – Mathematical Knowledge). The founding of premier specialized journals—most notably Educational Studies in Mathematics (ESM) in 1968 and the Journal for Research in Mathematics Education (JRME) in 1970—consolidated a scholarly identity characterized by methodological rigor and grounded in the structural determinants of mathematical knowledge [1-24].

1.2. From Behaviorism to Individual and Social Constructivism in Mathematics Research

With constructivism, the locus of research inquiry shifted from "observable learner behavior and rote learning outcomes" to "learners’ mental strategies and internal cognitive representations." This shift sparked groundbreaking research streams in:

- **Misconceptions and Alternative Frameworks:** Research demonstrated that students’ systematic errors do not reflect a cognitive vacuum, but rather coherent internal mental structures that require targeted didactic reconstruction.
- **APOS Theory (Action, Process, Object, Schema):** Developed by Ed Dubinsky and colleagues as an extension of Piaget’s reflective abstraction to explain how complex mathematical concepts (such as functions, calculus, and abstract algebra) evolve from an external procedural "Action" into an interiorized mental "Process," become encapsulated as an independent mathematical "Object" upon which higher-order operations can act, and are ultimately organized into a coherent cognitive "Schema" [25].

1.3. The French Didactic School: Theory of Didactical Situations, Conceptual Fields, and Didactic Transposition

The French Didactic School (Didactique des Mathématiques) represents one of the most fertile epistemological traditions in global mathematics education research, contributing three foundational theories that form cornerstones of contemporary scholarship:

- **Theory of Didactical Situations (TDS)** by Guy Brousseau: Distinguishes between explicitly directed didactic situations and "a-didactical situations" (situations adidactiques), wherein learners confront an authentic mathematical problem within a carefully engineered milieu without direct teacher prompting, progressing through phases of action, formulation, and validation before the teacher leads institutionalization. TDS also introduced the "didactic contract" (contrat didactique), capturing the implicit system of mutual expectations and obligations between teacher and students regarding mathematical knowledge [1].
- **Theory of Conceptual Fields** by Gérard Vergnaud: Posits that a mathematical concept is never constructed in isolation, but develops across an interconnected developmental field defined by a tuple of three sets: the set of situations that give the concept meaning, the set of operational invariants (concepts-in-action and theorems-in-action), and the set of symbolic, linguistic, and graphical representations [26].
- **Anthropological Theory of the Didactic (ATD) and Didactic Transposition** by Yves Chevallard: Traces the institutional transformations that mathematical knowledge undergoes as it moves from the "scholarly knowledge" (savoir savant) produced by professional mathematicians, to the "knowledge to be taught" (savoir à enseigner) codified in curriculum documents and textbooks, to the "knowledge actually taught" (savoir enseigné) in classrooms, and finally to the "learned knowledge" acquired by students—Catalyzing a rich global research tradition in textbook and institutional analysis [4].

1.4. Contemporary Cognitive Perspectives: Embodied Cognition and Educational Neuroscience in Mathematics Learning

Recent decades have witnessed the rise of the Embodied Cognition paradigm, pioneered in mathematics by scholars such as George Lakoff and Rafael Núñez in their seminal work *Where Mathematics Comes From*, alongside Luis Radford’s *Theory of Objectification* [5,3]. Their work demonstrates that even the most abstract mathematical ideas (such as infinity, sets, limits, and vector spaces) are grounded in spatial and sensorimotor conceptual metaphors rooted in bodily experience within the physical world.

This shift has converged with a surge in Educational Neuroscience and functional magnetic resonance imaging (fMRI) research led by cognitive neuroscientists such as [2]. Neuroimaging studies reveal the dynamic synergy between parietal brain networks (particularly the intraparietal sulcus, IPS) and prefrontal regions during mathematical processing, confirming that mathematical thinking is not an isolated symbolic manipulation, but a neurodynamic activity integrating spatial, visual, motor, and symbolic magnitude representations simultaneously.

Theoretical Paradigm	Pioneers and Founders	Conception of Mathematical Knowledge	Role of the Classroom Learner	Fundamental Research Question
Behaviorism	Thorndike, Skinner, Gagné	Cumulative hierarchy of skills and algorithmic associations linking stimuli and responses	Passive recipient practicing and mastering algorithmic steps through reinforcement	What is the impact of distributed practice and feedback on computational speed and accuracy?
Cognitive Constructivism	Piaget, von Glasersfeld, Dubinsky (APOS)	Active self-construction emerging from resolving cognitive disequilibrium and reflective abstraction	Explorer and thinker who coordinates mental actions to construct cognitive schemas	How does a student encapsulate mental actions and processes into abstract mathematical objects?
French Didactics (TDS / ATD / Conceptual Fields)	Guy Brousseau, Yves Chevallard, Gérard Vergnaud	Functionally constructed through problem situations and historically/institutionally transposed	Active agent within an a-didactical milieu, testing conjectures and negotiating the didactic contract	What are the characteristics of didactical situations that necessitate authentic mathematical proof?
Embodied Cognition & Educational Neuroscience	Lakoff & Núñez, Dehaene, Radford	Spatial and sensorimotor metaphorical structures rooted in the bodily and neural architecture	Embodied participant integrating physical gestures, artifacts, and digital tools into abstraction	How do spatial activities and sensorimotor gestures modulate mathematical neural pathways in the brain?

Table 1: Epistemological Comparison Matrix Across Major Theoretical Paradigms in Mathematics Education Research

Key Epistemological & Didactic Takeaways:

- ✓ The field of mathematics education has transitioned from dependence on general psychology to full epistemological autonomy with rigorous domain-specific frameworks and concepts.
- ✓ Contemporary theories converge in conceptualizing mathematical knowledge as individually constructed, socially and institutionally negotiated, and dynamically and neurobiologically embodied.

II. Contemporary Methodological Shifts and Micro-Investigative Research Tools

Contemporary Methodological Shifts & Microscopic Research Techniques (Design-Based Research [DBR], Lesson Study, Multimodal Learning Analytics, and Eye-Tracking)

1. Introduction to Methodological Shifts

Recent advances in mathematics education have not merely replaced theoretical frameworks; they have been accompanied by a parallel revolution in research methodologies and data collection and analysis techniques. For decades, classical pre-post quasi-experimental designs dominated the landscape, frequently treating the classroom as a "black box" focused solely on measuring terminal achievement outcomes without illuminating the complex micro-mechanisms through which mathematical understanding emerges in real time.

Addressing this limitation has given rise to frontier methodological movements, foremost among them: Design-Based Research (DBR), collaborative Japanese Lesson Study, and micro-analytic investigative technologies such as multi-angle video ethnography, eye-tracking, and Multimodal Learning Analytics (MMLA). This section examines these methodological shifts and the publication standards of leading international journals.

1.1. Design-Based Research (DBR) as a Leading Methodology

Design-Based Research (DBR) represents one of the most prominent methodological paradigms in contemporary mathematics education inquiry [6,27]. Rejecting the artificial dichotomy between theory generation and classroom practice, DBR involves engineering an innovative instructional intervention (e.g., a sequence of rich mathematical tasks, dynamic software environment, or learning module) and studying it within naturalistic classroom settings across iterative cycles that encompass:

- Preliminary didactic analysis and initial design grounded in explicit theoretical conjectures and Hypothetical Learning Trajectories (HLTs).
- Naturalistic classroom enactment accompanied by intensive, moment-by-moment qualitative and quantitative multimodal data collection.
- Retrospective and ongoing analysis leading to immediate refinement and redesign of the intervention conjectures across iterative cycles.
- Formulation of domain-specific Local Instruction Theories (LITs) that explain how mathematical understanding develops and how instructional design supports those learning trajectories.

1.2. Japanese Lesson Study as a Field-Based Research Movement

Lesson Study (Jugyuu Kenkyuu) has evolved from a foundational Japanese professional development practice into a globally recognized research methodology supported by dedicated international associations (such as the World Association of Lesson Studies, WALs) and scholarly venues [28].

This methodology relies on close collaboration between academic researchers and practicing teachers to jointly plan a "research lesson" centered on a specific mathematical learning challenge, enact and systematically observe the live lesson with fine-grained attention to students' real-time mathematical reasoning, and conduct an in-depth post-lesson reflective colloquium. Lesson Study has demonstrated exceptional power in generating actionable, tacit pedagogical content knowledge that remains inaccessible to survey questionnaires or isolated observations.

1.3. Micro-Investigative Techniques: Eye-Tracking and Multimodal Learning Analytics

Technological advances have equipped mathematics education researchers with unprecedented instruments for observing students' cognitive processes and problem-solving strategies at a microscopic scale:

- Eye-Tracking Technology: Enables precise monitoring of gaze trajectories, fixation durations, and saccadic movements while learners read mathematical word problems or inspect function graphs and geometric figures—accurately revealing cognitive differences between experts and novices and diagnosing visual attention allocation.
- Multimodal Learning Analytics (MMLA): Integrates high-definition video streams, audio transcripts, gesture and hand-movement tracking, physiological sensors, and touchscreen interaction logs, enabling a rich microgenetic understanding of kinesthetic visualization and mathematical discourse during collaborative problem solving.

1.4. Criteria for Methodological Rigor in Premier International Journals (JRME, ESM, ZDM)

Leading international journals in mathematics education enforce rigorous methodological standards that go far beyond surface descriptions or simple pre-post comparisons, requiring:

- Complete epistemological coherence connecting the research question, the theoretical lens, the research design, and the analytical units.
- Transparency of inferential chains and rigorous documentation of qualitative and quantitative coding procedures (e.g., open and axial coding protocols, audit trails, and inter-rater reliability indices).
- Substantive theoretical contribution rather than replicating stereotypical impact-measurement studies that merely confirm the obvious without illuminating underlying learning mechanisms.

Key Epistemological & Didactic Takeaways:

- ✓ Mathematics education research has moved beyond simplistic quantitative/qualitative binaries in favor of iterative Design-Based Research (DBR) and deep mixed-methods inquiry.
- ✓ Modern micro-investigative tools (eye-tracking and multimodal learning analytics) make it possible to observe the microgenetic formation of mathematical meaning in real time.

Chapter 2

Research Trends in Advanced Digital Curricula, Interactive Environments, and Artificial Intelligence

I. Artificial Intelligence Research, Interactive Technology, and Instrumental Transformation

Research Trends in Generative AI, Intelligent Tutoring Systems (ITS), Dynamic Geometry Software (DGS), and the Instrumental Approach

1. Introduction to Digital and Instrumental Transformation

Digital technology and artificial intelligence represent the most rapidly evolving frontier in contemporary mathematics education research agendas. Technology is no longer conceptualized merely as an alternative presentation medium or computational aid, but rather as a cognitive partner and epistemic medium that fundamentally reshapes the nature of mathematical activity.

Theoretically, the Instrumental Approach pioneered by Pierre Rabardel and Luc Trouche provides a rigorous foundation for understanding the relationship between technological artifacts and mathematical learning. It demonstrates that a physical or digital artifact becomes an effective psychological and epistemic instrument only through a dual developmental process known as Instrumental Genesis: instrumentalization (wherein the learner adapts and shapes the artifact's affordances) and instrumentation (wherein the artifact shapes and restructures the learner's cognitive utilization schemes) [7].

With the current explosion of Generative Artificial Intelligence (GenAI), Adaptive Intelligent Tutoring Systems (ITS), and Extended Reality (XR), critical research questions have intensified regarding the future of human mathematical reasoning, agency, and proof in an era of cognitive automation [19,20,29].

1.1. Instrumental Genesis and Orchestration in Mathematics Education

This perspective draws a fundamental distinction between an "artifact"—such as raw software code, a graphing calculator, or a computer—and an "instrument," which comprises the artifact integrated with the mental utilization schemes (*schèmes d'utilisation*) that the student constructs through purposeful mathematical activity.

Empirical didactic studies demonstrate that deploying environments such as GeoGebra or graphing calculators requires dedicated didactic time for instrumental genesis alongside deliberate classroom "instrumental orchestration" by the teacher; if a learner lacks adequate cognitive schemes, the software can become an epistemological obstacle rather than a bridge to conceptual understanding [7].

1.2. Research Trends in Dynamic Geometry Software (DGS) and Computer Algebra Systems (CAS)

Research on Dynamic Geometry Software (DGS) and Computer Algebra Systems (CAS) has matured far beyond demonstrating generic achievement gains to investigating the micro-mechanisms of mathematical exploration, invariant discovery, and conjecturing [8].

Contemporary studies focus on "guided dragging modalities," distinguishing systematically among wandering/random dragging, maintaining/guided dragging to discover geometric invariants, and dragging tests to validate conjectures—examining how dynamic

visual manipulation mediates the transition from empirical observation to rigorous deductive mathematical proof [8,18].

1.3. Generative and Adaptive Artificial Intelligence Research: Intelligent Tutoring Systems (ITS)

Mathematics education journals are witnessing an unprecedented wave of empirical and theoretical inquiry investigating the impact of Large Language Models (LLMs) and Adaptive Intelligent Tutoring Systems (ITS) across three critical dimensions [19,20,29]:

- Adaptive Problem Generation and Immediate Diagnostic Feedback: Investigating intelligent systems capable of diagnosing structural misconceptions and delivering step-by-step cognitive scaffolding without prematurely revealing the final answer [19,30].
- Proof Evaluation and Multi-Step Mathematical Reasoning: Auditing the capabilities and limitations of multimodal AI models in symbolic and visual mathematical reasoning, alongside the didactic risks of "mathematical hallucinations" [20,29].
- Cognitive Offloading and Epistemic Agency: Warning against the erosion of students' critical thinking, productive struggle, and cognitive autonomy when learners rely uncritically on automated solver tools [29,31].

1.4. Extended Reality (XR) and Immersive Environments in Spatial Abstraction Research

Augmented Reality (AR) and Virtual Reality (VR) research illuminates the teaching and learning of spatially demanding branches of mathematics, including 3D analytical geometry, multivariable calculus, vector fields, and topology [16].

Empirical and meta-analytic investigations demonstrate that learners' kinesthetic and spatial manipulation of 3D mathematical objects within immersive environments significantly reduces extraneous cognitive load and enhances mental rotation and cross-sectional visualization skills [16,31,32].

Target Technological Domain	Mainstream Research Topics	Adopted Theoretical Framework	Empirical and Didactic Findings
Dynamic Geometry & Graphing Software (GeoGebra / Desmos)	Exploratory dragging modalities, conjecture and proof generation, and synchronous linking of curves and algebraic equations	Instrumental Genesis & Orchestration Theory (Rabardel; Trouche) and TPACK Framework	Substantial gains in conceptual understanding of functions and conic sections and in mathematical conjecturing
Generative Artificial Intelligence (GenAI / LLMs)	Adaptive scaffolding generation, real-world problem posing, and diagnosis of conceptual errors vs. AI hallucinations	Cognitive Scaffolding Theory (Wood, Bruner, & Ross, 1976) and Self-Regulated Learning	Provides personalized real-time feedback while requiring pedagogical governance to prevent cognitive offloading
Adaptive Intelligent Tutoring Systems (ITS)	Psychometric and cognitive modeling of learner data and personalization of individual learning trajectories	Item Response Theory (IRT) and Cognitive Load Theory (Sweller et al.)	Closes foundational skill achievement gaps and reduces mathematics anxiety levels
Augmented, Virtual, and Extended Reality (AR / VR / XR)	3D stereoscopic manipulation, spatial vectors, and complex multivariable surfaces	Embodied Cognition Theory and Paivio's Dual Coding Theory (1986)	Significant leaps in spatial reasoning assessments and comprehension of 3D geometry and multivariable calculus

Table 2: Classification of Research Trends in AI Integration, Digital Environments, and Didactic Impact Levels

Key Epistemological & Didactic Takeaways:

- ✓ The instrumental approach underscores that merely possessing technology does not make it an intellectual instrument unless it is integrated with learners' cognitive schemes.
- ✓ AI research in mathematics education is maturing from initial technical fascination toward a critical examination of reasoning mechanisms and cognitive autonomy.

II. Contemporary Trends in Curriculum Research, STEM Integration, and Mathematical Modeling

Real-World Mathematical Modeling, Non-Routine Problem Solving & Posing, and Mathematical Literacy in Light of PISA

1. Introduction to Curriculum Integration and Mathematical Modeling

Contemporary curriculum research moves beyond teaching school mathematics as isolated topical islands, positioning mathematics instead as the analytical backbone of interdisciplinary Science, Technology, Engineering, and Mathematics (STEM/STEAM) education [23].

Recent scholarship investigates the "Mathematical Modeling Cycle" as an iterative dynamic process that begins with an open-ended real-world problem situation, simplifies and structures the context, formulates a mathematical model, solves the mathematical problem, interprets and validates the solution against real-world constraints, and informs decision-making—paired with problem-posing pedagogy and alignment with the OECD PISA Mathematical Literacy framework [32-34].

Key Epistemological & Didactic Takeaways:

- ✓ Real-world mathematical modeling and interdisciplinary STEM integration represent the contemporary pillars for cultivating critical thinking and complex problem solving.
- ✓ Global mathematics curricula are recalibrating their standards to align with the mathematical literacy and computational reasoning demands of PISA.

Chapter 3

Advanced Cognitive Trends, the Affective Domain, and the Sociopolitical Turn

I. Cognitive Processes Research and the Affective Domain in Mathematics Learning

Cognitive Processes & The Affective Domain in Mathematics Learning (Conceptual Understanding vs. Procedural Fluency, Early Algebra, Mathematics Anxiety, and Developmental Dyscalculia)

1. Introduction to Cognitive and Affective Processes

The mathematical mind is inseparable from the learner's psychological and affective state; mathematics is among the academic disciplines most capable of evoking intense emotions, ranging from the joy of discovery and intellectual passion to frustration, learned helplessness, and acute anxiety. Consequently, mathematics education research over the past two decades has forged a deep integration between fine-grained cognitive process studies and investigations of affective and neurocognitive variables.

Contemporary cognitive inquiry addresses enduring dilemmas—foremost among them the dialectical relationship between conceptual understanding and procedural fluency, and the developmental roots of early algebraic thinking in elementary school children. Simultaneously, premier journals devote substantial attention to the affective domain, examining mathematics anxiety, mathematical identity, growth mindset, and specific neurodevelopmental learning disabilities, particularly developmental dyscalculia.

1.1. Conceptual Understanding and Procedural Fluency: An Iterative Bidirectional Relationship

Contemporary empirical scholarship has resolved the long-standing "math wars" debate over whether concepts or procedures must strictly precede one another, establishing instead an "iterative bidirectional relationship" between the two [9].

Incremental gains in conceptual understanding support the invention and adoption of flexible computational procedures, while fluency with procedures reduces cognitive load and frees working memory capacity to reflect upon and deepen abstract structural concepts—confirming that divorcing the two in classroom instruction yields either fragile, unworkable understanding or brittle, rapidly forgotten rote memorization.

1.2. Early Algebra and Proportional Reasoning Research

Pioneering research by James Kaput, Maria Blanton, and David Carraher has demonstrated that elementary school children possess a natural capacity to reason algebraically and formulate mathematical generalizations years before encountering formal symbolic algebra [35].

Current research focuses on three foundational strands: generalizing arithmetic operations and structural properties, conceptualizing the equals sign ($=$) as a relational equivalence indicator rather than a unidirectional operational command ("compute the answer"), and cultivating proportional and functional reasoning as the primary cognitive gateway to advanced algebra, geometry, and physics [35].

1.3. The Affective Domain: Mathematics Anxiety, Mathematical Identity, and Growth Mindset

Mathematics Anxiety is one of the most extensively investigated affective phenomena in education. Neurocognitive and psychological studies demonstrate that acute mathematics anxiety triggers hyperactivity in the amygdala and pain-processing regions while disrupting prefrontal cortex executive networks, thereby taxing and paralyzing working memory resources during mathematical performance. Recent scholarship explores evidence-based pedagogical interventions, including cultivating a mathematical "growth mindset" grounded in the work, fostering positive "mathematical identity," and restructuring classroom and assessment environments into psychologically safe spaces that value errors as productive opportunities for conceptual growth and neural plasticity [10].

1.4. Specific Learning Disabilities: Developmental Dyscalculia and Early Interventions

Developmental Dyscalculia is a specific neurodevelopmental disorder affecting approximately 5% to 7% of learners, associated with core deficits in the Approximate Number System (ANS) and numerosity coding within the bilateral intraparietal sulcus (IPS) [11,2].

Pioneering research investigates early neurocognitive screening tools and adaptive digital remediation games designed to strengthen the mapping between symbolic numerical representations and non-symbolic spatial-quantitative magnitudes, while cautioning against conflating true developmental dyscalculia with general low achievement caused by inadequate instruction or mathematics anxiety [11].

Key Epistemological & Didactic Takeaways:

- ✓ Conceptual understanding and procedural fluency develop through a mutually reinforcing spiral; instruction that separates them undermines students' cognitive architecture.
- ✓ Mathematics anxiety is a genuine neurophysiological barrier that taxes working memory; addressing it requires reforming assessment practices and cultivating a growth mindset.

II. The Social and Political Turn in Mathematics Education Research

Ethnomathematics, Critical Mathematics Education, Equity and Language Diversity, and Classroom Mathematical Discourse

1. Introduction to the Social and Sociopolitical Turn

The late 1990s and early 2000s witnessed what has been termed the "Social Turn," followed by the "Sociopolitical Turn" in mathematics education research, articulated by leading scholars such as AS [12,14]. This paradigm challenged the prevailing epistemological myth that mathematics is a purely neutral, culture-free discipline detached from power, identity, and human contexts.

Grounded in the recognition that mathematics is a human cultural practice produced within historical contexts and often deployed as a gatekeeper distributing social opportunity and power, this perspective has generated vital research programs: ethnomathematics, critical mathematics education, linguistic and socioeconomic equity, and classroom discourse analysis.

1.1. Ethnomathematics and the Contributions of Cultural Diversity

Brazilian mathematician and philosopher Ubiratan D'Ambrosio founded the Ethnomathematics program to investigate and honor the mathematical ideas, algorithms, and spatial practices developed by diverse cultural groups (including artisans, indigenous communities, Arab-Islamic architectural geometry and tessellations, celestial navigation, and weaving traditions) [13].

Empirical studies demonstrate that integrating mathematics rooted in learners' local culture and civilizational heritage strengthens mathematical identity, elevates intrinsic motivation, and dismantles Eurocentric monopolies over the history and epistemology of mathematics [13].

1.2. Critical Mathematics Education (Ole Skovsmose)

Ole Skovsmose's Critical Mathematics Education conceptualizes mathematics as exercising a profound "formatting power" over contemporary society—shaping economic models, algorithmic decision-making, public policy, and environmental governance [14].

Research in this tradition aims to cultivate "mathemacy"—critical mathematical literacy for democratic citizenship—equipping learners not merely to execute equations, but to exercise the intellectual agency required to interrogate mathematical and statistical models and evaluate their ethical fairness and social bias [14].

1.3. Equity and Language Diversity Research in Mathematics Classrooms

The global equity agenda focuses on dismantling structural achievement and opportunity gaps in mathematics linked to socioeconomic status, gender, and linguistic diversity [12].

In particular, sociolinguistic studies examine the challenges of learning mathematics in a second or foreign language (such as English-medium mathematics instruction in Arab schools and universities), documenting the "double linguistic and cognitive burden" learners experience when simultaneously decoding foreign lexical structures and complex mathematical abstractions.

1.4. Sociology of Mathematics Classrooms and Discourse (Anna Sfard's Commognition Theory)

Classroom discourse analysis and Anna Sfard's Commognition Theory—which unifies cognition and interpersonal communication—conceptualize mathematics learning as a participationist shift in the learner's mathematical discourse, characterized by specialized word use, visual mediators, endorsed narratives, and discursive routines [15].

Empirical classroom studies contrast authoritarian teacher-led "Initiation–Response–Evaluation" (IRE) recitation patterns with dialogic "interactive inquiry communities," wherein students engage in mathematical argumentation, justify conjectures, and refine proofs through socially negotiated logical evidence [15].

Key Epistemological & Didactic Takeaways:

- ✓ Mathematics is not a detached, culturally neutral abstraction, but a human practice deeply intertwined with culture, social justice, and lived experience.
- ✓ Critical mathematical empowerment (Mathemacy) equips learners with the analytical agency to hold societal decisions and digital algorithms accountable.

Chapter 4

Research on Teacher Professional Knowledge, Critical Reading of Arab Scholarship, and the Future Agenda to 2035

I. Mathematics Teacher Professional Knowledge Research and Sustainable Professional Development

PCK, MKT, and TPACK Models, Teacher Beliefs, and Professional Learning Communities (PLCs)

1. Introduction to Teacher Preparation and Professional Knowledge Architecture

Hundreds of empirical studies and international policy reports concur that the teacher is the single most decisive school-level variable in achieving high-quality mathematical learning outcomes and overcoming barriers to abstraction. Consequently, mathematics teacher preparation and sustainable professional development stand at the forefront of global research agendas.

Conceptual frameworks modeling teacher knowledge have evolved remarkably over the past four decades—from Lee Shulman’s seminal Pedagogical Content Knowledge (PCK) paradigm, to Deborah Loewenberg Ball and her colleagues’ practice-based Mathematical Knowledge for Teaching (MKT) framework, to Punya Mishra and Matthew Koehler’s Technological Pedagogical Content Knowledge (TPACK) model [36-38]. This section analyzes these frameworks alongside research on teachers’ epistemological beliefs and professional learning communities.

1.1. The Mathematical Knowledge for Teaching (MKT) Framework by Deborah Loewenberg Ball and Colleagues

The MKT framework deconstructs the specialized mathematical knowledge demanded by the work of teaching into six fine-grained, intersecting domains organized across two major dimensions [37]:

- Subject Matter Knowledge (SMK): Encompasses Common Content Knowledge (CCK: solving mathematical problems accurately as any educated adult or mathematician would), Specialized Content Knowledge (SCK: uniquely mathematical knowledge required exclusively for teaching, such as unpacking the structural rationale of algorithms, evaluating unconventional student methods, and selecting mathematically sound representations), and Horizon Content Knowledge (HCK: awareness of how mathematical topics connect and extend across the broader disciplinary trajectory).
- Pedagogical Content Knowledge (PCK): Encompasses Knowledge of Content and Students (KCS: anticipating typical student misconceptions, errors, and developmental difficulties), Knowledge of Content and Teaching (KCT: sequencing instructional tasks and selecting appropriate didactic representations and examples), and Knowledge of Content and Curriculum (KCC: understanding curricular vertical alignment, standards, and instructional materials).

1.2. Research on Mathematics Teachers’ Beliefs and Classroom Impact

Foundational research demonstrates that a teacher’s deeply held epistemological beliefs about the "nature of mathematics" and the "nature of mathematics teaching and learning" act as cognitive filters governing classroom enactment, regardless of reformed textbooks or curricular mandates [39,40].

A teacher who views mathematics as a static body of fixed, ready-made rules and procedures will default to rote transmission even when equipped with state-of-the-art digital technology. Accordingly, contemporary professional development scholarship focuses on catalyzing belief change through reflective inquiry, video clubs, and collaborative analysis of student mathematical thinking [39,40].

1.3. Professional Learning Communities (PLCs) and Collaborative Peer Inquiry

Empirical evaluation studies consistently show that traditional, episodic "one-shot" training workshops detached from school contexts fail to produce sustainable change in classroom instruction.

Professional Learning Communities (PLCs), Communities of Practice, and school-based Lesson Study have emerged as powerful alternatives, wherein teachers collaborate systematically within their schools to examine student work artifacts, co-design inquiry lessons, and conduct peer observations—cultivating a sustainable culture of practitioner inquiry [28].

1.4. Mathematics Teacher Competencies in the Age of Artificial Intelligence and Digital Transformation

Contemporary mathematics education scholarship raises pressing questions regarding the evolving role of the teacher within generative AI-enhanced learning ecosystems and adaptive intelligent platforms [16-20].

The literature confirms that the teacher’s role is shifting decisively from a "knowledge transmitter and algorithmic demonstrator" to an "architect of didactic environments," an orchestrator of investigative mathematical discourse, and a steward of learners’ critical and ethical agency—necessitating a fundamental redesign of teacher education programs across faculties of education [20,29,38].

Conceptual Framework	Developers and Year	Core Dimensions and Components	Applied Research Focus	Common Measurement Methods and Tools
Pedagogical Content Knowledge (PCK)	Lee Shulman (1986)	Subject matter knowledge, pedagogical strategies, and knowledge of learners’ conceptions and preconceptions	How a teacher transforms specialized disciplinary knowledge into forms pedagogically accessible to diverse learners	Structured clinical interviews, lesson artifact analysis, and practitioner reflective logs
Mathematical Knowledge for Teaching (MKT)	Deborah Loewenberg Ball et al. (2008)	Subject Matter Knowledge (CCK, SCK, HCK) and Pedagogical Content Knowledge (KCS, KCT, KCC)	Identifying the precise mathematical reasoning and unpacking skills exclusive to the profession of mathematics teaching	Psychometric scenario-based multiple-choice instruments (LMT/MKT Measures) and classroom video coding rubrics
Technological Pedagogical Content Knowledge (TPACK)	Punya Mishra & Matthew J. Koehler (2006)	Triadic integration of Content Knowledge (CK), Pedagogical Knowledge (PK), and Technological Knowledge (TK)	Purposeful didactic orchestration of digital media and AI tools to serve specific mathematical concepts	Validated TPACK surveys, classroom observation protocols, and evaluation of technology-integrated lesson designs

Table 3: Comparison Matrix of Mathematics Teacher Knowledge Frameworks (PCK, MKT, TPACK) and Their Research Indicators

Key Epistemological & Didactic Takeaways:

- ✓ Mathematical Knowledge for Teaching (MKT) is a specialized professional knowledge base fundamentally distinct from the purely academic knowledge of a research mathematician.
- ✓ Effective professional development occurs job-embedded within classrooms through continuous communities of practice and Lesson Study, rather than isolated theoretical lectures.

II. A Critical Reading of Arab Research Reality and the Agenda for Future Priorities Through 2035

Meta-Analysis of Research Production, Methodological and Thematic Gaps, and Applied Guidelines for University Dissertations

1. Introduction to the Reality of Arab Research in Mathematics Education

The Arab world possesses substantial scholarly potential, with hundreds of faculties of education and curriculum departments producing thousands of master’s theses, doctoral dissertations, and peer-reviewed papers in mathematics education annually. Yet critical analytical reviews and meta-analyses reveal a persistent chasm between the quantitative volume of this output and its tangible impact on curriculum modernization or student achievement on national and international assessments such as TIMSS and PISA [21,23,24].

This gap stems largely from thematic repetition, over-reliance on formulaic quasi-experimental designs measuring surface achievement differences without probing cognitive mechanisms, and limited engagement with contemporary international didactic paradigms. This concluding section presents a candid critical reading of Arab mathematics education research and sets forth a strategic priority matrix and research roadmap through 2035, accompanied by practical guidelines for graduate researchers and academic supervisors.

1.1. Meta-Analysis of Mathematics Education Research in the Arab World: Strengths and Shortcomings

A systematic synthesis of dissertations and peer-reviewed studies published across Arab educational journals over the past decade highlights both notable strengths and structural limitations:

- Strengths: Significant quantitative productivity, strong interest in active learning strategies, and broad coverage across school mathematics strands and grade levels.
- Methodological and Thematic Deficiencies: Near-total dominance of classical pre-post quasi-experimental designs measuring "the effect of strategy X on achievement Y" without deep didactic theorization; the virtual absence of Design-Based Research (DBR), Lesson Study, microgenetic video analysis, and eye-tracking studies; and a scarcity of longitudinal investigations tracking the developmental growth of mathematical thinking across multiple school years.

1.2. Epistemological Gaps Between Local Production and Leading Global Trends (ICME, ESM, JRME)

Comparing local research output with frontier international venues (ICME, ESM, JRME, ZDM) reveals four critical knowledge gaps [21-24,27]:

- Scarcity of domain-specific didactic frameworks (such as French TDS, ATD, Conceptual Fields, and APOS theory) in favor of generic psychological theories.
- Limited authentic research on real-world mathematical modeling cycles and interdisciplinary STEM integration anchored in genuine industrial, environmental, and economic challenges [22,23].
- Paucity of interdisciplinary neurocognitive research on mathematical brain networks and developmental dyscalculia [11,2].
- Few rigorous investigations examining teacher instrumental orchestration and cognitive governance within Generative AI and adaptive tutoring ecosystems [19,20,29].

1.3. National Priorities Matrix and Proposed Future Research Agenda (2025–2035)

This volume recommends directing graduate dissertations and funded research grants toward seven strategic priority axes for the 2025–2035 decade:

- Generative AI and Adaptive Tutoring Systems Research: Designing and auditing intelligent platforms that deliver adaptive mathematical scaffolding while preserving students’ epistemic agency and preventing cognitive offloading.
- Design-Based Research (DBR): Engineering interactive learning modules for abstract mathematical concepts and documenting microgenetic student learning trajectories to build Local Instruction Theories.
- Mathematical Modeling and Computational Thinking Research: Integrating algorithmic coding, simulation, and data literacy into authentic real-world mathematical problem solving.
- Early Neurocognitive Screening and Remediation for Developmental Dyscalculia in primary school learners.
- School-Based Lesson Study and Professional Learning Communities for sustainable mathematics teacher development in Arab schools.
- Ethnomathematics Research connecting school mathematics curricula with Arab-Islamic architectural, geometric, and astronomical heritage.
- Curriculum Alignment and Assessment Research harmonizing national mathematics standards with OECD PISA Mathematical Literacy frameworks and National Development Visions.

1.4. An Applied Guideline for Master’s and Doctoral Researchers in Selecting and Designing Dissertations

The book offers four actionable guidelines for graduate researchers and academic supervisors to produce impactful dissertations with genuine scholarly contributions [25–40]:

- Move Beyond Formulaic Templates: Abandon repetitive titles of the form "The Effectiveness of a Proposed Program on Achievement," and instead investigate authentic didactic problems arising from how learners construct specific mathematical concepts.
- II. Ground Inquiry in Domain-Specific Didactic Theory: Construct theoretical frameworks rooted directly in mathematics education (e.g., TDS, ATD, APOS, Instrumental Genesis, Commognition, or MKT) rather than reciting generic educational theories detached from the mathematical structure.
- III. Exercise Methodological Boldness: Embrace Design-Based Research (DBR), Lesson Study, microgenetic protocol analysis, eye-tracking, and rigorous mixed-methods designs.
- IV. Pursue International Scholarly Dissemination: Publish findings in international peer-reviewed journals indexed in Scopus and Web of Science to amplify the Arab scholarly voice within the global mathematics education community.

Research Domain	Prevailing Reality in Arab Research	Leading Global Trends	Proposed Priorities for the Future Agenda (2025–2035)
Methodology & Data Collection Tools	Predominance of simple quasi-experimental designs (pre/post-tests and self-report questionnaires)	Design-Based Research (DBR), Lesson Study, Microgenetic Video Analysis, and Eye-Tracking	Expansion of iterative instructional design research, deep mixed methodologies, and multimodal inquiry
Technology & Digitalization	Survey studies and using software merely as aggregate presentation tools in comparative trials	Instrumental Genesis Theory, Generative AI, and Adaptive Intelligent Tutoring Systems (ITS)	Applied research on adaptive tutoring systems, AI pedagogical governance, and Extended Reality (XR)
Curriculum & Knowledge Integration	Fragmented studies within isolated school mathematics branches (algebra, geometry, arithmetic)	Math-centered STEM integration, real-world mathematical modeling, computational thinking, and data science	Developing integrative inquiry-based curricula uniting mathematical modeling, programming, and data analytics

Teacher Professional Development	Isolated training workshops and measuring competencies via self-assessment questionnaires	Fine-grained MKT and TPACK frameworks, Professional Learning Communities (PLCs), and Lesson Study	Implementing school-embedded professional development grounded in practice communities and live Lesson Study
Sociocultural, Affective & Neurocognitive Dimensions	Focus on general motivation and attitudes using brief quantitative scales	Ethnomathematics, Critical Mathematics Education, Mathematical Identity, Linguistic Equity, and Dyscalculia	Exploring mathematics in local heritage, remediating math anxiety and dyscalculia, and addressing bilingual challenges

Table 4: Matrix of Research Gaps Between Arab Production, Global Trends, and Future Agenda Priorities (2025–2035)

Key Epistemological & Didactic Takeaways:

- ✓ Advancing Arab scientific research in mathematics education requires moving from measuring surface achievement outputs to analyzing deep didactic processes.
- ✓ The 2035 Agenda establishes adaptive AI inquiry, Design-Based Research, mathematical modeling, and Lesson Study as urgent strategic priorities.

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