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Research Article

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Modeling The Drying Of An Unsaturated Deformable Porous Medium: Application To Packaging Kraft Paper

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Abstract

The main objective of this work is to model heat transfer in Kraft paper during drying. The model developed takes into account BIOT's assumptions regarding a uniform temperature environment; the paper used is assumed to be thermally thin (BIOT number Bi < 0.1). The results obtained are experimentally validated. They allowed us to show that the two drying phases of Kraft paper can be modeled by a thermal capacitor that charges upon contact with the drying cylinder (phase 1) and discharges into the free space (phase 2).

Keywords: Allium Roseum, Germination Characteristics, Seed Bank and Tunisia

1. Introduction

Kraft packaging paper is a porous, deformable, and unsaturated material composed of cellulose fibers [1]. Drying is an essential step in the manufacturing process, during which heat and mass transfer play a crucial role. This operation must be carefully controlled to preserve the paper's mechanical properties, which are highly sensitive to temperature and humidity variations [2,3].

Several recent studies have investigated heat and mass transfer in paper during drying [4,5]. The present work falls within this theme. It is based on Biot's hypotheses applied to a thin medium, considering paper as a material at a uniform temperature (Bi < 0.1). Measurements carried out on an industrial machine allowed for the experimental validation of the proposed model. The two drying phases of Kraft paper are thus interpreted as the behavior of a thermal capacitor, which charges upon contact with the drying cylinder and discharges into the free space. Figure 1 shows the two drying phases of KRAFT paper in the PM2 machine dryer at SOTIPAPIER.

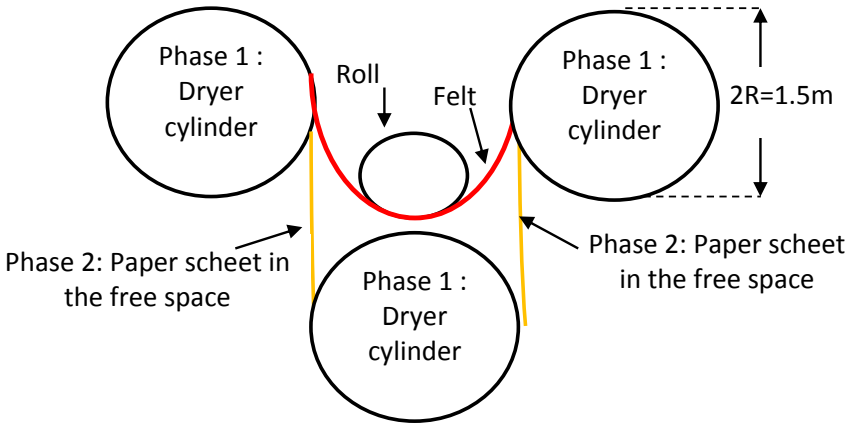

Figure 1: Drying Phases [6]

2. Theoretical study: Problem formulation

2.1 Hypotheses

In both drying phases, we assume:

• The surrounding environment is at a uniform temperature T0.

• The energy state of the paper at the beginning of the first phase is taken as the reference state.

• The paper is thermally thin: Uniform temperature environment (BIOT number < 0.1).

• The length L of the paper under study is equal to the length of the embracing arc.

• The width of the paper is ℓ = 3.16 m.

• The embracing angle is α = 120° (Figure 2).

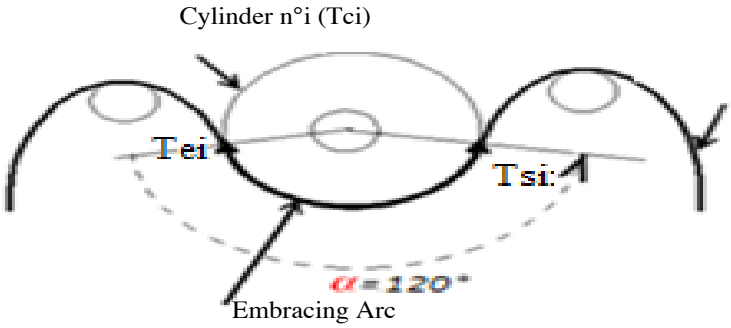

Figure 2: Embracing Arc

2.2 Thermal Balance Of The First Phase

The paper in contact with the drying cylinder n°i, at temperature Tci, receives heat and its temperature increases. The heat balance is written as:

$$\underbrace{\rho C_p V(T_{ci} - T_0)}_{\substack{\textit{Maximum energy output} \\ \textit{of the drying cylinder}}} = \underbrace{\rho C_p V \frac{dT}{dt}}_{\substack{\textit{Heat received by} \\ \textit{the paper sheet}}} + \underbrace{hs(T - T_0)}_{\substack{\textit{Heat lost to the} \\ \textit{surrounding environment}}} \quad (1)$$

We pose : $\theta = T-T_0$ et $\tau = \frac{\rho C_p V}{hS}$; Equation (1) becomes:

$$\theta_{ci} = \frac{d\theta}{dt} + \frac{1}{\tau}\theta \qquad (2)$$

The solution to equation (2) is of the form:

$$\theta(t) = A + B.\exp(-\frac{1}{\tau}t) \qquad (3)$$

Which gives, using boundary conditions ; $\theta(0)=0$ et $\theta(\infty)=\theta_c = T_c-T_0$:

$$\frac{T - T_0}{T_{ci} - T_0} = 1 - \exp\left[-\frac{1}{\tau}t\right] \qquad (4)$$

2.3 Thermal Balance Of The Second Phase

In the free space between the upper (n°i) and lower (n°i+1) cylinders, the paper at temperature TSI releases heat to the surrounding medium, at a uniform temperature T0, and its temperature decreases. The heat balance between t and t+dt is written as:

$$\underbrace{-\rho C_p V \frac{dT}{dt}}_{\text{Heat releeased by the paper sheet}} = \underbrace{hs(T - T_0)}_{\text{Heat transferred to the surrounding environment}} \qquad (5)$$

Either:

$$\frac{dT}{T - T_0} = -\frac{hs}{\rho C_p V}dt = -\frac{h}{\rho C_p e}dt \qquad (6)$$

Which gives, by integration:

$$\frac{T - T_0}{T_{si} - T_0} = \exp\left[-\frac{h}{\rho C_p e}t\right] \qquad (7)$$

2.4 Thermal Capacitor Model

Equations (4) and (7) show that the drying of Kraft paper can be interpreted as a thermal capacitor that charges during the first phase and discharges during the second phase (Figure 3). The time constant is:

$\tau = \underbrace{\left(\frac{1}{hS}\right)}_{\text{Thermal resistance}} \underbrace{(\rho C_p V)}_{\text{Thermal capacity}}$. The thermal capacitor model is shown in Figure 4

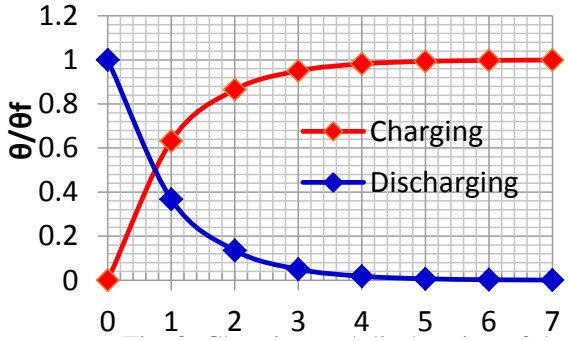

Figure 3: Charging and discharging of the thermal capacitor

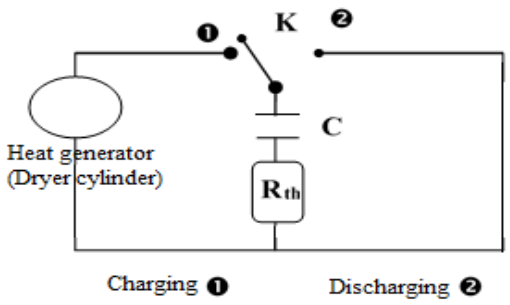

Figure 4: Thermal Capacitor Model

3 Experimental study

3.1 Material and Method

The experiment was conducted on kraft paper with a basis weight of 70 g/m² during drying. The experimental conditions were as follows:

• Machine speed: U1=260m.mn-1, U2=280m.mn-1, U3=310m.mn-1.

• The thickness of the sheet is assumed to be constant; e= 0.119 mm.

• The specific heat capacity of kraft paper is: Cpf = 1,34 kJ.kg-1.K-1.

• The kraft paper thermal conductivity is: λf = 1,34 W.m-1. °C-1. [7]

• The kraft paper density is: ρf = 1340 kg.m-3.

• Air characteristics at average temperature air-paper (30°C): Table 1

Density ρ (kg.m-3)	Dynamic viscosity	Specific heat capacity C (J.kg-1.K-1)	Thermal conductivity λ (W.m-1.K-1)	Prandtl. number Pr
1,177	1,85×10-5	1006	0,0262	0.708

Table 1: Air Characteristics at 30°C [8]

3.2 Measurements

The pressure is set at the press section of the SOTIPAPIER (TUNISIA) kraft paper manufacturing machine, and consequently, the moisture content x0, sch at the press outlet is set. The steam flow rate is then adjusted to ensure a fixed moisture content x1, sch at the dryer outlet. Using appropriate probes, the temperature TC of the drying cylinders, as well as those of the sheet Te and TS respectively, before and after its contact with these cylinders are measured. The measurements are shown in Table 2.

Cylinder n°	Tc	Te= T0	Ts	Cylinder n°	Tc	Te= T0	Ts
1	70	30,9	31,6	15	90,7	62,2	76,4
2	48,4	31,3	36,4	16	97,6	67	72,7
3	54,2	37,3	52,7	17	98,2	70,4	81,5
4	49,2	50	45,3	18	82,4	72,1	68,8
5	69	47,8	58,3	19	51,3	45,7	50,1
6	92,6	56,1	82,2	20	83,2	47,2	77,8
8	70,6	68,9	64,7	21	51,7	69,3	53,4
9	91	59,8	73,9	22	85	50,2	82,6
10	85,6	61,3	74,1	23	76,9	72,6	75
11	93,6	67,9	82,2	24	88,1	73,6	79,4
12	98,4	66,2	70,5	25	95	72,6	82,7
13	100,4	67,9	84,9	26	88,8	79,4	74,8
14	102,6	73,4	78,7				

Table 2: Temperature Variation of The Drying Sheets and Cylinders

3.3. Determination of the Convection Coefficient h

The second phase can be interpreted as gas at temperature T0 flowing at speed U over a large paper plate: length L=R.α, width l, and thickness e (Figure 5), in forced thermal convection. The dimensional analysis, aimed at determining the convectioncoefficient h, involves determining the REYNOLDS $\left(R_e = \frac{\rho U L_C}{\mu}\right)$ and NUSSELT($Nu = \frac{hL_C}{\lambda}$) numbers.

L_C=Volume/Section= $\frac{2.L.l.e}{l.e}$ /(l.e)=2L: Characteristic length:

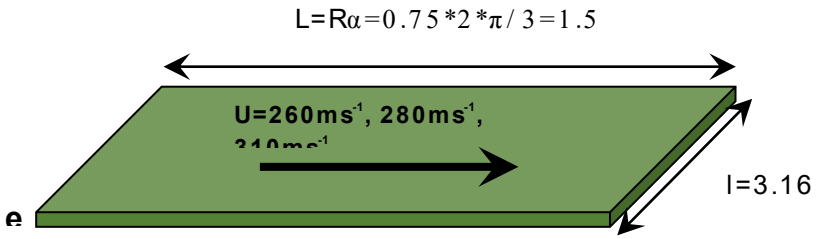

Figure 5: Equivalent System of the 2nd Drying Phase

- **REYNOLDS number** $\left(\mathbf{R_e = \frac{\rho U L_C}{\mu}}\right)$

Table 3 shows the REYNOLDS number values for the machine's three speeds.

U(m/mn)	260	280	310
Re	8,67.105	5,97.105	6,48.105

Table 3 : Reynolds Number Values

The regime is, therefore, turbulent given that Re > Rec = 3.10^5

- **NUSSELT number** $\left(\mathbf{Nu = \frac{hL_C}{\lambda}}\right)$ will be determined by the following formula since the regime is turbulent.

$$Nu = 0.029 \cdot Re^{4/5} \cdot Pr^{1/3} \quad (8)$$

Table 4 shows the NUSSELT number values for the machine's three speeds.

U(m/mn)	260	280	310
Nu	1454,88	1542,73	1673,51

Table 4: Reynolds Number Values

- **The convection coefficient h can be easily determined by: (h=λNu/L_C) .**

Table 5 shows the convection coefficient h values for the machine's three speeds.

U(m/mn)	260	280	310
h(W/m2K)	12,14	12,87	13,96

Table 5: Convection Coefficient h

For each thermal capacitor (cylinder n°i followed by a free space), the uniform temperature T_0 = Te will be taken according to measurements taken on the machine.

The results obtained for the three machine speeds allowed us to plot the curves in Figures 6, 7, and 8:

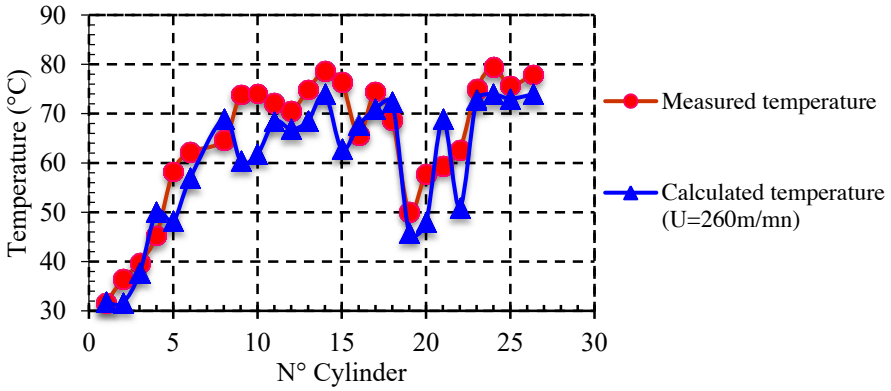

Figure 6: Measured and Calculated Temperatures TSI of the Paper at The Outlets of Cylinders n°i for U= 260 m/mn

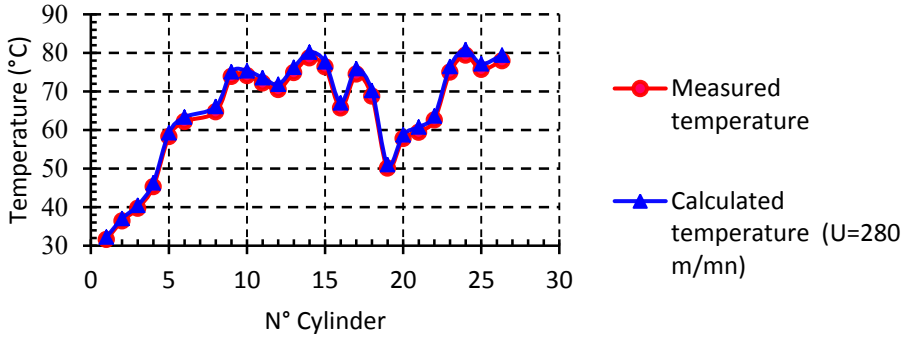

Figure 7: Measured and Calculated Temperatures TSI of the Paper at The Outlets of Cylinders n°i for U= 280 m/mn

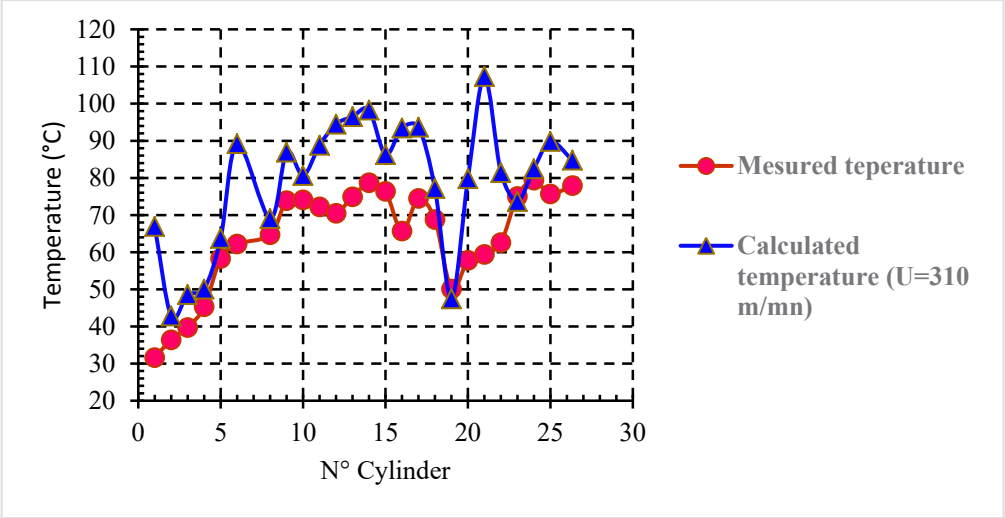

Figure 8: Measured and Calculated Temperatures TSI of the Paper at The Outlets of Cylinders n°i for U= 310 m/mn

The paper temperatures Tsi at the outlet of each cylinder were determined from the equation of the first phase. The inlet temperatures Tei can be obtained analogously using the equation of the second phase.

The curves obtained for the three speeds show good agreement between the experimental measurements and the values predicted by the thermal capacitor model. The average relative error is less than 5%, which confirms the reliability of the proposed model.

In particular, we observe that the speed U = 280 m/min provides the best match, which can be explained by:

- Higher measurement accuracy,
- More stable aerodynamic conditions in free space,

• A better established forced convection regime.

4. Advanced Validation

The model-experiment comparison shows an average relative error of less than 5%, highlighting the model's relevance. The speed U = 280 m/min offers the best accuracy, likely due to a stabilized aerodynamic regime, which promotes homogeneous forced convection. A more detailed analysis also shows that the thermal dynamics are strongly influenced by the uniformity of the air-temperature gradient in this configuration.

5.Conclusion

In this work, we modeled heat transfer in Kraft paper during the drying process, representing the two phases with a thermal capacitor model. The paper is thermally charged in contact with the cylinders (phase 1) and discharged into the free space (phase 2).

The model is based on Biot's assumptions, which are valid for a thermally thin material. Comparisons between theoretical results and experimental measurements show very good agreement, thus validating the relevance of the proposed model.

The best results were obtained at a speed of 280 m/min, suggesting that the model is particularly well-suited to optimal forced convection conditions.

Acknowledgment

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Nomenclature

Bi	BIOT number
C_{pf}	Paper specific heat capacity (kJ.kg-^{1}. K-)
e	Thickness paper sheet (m)
h	Convection coefficient (W/m^{2}K)
L_C	Characteristic length (m)
L	Embracing arc length (m)
ℓ	The sheet paper width (m)
Nu	NUSSELT number
Pr	Prandtl number
Re	REYNOLDS number
T0	Temperature of the surrounding environment (°C)
Tci	Temperature of cylinder n°i (°C)
Tei	Paper sheet temperature at the cylinder inlet n°i (°C)
Tsi	Paper sheet temperature at the cylinder outlet n°i (°C)
U	Machine speed (m/s)
α	embracing angle (rad)
τ	Time constant (s)
λ_f	Paper thermal conductivity (W.m-$^{1.°}$C^{-1})
ρf	Paper density (kg.m^{-3})

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