ISSN: 2997-6170

Space Science Journal

Research Article

Maxwell's Octonion Equations

Vadim Sovetov*

Doctor of Technical Since

*Corresponding Author
Sovetov V, Doctor of Technical Since.

Submitted: 2025, Dec 05; **Accepted:** 2026, Jan 05; **Published:** 2026, Jan 19

Citation: Sovetov, V . (2026). Maxwell's Octonion Equations. *Space Sci J, 3*(1), 01-14.

Abstract

Analytically, Maxwell's equations for the octonion are obtained. It is shown that when using shortened notations for the scalar and vector products of the Hamiltonian operator on the electric and magnetic field vectors, the equations have the same form as Maxwell's equations for a quaternion. This means that the equations describe the same physical phenomena, only for spaces of different dimensions. According to the Cauchy-Riemann conditions (CRC) and, accordingly, the law of conservation of energy, an electron cannot transition to an arbitrary state, but only makes quantum jumps to those locations in space where it conserves energy. Consequently, an electron moves in space along an orbit. The resulting Maxwell equations contain Gauss's law as a scalar part, which is related to the imaginary parts by CRC and, therefore, a change in the scalar part determines the value of the imaginary parts of the equations. A quaternion has 4D dimensions, while an octonion has 8D dimensions. It is shown that the octonion matrix can be decomposed into two non-intersecting matrices in 8D space. A quaternion can be either single-frequency or three-frequency, an octonion can be single-frequency or seven-frequency. Using the formula for representing the octonion matrix through two quaternion matrices, the corresponding images of the change in the scalar part of the seven-frequency octonion for 4 quaternions are shown.

Keywords: Maxwell's Equations, Quaternion, Octonion, Electrodynamic, Circulation, Rotor

1. Introduction

Maxwell's equations demonstrated the possibility of the formation and emission of electromagnetic waves, laid the foundations of electrodynamics and served as an impetus for the development of radio communications and other electronic devices and systems [1]. However, at present, with the construction of global Internet networks and global mobile networks, the need for communication channel capacity has increased significantly. For example, the new 5G next-generation mobile communications standard requires downlink speeds of up to 20 Gbps and subscriber speeds of up to 100 Mbps. The transmission speed in 6G is expected to increase from 1 Tbit/s. The use of traditional modulation schemes requires a transition to a higher wave range (more than 20 GHz), which is associated with an increase in the power of transmitting devices or a decrease in the communication distance. To increase throughput in 6G, it is planned to use the terahertz range from 300 GHz to 3 THz. Also, a natural solution to the problem of increasing the energy distance between information symbols while maintaining the occupied frequency band and, accordingly, the channel capacity, is the transition from a two-dimensional signal space to a multidimensional one. Currently, methods of transmitting information using the Multiple-Input Multiple-Output (MIMO) scheme have become very popular. It has been theoretically shown that the MIMO scheme can increase the communication channel capacity by N times, where N is the number of inputs equal to the number of outputs, i.e. MIMO N×N [2]. To increase throughput in 5G and 6G, massive and ultra-massive MIMO are expected to be used. Thus, to increase the capacity of global communication systems, it is necessary to increase the dimensionality of the signal space. Maxwell's equations use a vector representation of magnetic and electrical intensities without specifying their dimensions. Maxwell's equation for the quaternion is known [3]. The aim of this article is to obtain Maxwell's equation for the octonion.

2. Materials and Methods for Solving the Problem

Hypercomplex numbers are obtained using the doubling procedure. Thus, a quaternion is obtained from two complex numbers, an octonion from two quaternions, a sedenion from two octonions, etc. Since hypercomplex numbers are formed from one scalar and other imaginary numbers that are orthogonal to each other and, therefore, they can be considered as spatial coordinates, then when using them one can expect the formation of spaces of high dimension, multiples of 2^n, where $n = 1,2,3 \ldots$.

The octonion or octave in algebraic form can be represented as:

$$o = se + xi + yj + zk + s_1e_1 + x_1i_1 + y_1j_1 + z_1k_1 \,. \tag{1}$$

where $s, e, x, y, z, s_1, x_1, y_1, z_1$ – real numbers, $i, j, k, e_1, i_1, j_1, k_1$ - imaginary units.

Unlike complex numbers and quaternions, octaves do not have the property of associativity [4]. The multiplication operations of imaginary units of the octonion are shown in Table 1.

Table 1. Operations of multiplication of imaginary units of the octonion.

$$\begin{array}{c|cccc|cccc}
\times & e & i & j & k & e_1 & i_1 & j_1 & k_1 \\
\hline
e & e & i & j & k & e_1 & i_1 & j_1 & k_1 \\
i & i & -e & k & -j & i_1 & -e_1 & -k_1 & j_1 \\
j & j & -k & -e & i & j_1 & k_1 & -e_1 & -i_1 \\
k & k & j & -i & -e & k_1 & -j_1 & i_1 & -e_1 \\
\hline
e_1 & e_1 & -i_1 & -j_1 & -k_1 & -e & i & j & k \\
i_1 & i_1 & e_1 & -k_1 & j_1 & -i & -e & -k & j \\
j_1 & j_1 & k_1 & e_1 & -i_1 & -j & k & -e & -i \\
k_1 & k_1 & -j_1 & i_1 & e_1 & -k & -j & i & -e
\end{array} . \tag{2}$$

Let us write the exponential function of the octonion (1):

$$\mathrm{e}^o = \mathrm{e}^{se+xi+yj+zk+s_1e_1+x_1i_1+y_1j_1+z_1k_1} = \mathrm{e}^{se}\,\mathrm{e}^{xi}\,\mathrm{e}^{yj}\,\mathrm{e}^{zk}\,\mathrm{e}^{s_1e_1}\,\mathrm{e}^{x_1i_1}\,\mathrm{e}^{y_1j_1}\,\mathrm{e}^{z_1k_1} \,. \tag{3}$$

Let us denote the radius of rotation in 8D space as

$$\mathrm{e}^s = r = \sqrt{s^2 + x^2 + y^2 + z^2 + s_1^2 + x_1^2 + y_1^2 + z_1^2} \,.$$

The radius of rotation is equal to the modulus of the octonion |o|.

According to Euler's formula, we write expression (3) in polar representation:

$$\begin{aligned}\mathrm{e}^o = \mathrm{e}^{se}\left(\cos x + i\sin x\right)\left(\cos y + j\sin y\right)\left(\cos z + k\sin z\right)\times \\ \times\left(\cos s_1 + e_1\sin s_1\right)\left(\cos x_1 + i_1\sin x_1\right)\left(\cos y_1 + j_1\sin y_1\right)\left(\cos z_1 + k_1\sin z_1\right) .\end{aligned} \tag{4}$$

Let us represent real functions with imaginary units in (4) as functions of time and write them in the form of angular frequencies [3]:

$$x(t) = \omega_i t\,,\; y(t) = \omega_j t\,,\; z(t) = \omega_k t\,,\; s_1(t) = \omega_{e_1} t\,,\; x_1(t) = \omega_{i_1} t\,,\; y_1(t) = \omega_{j_1} t\,,\; z_1(t) = \omega_{k_1} t\,,$$

where ω_i, ω_j, ω_k, ω_{e_1}, ω_{i_1}, ω_{j_1}, ω_{k_1} - angular frequencies on orthogonal imaginary coordinate axes $i, j, k, e_1, i_1, j_1, k_1$.

Let us represent the octonion (1) as an 8×8 matrix [5]:

$$\mathbf{O}=\begin{bmatrix} s & x & y & z & s_1 & x_1 & y_1 & z_1 \\ -x & s & -z & y & -x_1 & s_1 & z_1 & -y_1 \\ -y & z & s & -x & -y_1 & -z_1 & s_1 & x_1 \\ -z & -y & x & s & -z_1 & y_1 & -x_1 & s_1 \\ -s_1 & x_1 & y_1 & z_1 & s & -x & -y & -z \\ -x_1 & -s_1 & z_1 & -y_1 & x & s & z & -y \\ -y_1 & -z_1 & -s_1 & x_1 & y & -z & s & x \\ -z_1 & y_1 & -x_1 & -s_1 & z & y & -x & s \end{bmatrix}. \tag{5}$$

We decompose matrix (5) into basis matrices [6]:

$$\mathbf{E}=\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix},\ \mathbf{I}=\begin{bmatrix} 0&1&0&0&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \\ 0&0&0&-1&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&-1&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&-1&0 \end{bmatrix}, \tag{6}$$

$$\mathbf{J}=\begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \\ 0&-1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&-1&0 \\ 0&0&0&0&0&0&0&-1 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \end{bmatrix},\ \mathbf{K}=\begin{bmatrix} 0&0&0&1&0&0&0&0 \\ 0&0&-1&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&-1 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&-1&0&0 \\ 0&0&0&0&1&0&0&0 \end{bmatrix},$$

$$\mathbf{E}_1=\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ -1&0&0&0&0&0&0&0 \\ 0&-1&0&0&0&0&0&0 \\ 0&0&-1&0&0&0&0&0 \\ 0&0&0&-1&0&0&0&0 \end{bmatrix},\ \mathbf{I}_1=\begin{bmatrix} 0&0&0&0&0&1&0&0 \\ 0&0&0&0&-1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&-1&0 \\ 0&1&0&0&0&0&0&0 \\ -1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&-1&0&0&0&0&0 \end{bmatrix},$$

$$\mathbf{J}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{K}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The basis matrices are orthogonal [6]. The sum of the matrices will look like:

$$\mathbf{O} = \mathbf{E} + \mathbf{I} + \mathbf{J} + \mathbf{K} + \mathbf{E}_1 + \mathbf{I}_1 + \mathbf{J}_1 + \mathbf{K}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 \end{bmatrix}. \quad (7)$$

We write the octonion (5) through the sum of the basis matrices (6) as

$$\mathbf{O} = s\mathbf{E} + x\mathbf{I} + y\mathbf{J} + z\mathbf{K} + s_1\mathbf{E}_1 + x_1\mathbf{I}_1 + y_1\mathbf{J}_1 + z_1\mathbf{K}_1. \quad (8)$$

The determinant of matrix (5) is equal to $|\mathbf{O}| = \left(s^2 + x^2 + y^2 + z^2 + s_1^2 + x_1^2 + y_1^2 + z_1^2\right)^4$. The octonion matrix (5) will be orthogonal when normalizing the octonion in algebraic form (2) modulo |o|.

The matrix representation (8) of the octonion can be split into two spatially separated matrices [6]. The first matrix is formed by the basis matrices **E, I, J$_1$, K$_1$** and, accordingly, has the structure:

$$\mathbf{E} + \mathbf{I} + \mathbf{J}_1 + \mathbf{K}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \quad (9)$$

The second matrix is formed by the basis matrices **J, K, E$_1$, I$_1$** and has a structure:

$$\mathbf{E}_1+\mathbf{J}+\mathbf{K}+\mathbf{I}_1=\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 \end{bmatrix}. \tag{10}$$

As can be seen from the structures of matrices (9) and (10), their elements do not intersect when superimposed and, as a whole, form the matrix of the sum of all basis matrices (7). This property of octonion matrices helps to simplify receiving devices [6]. Let us write table (2) with a matrix representation of its elements as Table 2. Operations of multiplication of basis matrices of the octonion.

×	**E**	**I**	**J**	**K**	$\mathbf{E}_1$	$\mathbf{I}_1$	$\mathbf{J}_1$	$\mathbf{K}_1$
E	**E**	**I**	**J**	**K**	$\mathbf{E}_1$	$\mathbf{I}_1$	$\mathbf{J}_1$	$\mathbf{K}_1$
I	**I**	–**E**	**K**	–**J**	$\mathbf{I}_1$	$-\mathbf{E}_1$	$-\mathbf{K}_1$	$\mathbf{J}_1$
J	**J**	–**K**	–**E**	**I**	$\mathbf{J}_1$	$\mathbf{K}_1$	$-\mathbf{E}_1$	$-\mathbf{I}_1$
K	**K**	**J**	–**I**	–**E**	$\mathbf{K}_1$	$-\mathbf{J}_1$	$\mathbf{I}_1$	$-\mathbf{E}_1$
$\mathbf{E}_1$	$\mathbf{E}_1$	$-\mathbf{I}_1$	$-\mathbf{J}_1$	$-\mathbf{K}_1$	–**E**	**I**	**J**	**K**
$\mathbf{I}_1$	$\mathbf{I}_1$	$\mathbf{E}_1$	$-\mathbf{K}_1$	$\mathbf{J}_1$	–**I**	–**E**	–**K**	**J**
$\mathbf{J}_1$	$\mathbf{J}_1$	$\mathbf{K}_1$	$\mathbf{E}_1$	$-\mathbf{I}_1$	–**J**	**K**	–**E**	–**I**
$\mathbf{K}_1$	$\mathbf{K}_1$	$-\mathbf{J}_1$	$\mathbf{I}_1$	$\mathbf{E}_1$	–**K**	–**J**	**I**	–**E**

(11)

When representing octonions in matrix form, it is necessary to take into account that, due to the absence of associativity of octonions in the hypercomplex representation, when multiplying the basis matrices, the signs indicated in table (2) may not coincide with the signs in table (11) [5].

We will represent the model of the information transmission system in the form of a dynamic equation in the state space [6]:

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t), \tag{12}$$

where **A** – state transition matrix, $\mathbf{x}(t)$ - state vector, $\dot{\mathbf{x}}(t)$ - time derivative of the state vector.

The state transition matrix in (12) is determined by the imaginary part of the octonion matrix (5) in which the elements at the imaginary parts x, y, z, s_1, x_1, y_1, z_1 replaced by angular frequencies: ω_i, ω_j, ω_k, ω_{e_1}, ω_{i_1}, ω_{j_1}, ω_{k_1} on orthogonal spatial coordinate axes **I, J, K,** $\mathbf{E}_1$, $\mathbf{I}_1$, $\mathbf{J}_1$, $\mathbf{K}_1$ in 8D space:

$$\mathbf{A}=\begin{bmatrix} 0 & \omega_i & \omega_j & \omega_k & \omega_{e_1} & \omega_{i_1} & \omega_{j_1} & \omega_{k_1} \\ -\omega_i & 0 & -\omega_k & \omega_j & -\omega_{i_1} & \omega_{e_1} & \omega_{k_1} & -\omega_{j_1} \\ -\omega_j & \omega_k & 0 & -\omega_i & -\omega_{j_1} & -\omega_{k_1} & \omega_{e_1} & \omega_{i_1} \\ -\omega_k & -\omega_j & \omega_i & 0 & -\omega_{k_1} & \omega_{j_1} & -\omega_{i_1} & \omega_{e_1} \\ -\omega_{e_1} & \omega_{i_1} & \omega_{j_1} & \omega_{k_1} & 0 & -\omega_i & -\omega_j & -\omega_k \\ -\omega_{i_1} & -\omega_{e_1} & \omega_{k_1} & -\omega_{j_1} & \omega_i & 0 & \omega_k & -\omega_j \\ -\omega_{j_1} & -\omega_{k_1} & -\omega_{e_1} & \omega_{i_1} & \omega_j & -\omega_k & 0 & \omega_i \\ -\omega_{k_1} & \omega_{j_1} & -\omega_{i_1} & -\omega_{e_1} & \omega_k & \omega_j & -\omega_i & 0 \end{bmatrix}. \tag{13}$$

As is known, the functions of the fundamental matrix $\mathbf{\Phi}(\omega_i, \omega_j, \omega_k, \omega_{e_1}, \omega_{i_1}, \omega_{j_1}, \omega_{k_1}, t)$ for combination frequencies are divided into two frequency groups [6]. The first group of combination frequencies includes frequencies with numbers: 1 2 5 6 11 12 15 16 19 20 23 24 25 26 29 30 33 34 37 38 43 44 47 48 51 52 55 56 57 58 61 62. The second group includes frequencies with numbers: 3 4 7 8 9 10 13 14 17 18 21 22 27 28 31 32 35 36 39 40 41 42 45 46 49 50 53 54 59 60 63 64. The first group includes frequencies that form elements of single-frequency matrices corresponding to the basis matrices (9). The second group includes frequencies that form elements of single-frequency matrices corresponding to the basis matrices (10). The formulas for the elements of the fundamental matrix are presented in [6].

3. Technique of Obtaining Maxwell's Octonion Equations

When deriving Maxwell's equations for the octonion, we will use the method for obtaining Maxwell's equations for the quaternion [3]. Let's multiply two octonions in algebraic form (1):

$$qp = \left(q_0e + q_1i + q_2j + q_3k + q_4e_1 + q_5i_1 + q_6j_1 + q_7k_1\right)\left(p_0e + p_1i + p_2j + p_3k + p_4e_1 + p_5i_1 + p_6j_1 + p_7k_1\right).$$

We remove the scalar terms from the resulting expression:

$$\begin{aligned}
&q_1p_0i + q_2p_0j + q_3p_0k + q_4p_0e_1 + q_5p_0i_1 + q_6p_0j_1 + q_7p_0k_1 + \\
&+q_0p_1i - q_2p_1k + q_3p_1j - q_4p_1i_1 + q_5p_1e_1 + q_6p_1k_1 - q_7p_1j_1 + \\
&+q_0p_2j + q_1p_2k - q_3p_2i - q_4p_2j_1 - q_5p_2k_1 + q_6p_2e_1 + p_2q_7i_1 + \\
&+q_0p_3k - q_1p_3j + q_2p_3i - q_4p_3k_1 + q_5p_3j_1 - q_6p_3i_1 + q_7p_3e_1 + \\
&+q_0p_4e_1 + q_1p_4i_1 + q_2p_4j_1 + q_3p_4k_1 - q_5p_4i - q_6p_4j - q_7p_4k + \\
&+q_0p_5i_1 - q_1p_5e_1 + q_2p_5k_1 - q_3p_5j_1 + q_4p_5i + q_6p_5k - q_7p_5j + \\
&+q_0p_6j_1 - q_1p_6k_1 - q_2p_6e_1 + q_3p_6i_1 + q_4p_6j - q_5p_6k + q_7p_6i + \\
&+q_0p_7k_1 + q_1p_7j_1 - q_2p_7i_1 - q_3p_7e_1 + q_4p_7k + q_5p_7j - q_6p_7i
\end{aligned} \tag{14}$$

Let us denote the pure octonions as q and p and write the pure octonions as vectors:

$$\mathrm{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 \end{bmatrix}^{\mathrm{T}},\ \mathrm{p} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 \end{bmatrix}^{\mathrm{T}} \tag{15}$$

The scalar product of vectors (15) will be equal to

$$\mathrm{q{\cdot}p} = q_1p_1 + q_2p_2 + q_3p_3 + q_4p_4 + q_5p_5 + q_6p_6 + q_6p_6.$$

We write the removed scalar part of the product qp as

$$q_0p_0 - \left(q_1p_1 + q_2p_2 + q_3p_3 + q_4p_4 + q_5p_5 + q_6p_6 + q_6p_6\right) = q_0p_0 - \mathrm{q{\cdot}p}. \tag{16}$$

Let us isolate from the sum (14) the terms with imaginary numbers multiplied by p_0 and q_0 and write them using the notation (15):

$$\begin{aligned}
&p_0\,\mathrm{q} = p_0\begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 \end{bmatrix}^{\mathrm{T}}, \\
&q_0\,\mathrm{p} = q_0\begin{bmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 \end{bmatrix}^{\mathrm{T}}.
\end{aligned} \tag{17}$$

Let's combine the remaining terms in (14) for the same imaginary numbers:

$$\begin{aligned}
&i\left[(q_2p_3 - q_3p_2) + (q_4p_5 - q_5p_4) + (q_7p_6 - q_6p_7)\right] + \\
&+j\left[(q_3p_1 - q_1p_3) + (q_4p_6 - q_6p_4) + (q_5p_7 - q_7p_5)\right] +
\end{aligned}$$

$$
\begin{aligned}
&+k\left[(q_1p_2-q_2p_1)+(q_4p_7-q_7p_4)+(q_6p_5-q_5p_6)\right]+\\
&+e_1\left[(q_5p_1-q_1p_5)+(q_6p_2-q_2p_6)+(q_7p_3-q_3p_{71})\right]+\\
&+i_1\left[(q_1p_4-q_4p_1)+(p_2q_7-q_2p_7)+(q_3p_6-q_6p_3)\right]+\\
&+j_1\left[(q_1p_7-q_7p_1)+(q_2p_4-q_4p_2)+(q_5p_3-q_3p_5)\right]+\\
&+k_1\left[(q_6p_1-q_1p_6)+(q_2p_5-q_5p_2)+(q_3p_4-q_4p_3)\right]
\end{aligned}
\tag{18}
$$

Members (18) correspond to circulation along imaginary (spatial) coordinate axes $i, j, k, e_1, i_1, j_1, k_1$. We use the notation (15) and write (18) as a vector product of pure octonions:

$$
\mathrm{q}\times\mathrm{p}\,. \tag{19}
$$

Thus, we combine the obtained expressions (16), (17) and (19) of the product qp and write them in the form:

$$
qp=\left(q_0p_0-\mathrm{q}\cdot\mathrm{p}\ \right)+\left(q_0\,\mathrm{p}+\,p_0\,\mathrm{q}+\mathrm{q}\times\mathrm{p}\right). \tag{20}
$$

Equation (20) for the octonion corresponds in form to the equation for the quaternion [3]. To obtain Maxwell's equations for the octonion, we use the matrix representation of the partial derivatives of the octonion:

$$
\frac{df(\mathrm{o})}{d\,\mathrm{o}}=\begin{bmatrix}
\partial_s & \partial_x & \partial_y & \partial_z & \partial_{s_1} & \partial_{x_1} & \partial_{y_1} & \partial_{z_1}\\
-\partial_x & \partial_s & -\partial_z & \partial_y & -\partial_{x_1} & \partial_{s_1} & \partial_{z_1} & -\partial_{y_1}\\
-\partial_y & \partial_z & \partial_s & -\partial_x & -\partial_{y_1} & -\partial_{z_1} & \partial_{s_1} & \partial_{x_1}\\
-\partial_z & -\partial_y & \partial_x & \partial_s & -\partial_{z_1} & \partial_{y_1} & -\partial_{x_1} & \partial_{s_1}\\
-\partial_{s_1} & \partial_{x_1} & \partial_{y_1} & \partial_{z_1} & \partial_s & -\partial_x & -\partial_y & -\partial_z\\
-\partial_{x_1} & -\partial_{s_1} & \partial_{z_1} & -\partial_{y_1} & \partial_x & \partial_s & \partial_z & -\partial_y\\
-\partial_{y_1} & -\partial_{z_1} & -\partial_{s_1} & \partial_{x_1} & \partial_y & -\partial_z & \partial_s & \partial_x\\
-\partial_{z_1} & \partial_{y_1} & -\partial_{x_1} & -\partial_{s_1} & \partial_z & \partial_y & -\partial_x & \partial_s
\end{bmatrix}
\begin{bmatrix} p\\ u\\ v\\ w\\ p_1\\ u_1\\ v_1\\ w_1 \end{bmatrix}. \tag{21}
$$

Let us write expression (21) in terms of basis matrices (6) as

$$
\frac{df(\mathrm{o})}{d\,\mathrm{o}}=\left(\mathbf{E}\partial_s+\mathbf{I}\partial_x+\mathbf{J}\partial_y+\mathbf{K}\partial_z+\mathbf{E}_1\partial_{s_1}+\mathbf{I}_1\partial_{x_1}+\mathbf{J}_1\partial_{y_1}+\mathbf{K}_1\partial_{z_1}\right)\mathbf{p}=\mathbf{Op}, \tag{22}
$$

where $\mathbf{p}$ is a vector of functions of the octonion (2) $\mathbf{p}=f(\mathrm{o})=\begin{bmatrix}p & u & v & w & p_1 & u_1 & v_1 & w_1\end{bmatrix}^{\mathrm{T}}$ and $\mathbf{O}$ is the matrix of partial derivatives. The derivative of an octonion with respect to its conjugate octonion is calculated as

$$\frac{df(o)}{d\bar{o}} = \begin{bmatrix} \partial_s & -\partial_x & -\partial_y & -\partial_z & -\partial_{s_1} & -\partial_{x_1} & -\partial_{y_1} & -\partial_{z_1} \\ \partial_x & \partial_s & \partial_z & -\partial_y & \partial_{x_1} & -\partial_{s_1} & -\partial_{z_1} & \partial_{y_1} \\ \partial_y & -\partial_z & \partial_s & \partial_x & \partial_{y_1} & \partial_{z_1} & -\partial_{s_1} & -\partial_{x_1} \\ \partial_z & \partial_y & -\partial_x & \partial_s & \partial_{z_1} & -\partial_{y_1} & \partial_{x_1} & -\partial_{s_1} \\ \partial_{s_1} & -\partial_{x_1} & -\partial_{y_1} & -\partial_{z_1} & \partial_s & \partial_x & \partial_y & \partial_z \\ \partial_{x_1} & \partial_{s_1} & -\partial_{z_1} & \partial_{y_1} & -\partial_x & \partial_s & -\partial_z & \partial_y \\ \partial_{y_1} & \partial_{z_1} & \partial_{s_1} & -\partial_{x_1} & -\partial_y & \partial_z & \partial_s & -\partial_x \\ \partial_{z_1} & -\partial_{y_1} & \partial_{x_1} & \partial_{s_1} & -\partial_z & -\partial_y & \partial_x & \partial_s \end{bmatrix} \begin{bmatrix} p \\ u \\ v \\ w \\ p_1 \\ u_1 \\ v_1 \\ w_1 \end{bmatrix}. \tag{23}$$

Using basis matrices, the derivative of an octonion with respect to its conjugate octonion is:

$$\frac{df(\mathrm{o})}{d\bar{\mathrm{o}}} = \left(\mathbf{E}\partial_s - \mathbf{I}\partial_x - \mathbf{J}\partial_y - \mathbf{K}\partial_z - \mathbf{E}_1\partial_{s_1} - \mathbf{I}_1\partial_{x_1} - \mathbf{J}_1\partial_{y_1} - \mathbf{K}_1\partial_{z_1}\right)\mathbf{p} = \mathbf{O}^{\mathrm{T}}\mathbf{p}. \tag{24}$$

Expression (23) differs from (22) in that the transposed matrix is used as the matrix of partial derivatives. The Hamiltonian operator for the octonion can be represented as a sum of partial derivatives with respect to the octonion for various imaginary numbers:

$$\nabla_o = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} + e_1\frac{\partial}{\partial s_1} + i_1\frac{\partial}{\partial x_1} + j_1\frac{\partial}{\partial y_1} + k_1\frac{\partial}{\partial z_1}. \tag{25}$$

The Hamilton operator (25) in vector representation will have the form:

$$\nabla_{\mathbf{o}} = \begin{bmatrix} \partial x & \partial y & \partial z & \partial s_1 & \partial x_1 & \partial y_1 & \partial z_1 \end{bmatrix}^{\mathrm{T}}. \tag{26}$$

We will write the CRC based on the equality of expression (23) to zero:

$$\begin{aligned}
\frac{\partial p}{\partial s} &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{\partial p_1}{\partial s_1} + \frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} + \frac{\partial w_1}{\partial z_1}, \\
\frac{\partial p}{\partial x} &= -\frac{\partial u}{\partial s} - \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{\partial p_1}{\partial x_1} + \frac{\partial u_1}{\partial s_1} + \frac{\partial v_1}{\partial z_1} - \frac{\partial w_1}{\partial y_1}, \P \\
\frac{\partial p}{\partial y} &= \frac{\partial u}{\partial z} - \frac{\partial v}{\partial s} - \frac{\partial w}{\partial x} - \frac{\partial p_1}{\partial y_1} - \frac{\partial u_1}{\partial z_1} + \frac{\partial v_1}{\partial s_1} + \frac{\partial w_1}{\partial x_1}, \P \\
\frac{\partial p}{\partial z} &= -\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \frac{\partial w}{\partial s} - \frac{\partial p_1}{\partial z_1} + \frac{\partial u_1}{\partial y_1} - \frac{\partial v_1}{\partial x_1} + \frac{\partial w_1}{\partial s_1}, \P \\
\frac{\partial p}{\partial s_1} &= \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial y_1} + \frac{\partial w}{\partial z_1} - \frac{\partial p_1}{\partial s} - \frac{\partial u_1}{\partial x} - \frac{\partial v_1}{\partial y} - \frac{\partial w_1}{\partial z}, \P \\
\frac{\partial p}{\partial x_1} &= -\frac{\partial u}{\partial s_1} + \frac{\partial v}{\partial z_1} - \frac{\partial w}{\partial y_1} + \frac{\partial p_1}{\partial x} - \frac{\partial u_1}{\partial s} + \frac{\partial v_1}{\partial z} - \frac{\partial w_1}{\partial y}, \P \\
\frac{\partial p}{\partial y_1} &= -\frac{\partial u}{\partial z_1} - \frac{\partial v}{\partial s_1} + \frac{\partial w}{\partial x_1} + \frac{\partial p_1}{\partial y} - \frac{\partial u_1}{\partial z} - \frac{\partial v_1}{\partial s} + \frac{\partial w_1}{\partial x}, \P \\
\frac{\partial p}{\partial z_1} &= \frac{\partial u}{\partial y_1} - \frac{\partial v}{\partial x_1} - \frac{\partial w}{\partial s_1} + \frac{\partial p_1}{\partial z} + \frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} - \frac{\partial w_1}{\partial s}. \P
\end{aligned} \tag{27}$$

Let us represent expression (23) in vector form as a product $\mathbf{O}^{\mathrm{T}}\mathbf{p}$, where the matrix $\mathbf{O}^{\mathrm{T}}$ is the matrix of the derivative with respect to time and the spatial coordinate axes of the octonion function with respect to the conjugate octonion (24):

$$\mathbf{O}^{\mathrm{T}}\mathbf{p} = \begin{bmatrix} \partial_{s,t}p-(\partial_{x,t}u+\partial_{y,t}v+\partial_{z,t}w+\partial_{s_1,t}p_1+\partial_{x_1,t}u_1+\partial_{y_1,t}v_1+\partial_{z_1,t}w_1) \\ (\partial_{x,t}p+\partial_{s,t}u)+(\partial_{z,t}v-\partial_{y,t}w)+(\partial_{x_1,t}p_1-\partial_{s_1,t}u_1)+(\partial_{y_1,t}w_1-\partial_{z_1,t}v_1) \\ (\partial_{y,t}p+\partial_{s,t}v)+(\partial_{x,t}w-\partial_{z,t}u)+(\partial_{y_1,t}p_1-\partial_{s_1,t}v_1)+(\partial_{z_1,t}u_1-\partial_{x_1,t}w_1) \\ (\partial_{z,t}p+\partial_{s,t}w)+(\partial_{y,t}u-\partial_{x,t}v)+(\partial_{z_1,t}p_1-\partial_{y_1,t}u_1)+(\partial_{x_1,t}v_1-\partial_{s_1,t}w_1) \\ (\partial_{s_1,t}p+\partial_{s,t}p_1)+(\partial_{x,t}u_1-\partial_{x_1,t}u)+(\partial_{y,t}v_1-\partial_{y_1,t}v)+(\partial_{z,t}w_1-\partial_{z_1,t}w) \\ (\partial_{x_1,t}p+\partial_{s,t}u_1)+(\partial_{s_1,t}u-\partial_{z_1,t}v)+(\partial_{y_1,t}w-\partial_{x,t}p_1)+(\partial_{y,t}w_1-\partial_{z,t}v_1) \\ (\partial_{y_1,t}p+\partial_{s,t}v_1)+(\partial_{z_1,t}u-\partial_{x_1,t}w)+(\partial_{s_1,t}v-\partial_{y,t}p_1)+(\partial_{z,t}u_1-\partial_{x,t}w_1) \\ (\partial_{z_1,t}p+\partial_{s,t}w_1)+(\partial_{x_1,t}v-\partial_{y_1,t}u)+(\partial_{s_1,t}w-\partial_{z,t}p_1)+(\partial_{x,t}v_1-\partial_{y,t}u_1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (28)$$

According to CRC, the resulting vector (28) must be equal to 0, since the derivatives are taken with respect to the conjugate quaternion and are orthogonal to the directions of change of the octonion functions.

Then, taking into account the notations (16), (19), (26) and CRC (27), we write (28) as

$$\partial_{s,t}p-\nabla_{\mathbf{o}}\cdot \mathrm{p} = 0\,. \text{ – scalar equation,} \qquad (29)$$

$$\nabla_{\mathbf{o}}p+\partial_{s,t}\,\mathrm{p}=\nabla_{\mathbf{o}}\times \mathrm{p} \text{ - vector equation.} \qquad (30)$$

The derivatives of the scalar part of a function of the octonion p with respect to imaginary coordinates $\nabla_{\mathbf{o}}p$ will be a scalar vector. The derivative of the imaginary parts of a function of a quaternion along a scalar axis $\partial_{s,t}\,\mathrm{p}$ will also be a scalar vector. However, since the scalar coordinate axis is orthogonal to all imaginary coordinate axes, the terms located on the left side of vector (30) also form 7 circulations on the corresponding planes of the spatial coordinate axes. The remaining three terms in each row of vector (28) form vector sums of the circulations of three different and orthogonal planes. Due to the orthogonality of the coordinate axes of the planes, the circulations are added up vectorially. According to CRC, the left-hand circulation must be equal to the vector sum of the three right-hand circulations.

The first line of equation (29) can be written for the electric field vector in the form of the Hamiltonian operator (26) for a pure octonion $\mathrm{E}=\begin{bmatrix} E_u & E_v & E_w & E_{p_1} & E_{u_1} & E_{v_1} & E_{w_1} \end{bmatrix}^{\mathrm{T}}$ and the derivative along the scalar coordinate axis of the scalar part of the octonion p_{E}, representing the charge density $\rho_q=\partial_{s,t}p_E$ in the form of a scalar equation:

$$\nabla_{\mathbf{o}}\cdot \mathrm{E} = \rho_q\,. \qquad (31)$$

The left side of the remaining 7 lines (28) represents the vector equation and is equal to the derivatives of the scalar part of the octonion p_{E} along the imaginary axes in sum with the derivatives along the scalar axis of the pure octonion E. According to CRC, the left side is equal to the circulation for a pure octonion:

$$\nabla_{\mathbf{o}}p_{\mathrm{E}}+\partial_{s,t}\,\mathrm{E}=\nabla_{\mathbf{o}}\times \mathrm{E}\,. \qquad (32)$$

Expression (31) represents the scalar part of the octonion, and equation (32) represents the vector part. We write similar equations in the form of a scalar and vector part for the magnetic intensity of a pure octonion $\mathrm{H}=\begin{bmatrix} H_u & H_v & H_w & H_{p_1} & H_{u_1} & H_{v_1} & H_{w_1} \end{bmatrix}^{\mathrm{T}}$:

$$\nabla_{\mathbf{o}}\cdot \mathrm{H} = \rho_m \text{ – scalar equation,} \qquad (33)$$

$$\nabla_{\mathbf{o}}p_{\mathrm{H}}+\partial_{s,t}\,\mathrm{H}=\nabla_{\mathbf{o}}\times \mathrm{H} \text{ – vector equation.} \qquad (34)$$

Thus, from Maxwell's equations for the octonion (31), (32), (33), (34), it is clear that they are similar in form to the corresponding Maxwell's equations for the quaternion [3]. This fact suggests that the equations reflect the same physical processes, only in spaces of different dimensions. We also note that the classical Maxwell equations are also described in vector form, but the dimension of space is not specified [1]. In our case, the dimension of the space corresponds to the dimension of hypercomplex numbers, i.e. 4D for the quaternion and 8D for the octonion.

3.1. Separation of Maxwell's equations for the octonion in space

Let us represent the octonion in the form of quaternions. Let us consider the octonion (7) and divide it into quaternion matrices [5]:

$$\mathbf{P}=\begin{bmatrix}-1 & 1 & 1 & 1\\ -1 & -1 & 1 & -1\\ -1 & -1 & -1 & 1\\ -1 & 1 & -1 & -1\end{bmatrix}\quad \mathbf{Q}=\begin{bmatrix}1 & -1 & -1 & -1\\ 1 & 1 & 1 & -1\\ 1 & -1 & 1 & 1\\ 1 & 1 & -1 & 1\end{bmatrix}\quad -\mathbf{P}^{\mathrm{T}}=\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 1 & 1 & -1\\ -1 & -1 & 1 & 1\\ -1 & 1 & -1 & 1\end{bmatrix}$$

$$\mathbf{Q}^{\mathrm{T}}=\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 1 & -1 & 1\\ -1 & 1 & 1 & -1\\ -1 & -1 & 1 & 1\end{bmatrix}$$

Let's imagine the octonion in the form of 4 quaternion matrices:

$$\mathbf{O}=\begin{bmatrix}\mathbf{Q}^{\mathrm{T}} & -\mathbf{P}^{\mathrm{T}}\\ \mathbf{P} & \mathbf{Q}\end{bmatrix} \tag{35}$$

The quaternion matrices on the diagonals are orthogonal due to the orthogonality of quaternion matrices by definition: $\mathbf{QQ}^{\mathrm{T}}=\mathbf{E}$ and $\mathbf{PP}^{\mathrm{T}}=\mathbf{E}$.

Since the octonion (7) is also represented as the sum of two spatially separated matrices (9) and (10), the state transition matrix (13) and the partial derivative matrices in (21) and (23) can also be represented as the sum of two non-intersecting matrices $\mathbf{E}-\mathbf{I}-\mathbf{J}_1-\mathbf{K}_1$ and $-\mathbf{E}_1-\mathbf{J}-\mathbf{K}-\mathbf{I}_1$. Let us write the matrix of partial derivatives $\mathbf{O}^{\mathrm{T}}$ for the first combination of basis matrices:

$$\begin{array}{cccccccc} s & x & y & z & s_1 & x_1 & y_1 & z_1 \end{array}$$

$$\mathbf{E}-\mathbf{I}-\mathbf{J}_1-\mathbf{K}_1=\begin{bmatrix}\partial_{s,t}p & -\partial_{x,t}u & 0 & 0 & 0 & 0 & -\partial_{y_1,t}v_1 & -\partial_{z_1,t}w_1\\ \partial_{s,t}u & \partial_{x,t}p & 0 & 0 & 0 & 0 & -\partial_{y_1,t}w_1 & \partial_{z_1,t}v_1\\ 0 & 0 & \partial_{y,t}p & \partial_{z,t}u & \partial_{s_1,t}v_1 & \partial_{x_1,t}w_1 & 0 & 0\\ 0 & 0 & -\partial_{y,t}u & \partial_{z,t}p & \partial_{s_1,t}w_1 & -\partial_{x_1,t}v_1 & 0 & 0\\ 0 & 0 & -\partial_{y,t}v_1 & -\partial_{z,t}w_1 & \partial_{s_1,t}p & \partial_{x_1,t}u & 0 & 0\\ 0 & 0 & -\partial_{y,t}w_1 & \partial_{z,t}v_1 & -\partial_{s_1,t}u & \partial_{x_1,t}p & 0 & 0\\ \partial_{s,t}v_1 & \partial_{x,t}w_1 & 0 & 0 & 0 & 0 & \partial_{y_1,t}p & -\partial_{z_1,t}u\\ \partial_{s,t}w_1 & -\partial_{x,t}v_1 & 0 & 0 & 0 & 0 & \partial_{y_1,t}u & \partial_{z_1,t}p\end{bmatrix}.$$

The second matrix is formed by the basis matrices $\mathbf{J}$, $\mathbf{K}$, $\mathbf{E}_1$, $\mathbf{I}_1$ and has the structure:

$$
-\mathbf{E}_1-\mathbf{J}-\mathbf{K}-\mathbf{I}_1=\begin{array}{c} \begin{matrix} s & x & y & z & s_1 & x_1 & y_1 & z_1 \end{matrix} \\ \begin{bmatrix} 0 & 0 & -\partial_{y,t}v & -\partial_{z,t}w & -\partial_{s_1,t}p_1 & -\partial_{x_1,t}u_1 & 0 & 0 \\ 0 & 0 & \partial_{y,t}w & -\partial_{z,t}v & \partial_{s_1,t}u_1 & -\partial_{x_1,t}p_1 & 0 & 0 \\ \partial_{s,t}v & -\partial_{x,t}w & 0 & 0 & 0 & 0 & -\partial_{y_1,t}p_1 & -\partial_{z_1,t}u_1 \\ \partial_{s,t}w & \partial_{x,t}v & 0 & 0 & 0 & 0 & \partial_{y_1,t}u_1 & -\partial_{z_1,t}p_1 \\ \partial_{s,t}p_1 & -\partial_{x,t}u_1 & 0 & 0 & 0 & 0 & \partial_{y_1,t}v & \partial_{z_1,t}w \\ \partial_{s,t}u_1 & \partial_{x,t}p_1 & 0 & 0 & 0 & 0 & -\partial_{y_1,t}w & \partial_{z_1,t}v \\ 0 & 0 & \partial_{y,t}p_1 & -\partial_{z,t}u_1 & -\partial_{s_1,t}v & \partial_{x_1,t}w & 0 & 0 \\ 0 & 0 & \partial_{y,t}u_1 & \partial_{z,t}p_1 & -\partial_{s_1,t}w & -\partial_{x_1,t}v & 0 & 0 \end{bmatrix} \end{array}.
$$

Let's remove the zero elements and group the remaining elements into 4 matrices:

$$
\begin{bmatrix} \partial_{s,t}p & -\partial_{x,t}u & -\partial_{y_1,t}v_1 & -\partial_{z_1,t}w_1 \\ \partial_{s,t}u & \partial_{x,t}p & -\partial_{y_1,t}w_1 & \partial_{z_1,t}v_1 \\ \partial_{s,t}v_1 & \partial_{x,t}w_1 & \partial_{y_1,t}p & -\partial_{z_1,t}u \\ \partial_{s,t}w_1 & -\partial_{x,t}v_1 & \partial_{y_1,t}u & \partial_{z_1,t}p \end{bmatrix} \begin{bmatrix} \partial_{y,t}p & \partial_{z,t}u & \partial_{s_1,t}v_1 & \partial_{x_1,t}w_1 \\ -\partial_{y,t}u & \partial_{z,t}p & \partial_{s_1,t}w_1 & -\partial_{x_1,t}v_1 \\ -\partial_{y,t}v_1 & -\partial_{z,t}w_1 & \partial_{s_1,t}p & \partial_{x_1,t}u \\ -\partial_{y,t}w_1 & \partial_{z,t}v_1 & -\partial_{s_1,t}u & \partial_{x_1,t}p \end{bmatrix}, \quad (36)
$$

$$
\begin{bmatrix} \partial_{s,t}v & -\partial_{x,t}w & -\partial_{y_1,t}p_1 & -\partial_{z_1,t}u_1 \\ \partial_{s,t}w & \partial_{x,t}v & \partial_{y_1,t}u_1 & -\partial_{z_1,t}p_1 \\ \partial_{s,t}p_1 & -\partial_{x,t}u_1 & \partial_{y_1,t}v & \partial_{z_1,t}w \\ \partial_{s,t}u_1 & \partial_{x,t}p_1 & -\partial_{y_1,t}w & \partial_{z_1,t}v \end{bmatrix} \begin{bmatrix} -\partial_{y,t}v & -\partial_{z,t}w & -\partial_{s_1,t}p_1 & -\partial_{x_1,t}u_1 \\ \partial_{y,t}w & -\partial_{z,t}v & \partial_{s_1,t}u_1 & -\partial_{x_1,t}p_1 \\ \partial_{y,t}p_1 & -\partial_{z,t}u_1 & -\partial_{s_1,t}v & \partial_{x_1,t}w \\ \partial_{y,t}u_1 & \partial_{z,t}p_1 & -\partial_{s_1,t}w & -\partial_{x_1,t}v \end{bmatrix}.
$$

Thus, we obtained 8 equations (28), which, according to their structure, can be divided into 4 groups of 4 elements (35) or (36). Representing the 8D octonion by four 4D matrices allows us to simplify calculations and also to clearly represent the octonion functions in 3D space.

3.2. Gauss's Law for the Octonion

Gauss's law for charge states that the charge density is equal to the divergence of the electric field [1]. The octonion has 8 spatial coordinates with 7 imaginary axes and 1 scalar axis. A charge q has a particle that exists only in more complex particles or bodies. In space, the octonion charge q, according to Gauss's law, is simply calculated from the wave strength. According to (31), the charge density (divergence) is calculated as the scalar product of the field strength vector in the form of a pure octonion E with the Hamiltonian operator in the vector representation (26). Since it is necessary to take into account the permittivity ε_0 in space, the electric flux is used to calculate the charge density $D = \varepsilon_0 E$. The expression for the divergence of the electrical field is obtained from the first equation of CRC (27):

$$
\operatorname{div} E = \left(\partial_{x,t}u+\partial_{y,t}v+\partial_{z,t}w+\partial_{s_1,t}p_1+\partial_{x_1,t}u_1+\partial_{y_1,t}v_1+\partial_{z_1,t}w_1\right)=\partial_{s,t}p\,. \quad (37)
$$

As can be seen from formula (37), the charge density changes in accordance with changes in the octonion functions of the electrical field strength. It is impossible to represent the charge values at different moments in time in 8-dimensional space. However, as shown in section 3.1, the octonion matrix can be represented as 4 quaternions (35).

The seven frequency functions of the octonion are presented in [6]. As the initial state we consider the vector $\mathbf{x}(0)=[-1\ \ 1\ \ -1\ \ -1\ \ 1\ \ -1\ \ -1\ \ 1]$. From 7 reference frequencies we obtain 64 combination frequencies. Figure 1 shows 8 signals in 7 spatial coordinates and one scalar axis, formed by summing 64 different combination frequencies when multiplying a multi-frequency fundamental matrix by an information vector .

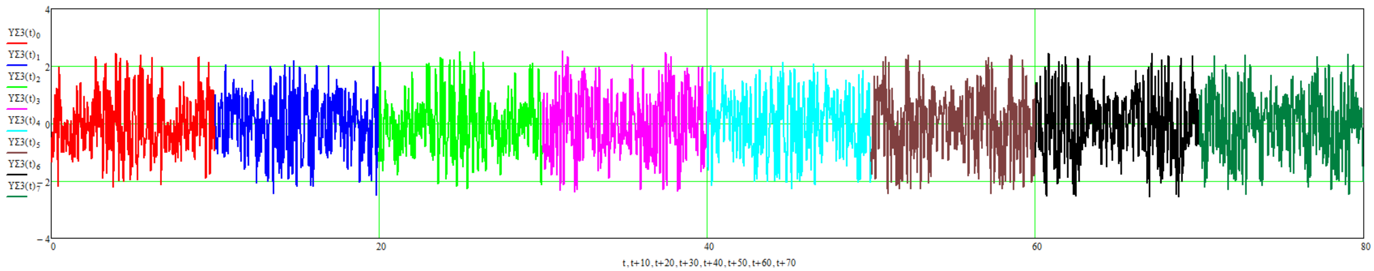

Figure 1: Signals formed by the sum of combination frequencies along one scalar axis and 7 spatial

As can be seen from the graphs, thanks to CRC, the sum of multi-frequency signals does not create values with large amplitudes, i.e. there is no peak factor.

Figures 1, 2, 3, 4 show the rotation trajectories of the scalar part of the octonion in the form of 3-dimensional trajectories for the constituent matrices of quaternions (35).

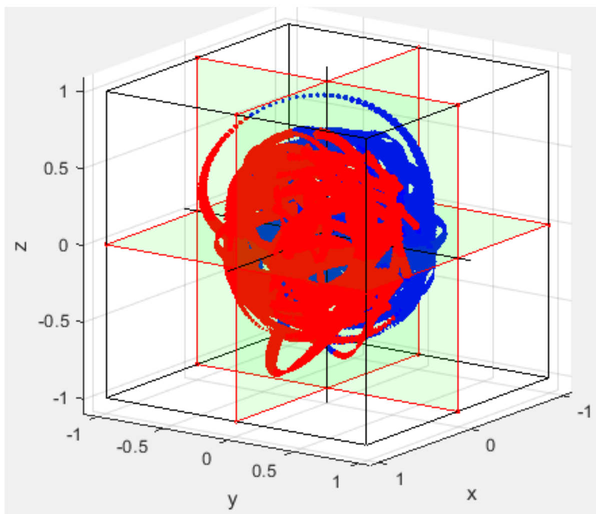

Figure 2: Quaternion $\mathbf{Q}^T$ (35)

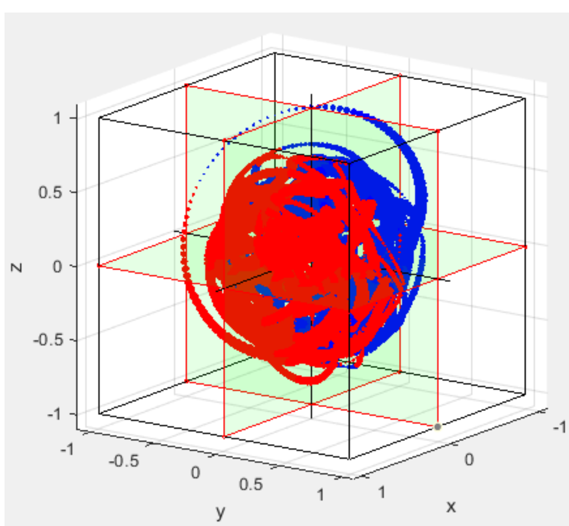

Figure 3: Quaternion $-\mathbf{P}^T$ (35)

The charge value is represented as a point mass calculated using formula (37). The positive charge value is shown in red and the negative charge value is shown in blue.

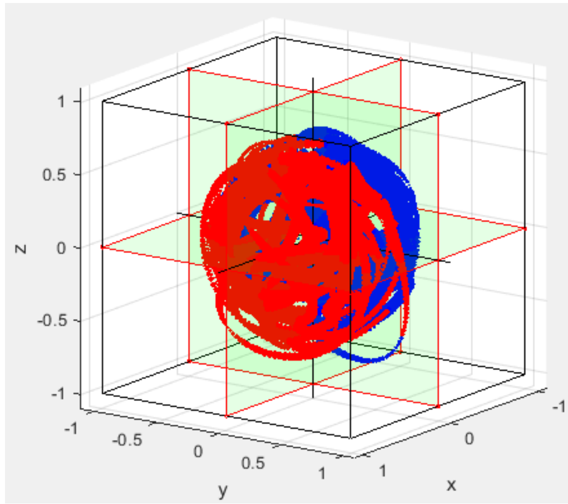

Figure 4: Quaternion **P** (35)

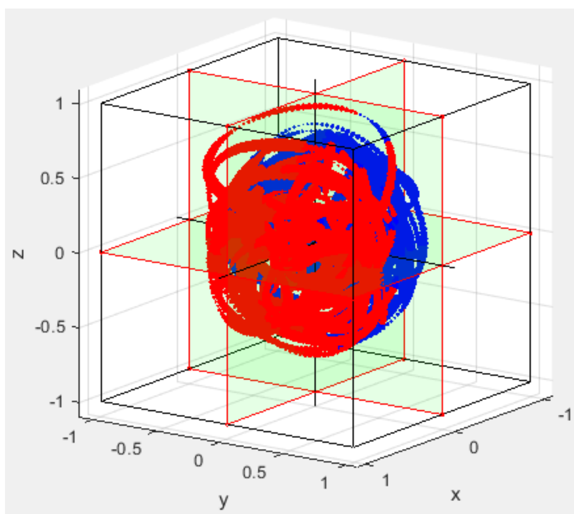

Figure 5: Quaternion **Q** (35)

Note that the charges are located on the scalar axis of the quaternion. In figures 2, 3, 4, 5, for clarity, the charges are arranged in accordance with the values of the oscillation amplitude of the total signals of the 7 imaginary parts shown in figure 1. The existing excesses of the radius of rotation of the average value have, as can be seen from the figures, small values of the scalar part and are depicted as small point masses. This means that the total energy has a constant value.

4. Conclusion

The resulting Maxwell equations for the octonion describe electromagnetic interactions in 8-dimensional space. It is shown that the 8D space of the octonion can be divided into 4 spaces of the 4D quaternion. This capability allows for the simplification of antenna implementation and the creation of large-scale MIMO systems.

References

1. Hayt, W., & Buck, J. (2012). Engineering electromagnetics. Copyright @ 2001 The McGraw Companies.
2. Telatar, E. (1999). Capacity of multi-antenna Gaussian channels. *European transactions on telecommunications, 10*(6), 585-595.
3. Vadim Sovetov. (2025). Maxwell's Quaternion Equations. Space Sci J, 2(1), 01-13.
4. Baez, J. C. (2001). The Octonions. Department of Mathematics University of California Riverside CA 92521 email: baez@ math.

ucr. edu.
5. Yongge Tian: Matrix representations of octonions and their applications, Adv. Applied Clifford Algebras 10,1 (2000) 67-91.
6. Sovetov, V. (2024). The MIMO Data Transfer Line with Seven-Frequency Octonion Carrier. *Eng OA, 2*(3), 01-23.