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Mathematical Analysis of The Validity Limit of Newton's Third Law

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Abstract

A mechanical system involving electromagnetic forces has been analyzed. By deducing the magnetic field generated by a point charge, the force exerted by that particle on another particle of the same type has been obtained. Similarly, the response of the latter was calculated, showing that both forces do not have the same direction, which constitutes a violation of Newton's third law. Similarly, the magnitudes of both forces were calculated, resulting in different values.

Keywords: Newton's third law, Charged Particles, Electromagnetic Field, Conservation of Linear Momentum

1. Introduction

Magnetic forces originate from the motion of charged particles, which is the most notable evidence of the link between magnetic and electrical phenomena. The concept of a magnetic field is used to describe the magnetic influence of electric currents and magnetic materials. It is verifiable that: magnetic fields appear in the vicinity of currents; the magnetic field produced by one current can interact with another current; the force of interaction between current-carrying conductors is perpendicular to the currents and acts in the plane containing them; and the magnetic field caused by a current-carrying wire decreases with distance from the wire (the interaction decreases as the wires move further apart).

The origin of electromagnetic forces lies in the atomic or molecular order of substances known as ferromagnetic. Ferromagnetic materials, in their structure, are divided into magnetic domains separated by surfaces known as Bloch walls [1-4]. If the magnetic domains are disordered, the substance is not ferromagnetic, but if the magnetic domains are ordered, then the substance is ferromagnetic. A substance that is not ferromagnetic can also acquire magnetism, for example, when rubbing the tip of a pair of scissors, a knife, or a screwdriver with a magnet, they acquire the properties of ferromagnetic substances, behaving like small magnets, but the attractive power of these metal pieces is not very strong.

In physics, magnetism is an extremely relevant aspect in the management of electrical and magnetic forces, since the relationship between them is important in the field that involves studies related to the movement of electrical charges, conservation of charge, conductors and insulators, frictional charging and induction, among others. The concatenation of these phenomena is demonstrated in a simple example: ordinary gases are insulators, but ionized gases are good conductors, The air around us, for example, is an insulator, but the ionized air that forms in lightning is a good conductor.

The magnetic force becomes relevant in this work because it is a force that does not comply with Newton's third law [5,6]. The case of 2 electric charges in relative motion is a classic case in which this law is not valid [7]. Specifically, the force that particle 1 exerts on 2 is not equal in magnitude to that of 2 on 1, and their directions do not coincide [8,9]. Even though it is often said that the Lorentz force violates this law, it is enough to say that the magnetic force violates it, since it is precisely the magnetic part of the Lorentz force that prevents the action-reaction principle from being fulfilled exactly. The structure of this article is as follows: this introduction constitutes

Section 1, in Section 2 the modules, direction and sense of the magnetic forces exerted between 2 particles are analyzed; a discussion on the results obtained is carried out in Section 3, and in Section 4 the most relevant conclusions of this article are stated.

2. Analysis

Newton's original formulation of the third law implies that action and reaction, in addition to being equal in magnitude and opposite, are collinear However, there are systems that do not comply with this law, for example, a system in which electromagnetic forces are present [10-13]. In particular, the magnetic component of the Lorentz force does not comply with this law. We will demonstrate this below. The law that describes the interaction between currents is the following [14].

$$d\vec{F} = Id\vec{l} \times \vec{B} \tag{2.1}$$

where the magnetic force differential acts on the current element $Id\vec{l}$ under the action of the external magnetic field $\vec{B}$, while the magnetic force acting on charge carriers (electrons, protons, alpha particles, etc.) obeys the law [15,16].

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2.2}$$

where q is the charge and $\vec{v}$ is the velocity of the carriers.

The contributions to the magnetic field made by an electric current flowing through a conductor, in the position described by the vector $\vec{r}$, are described by the relation [17,18].

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \tag{2.3}$$

Which is the expression known as the differential Biot-Savart law.

The total magnetic field in $\vec{r}$ is obtained by integrating all the current elements $Id\vec{l}$:

$$\vec{B}(\vec{r}) = \int d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \tag{2.4}$$

If the case under consideration includes stationary currents circulating through a circuit, the integral is taken for a closed path, since if the conductor is open there is no circulation of current, that is, for the latter to occur the circuit must be closed.

Equation 2.4 constitutes the Biot-Savart integral law. Written this way, it is not applicable to determining the magnetic field inside the conductor through which the current flows. In this law, the conductor is a line, and it cannot be used to determine the field inside the conductor, since at these points the denominator is zero and the expression remains undetermined. To use the Biot-Savart integral in cases of non-filiform conductors and points inside the conductor, it must be generalized. For this purpose, we start from the relationship [19].

$$I = \iint \vec{J} \cdot d\vec{S} \tag{2.5}$$

Substituting Equation 2.5 into Equation 2.4 gives

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \left(\iint \vec{J} \cdot d\vec{S} \right) \frac{d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \tag{2.6}$$

The vector $\vec{J}$ is parallel to the element of length $d\vec{l}$, therefore, the angle between the directions of the current density vector and $d\vec{S}$ is the same as that established between the directions of the vectors $d\vec{l}$ and $d\vec{S}$. In this way, it is valid to write Equation 2.6 in the following form

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int\left(\iint d\vec{l}\cdot d\vec{S}\right)\frac{\vec{J}\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \tag{2.7}$$

Because the current density vector flux is calculated along the conductor's cross-section, the dot product of the vector related to the length element and the oriented surface element is equivalent to the volume element of the conductor. Thus

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint\frac{\vec{J}(r')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dV' \tag{2.8}$$

where the volume element has been prioritized because it is related to the variable r' that describes the different points inside the conductor.

Equation 2.8 is used to determine the magnetic field at any point in space, including the interior of a conductor. However, it cannot be used to calculate the field generated by a single particle. Equation 2.3, from which we derived Equation 2.8, only makes sense as one element of a sum performed over a continuous set; such a sum represents the magnetic induction of a loop or electric circuit. The continuity equation

$$\nabla\cdot\vec{J} = 0 \tag{2.9}$$

is not satisfied by an isolated current element $Id\vec{l}$, the current would come out of nowhere and vanish after traveling the length $d\vec{l}$. An approximate solution is obtained by taking into account that the current is actually moving charge, so the $Id\vec{l}$ can be replaced by $q\vec{v}$, where *q* is the charge and $\vec{v}$ is its velocity cite [20]. The field generated by such a charge would be:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \tag{2.10}$$

This expression is an approximation because, when starting from the equation valid for direct currents and attempting to transform it for point charges, the interactions between charges have been neglected. It depends on time and, furthermore, is valid only for charges whose velocities are small compared to that of light, and whose accelerations can be neglected. It can be seen that Equations 2.10 and 2.3 are related in the same way as Equations 2.2 and 2.1.

Using Equations 2.2 and 2.10 we can determine the force exerted by one charged particle on another. Considering 2 point particles with charges q_1 and q_2, and velocities $\vec{v}_1$ and $\vec{v}_2$ the force exerted by particle 1 on particle 2 is

$$\vec{F}_{12} = q_2\vec{v}_2\times\vec{B}_1 = q_2\vec{v}_2\times\frac{\mu_0}{4\pi}\frac{q_1\vec{v}_1\times\hat{u}_{12}}{d^2} = \frac{\mu_0 q_2 q_1}{4\pi}\frac{\vec{v}_2\times(\vec{v}_1\times\hat{u}_{12})}{d^2} \tag{2.11}$$

where *d* is the distance between the 2 particles and $\hat{u}_{12}$ is the unit vector that goes from particle 1 to 2.
Similarly, the force exerted by particle 2 on 1 is

$$\vec{F}_{21} = q_1\vec{v}_1\times\frac{\mu_0}{4\pi}\frac{q_2\vec{v}_2\times\hat{u}_{21}}{d^2} = \frac{\mu_0 q_2 q_1}{4\pi}\frac{\vec{v}_1\times(\vec{v}_2\times\hat{u}_{21})}{d^2} = \frac{\mu_0 q_2 q_1}{4\pi}\frac{\vec{v}_1\times[\vec{v}_2\times(-\hat{u}_{12})]}{d^2} \tag{2.12}$$

$\hat{u}_{21}$ is the unit direction vector that goes from particle 2 to 1, so: $\hat{u}_{21} = -\hat{u}_{12}$. If we use the vector identity

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\ vecb - (\vec{a}\cdot\vec{b})\vec{c} \tag{2.13}$$

Equations 2.11 and 2.12 take the form

$$\vec{F}_{12} = \frac{\mu_0 q_2 q_1}{4\pi d^2}[(\vec{v}_2\cdot\hat{u}_{12})\vec{v}_1 - (\vec{v}_2\cdot\vec{v}_1)\hat{u}_{12}] = \frac{\mu_0 q_2 q_1 v_2}{4\pi d^2}(\vec{v}_1\cos\alpha - \hat{u}_{12}v_1\cos\beta) \tag{2.14}$$

$$\vec{F}_{21} = \frac{\mu_0 q_2 q_1}{4\pi d^2}[-(\vec{v}_1 \cdot \hat{u}_{12})\vec{v}_2 + (\vec{v}_2 \cdot \vec{v}_1)\hat{u}_{12}] = \frac{\mu_0 q_2 q_1 v_1}{4\pi d^2}(\hat{u}_{12} v_2 \cos\beta - \vec{v}_2 \cos\gamma) \tag{2.15}$$

where we have identified as α, β and γ the angles formed between the vectors $\vec{v}_2$ and $\hat{u}_{12}$, $\vec{v}_2$ and $\vec{v}_1$, and $\vec{v}_1$ and $\hat{u}_{12}$, respectively. The moduli of the vectors $\vec{v}_1$ and $\vec{v}_2$ have been denoted as v_1 and v_2, respectively.

It can be seen that the first force lies in the plane formed by $\hat{u}_{12}$ and $\vec{v}_1$, and the second in the plan formed by $\hat{u}_{12}$ and $\vec{v}_2$ Therefore, these forces do not always lie on the same line. Furthermore, they do not point in the direction of the load they act on; it can be seen that they are not collinear with the $\hat{u}_{12}$ vector. A schematic representation is shown in Figure 2 as an example. q_1

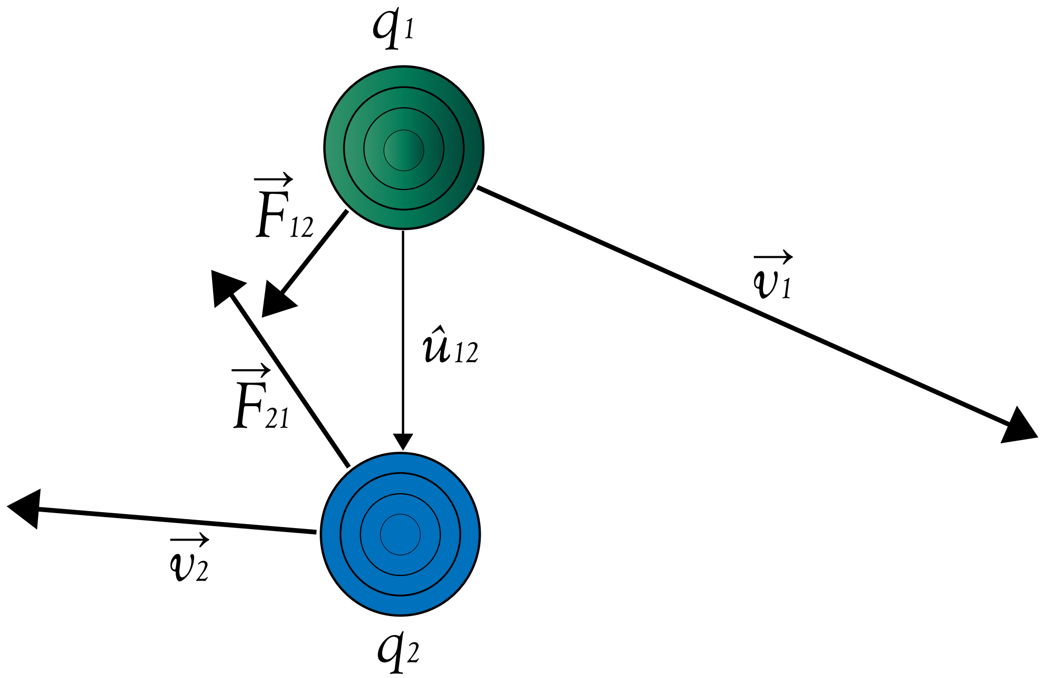

Figure 2.1: Representation of 2 Charged Particles Interacting in Motion

Let us now calculate the moduli of $\vec{F}_{12}$ and $\vec{F}_{21}$ from Equations 2.14 and 2.15.

$$F12 = \frac{\mu_0 |q_2||q_1| v_2}{4\pi d^2}|\vec{v}_1 \cos\alpha - \hat{u}_{12} v_1 \cos\beta| \tag{2.16}$$

$$F_{21} = \frac{\mu_0 |q_2||q_1| v_1}{4\pi d^2}|\hat{u}_{12} v_2 \cos\beta - \vec{v}_2 \cos\gamma| \tag{2.17}$$

To calculate $|\vec{v}_1 \cos\alpha - \hat{u}_{12} v_1 \cos\beta|$ and $|\hat{u}_{12} v_2 \cos\beta - \vec{v}_2 \cos\gamma|$, let's first obtain their squares, and then find the square roots of those values.

$$\begin{aligned} |\vec{v}_1 \cos\alpha - \hat{u}_{12} v_1 \cos\beta|^2 &= v_1^2 \cos^2\alpha + v_1^2 \cos^2\beta - 2 v_1 \vec{v}_1 \cdot \hat{u}_{12} \cos\alpha \cos\beta \\ &= v_1^2(\cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta\cos\gamma) \end{aligned} \tag{2.18}$$

$$\begin{aligned} |\hat{u}_{12} v_2 \cos\beta - \vec{v}_2 \cos\gamma|^2 &= v_2^2 \cos^2\beta + v_2^2 \cos^2\gamma - 2 v_2 \hat{u}_{12} \cdot \vec{v}_2 \cos\beta \cos\gamma \\ &= v_2^2(\cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma) \end{aligned} \tag{2.19}$$

where we have taken into account that for any vector $\vec{a}$ it is true that:

$$\vec{a}^2 = \vec{a} \cdot \vec{a} = a \cdot a \cdot \cos(0) = a^2 \tag{2.20}$$

So we have that:

$$|\vec{v}_1 \cos\alpha - \hat{u}_{12} v_1 \cos\beta| = v_1 \sqrt{\cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta\cos\gamma} \tag{2.21}$$

$$|\hat{u}_{12} v_2 \cos\beta - \vec{v}_2 \cos\gamma| = v_2 \sqrt{\cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma} \tag{2.22}$$

This implies that:

$$F12 = \frac{\mu_0 |q_2||q_1| v_2 v_1}{4\pi d^2} \sqrt{\cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta\cos\gamma} \tag{2.23}$$

$$F_{21} = \frac{\mu_0 |q_2||q_1| v_2 v_1}{4\pi d^2} \sqrt{\cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma} \tag{2.24}$$

We see that the magnitudes of both forces are not equal in general, they only take the same value when $\cos^2\alpha = \cos^2\gamma$. The latter is true when $\alpha = \gamma$, $\alpha = \pi - \gamma$, $\alpha = \pi + \gamma$ *or* $\alpha = 2\pi - \gamma$. These are the cases that correspond to the particles moving in directions that form the same angle with the vector that goes from one to the other, regardless of the direction. For other cases, the magnitudes of these forces are different and therefore do not comply with Newton's third law.

3. Discussion

The results obtained in Equations 2.23 and 2.24 state that both particles must move in the same direction or, in general, in symmetrical directions with respect to the vector that goes from one to the other, regardless of the sense, so that the magnetic forces exerted on each other are of equal magnitude. Equations 2.14 and 2.15 state that only if both particles move in the same direction and sense, the forces involved have the same direction and opposite sense. This means that the magnetic forces exerted by 2 charged particles in motion have the same magnitude, direction and opposite sense only when the latter move towards the same region of space in parallel directions, and the speed with which each one does so may be different.

The case analyzed in this article demonstrates the non-compliance with Newton's third law. Specif- ically, the results obtained here show that this law has a limit of validity. While it may be observed in some systems, it is not observed in others involving electromagnetic forces; although it could well be argued that in the presence of moving charged particles, it has a limit of validity.

To prove the violation of Newton's third law, experiments have even been proposed and situations have even been found in which its non-compliance is convenient [21-23].

Beyond the mathematical analysis carried out in this work, a physical analysis of the problem of moving charged particles also leads us to the conclusion that Newton's third law does not generally hold. The essence of the explanation is that the speed of electromagnetic interaction, which is the speed of light, is not infinite, so the forces exerted are not felt instantaneously. Because of this delay, the forces exerted in response will not be equal and opposite to the original forces.

However, everything seen so far does not violate the principle of conservation of linear momentum, since in the case analyzed, the particles emit electromagnetic radiation, which in turn will have an associated linear momentum. While the sum of the linear momentum of both particles will not remain constant, the following magnitude will:

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_{em} = 0 \tag{3.1}$$

where the third summand is a magnitude associated with the field [24]. For this reason, studies related to the dynamics of charged particles in the presence of magnetic fields and specifi- cally, with the collision of two charged particles take into account both the momentum of the particles and that of the field. In fact, when the linear momentum of a particle is calculated, it is found that there is a part of it that is not in its velocity, but in the field. The latter is evidence that part of the linear momentum of the particle is stored in the magnetic field outside the particle [25-28].

Equation 3.1 is the reason why physicists were forced to think of force fields as real things, not just as mathematical devices for calculating forces. This has set the tone for the development of numerous field theories [29-32]. It is these same field theories that have solved this problem of the violation of Newton's third law: charged particles interact with each other through the field; particle-field and field-particle interactions do obey Newton's third law. Thus, in the complete particle+field system, Newton's third law is obeyed [33].

Despite all that has been analyzed, it is not correct to claim that there were errors in Newton's laws. Newton's three laws, along with the Law of Universal Gravitation, form the axioms of a mechanical theory of unprecedented success. Their confluence with the development of differential and integral calculus achieved prodigious developments, such as those achieved by Leverrier, which allowed him to predict the existence of Neptune, affecting the orbit of Uranus. With the developments of Lagrange and Hamilton a glorious cycle in the history of science closed: there was a theory of the cosmos that explained almost everything appreciable [34-36]. It is one of the great resounding successes of human science [37-40].

The problem is that the scenario proposed by Newton is a model, not reality. The one proposed by Einstein is another model, nor is it reality. But it predicts and explains everything that Newton's model predicts and explains, and it also succeeds where Newton fails: at speeds close to the speed of light and in the presence of intense gravitational fields.

That being the case, we can't call it an error: NASA continues to use Newtonian theory in its orbital calculations whenever possible (which is almost always). Quite simply, the Einsteinian model is more faithful to reality, often with one serious drawback: it's much more analytically complicated.

4. Conclusion

In this work, a mathematical analysis of the observance of Newton's third law for two interacting charged particles has been performed. The results provide evidence that this law is not generally observed. The directions of action and reaction do not coincide; the analysis shows that they are not even in the same plane in several cases. The magnitudes of the forces depend on the angles formed between the directions of motion and the vector running from particle to particle, which means that they only have the same value in specific situations [41].

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