

Lunar Motion Theory (Three-Body Problem)

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Unlike the two-body problem, the three-body problem has no general analytical solution. The differential equations describing the motion of three mutually gravitating bodies are non-integrable and cannot be solved to yield explicit formulas for determining the position of the bodies depending on time. Under most initial conditions, a dynamical system of three orbiting bodies exhibits chaotic behavior; and the only viable approach for predicting their trajectories is to use numerical methods.

Abstract

Attempts to construct a definitive Lunar Motion Theory have persisted continuously since the era of Isaac Newton.

Modeling lunar orbital dynamics while accounting for the gravitational perturbations of the Earth, the Sun, and other planetary bodies is a most formidable challenge in astronomy.

The article presents a physical model and calculation of lunar motion, built on the laws set forth in the article *Laws of Astronomy after I. Kepler and I. Newton* <https://disk.yandex.ru/i/212qCAN044YZEg>. The results show that lunar motion is formed by a complex combination of Earth and Sun gravitational attraction components, acting on the Moon in the direction of the planetary system's motion.

We calculated lunar motion according to the presented physical model (*Viktor's calculation spreadsheet - Google Sheets*).

The calculation results for the interacting Earth-Sun-Moon system agree with the observed motion (published data) and confirm the presented **Lunar Motion Theory**.

The calculation can include any number of objects (planets) that influence lunar motion.

1. Introduction

Today, astronomers present the Moon's motion around the Earth as a combination of several movements:

- revolution around the Earth in an elliptical orbit,
- rotation of the lunar orbital plane; its nodes shift westward, making a full turn in 18.6 years,
- rotation of the lunar orbit's major axis with a period of 8.8 years (occurring opposite the nodal movement),
- periodic changes in lunar orbital inclination with respect to the ecliptic from 90° 59' to 50° 19',
- periodic changes in lunar orbital size: perigee from 356.41 to 369.96 thousand km, apogee from 404.18 to 406.74 thousand km,
- the Moon's gradual recession from the Earth due to tidal

acceleration (approximately 4 cm per year).

Some researchers add a number of other motions to this list.

Since each orbital element experiences a multitude of perturbations with different periods and amplitudes, the actual motion of the Moon remains unusually complex, and its study is a most difficult problem of celestial mechanics.

Calculations of lunar motion use expressions with tens of thousands of terms (coefficients and arguments for trigonometric functions), and no limit exists to their number if higher accuracy is required.

2. Lunar Motion Theory

The lunar movements listed above are not independent motions whose combination determines (shapes) the final trajectory of the Moon. Instead, these movements themselves result from the action of hidden, currently unrecognized interactions.

The article *Laws of Astronomy after I. Kepler and I. Newton* (link to article) describes previously unknown physical interactions that shape the observed planetary orbits and explain the complex nature of lunar motion.

Relative to the Earth, the Moon moves in two planes: it rotates around the Earth in the equatorial plane and, in the form of periodic oscillations, moves in the direction of the Earth-Moon planetary system motion (Figure 1).

At the same time, the Moon, together with the Earth, also moves in two planes relative to the Sun: it rotates in the celestial equator plane and, in the form of periodic oscillations, moves in the direction of the solar system motion (Figure 1).

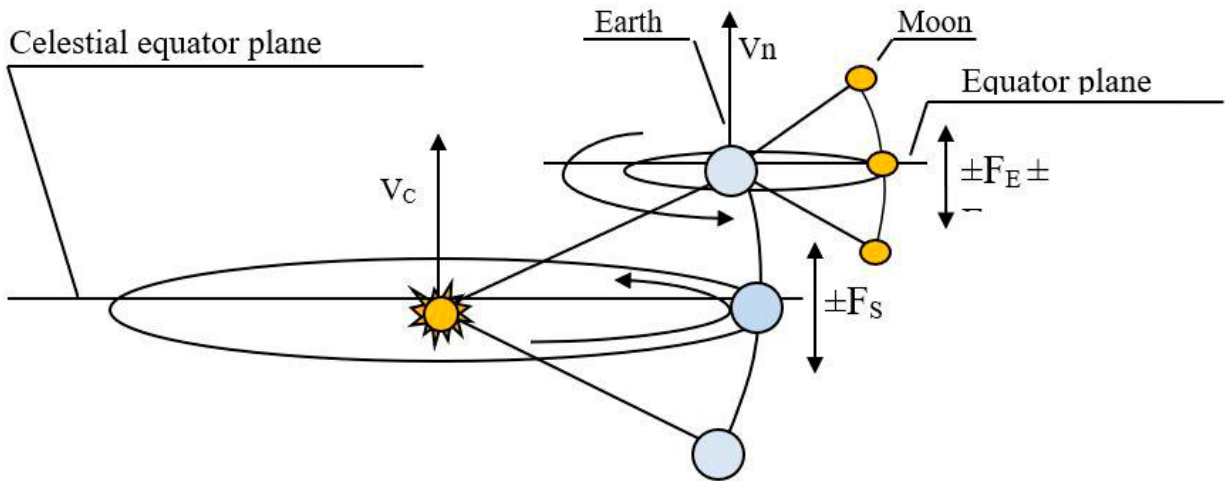


Figure 1: Physical Model of Planetary Motion. For Illustrative Purposes, The Earth Relative to The Sun and The Moon Relative to The Earth Are Shown from One Side Only.

The combination of these motions shapes the observed planetary orbit.

One can confidently assume that planetary rotation in the equatorial plane is uniform.

According to *Laws of Astronomy after I. Kepler and I. Newton*, planetary cyclic oscillations in the direction of the planetary system motion play the primary role in shaping the observed orbit. These oscillations occur under the influence of the gravitational component F of the central body's attraction.

Calculating this motion provides insight into the declination at any given moment, into the dynamics of declination changes, and into the inclination angle of the planet's rotational plane. It also provides insight into the duration of planetary cycles and the planet's velocity in the direction of the planetary system motion.

Superimposing the planet's cyclic oscillations along the planetary system's motion vector and the planet's uniform motion in the equatorial plane makes it possible to construct the observed planetary orbit, determine the planet's orbital velocity along the observed orbit at any given moment, and track its velocity change dynamics.

When calculating the Moon's oscillatory motion along the planetary system's motion vector, one must simultaneously account for the Earth's oscillatory motions relative to the Sun, the Moon's oscillatory motions relative to the Sun, and the Moon's oscillatory motions relative to the Earth.

We calculate the Moon's oscillatory motion along the planetary system's motion vector, as well as the resulting lunar declination, using the same methodology as the Earth's declination calculation. The final lunar acceleration (for the Sun, Earth, and Moon system) is the sum of the acceleration arising from the Earth's gravitational attraction component F_E and the Sun's gravitational attraction component F_S , both acting along the planetary system's motion vector (Figure 2).

At the same time, these force vectors periodically either coincide or point in opposite directions. At certain moments, one or both forces can equal zero.

The complex combination of these forces, which constantly change in both magnitude and direction, determines the complex nature of lunar motion.

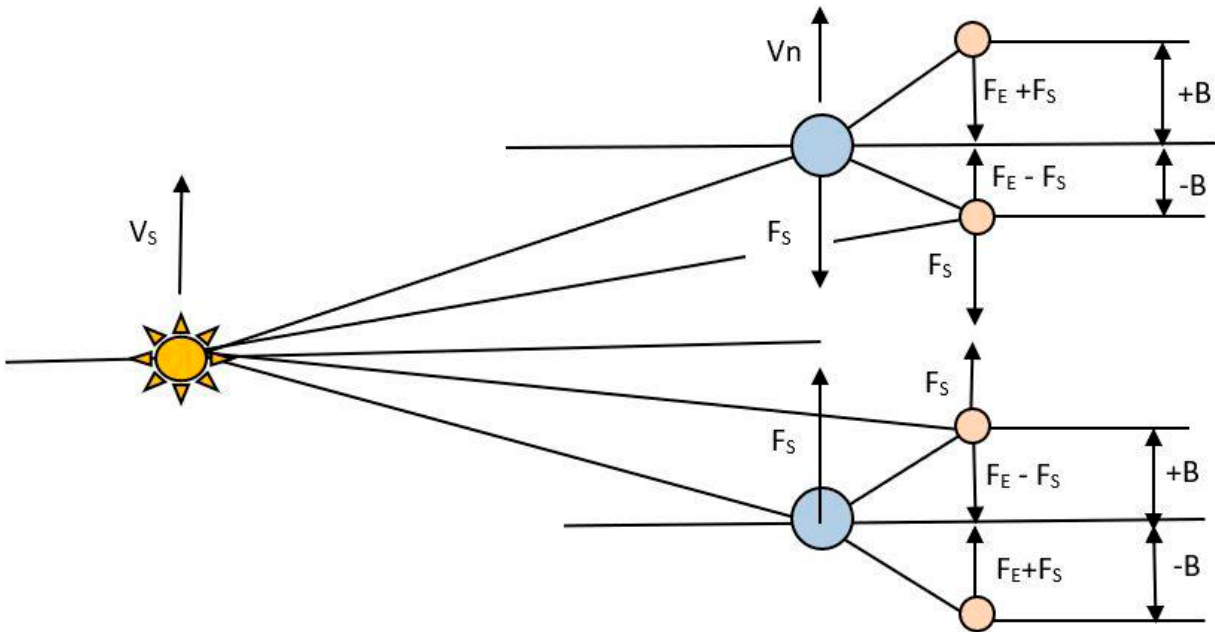


Figure 2: Complex Combination of Gravitational Component Actions on The Moon. For Illustrative Purposes, The Earth Relative to The Sun and The Moon Relative to The Earth Are Shown from One Side Only.

The calculation used a time step of 3,600 seconds. Within this interval, the physical parameters shaping lunar motion are assumed to be constant.

(All presented calculations are non-definitive and require refinement to account for gravitational interactions with other planets and various other perturbations...).

For calculating the lunar cycles:

- The Sun-Moon distance is assumed to be equal to the Sun-Earth distance;
- The Earth-Moon distance is calculated;
- The lunar oscillations relative to the Sun, driven by the gravitational component F_{Sun} , mirror the Earth oscillations.

Sequence for calculating the Moon cycles:

1. Calculate the instantaneous lunar acceleration driven by the Earth's gravitational force (F_{Earth}).
2. Calculate the velocity increment ($\pm$) along the planetary system's trajectory over time step Δt , induced by the Earth's gravitational force (F_{Earth}).
3. Calculate the net velocity increment ($\pm$) along the solar system's trajectory over time step Δt , combining the gravitational influences of the Earth (F_{Earth}) and the Sun (F_{Sun}).

4. Determine the instantaneous lunar velocity along the solar system's trajectory.
5. Calculate the distance traveled by the Moon along the solar system's trajectory over time step Δt .
6. Calculate the displacement differential between the Moon and the Sun along the solar system's trajectory over time step Δt .
7. Determine the Moon's distance B_n from the celestial equatorial plane.
8. Compute $\sin \delta$ ($\sin \delta = B_n / 382000 \text{ km}$).
9. Compute the lunar declination.

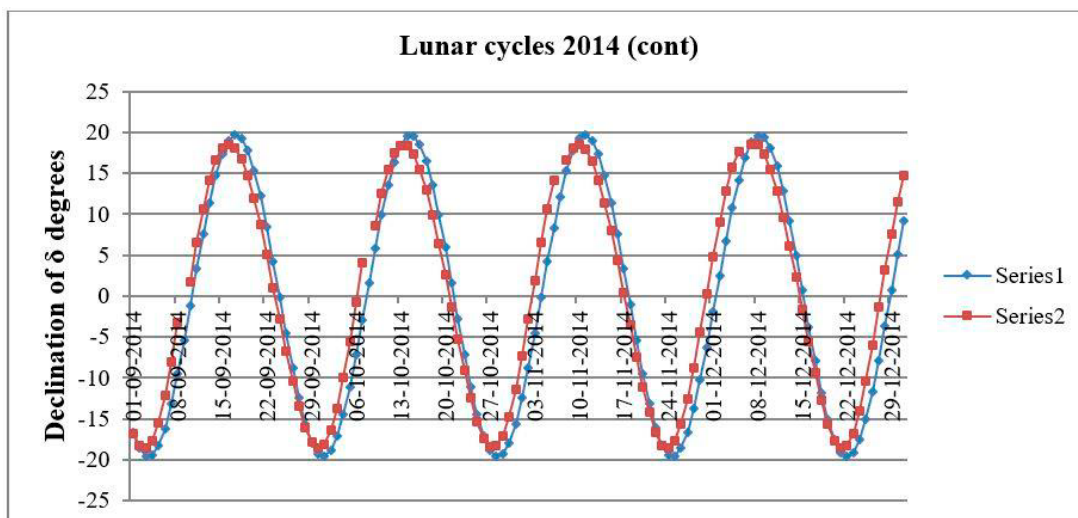
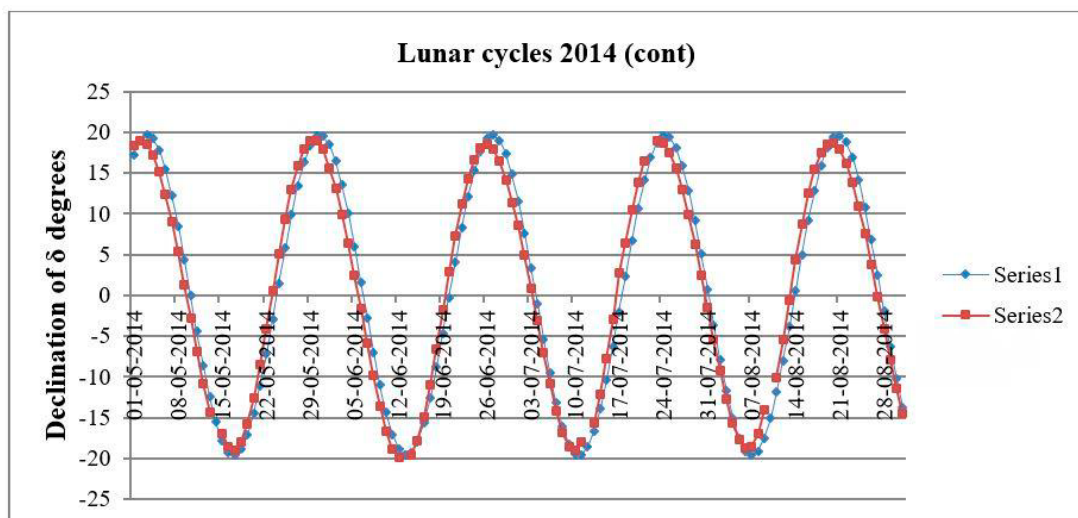
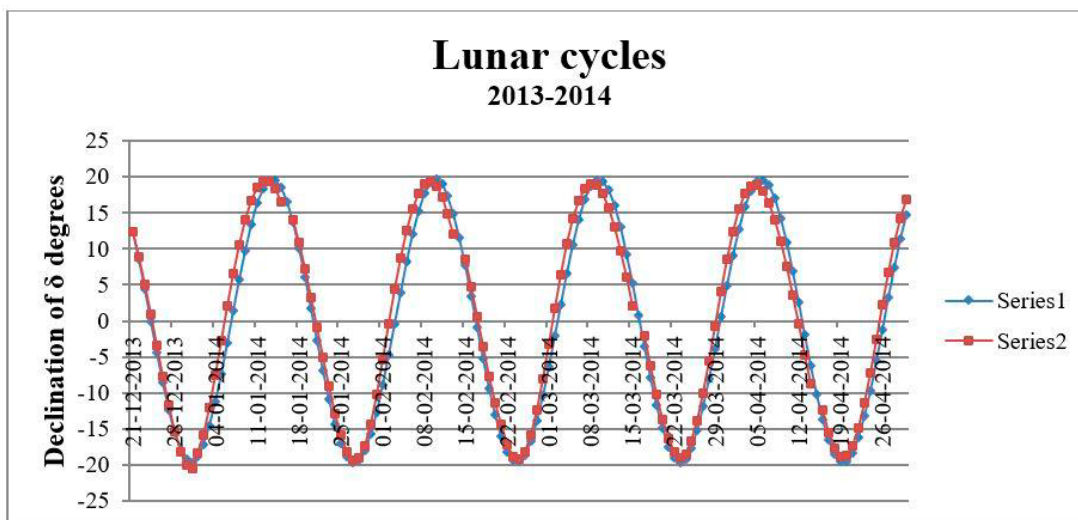
Iterate the calculation for the subsequent time step Δt .

Graph 1 presents the calculated lunar declination (motion) shaped by cyclic lunar oscillations along the planetary system's motion vector. These oscillations occur under the influence of only the Sun and Earth gravitational attraction components F_s and F_E – **Series 1**.

Lunar declination (motion) data from published sources (Alexandrovich. Ephemerides of the Sun, Moon 2013–2014) is overlaid onto this graph – **Series 2**.

(The calculation table data can also be used to plot graphs showing changes in other motion parameters).

Graph 1: Lunar Cycles Under the Influence of The Sun and Earth Gravitational Attraction Components F_s and F_e Acting Along the Planetary System's Motion Vector.



The graphs show that the lunar motion sine wave (blue), plotted based on the Lunar Motion Theory, mirrors the sine wave (red) plotted from published source data.

The calculations demonstrate and confirm that cyclic lunar oscillations along the planetary system's motion vector, occurring under the influence of the Sun and Earth gravitational attraction components, shape lunar motion and the observed orbit.

As noted in the introduction, contemporary lunar motion calculations use a large number of coefficients and trigonometric function arguments. In contrast, this calculation uses essentially

just two variables: The Sun and Earth gravitational attraction components FS and FE acting on the Moon along the planetary system's motion vector.

The calculated lunar declination, its rate of change, and the lunar cycle durations shaped by the Moon's oscillatory motion along the planetary system's motion vector agree with observed values and published source data.

Graph 2 presents a chart showing the changes in the Moon's velocity Vn along the solar system's motion vector.

Graph 2: The Graph Displays the Complex Nature of Lunar Motion Along the Planetary System's Motion Vector.

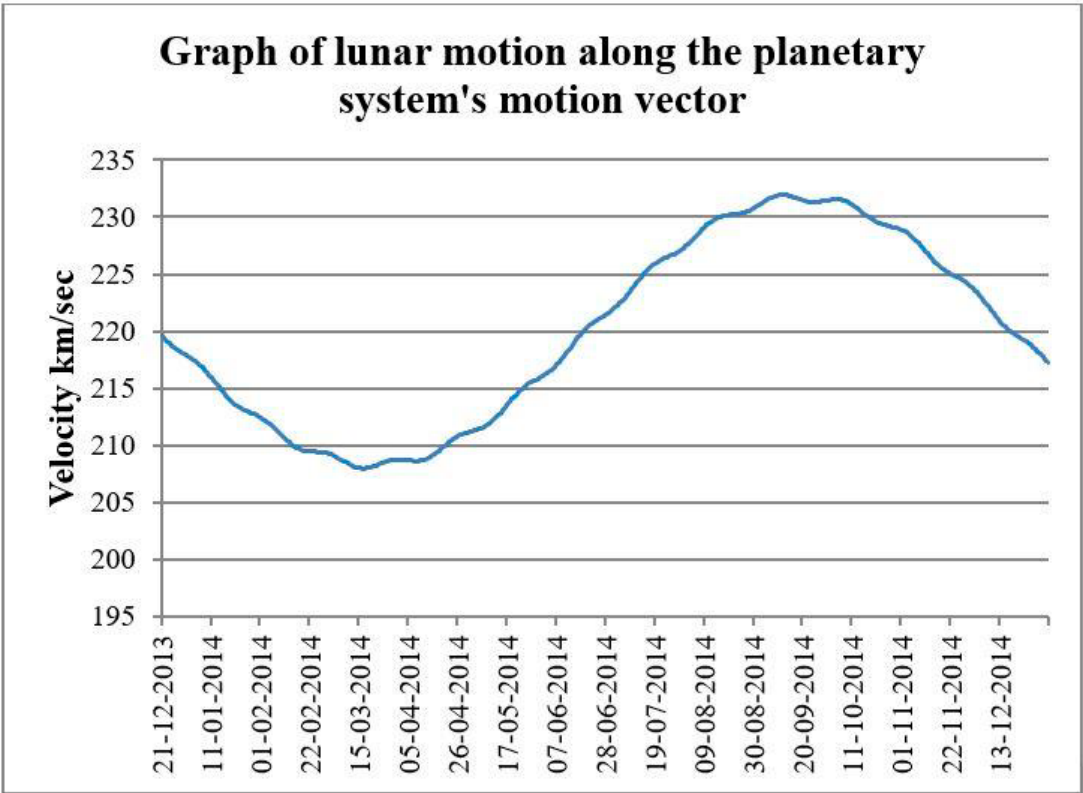


Figure 3: The Plane of Rotation of The Moon.

The plane of rotation of the Moon has an S-shape, like the plane of rotation of the Earth, Figure 4.

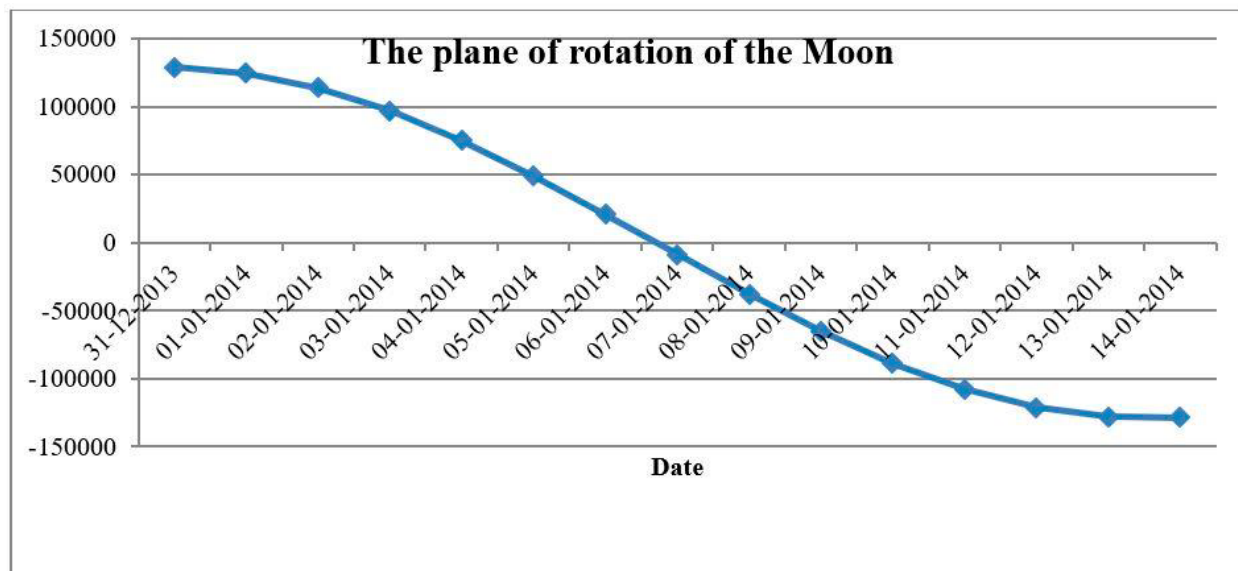


Figure 4: The Plane of Rotation of The Moon has an S-shape, Like the Plane of Rotation of The Earth.

Conclusion

The lunar cycle calculations confirm the presented Lunar Motion Theory, which, together with the applied calculation methodology, can incorporate any number of bodies to account for their effects on lunar motion character.

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