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Research Article

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Liquids in Motion: *An Investigation into Newton's Model for Viscosity*

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Abstract

Newton's model of viscosity in liquids employs a fixed plate and a parallel moving plate separated by a gap. The moving plate takes a conceptual liquid layer along with it and the stationary plate holds a similar liquid layer in place. This creates shear between the layers, and the differences in speeds between layers was assumed to be an arithmetic progression to zero. Experimental results and logic indicate: no zero velocity in such a system, relative speeds are actually in geometric progression, and that therefore Newton's model is incorrect.

Dimensional analyses of a rotating cylinder and a moving plate in viscous liquids both give the dimensions of viscosity as $M/(L^2.T)$, whereas the currently accepted unit of viscosity, the poise, has dimensions $M/(L.T)$. Therefore, the poise has been incorrectly assigned to viscosity, and existing formulas lack predictive power.

Introduction

When I started developing an improved type of rotary viscometer, I did not know it would lead me to investigate Isaac Newton's work on the subject of viscosity. Rotary viscometers generally rotate a disc or cylinders immersed in the viscous medium being studied, and compare the torque (twisting force) with that obtained using viscous oil standardised by capillary viscometry. Current models produce results that are quite dependent on the exact geometry of each instrument. Since I wanted "clean" viscosimetric and rheological results that were independent of the geometry of the viscometer, I designed a variant on the general pattern. The work led me to construct and prove an alternative model to Newton's, and to establish and prove a different relationship between the torque in a cylindrical viscometer and its radius.

Background

To produce results, my design needed the mathematics behind it that would be used to convert torque and the dimensions and speed of the instrument into the preferred unit of viscosity, the centipoise or milli-Pascal-second. It was apparent that the torque must be directly proportional to the area of the cylinder (which is effectively topless and bottomless in my design), the speed of rotation, the radius of the cylinder (or a function thereof) and the viscosity of the liquid. Viscosity should be calculable from those measurements, yet no online search revealed a formula. Also, a very thorough scan of the literature indicated that the size of the container was believed to determine the shear pattern set up by a rotating cylinder in a viscous liquid, and therefore control torque. It became obvious that container perimeter effects could not contribute anything of importance to the torque on the cylinder, since the shear force at the perimeter was so small as to be negligible if the container was large enough. These considerations bring into doubt the models used to develop the equations of viscosity.

To resolve these, I carried out a Dimensional Analysis of my system, which simulates a topless and bottomless cylinder rotating in a viscous liquid and being driven by a contact-less drive shaft (see Figure 1). Dimensional Analysis is a method to verify a formula, and describes a system in dimensions of Length (L), Mass (M) and Time (T). Constants that apply as factors to a formula are dimensionless, essentially having a value of 1. Dimensions carry through a calculation and can be multiplied, divided and raised to powers, but different dimensions cannot be added or subtracted.

Published dimensions for dynamic viscosity, η or μ, include those derived from the Poiseuille equation (and as published on the Wiki Books page), and are M/LT. Wikipedia gives *force×time/area*, (Pascal-seconds) which produces *mass×accelerationtime/area,* which $= M(L \div T^2) \times T/L^2$ and also gives M/LT.

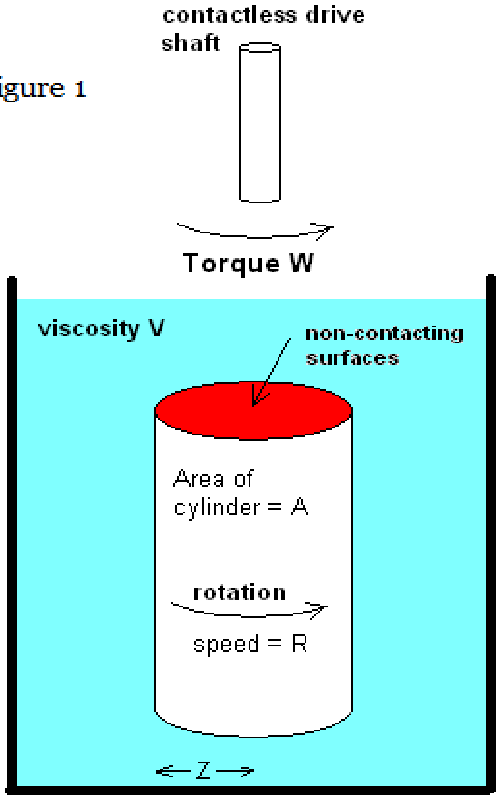
This is consistent with the units of dynamic viscosity (mPa.s or centipoise) as measured by rotating disc or cylinder viscometers such as the Brookfield, which are calibrated by standard oil that is measured by capillary viscometry as kinematic viscosity (centistokes) and converted into dynamic via the density. The following analysis, based on my design, obtains a different result.

Dimensional Analysis (Mass, Length, Time) of Viscosity for Rotating Cylinder

Viscometry Based on *ab Initio* Properties

Consider an ideal cylinder with no top or bottom circular ends, with only its outside surface being immersed in a viscous liquid having a linear shear/stress relationship (known commonly as "Newtonian flow"), and being rotated about its axis by a remotely attached drive shaft, as in the diagram below. The container (not drawn to scale) is so large that the liquid's velocity against the walls is negligibly small, and thus contribution to torque from the container is negligible.

Figure 1



Torque, W, is experienced by the drive motor to overcome the viscosity of the fluid. The energy the drive motor imparts to the system is absorbed by the internal friction of the liquid and converted to heat.

W ∝ area A. (Twice the area, for example, will generate twice the torque, for constant cylinder radius.) Also, W viscosity V (conventionally, η or μ.)

Also, W ∝ revolutions per second, R. Twice the rotational speed, for example, will generate twice the torque, since the rate of shear is doubled. This occurs only with liquids with so-called "Newtonian flow", where the applied force is proportional to the rate of shear. Pastes, slurries, dispersions and many polymer solutions do not conform to Newtonian flow.

In considering how the radius of the cylinder affects the torque, we keep all other parameters the same, especially the area, and just vary the radius. So, if we double the radius, we must halve the height of the cylinder and double the circumference. Hence, the length of liquid moved against the circumference in the same time interval also doubles, and hence we double the resistance due to the liquid and double the torque. (See the following paragraph "Why the length of liquid the cylinder surface traverses matters" for a full argument supporting this concept.) Also, if we double the radius, we double the length parameter of the torque (torque is length times force) and hence double the torque a second time, each doubling having a different source.

Therefore W ∝ square of the radius of the cylinder, Z2 (see experimental proof of this below).

The constants of proportionality can be combined into one, J. (In deriving a unit for viscosity, J becomes 1 and the S.I. Units of Mass (kg), Length (metre) and Time (seconds) are substituted into Equation 1 below to obtain the viscosity.)

The dimensions of each of W, J, A, Z and R can now be combined to obtain the dimensions for V.

∴ $W = J.A.V.R.Z^2$, therefore ……. **Equation 1**

Torque, W, is defined as force F × length L, the same as for leverage.

∴ W = F.L but since force F = mass × acceleration, W = mass × acceleration × length.

The dimensions of acceleration are $\frac{L}{T^2}$.

∴ the dimensions of torque, W, are $\frac{M.L.L}{T^2} = \frac{M.L^2}{T^2}$

J is a constant and therefore dimensionless.

A is area and its dimensions are L^2.

R is rotation speed which is revolutions per second and its di-

mension is T^{-1} or 1/T.
Z is a length with dimensions L. $\therefore Z^2$ has dimensions L^2.
Dimensions can be multiplied together to give the overall value.

$\therefore$ dimensions for viscosity $V = \frac{W}{A.R.Z^2} = \frac{M.L^2}{T^2} \div L^2 \div T^{-1} \div L^2 = \frac{M.L^2T}{L^4.T^2} = \frac{M}{L^2.T}$

This is significantly different from the currently accepted result M/(L.T), so it required a second analysis.

"Unwrapping" a Cylinder into a Plane

As a check on this derivation, consider as a thought experiment "unwrapping" the cylinder into a solid rectangle of sides 2π.Z by l, thus maintaining the area A, which must remain single-surfaced in this imaginary model to maintain the area in contact with the liquid. Further consider making the rectangle infinitesimally thin and attaching two infinitesimally thin wires to two adjacent corners, then dragging the rectangle parallel to its plane through the viscous liquid in a very large container. Further consider the force necessary to do this, against the viscous drag of the liquid, which is, due to energy loss, not transmitted fully to the edge of the container but remains fairly local to the rectangle. It will not matter to this force whether the rectangle is dragged by its long side or its short, since it is only area that contacts the liquid.

Let F = force
Let x = velocity of rectangle. All other parameters are as previous example.

$$F = J.A.V.x$$

$$\therefore V = \frac{F}{J.A..x}$$

The dimensions of F (= mass × acceleration) are $\frac{M.L}{T^2}$

The dimensions of x (= length ÷ time) are $\frac{L}{T}$

$\therefore V = \frac{M.L}{T^2} \div L^2 \div \frac{L}{T} = \frac{M.L}{T^2.L^2} \times \frac{T}{L} = \frac{M.L..T}{T^2.L^3} = \frac{M}{T.L^2}$, which is the same as obtained for the cylinder. This verifies the analysis, and suggests the current dimensions for viscosity of M/(L.T) as incorrect. It requires experimental proof, which follows.

Possible Objections to the Unwrapped Cylinder Model

An academic in the subject claims that edge effects are dominant in such a model, which I refute. Consider then, changing the shape of the unwrapped plane into a circular disc. This is the shape with the minimum edge while keeping the area the same. While, it may be argued, there may still be edge (circumferential) effects, we should be able to remove these mathematically by measuring them experimentally and then increasing the size of the disc. As the radius of the circle increases, the circumference maintains a linear relationship with the radius; that is 2π.r. But the area increases according to π.r2, so circumferential effects will reduce as the circle radius increases, and we can entirely eliminate their contribution to measured force via a mathematical treatment (which is outside the scope of this article) or by simply increasing the area sufficiently. This will leave the shear rate as proportional to the area of the rectangle or disc, removing any edge contribution.

Why the Length of Liquid the Cylinder Surface Traverses Matters?
Going back to the unwrapped cylinder in the form of a rectangle, it will have a length of 2π.r and a height of h, the height of the cylinder, and move through the liquid in the direction of the length. As it derives from a rotating cylinder, it must move through its length in the time the cylinder took to rotate once, a distance of 2π.r . If we now keep the rectangular area the same by halving (say) its height and doubling that length, while still moving in the direction of that length over the same time period to correspond with constant rotational period, the rectangle must move twice as fast. This is why there are two contributions to torque from the radius, and therefore proportionality to radius squared.

Experimental Proof of the Relationship Between Torque and Radius (or diameter) of Cylinder

Pure cylindrical data cannot be obtained from current cylinder viscometers because the circular top and bottom of those cylinders produces torque in a viscous liquid. In a liquid of "Newtonian" viscosity, this torque contributes to the overall torque in a predictable and acceptable way, because the different shear rates at the centre and perimeter of these circular areas produce torque that is entirely proportional to the viscosity, and just add in to the overall torque that is calibrated into viscosity when the liquid's viscosity is measured by tube viscometry. It is only when measuring liquids with non-Newtonian flow that the different shear rates near the centre of the circles and at the perimeter contribute in an unpredictable way to the overall torque. This is because the torque is not linear with respect to shear rate for non-Newtonian liquids. Either "shear thinning" takes place when such a liquid (for example, a polymer solution) is sheared faster than before with a rotating spindle, or "dilatency" occurs, the latter being a thickening with faster shear such as experienced by cornstarch dispersions. Since I wanted to use purely cylindrical torque data to actually show that a liquid was non-Newtonian, and to derive a torque/shear rate equation from that data, I needed to remove the effect of the circles.

To achieve this requires a simple partial immersion of the cylinder in the liquid, so that some of the cylinder is above the liquid. If the torque readings are taken at the same rotation speed and with different lengths of the cylinder immersed, and if those lengths can be accurately measured, the constant torque contribution of the circular bottom end can be eliminated graphically. I carried this out experimentally using cylindrical wooden dowelling rods mounted on the bottom of a disk spindle and attached to a dial viscometer. With the bottom of the dowelling cylinder resting on the surface of a viscous liquid, and with a pointer attached to the viscometer, the height of the cylinder above the bench surface can be read off from a vertical rule held alongside. When the viscometer is lowered, the cylinder enters the liquid and the surface of the liquid rises due to displacement. Thus, the actual immersed length of the cylinder can be easily calculated from the diameter of the cylinder and internal diameter of the vessel, according to the formula:

$P_2 = P_1\left(1+\dfrac{d^2}{D^2-d^2}\right)$ where P_2 = immersed length of cylinder, P_1 = distance of base of cylinder below liquid surface, d = diameter of cylinder, D = internal diameter of vessel:- **Equation 2**

A non-crystallising aqueous solution of glycerol, sucrose and dextrose of viscosity 357.5 centipoise at 17.5^{0}C was prepared and verified for Newtonian flow. Five wooden cylinders, appropriately weighted to prevent floating, were attached in turn to a disk probe of the dial viscometer NDJ-1. These were lowered into the solution and rotated at 12 revolutions per minute (rpm) while the percentage torque was read off from the dial. With thought for the limitations of accuracy of the equipment, multiple readings were taken at different immersion depths and a straight line plotted of % torque against immersion depth, generating a tolerable set of data in each case.

Table 1

Side scale	**1.8325 cm probe**		**1.627 cm probe**		**1.4225 cm probe**		**1.215 cm probe**		**1.026 cm probe**	
Start height 18.6 cm	**Immersion, cm ***	**% torque**	**Immersion, cm**	**% torque**	**Immersion, cm**	**% torque**	**Immersion, cm**	**% torque**	**Immersion, cm**	**% torque**
18	0.619	11.5	0.615	8.5	0.611	5	0.608	3.9	0.606	2.5
17.5	1.135	16.5	1.128	12.2	1.121	7.75	1.115	6	1.111	4
17	1.651	21.5	1.640	16.1	1.631	10.5	1.622	8	1.616	5.3
16.5	2.167	26.4	2.153	19.9	2.140	13.5	2.129	10.2	2.121	6.6
16	2.683	31	2.665	23.5	2.650	16.5	2.636	12.5	2.626	8
15.5	3.199	36	3.178	27.5	3.159	19.9	3.143	14.7	3.131	9.4
15	3.715	41	3.690	31.4	3.669	23.3	3.650	16.5	3.635	10.5
14.5	4.231	46.5	4.203	34.9	4.178	26.5	4.157	19	4.140	12
14	4.747	50.9	4.715	38.5	4.688	29.3	4.664	21	4.645	12.8
13.5	5.263	55.5	5.228	42	5.197	31.9	5.171	23	5.150	14.8
13	5.779	60.1	5.741	45.6	5.707	34.3	5.678	24.6	5.655	16.1
12.5	-	-	6.253	49	6.216	36.5	6.184	26.3	6.160	17.4

* see Equation 2

The slope of each line is measured in % torque per cm immersion, and these data were plotted against probe diameter in each case, plus an extra point for zero torque at zero diameter. The following graph was obtained:

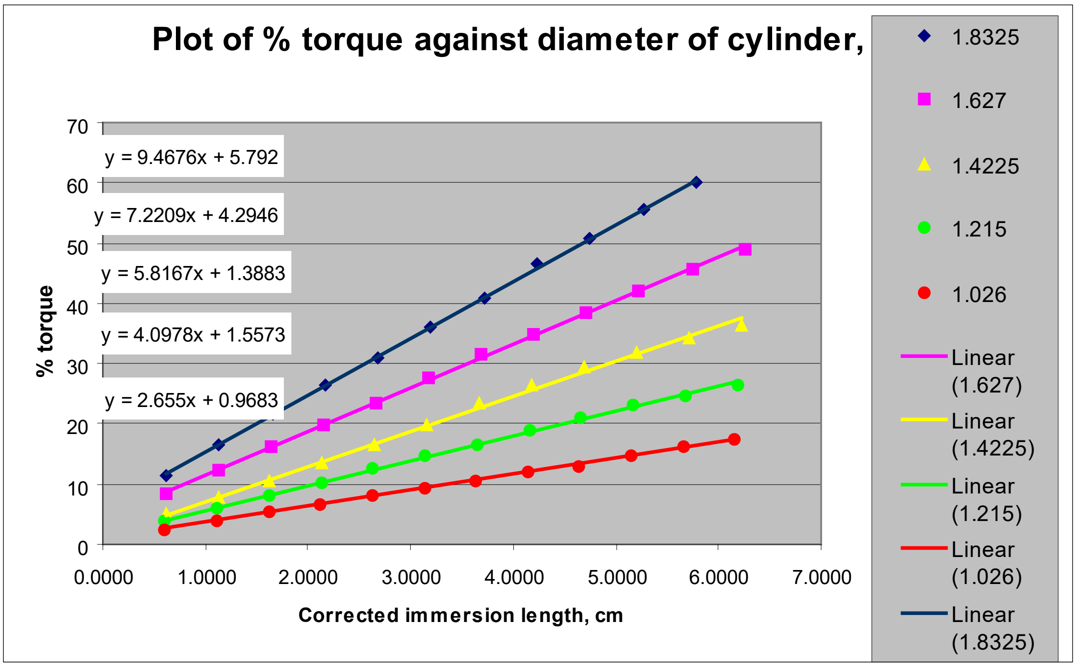


Figure 2

The slope of each line is in % torque per centimetre of immersion, and these were plotted against probe diameter to produce the following curve:

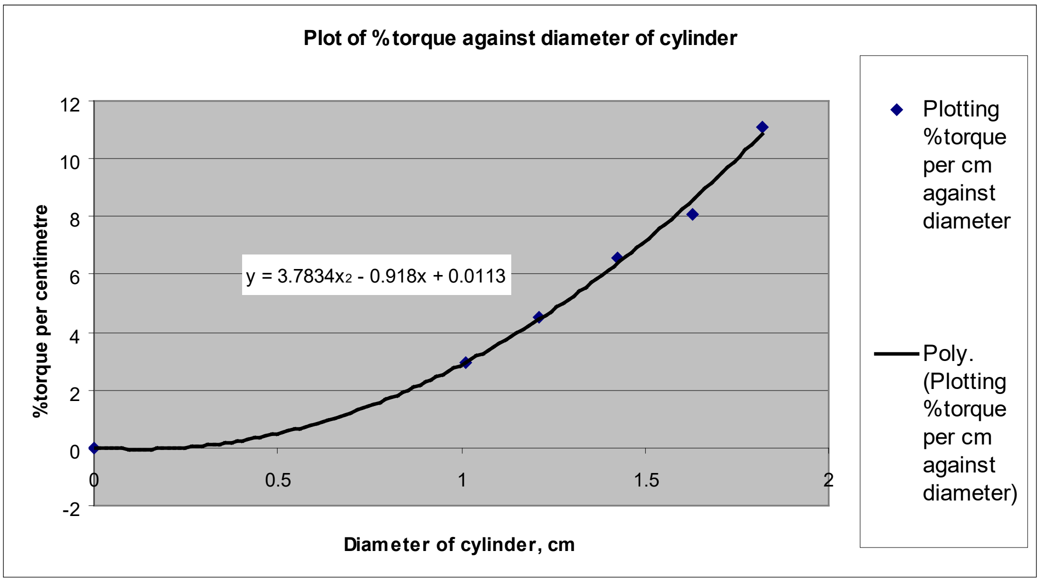


Figure 3

While there is a small term in x, this is most probably partly due to the cylinders being off-centre and partly to the curve matching, and will be easy to remedy if properly machined cylindrical probes were substituted when the experiment is duplicated in another laboratory: a cost which I found necessary to avoid. Nevertheless, most of the contribution to torque is from the square of the diameter. The relationship can be visualised better by plotting torque against the square of the diameter directly:

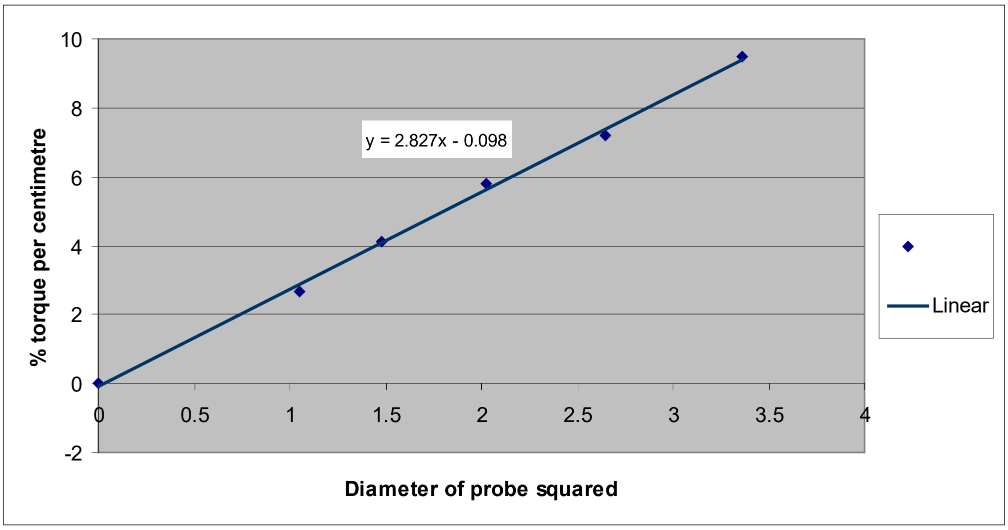


Figure 4

2nd verification using industry-standard spindles (Ametek Brookfield)

In a similar treatment, the expected torque results for three industry-standard spindles can be plotted to show the relationship of diameter to torque. Brookfield publish data for their spindles in the publication "More Solutions to Sticky Problems", available online.

Table 2

Take the case of spindle #61 LV, which has a quoted factor of ×72 at 1 rpm. A 100% torque reading is obtained with a 7200cP liquid. Its length is 65.10 mm. If a version of the #66 LV CYL spindle with a new length of 65.10 mm is immersed in this liquid, it will produce a lower torque. The appropriate factor for the lengthened #66 LV CYL spindle will be 330×53.95÷65.10 because the new version is longer than the original. The new version's factor is thus 273.48, and the torque reading will be 7200÷273.48, which is 26.33% on the dial.

Brookfield spindle description	Brookfield factor (to be divided by revolutions per minute)	Actual immersion length of spindle, mm	Diameter, mm (converted from inches in publication)
#61 LV	72	65.10	18.842
#66 LV CYL	330	53.950	10.254
#64 LV	6000	31.1404	3.175
#65 LV	12×10^3	13.6144	3.175
#7 RV/H	3750	50.368	3.175

Similarly, the #7 RV/H spindle with a quoted factor of 3750 and length of 50.368 mm will obtain a factor of 2901.38 for a 65.10 mm device and produce a dial reading of 7200÷2901.38 = 2.4816%. When these data, with the (0,0) point included are plotted in *Excel* the following graph is obtained:

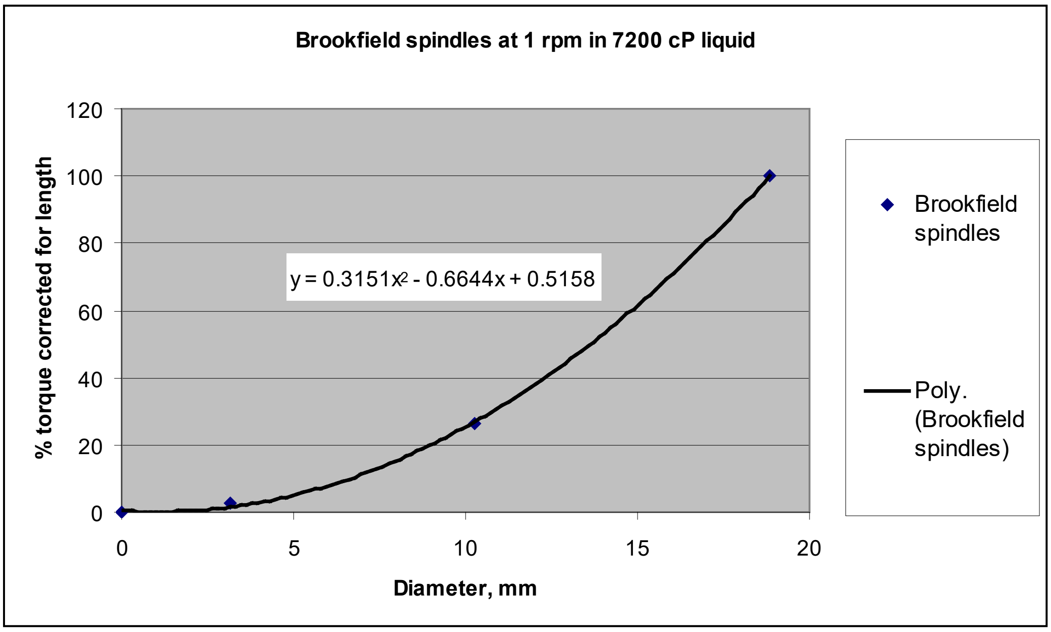
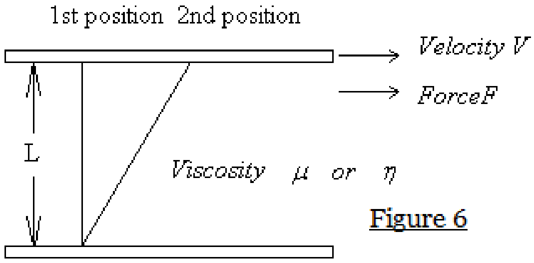


Figure 5

Since the x^2 values are much higher than in the previous example, the contribution of the x^2 term is much higher than the x term, where the latter may arise from the shaft and circular bases and tops of the cylinders. Nevertheless, there is a clear fit between the squares of the diameters and the torque.

Newton's Model for Viscosity

I decided to look again at Newton's original model, to which there are many published references:



In this model, the top surface moves to the right, carrying the viscous liquid with it. The velocity of the liquid was assumed by Newton to decrease to zero at the bottom surface via a series of conceptual "steps" in an arithmetic progression, and all current calculations of viscosity are based on this model; most importantly, the physical dimensions.

A position of zero velocity is exactly what is described in many diagrams of liquid flows (see "No Slip Condition in Wikipedia).

Newton's model looks suspiciously like the model for an elastic solid, stretching to the right and remaining anchored to the lower surface. In an elastic solid, there are lines of atoms joining the top and bottom surfaces, and they stay present even after movement. This is not true for an element of liquid, where there are no lines of atoms linking the layers. Also, according to my understanding, there is no point of zero movement but there is just the transmission of less and less movement the further away from the moving surface and into the bulk of the liquid one gets.

Looking at Newton's model another way, one can understand that in reality, it requires a rectangular box with transparent sides:

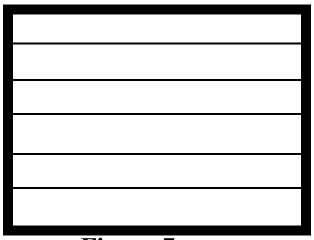


Figure 7

The box contains a viscous liquid, designated by the thin lines. If the top surface of the box is pushed to the right, the following shape is obtained:

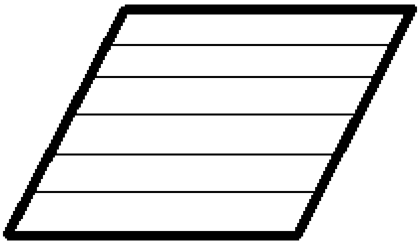


Figure 8

With this container, all of the liquid is pushed to the right, and flows that way. But for the liquid to move, movement of the sides of the box is required in order to push the liquid. In Newton's model, there are no sides to his box, and therefore the liquid cannot move the way he intended, in this linear fashion, even as a very small "element" of a larger system. The liquid can only move when the top of the box transmits some motion to the layer of liquid underneath, then that layer transmits movement to the layer beneath it, and so on.

A proposed New Model

What must be the case around a rotating cylinder in a "Newtonian" liquid is that a series of conceptual concentric, infinitesimally thin rotating cylinders of liquid are present, with the rotational speed of each being in a *geometric* series as we get further away from the driving cylinder. (See example below.)

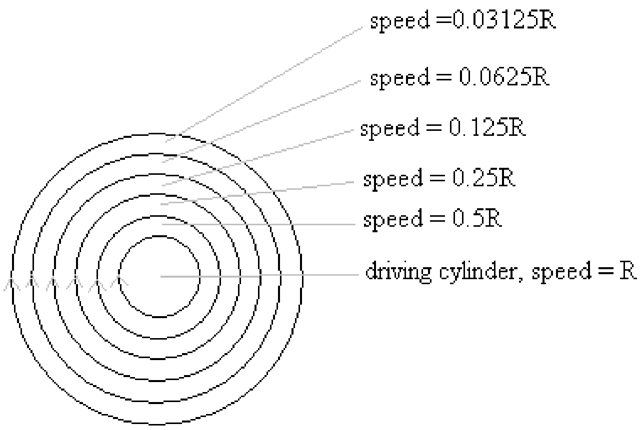


Figure 9

In this thought experiment, each notional cylinder must impart exactly the same relative motion at a ratio of 0.5 to its neighbour, or whatever fraction pertains to the substance in question. Clearly, a thin liquid will experience a fraction approaching unity, and the velocity of the cylinders will gradually diminish the further away from the driving cylinder one gets. In this case, the liquid will move due to its own kinetic energy, resulting from its mass and velocity. (It should be noted that when measuring low viscosity liquids, a much larger container is needed to avoid drag of the liquid against the container's inner wall, because much less energy is being absorbed and velocity will be higher at the wall, resulting in extra torque at the spindle.)

A liquid with high viscosity will experience a low fraction, so that the velocities drop quickly and kinetic energy is reduced. A material with a non-Newtonian flow behaviour will set up conceptual concentric cylinders where the velocity of each cylinder will not decrease according to a geometric series but by some other function that can go to zero.

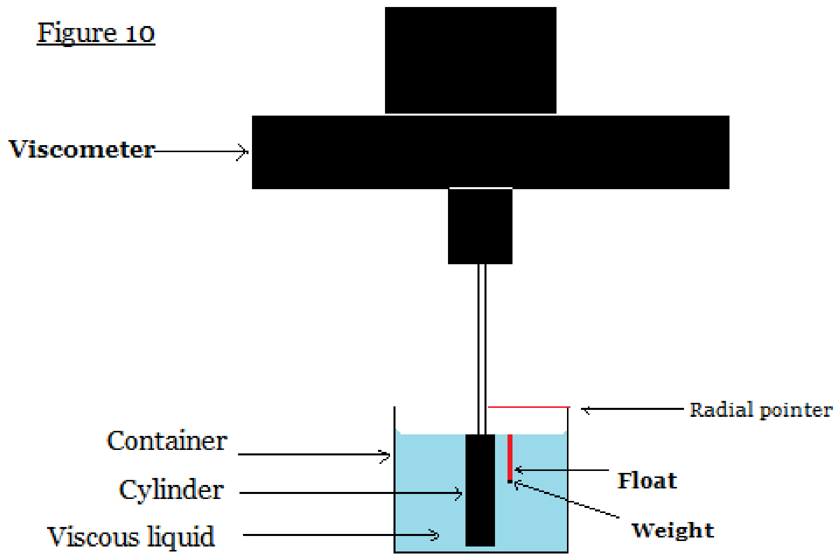
Published work on Rotating Cylinders

G.I. Taylor F.R.S studied liquid flow between cylinders and *published in Philosophical Transactions of the Royal Society of London, Series A, Vol.223 Isue 605-615 VIII* in 1922 in an article entitled "Stability of a Viscous Liquid contained between Two Rotating Cylinders", but this study was concerned with the onset of instability of flow. The same subject was later worked on by Katherine J. Asztalos and Jorge Pulpeiro Gonzalez in a piece entitled "Stability Analysis of Taylor-Couette flow, May 5th 2017, but there was no experimental work described, just mathematical analysis. My geometric series model required testing in the following experiment:

Experiment to Find the Velocities of Concentric Circles in A Viscous Liquid Driven by A Rotating Cylinder

The following apparatus was constructed:

Figure 10



The dimensions of the apparatus are as follows:
Cylindrical tank height = 10.9 cm internal. PMMA (clear acrylic).
Internal diameter of tank = 10.4 cm.
Float = 4.8 cm, made of wood and weighted at bottom, to float vertically.
Cylindrical probe diameter = 1.86cm.
Viscometer is a Chinese-made NDJ-1, capable of 6, 12, 30 and 60 rpm, and was checked for accuracy of speed.

Composition of Viscous, Non-Crystallising, Liquid

Glycerol	360g
Sucrose	210g
Dextrose	290g
Water	181.75g

Total in container 1041.75g. The composition is heated to clarity with stirring, cooled, and water loss replaced. Viscosity = 439cP at 15.7ºC. It is still somewhat hygroscopic.

Method

The objective of the method is to measure the velocities of a stirred liquid across the space between the driving cylinder and the wall of the vessel. A float is placed in various measured positions relative to the centre, and its time for one revolution measured. The viscometer probe is lowered into the liquid so that the top of the probe is just below the surface of the liquid. This can be verified by viewing a straight, vertical object by reflection from the surface, where any meniscus can be easily seen. The objective is that no strong meniscus further out than about 1cm is obtained when the probe is in position. Floats will rise into an elevated surface, according to the so-called "Cheerios Effect" as described by Dominic Vella and L. Mahadevan, arXiv:cond-mat/0411688v3, 25/6/2007. Since that surface is sloping in a meniscus, any floating object will also move towards the object generating the meniscus.

The cylindrical probe was centred in the cylindrical vessel by checking the distance of the probe from the vessel wall using dividers, and moving the viscometer until all distances equal. A pointer was attached radially to the top of the vessel, and the time for one revolution recorded for each position of the float.

In the current set-up, the viscometer is run at 6 revolutions per minute (rpm). The float is initially placed as near as possible to the centre and the distance measured. The motor is run, with the viscometer disc locked and the spring ineffective, and the distance of the float from the probe is measured with dividers after one complete revolution, to an accuracy of 0.5mm. This act is carried out after moving the float outwards for each position. The distance from the centre = distance from viscometer probe + ½ diameter of probe. The effect of centripetal force on the float as it was moved around in a circle was found to be negligible. The following results were obtained:

Table 3

Distance from centre, cm	Time (t), seconds	Rpm (=60/t)	Distance from centre, cm	Time (t), seconds	Rpm (=60/t)	Distance from centre, cm	Time (t), seconds	Rpm (=60/t)
1.38	24.91	2.4087	2.38	79.16	0.7580	3.48	221.76	0.2706
1.53	32.83	1.8276	2.58	102.15	0.5874	3.58	282.28	0.2126
1.83	41.78	1.4361	2.78	123.95	0.4841	3.93	456.05	0.1316
2.03	54.85	1.0939	2.93	137.51	0.4363			
2.18	70.32	0.8532	3.08	152.29	0.3940			

When plotted in *Excel*, the following graph was obtained:

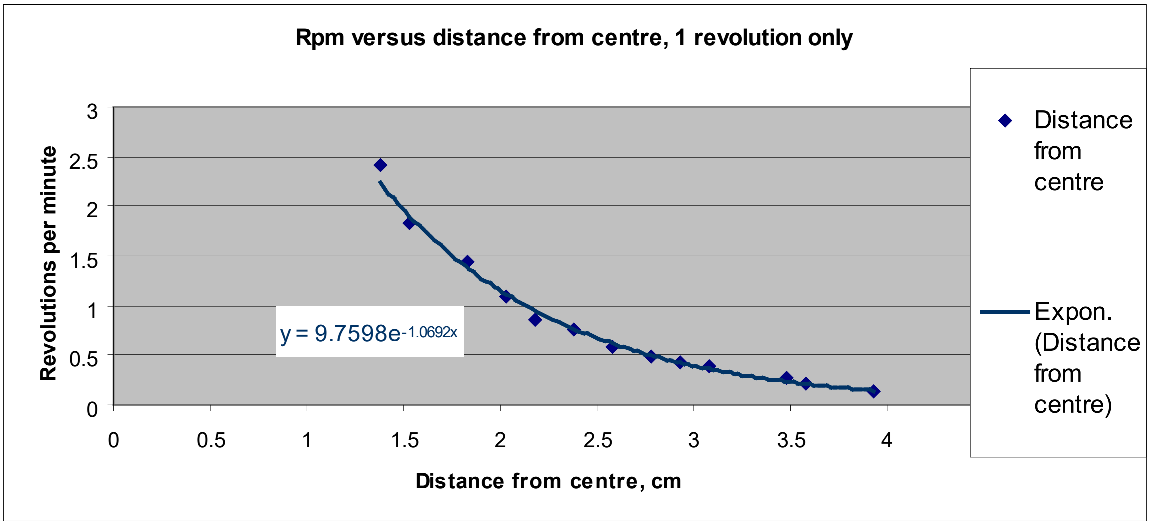


Figure 11

The points follow an exponential curve quite closely, and it corresponds to a geometric series of value decay over distance, with a common constant of decay per unit distance.

This can be seen in a better presentation using a logarithmic scale for rpm, when it can be compared to a decay curve with a factor of approximately 0.35 per centimetre of distance:

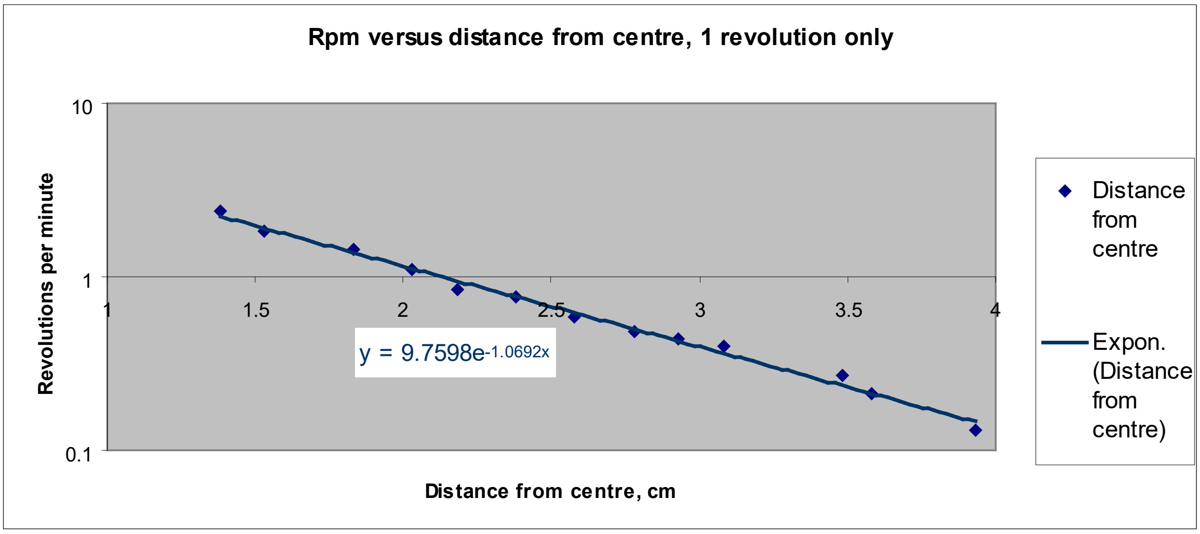


Figure 12

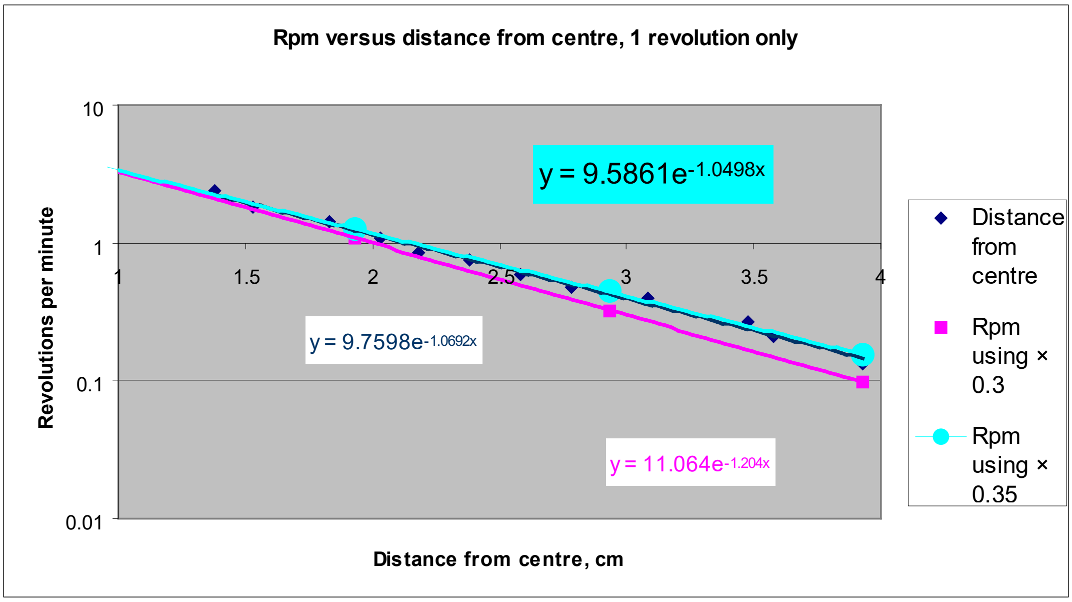


Figure 13

It is interesting to note that when these lines are extended backwards to 0.93 = the radius of the driving cylinder, (by using the Excel equations), the calculated rpm is only 3.611: rather lower than the rotation speed of 6 rpm for the cylinder. This does indicate slip between the cylinder, a highly polished stainless steel with a high surface energy, and the liquid, which also has a high surface tension. It will be interesting to test mineral oils and viscous liquids such as polyisobutylenes, which will have lower surface tension and should wet out a metal surface more energetically. These further improvements are outside the scope of this thesis.

Also, when extending the lines to reach the wall of the vessel, 5.20cm from the centre, the radial velocity = 0.03757 rpm, which calculates to 0.2046 mm/sec at the wall. This is lower than the observed drainage time of the syrup from the PMMA wall under the small force of gravity.

Conclusion

The currently accepted model for movement of liquid next to a surface states that the decay in movement with distance takes place along an arithmetic series, according to the doctrine set up by Newton and his model (discussed previously). It has been shown elsewhere in ordinary viscometry using liquids flowing under their own weight, that it is difficult to demonstrate any slip. Therefore, there was established a "No Slip principle" (see Wikipedia entry). Therefore, the velocity of the liquid has been assumed to be the same as that of the driving surface and zero at the stationary surface, the wall of the vessel. The work here described disproves the currently held views on both slip and the decay of flow with distance.

Notes on the Concentric Cylinder Model

The conceptual concentric cylinder model is simplified here. For a fully immersed cylinder, the set of concentric cylinders will also be immersed and there will be static liquid above and below them, impinging on their freedom to rotate. These static layers will, of course, begin to rotate in their turn, will consume energy and contribute to torque. (A delay in a stable viscometer reading is often seen at the start of a test, which corresponds with the concentric cylinders of liquid and the top and bottom circles of liquid reaching equilibrium velocity.)

These energetic effects are downstream from the driving force, so do not affect the model, all downstream energy being absorbed by the liquid. My current model does rely on fairly viscous liquids, where all effects take place within the timescales of the experiment. Low viscosity liquids (e.g. water) can sustain rapid movements and vortices containing kinetic energy, which more viscous liquids cannot sustain.

Poiseuille's own derivation relating to liquid passing through a capillary tube also suffers from an assumption of arithmetic series in the laminar flows between the centre of the tube and the wall. Here, the force driving the fluid down is pressure from the column of liquid. The current belief is that there is zero liquid velocity at the tube wall, in accordance with the "no slip" principle (see "No slip" in Wikipedia). Anyone who has ever carried out a titration in a burette knows that there is no appreciable delay in the liquid at the tube wall descending, although there may well be a layer of liquid a few molecules thick remaining attached to the wall. A velocity approaching zero at the wall suggests a drain time approaching infinity. Therefore, the velocity cannot be zero at the tube wall and the effect of meniscus pressure must be considered. (I speculate that the velocity of liquid at the tube wall is a function of its viscosity, the surface energy of the wall material and the surface tension of the liquid.) It would be preferable, for capillary studies of viscosity, to derive a formula based on the assumption that the shear rates also follow a geometric series, rather than the arithmetic series that has been used to create the original formula, to obtain the correct dimensions. Then, results from capillary viscometry can be used for calibration in rotating cylinder viscometry by a mathematical treatment only. To study this properly, the surface tension of the liquid and the surface energy of the capillary tube (normally specially cleaned glass) must be incorporated into the equations.

Conclusions

1. Study of a rotating cylinder in a viscous liquid has shown that container size cannot be an issue or be part of the formula for calculation of torque in large volumes of liquid, (although close-fitting concentric cylinders with small quantities of liquid have been useful in the study of liquid flow).
2. No formula previously existed for calculating the force required to move a thin rectangle through a viscous liquid at a known speed and parallel to its plane. Thus, the model physicists have relied on since the beginning lacks the predictive ability that should be available, given known parameters.
3. A mathematical study of the pattern of shear forces based on geometric progressions is needed for rotational viscometry. This must incorporate values for the surface energies of the liquids and solids involved, since the materials used to make viscometer probes and containers are likely to affect the results.
4. The currently accepted dimensions of viscosity are in error. Liquids used as standards in the industry can be re-measured using the pure cylinder viscometry here.
5. A new unit for viscosity should be created, and given a name. My suggestion is skel, being derived from vi(sk)osity and Bel, the unit of sound intensity. A national competition to name the unit would generate interest.
6. Work is needed to reproduce the experiments herein under more controlled conditions than I could provide.