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Research Article

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Linearity of the Position of Prime Numbers. The Order of Prime and Composite Numbers

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Abstract

In this paper it was shown that all prime numbers lie on 96 half-lines. At the same time, it was shown that if a given number does not lie on any of the above half-lines, then it is a composite number. A corresponding linear mathematical relationship was also derived and it was shown that all prime numbers must satisfy it. If a given number does not satisfy the above dependence, then it is a composite number. One linear equation and 8 numbers are enough to carry out all of the above-mentioned analyses. In addition, the astonishing cyclical nature of the occurrence of composite numbers in the set of natural numbers has been demonstrated.

Keywords: Prime Numbers, Ulam Spiral, Sieve of Eratosthenes.

1. Introduction

A certain regularity of the distribution of prime numbers was also discovered [1-3].

There are many articles in the scientific literature suggesting that the prime numbers are chaotic or has a fractal structure [4-10]. However, as shown below, the distribution of prime numbers is very ordered.

2. The Methods

We will treat successive natural numbers (n) as successive degrees of angle. We mark the location of these numbers in the (x, y) plane in a trigonometric way:

$$x(n) = n * \cos\left(n\frac{\pi}{180}\right) \quad (1)$$

$$y(n) = n * sin\left(n\frac{\pi}{180}\right). \quad (2)$$

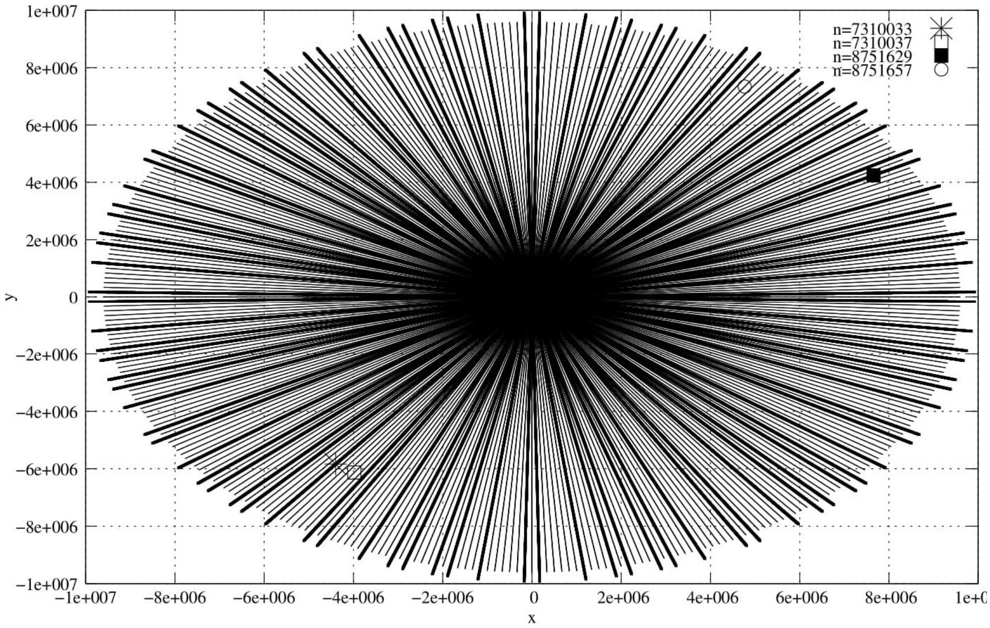

Figure 1: Location of prime and composite numbers

On the graph, we will get a cyclically repeating set of 360 half-lines (thin and thick lines in Fig. 1). In the above graph, we will also mark the location of the prime numbers (*pn*) in the same way:

$$x(pn) = pn * \cos\left(pn\frac{\pi}{180}\right) \tag{3}$$

$$y(pn) = pn * sin\left(pn\frac{\pi}{180}\right). \tag{4}$$

As a result, we will obtain a cyclic bundle of 96 half-lines (thick lines on Fig. 1). 660000 prime numbers were used to draw the thick lines. Looking at Figure 1, we notice that the distribution of thick lines repeats cyclically every 30th. The first distribution contains the following sequence of 8 numbers:

p^n_{basic} = (1,7,11,13,17,19,23,29). (5)

For obvious reasons, this sequence does not contain primes 2, 3, and 5.

Any prime number (greater than 30) therefore satisfies the following relationship:

$pn = pn_0 + n * 30$ (6)

where pn_0 is one of the numbers in the set p^n_{basic}, and n is any natural number.

This research has discovered an amazing property. It turns out that composite numbers, which do not lie on thick half-lines (Fig. 1) and do not satisfy equation (6), appear cyclically in a strictly defined rhythm (crosses on the Fig. 2): 3, 5, 1, 5, 3, 1, 3, 1, etc. That is, first three consecutive composite numbers appear, then five, then one, then five, then three, then one, then three, and finally one composite number. This cycle repeats endlessly. Each of these "boxes" is separated by only one number lying on the thick half-lines and satisfying equation (6). For example, starting with the number 50, we have the following cycle: 50, 51, 52, (53), 54, 55, 56, 57, 58, (59), 60, (61), 62, 63, 64, 65, 66, (67), 68, 69, 70, (71), 72, (73), 74, 75, 76, (77), 78, (79)... etc. The above cycle starts again with the number 80. The numbers in parentheses lie on the thick half-lines and satisfy equation (6), while the other numbers lie on thin half-lines, do not satisfy equation (6) and are only composite numbers.

This phenomenon is also shown in Fig. 2, where crosses mark composite numbers that do not meet equation (6) and lie only on thin half-lines. Circles indicate numbers that meet equation (6) and lie on thin half-lines. Individual packages contain consecutive numbers. So, for example, the first box on the example above is the set (50, 51, 52). It should be noted that the numbers in parentheses (circles in Fig. 2) are prime numbers (except for the number 77). The remaining numbers (marked with crosses) are only composite numbers. In each subsequent cycle, the situation is identical, i.e. numbers not in parentheses numbers in parentheses are only composite numbers, while underlined numbers can be both prime and composite numbers. It should be added that in the vast majority of cases in a given cycle the numbers are only marked with prime numbers. Using the above phenomenon, one can immediately determine any large composite number.

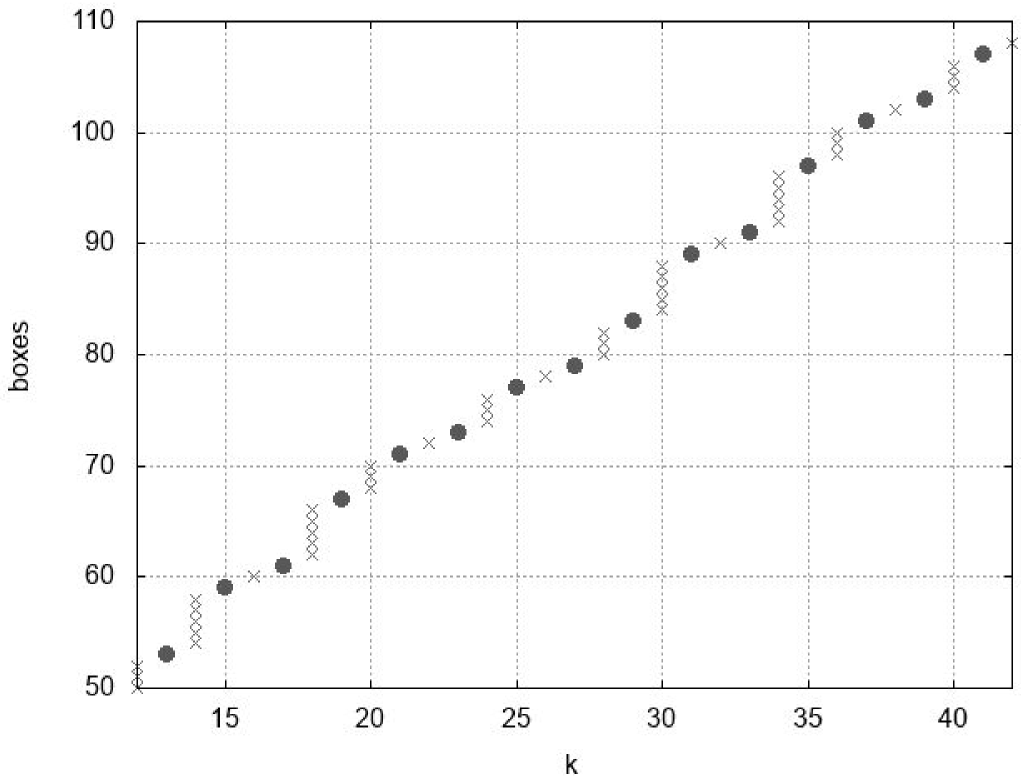

Figure 2: The cyclicality of composite numbers.

3. Summary

To sum up, it should be stated that if a given natural number does not satisfy the dependence (6), it means that it is definitely a composite number. If, on the other hand, a given natural number is a prime number, it must satisfy equation (6). The graphical interpretation of this statement is that if a given natural number does not lie on any thick ray, it means that it is definitely a composite number. If, on the other hand, a given natural number is a prime number, it must lie on one of the ray lines marked in Fig. 1 with a thick line. Thus, all prime numbers are: 2, 3, 5 and all others determined by the formula (6). There are no others. It can therefore be said that the base of prime numbers is 8 numbers from the set (5). It is said that the "atoms" of natural numbers are prime numbers. The above analysis shows that the "atoms" of prime numbers are set (5). It can therefore be said that the "atoms" of all natural numbers are 8 numbers from the set (5).

An extremely interesting phenomenon is the cyclicality of the occurrence of composite numbers in the form of "boxes" according to the scheme: 3, 5, 1, 5, 3, 1, 3, 1. Each of these boxes is separated by only one number that meets the equation (6) and lies on the thick half-line. This makes it possible to immediately determine any large composite number.

4. Conclusions

Conclusion 1. All prime numbers (except 2, 3 and 5) lie on 96 thick half-lines and satisfy equation (6). This is a necessary condition for a natural number to be prime.

Conclusion 2. Natural numbers that do not lie on thick half-lines and do not satisfy dependence (6) are composite numbers. This is a sufficient condition for a given natural number to be a composite number.

Examples

Example 1. Take the number 7310033. It is a prime number and it lies on the half-lines marked with a thick line. The coordinates of its location are (-4399287.68; -5838051.93) (asterisk).

Example 2. Take the number 7310037. It does not lie on any half-lines marked with a thick line, which means that it is definitely a composite number. The coordinates of its location are (-3981331.5; -6130712.88) (bright square). The same conclusions are reached using dependence (6), which after transformation has the following form:

$$n = \frac{pn - pn_0}{30}. \qquad (7)$$

Thus, if a given number *pn* is prime, then there exists a number pn_0 in (5) for which the right-hand side of (7) is an integer. This is confirmed by the first example:

$$\frac{7310033 - 23}{30} = 243667. \qquad (8)$$

In the second example, i.e. for *pn*=7310037, there is no number pn_0 in (5) for which the right-hand side of (7) is an integer. So this number is a composite number.

Similar results are also obtained for all other natural numbers. Let's take some more examples and let them be the numbers 8751629 and 8751657. The first one is a prime number and lies on the fat line. Its location coordinates are (7654347.19; 4242873.93) (black square). The second number is not on the fat line. It is therefore a composite number and its location coordinates are (4766494.02; 7339757.15) (circle). In the first case, from equation (7) we get the value:

$$\frac{8751629-29}{30} = 291720. \qquad (9)$$

In the second case, there is no number pn_0 for which the right-hand side of (7) would be an integer (a composite number).

Final Comment

As shown above, the distribution of prime numbers has a unique order. Unlike, for example, Ulam's Spiral, all of them are located on 96 strictly defined half-lines, and all these numbers unambiguously satisfy the linear equation (6). Equally, this is based on only 8 numbers from the set (5). The presented method is much simpler and much faster, for example, than the Sieve Eratosthenes method [11]. It does not require any iteration or recursion. The result is obtained in one calculation step according to equation (6) or graphically, by determining the position of the number in the diagram in Fig. 1. Let the final conclusion be that the necessary condition for a given number to be a prime number is that it lies on one of the 96 half-lines and satisfies equation (6). On the other hand, a sufficient condition for a given number to be a composite number is that it does not lie on any thick half-lines and does not satisfy equation (6). In other words, if a given natural number is to be a prime number, it must lie on one of the thick half-lines and satisfy equation (6). However, if a given number does not lie on any thick half-lines, it means that it is definitely a composite number.

This whole article can be reduced to one sentence: if a given number lies on the thin half-line, then it is definitely a composite number and every prime number lies on the thick half-line. Or (which is equivalent): if a given number does not satisfy equation (6), then it is definitely a composite number and every prime number satisfies equation (6).

This work can also be found on the following open scientific social networking sites [12-16].

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