ISSN: 2639-0108

Research Article

Advances in Theoretical & Computational Physics

Ion-Acoustic Solitary Wave Solutions of Three-Dimensional Zakharov-Kuznetsov-Burgers Equation for Dust Ion Acoustic Waves and Their Applications

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Submitted: 01 Dec 2018; **Accepted**: 13 Dec 2018; **Published**: 02 Jan 2019

Abstract

Modified extended mapping method is further modified to discover traveling wave solutions of non- linear complex physical models, arising in various fields of applied sciences. The method is applied to three-dimensional Zakharov-Kuznetsov-Burgers equation in magnetized dusty plasma. Consequently different kinds of families of exact traveling wave solutions that represent electric field potential, electric and magnetic fields are fruitfully surveyed, with the help of Mathematica. The obtained novel exact traveling wave solutions are in different forms such as bright and dark solitary wave, periodic solitary wave, dark and bright soliton, etc., that are represented in the forms of trigonometric, hyperbolic, exponential and rational functions. The properties of some of the novel traveling wave solutions are shown by figures. The obtained results exhibit the effectiveness, power and exactness of the method that can be used for many other nonlinear problems.

Keywords: Modified Extended Mapping Method, 3-D Zakharov-Kuznetsov-Burgers Equation, Exact and Solitary Wave Solutions, Electric Field Potential, Electric and Magnetic Fields, Graphical Representation

Introduction

The mechanism and dynamics of many important phenomena in various fields of science such as physics, chemistry, biology, fluid dynamics, optical fibers, plasma and various other fields of engineering are designated by nonlinear partial or ordinary differential equations [1-6]. Exploration of the exact traveling wave solutions of these physical models provide better knowledge, uses and realize the mechanism. Various solutions of nonlinear evolution equations, associated with different mathematical methods, are very critical in the above mentioned fields. A number of techniques have been investigated to procure exact solutions of nonlinear physical problems. Some of the important methods are Modified extended mapping method [7], Backlund transformation [8], homogeneous balance method [9], tanh and extended-function method [10, 11], Exp-function method [12], modified simple equation method [13-15], direct algebraic method [16], modified extended tanh-function method [17, 18], and many others [19-22]. By means of techniques, various researchers formulate traveling wave solutions for different types of nonlinear partial differential equations [23-27].

To understand many space and astrophysical phenomena, and many industrial and physical applications, the study of the nonlinear wave phenomena in plasmas containing heavy dust particles is important. In the authors studied the behavior of the nonlinear waves in magnetized dusty plasma and derived Zakharov- Kuznetsov-Burgers (ZKB) equation in. Also the traveling wave solutions are discussed in detail. In this paper we further modify "modified extended mapping method" given in [7] to explore various families of exact traveling solutions of the nonlinear three-dimensional Zakharov-Kuznetsov-Burgers equation in.

Description of the method

Consider a non-linear physical model in the form a partial differential equation in (3+1) independent variables x, y, z and t as

$$U(\varphi, \varphi_t, \varphi_x, \varphi_y, \varphi_z, \varphi_{tx}, \varphi_{ty}, \ldots, \varphi_{xy}, \ldots, \varphi_{xx}, \ldots) = 0, \quad (1)$$

where U is a polynomial function in the dependent variable $\varphi(x, y, z, t)$ and its derivatives with respect to independent variables x, y, z and t. The main steps of the proposed method are the following.

Step 1: Using the traveling wave transformation $\varphi(x, y, z, t) = \varphi(\theta)$, with $\theta = kx + ly + mz - \omega t$, where k, l, m and ω are wave numbers and frequency, equation (1) reduces to ordinary differential equation

$$V(\varphi, \varphi', \varphi'', \ldots), \quad (2)$$

where V is a polynomial in $\varphi(\theta)$ and its derivatives containing linear and nonlinear terms.

Step 2: We seek for the solution of Eq. (2) in the following generalized ansätze

$$\varphi(\theta) = \sum_{j=0}^{n} a_j G^j(\theta) + \sum_{j=2}^{n} c_j G^{j-2}(\theta) G'(\theta) + \sum_{j=-1}^{-n} d_{-j} G^j(\theta) G'(\theta), \quad (3)$$

where a_j, c_j and d_j for $j = 1; 2\ldots, n$ are arbitrary constants and the values of $G(\theta)$ and $G'(\theta)$ satisfy

$$G'(\theta) = \sqrt{\beta_0 + \beta_1 G(\theta) + \beta_2 G^2(\theta) + \beta_3 G^3(\theta) + \beta_4 G^4(\theta)}, \quad (4)$$

where β_i's are constants to be determined such that $\beta_n \neq 0$. By choosing different values of β_j for ($j = 0, 1,\ldots,6$), this auxiliary equation has many kinds of solutions.

Step 3: Balancing the nonlinear term and the highest order derivative term in equation (2), give the value of the positive integer n in equation (3).

Step 4: Substituting equation (3) along with equation (4) into equation (2) and bring together all the coefficients of the same power $G^j(\theta)$ and put them equal to zero, we get a system of algebraic equations. The values of all the parameters and constants are obtained by Solving this system of algebraic equations.

Step 5: Substitute these values and $G(\theta)$ into equation (3), different kinds of solutions of equation (1) are obtained.

Application of the method to three dimensional Zakharov-Kuznetsov- Burgers equation

Consider the three dimensional Zakharov-Kuznetsov-Burgers equation

$$\varphi_t + a\varphi\varphi_x + b\varphi_{xxx} + c\varphi_{xyz} + c\varphi_{zzx} + d\varphi_{xx} = 0, \quad (5)$$

where a, b, c and d are arbitrary constants. Consider the traveling wave transformation $\varphi(x, y, z, t) = \varphi(\theta)$, $\theta = kx + ly + mz - \omega t$, where k, l, m and ω are numbers and frequency to be determined later, the ZKB equation becomes

$$-\omega\varphi' + ak\varphi\varphi' + dk^2\varphi'' + (bk^3 + ckl^2 + ckm^2)\,\varphi''' = 0 \quad (6)$$

Balancing the highest order derivative term φ''' and nonlinear term $\varphi\varphi'$ appearing in (6) we obtain ($n = 2$). Putting $n = 2$ in equation (3), we get the solution of equation (6) in the form

$$\varphi(\theta) = a_0 + a_1 G(\theta) + a_2 G^2(\theta) + c_2 G'(\theta) + \left(\frac{d_1}{G(\theta)} + \frac{d_2}{G^2(\theta)}\right) G'(\theta). \quad (7)$$

Following the step 4, substituting equation (7) into equation (6) and bring together all the terms having the same power $G^J(\theta)$, a system of algebraic equations is obtained. By solving this system of equations, different sets of values for the parameters a_j, c_j, d_j and for the numbers and frequency are obtained. Substituting these values in equation (7), different families of solutions of equation (6) are obtained as mentioned below.

Family 1: $\beta_0 = 0, \beta_1 = 0, \beta_4 = 0$:

Following the step 4, the following sets of parameters values are obtained.

Set 1:

$$a_1 = 0, a_2 = 0, c_2 = 0, d_1 = -\frac{dk}{a}, d_2 = 0, m = \pm\sqrt{\frac{-bk^2 - cl^2}{c}}, \omega = aa_0k.$$

Set 2:

$$a_1 = \pm\frac{3\beta_3 dk}{5a\sqrt{\beta_2}}, a_2 = 0, c_2 = 0, d_1 = -\frac{6dk}{5a}, d_2 = 0, m = \pm\sqrt{\frac{-\frac{5bk^2}{c} - \left(5l^2 \mp \frac{dk}{\sqrt{\beta_2}c}\right)}{5}}, \omega = aa_0k.$$

Substituting the first set of parameters values, along with two different values of G(θ) from equation (4), in equation (7), we obtain the following solution.

$$\varphi_{11} = a_0 - \frac{\sqrt{-\beta_2}dk\tan\left(\frac{1}{2}\sqrt{-\beta_2}\theta\right)}{a}, \quad \beta_2 < 0. \quad (8)$$

For the second set we have the following solution.

$$\varphi_{12} = \frac{5aa_0 + 3\sqrt{\beta_2}dk\left(\sinh\left(\sqrt{\beta_2}\theta\right) - 1\right)\operatorname{sech}^2\left(\frac{\sqrt{\beta_2}\theta}{2}\right)}{5a}, \quad \beta_2 > 0. \quad (9)$$

The electric and magnetic fields are governed by the position and motion of the electrons and positrons in a magnetized, electron-positron dusty plasma. The electric field $\vec{E}$ points from the high to low electric potential regions. The electric field generated by a collection of fixed charges is the gradient of the scalar potential or voltage φ, formulated as

$$\vec{E} = -\nabla\varphi = -\frac{\partial\varphi}{\partial x}\hat{x} - \frac{\partial\varphi}{\partial y}\hat{y} - \frac{\partial\varphi}{\partial z}\hat{z}. \quad (10)$$

Using the formula (10) the electric field of the electric potential φ_{11} is formulated as

$$\vec{E}_{11} = -\frac{\beta_2 dk\sec^2\left(\frac{1}{2}\sqrt{-\beta_2}\theta\right)}{2a}.\left(k\hat{x} + l\hat{y} + m\hat{z}\right). \quad (11)$$

According to the Faraday's law, the time-changing magnetic field Supplements a spatially changing, non-conservative electric field, and vice versa. The Maxwell-Faraday equation is given by

$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}. \quad (12)$$

Where the $\nabla\times$ is curl operator and $\vec{B}$ is the magnetic field. Using Maxwell-Faraday equation (12), the magnetic field for the electric potential φ_{11} is formulated as

$$\vec{B}_{11} = \frac{\beta_2 dk(k-m)\sec^2\left(\frac{1}{2}\sqrt{-\beta_2}\theta\right)}{2a\omega}.\left(-l\hat{x} + \hat{y}(k+m) - l\hat{z}\right). \quad (13)$$

The graphical representations of the electric potential φ_{11} and its electric and magnetic fields are shown in the figure below.

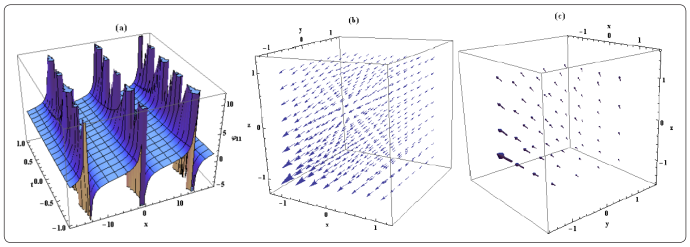

Figure 1: (a) is the periodic solitary wave solution Φ_{11}, (b) is its electric field $\vec{E}_{11}$ and (c) is the magnetic field B_{11}.

Family 2: $\beta_0 = 0, \beta_1 = 0, \beta_2 = 0$: In this family we have the following **set 1**.

$$a_1 = -\frac{\sqrt{\beta_4}dk}{a}, a_2 = 0, c_2 = 0, d_1 = -\frac{dk}{a}, d_2 = 0, m = -\frac{\sqrt{-bk^2-cl^2}}{\sqrt{c}}, \omega = aa_0k.$$

The only solution related to this set of parameters values is

$$\varphi_{21} = \frac{aa_0\beta_3\theta - 2aa_0\sqrt{\beta_4} + 2\beta_3 dk}{a\beta_3\theta - 2a\sqrt{\beta_4}}, \quad \beta_4 > 0. \tag{14}$$

Using formulas (10) and (12) the electric and magnetic fields of the electric potential φ_{21} are formulated as

$$\vec{E}_{21} = \frac{2\beta_3^2 dk}{a\left(\beta_3\theta - 2\sqrt{\beta_4}\right)^2}.\left(k\hat{x} + l\hat{y} + m\hat{z}\right). \tag{15}$$

$$\vec{B}_{21} = \frac{2\beta_3^2 dk(k-m)}{a\omega\left(\beta_3\theta - 2\sqrt{\beta_4}\right)^2}.\left(-\hat{y}(k+m) + l\hat{x} + l\hat{z}\right). \tag{16}$$

Graphically the electric potential φ_{21} and its electric field and magnetic field are shown in the figure below.

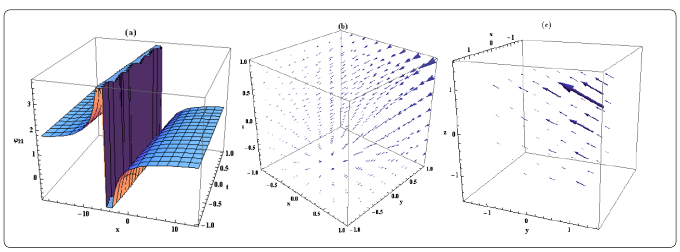

Figure 2: (a) is the soliton-like solution φ_{21}, (b) is the electric field $\vec{E}_{21}$ and (c) is it's magnetic field $\vec{B}_{21}$.

Family 3: $\beta_0 = 0, \beta_1 = 0, \beta_2 = 0$: In this family we have three sets of parameters values.

Set 1:

$$a_1 = \pm\frac{\sqrt{\beta_4}dk}{a}, a_2 = 0, c_2 = 0, d_1 = -\frac{dk}{a}, d_2 = 0, m = \pm\frac{\sqrt{-bk^2-cl^2}}{\sqrt{c}}, \omega = aa_0k.$$

Set 2:

$$a_1 = 0, a_2 = \frac{6\beta_4 dk}{5a\sqrt{\beta_2}}, b_1 = 0, b_2 = 0, c_2 = 0, d_1 = -\frac{12dk}{5a}, d_2 = 0, m = \pm\frac{\sqrt{-\frac{10bk^2}{c} - \frac{dk}{\sqrt{\beta_2}c} - 10l^2}}{\sqrt{10}}, \omega = aa_0k.$$

Set 3:

$$a_1 = \pm\frac{6\sqrt{\beta_4}dk}{5a}, a_2 = \pm\frac{6\beta_4 dk}{5a\sqrt{\beta_2}}, c_2 = \pm\frac{6\sqrt{\beta_4}dk}{5a\sqrt{\beta_2}}, d_1 = -\frac{6dk}{5a}, d_2 = 0, m = \pm\frac{\sqrt{-\frac{5bk^2}{c} + \frac{dk}{\sqrt{\beta_2}c} - 5l^2}}{\sqrt{5}}, \omega = aa_0k.$$

For the first set there is only one solution.

$$\varphi_{31} = \frac{aa_0 - \sqrt{-\beta_2}dk\tan\left(\sqrt{-\beta_2}\theta\right) + \sqrt{-\frac{\beta_2}{\beta_4}}\sqrt{\beta_4}dk\sec\left(\sqrt{-\beta_2}\theta\right)}{a}, \quad \beta_2 < 0, \beta_4 > 0. \tag{17}$$

The following is the solution related to the second set.

$$\varphi_{32} = \frac{5aa_0 + 6\sqrt{\beta_2}dk\left(\sinh\left(2\sqrt{\beta_2}\theta\right) - 1\right)\operatorname{sech}^2\left(\sqrt{\beta_2}\theta\right)}{5a}, \quad \beta_2 > 0, \beta_4 < 0. \tag{18}$$

Now, let's formulate the electric and magnetic field of the electric potential φ_{32} and show them graphically.

$$\vec{E}_{32} = -\frac{12\beta_2 dk\left(\tanh\left(\sqrt{\beta_2}\theta\right) + 1\right)\operatorname{sech}^2\left(\sqrt{\beta_2}\theta\right)}{5a}.\left(k\hat{x} + l\hat{y} + m\hat{z}\right). \tag{19}$$

$$\vec{B}_{32} = \frac{12\beta_2 dk(k-m)\tanh\left(\sqrt{\beta_2}\theta\right)\left(\tanh^2\left(\sqrt{\beta_2}\theta\right) + \tanh\left(\sqrt{\beta_2}\theta\right) - 1\right)}{5a\omega}.\left(-\hat{y}(k+m) + l\hat{x} + l\hat{z}\right). \tag{20}$$

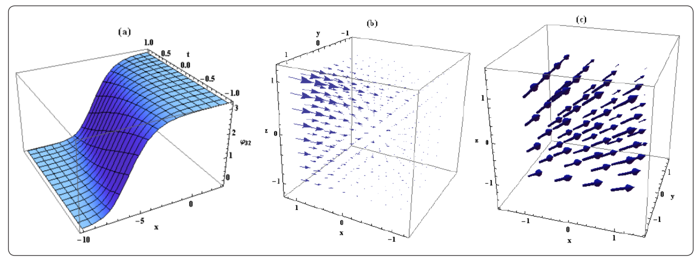

Figure 3: (a) is the soliton-like solution φ_{32}, (b) is the electric field E_{32} and (c) is the related magnetic field B_{32}.

Family 4: $\beta_0 = 0, \beta_1 = 0$

The following are the sets of parameters values.

Set 1:

$$a_1 = \pm\frac{\sqrt{\beta_4}dk}{a}, a_2 = 0, c_2 = 0, d_1 = -\frac{dk}{a}, d_2 = 0, m = \pm\frac{\sqrt{-bk^2-cl^2}}{\sqrt{c}}, \omega = aa_0k.$$

Set 2:

$$a_1 = \pm\frac{3dk\left(\frac{\beta_3}{\sqrt{\beta_2}} \pm 2\sqrt{\beta_4}\right)}{5a}, a_2 = \pm\frac{6\beta_4 dk}{5a\sqrt{\beta_2}}, c_2 = \pm\frac{6\sqrt{\beta_4}dk}{5a\sqrt{\beta_2}}, d_1 = -\frac{6dk}{5a}, d_2 = 0,$$

$$m = \pm\frac{\sqrt{-\frac{5bk^2}{c} - \frac{dk}{\sqrt{\beta_2 c}} - 5l^2}}{\sqrt{5}}, \omega = aa_0k.$$

Consider the second set of parameters related to which has six solutions in which one of them is mentioned below. The other five solutions can be formulated by the same way.

$$\begin{aligned}\varphi_{41} =& \frac{\operatorname{sech}^2\left(\sqrt{\beta_2}\theta\right)}{10a\left(\sqrt{\delta_2} - \beta_4\operatorname{sech}\left(\sqrt{\beta_2}\theta\right)\right)^2}\Bigg(10aa_0\left(\beta_4 - \sqrt{\delta_2}\cosh\left(\sqrt{\beta_2}\theta\right)\right)^2 \\ &- 24\sqrt{\beta_4}\beta_2 dk\left(\sqrt{\delta_2}\left(\sinh\left(\sqrt{\beta_2}\theta\right) - \cosh\left(\sqrt{\beta_2}\theta\right)\right) + \beta_4\right) + \\ &12\sqrt{\beta_2}dk\left(\beta_3 - \sqrt{\delta_2}\sinh\left(\sqrt{\beta_2}\theta\right)\right)\left(\beta_4 - \sqrt{\delta_2}\cosh\left(\sqrt{\beta_2}\theta\right)\right) \cdot \\ &- 48\beta_4\beta_2^{3/2}dk\Bigg), \quad \delta_2 > 0.\end{aligned} \tag{21}$$

The electric field and magnetic field of the electric potential φ_{41} are formulated below and are shown graphically.

$$\vec{E_{41}} = \frac{6\beta_2 d\sqrt{\delta_2} k \operatorname{sech}^3\left(\sqrt{\beta_2}\theta\right)}{5a\left(\beta_4 \operatorname{sech}\left(\sqrt{\beta_2}\theta\right) - \sqrt{\delta_2}\right)^3} \left(\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{\delta_2}\left(-\sinh\left(2\sqrt{\beta_2}\theta\right) + \right.\right.\right.$$

$$\left.\cosh\left(2\sqrt{\beta_2}\theta\right) - 3\right) + 2\beta_4\left(\sinh\left(\sqrt{\beta_2}\theta\right) + \cosh\left(\sqrt{\beta_2}\theta\right)\right)\Big) +$$

$$\left(\sqrt{\delta_2}\cosh\left(\sqrt{\beta_2}\theta\right) - \beta_4\right)\left(\beta_3 \sinh\left(\sqrt{\beta_2}\theta\right) - \beta_4 \cosh\left(\sqrt{\beta_2}\theta\right) + \sqrt{\delta_2}\right)$$

$$+ 8\beta_2\beta_4 \sinh\left(\sqrt{\beta_2}\theta\right)\Big).k\hat{x} + l\hat{y} + m\hat{z}. \quad (22)$$

$$\vec{B_{41}} = \frac{6\beta_2 d\sqrt{\delta_2} k(k-m)}{\left(5a\omega\left(\beta_4 - \sqrt{\delta_2}\cosh\left(\sqrt{\beta_2}\theta\right)\right)^3\right)} \left(\sqrt{\beta_2}\sqrt{\beta_4}\left(\sqrt{\delta_2}\left(-\sinh\left(2\sqrt{\beta_2}\theta\right) + \right.\right.\right.$$

$$\left.\cosh\left(2\sqrt{\beta_2}\theta\right) - 3\right) + 2\beta_4\left(\sinh\left(\sqrt{\beta_2}\theta\right) + \cosh\left(\sqrt{\beta_2}\theta\right)\right)\Big) +$$

$$\left(\sqrt{\delta_2}\cosh\left(\sqrt{\beta_2}\theta\right) - \beta_4\right)\left(\beta_3 \sinh\left(\sqrt{\beta_2}\theta\right) - \beta_4 \cosh\left(\sqrt{\beta_2}\theta\right) + \sqrt{\delta_2}\right) +$$

$$8\beta_2\beta_4 \sinh\left(\sqrt{\beta_2}\theta\right)\left(-\hat{y}(k+m) + l\hat{x} + l\hat{z}\right).$$

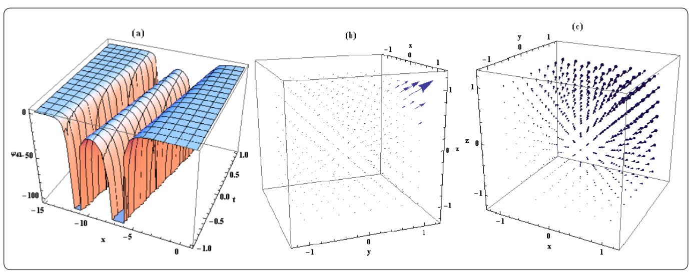

Figure 4: (a) is the soliton-like solution φ_{41}, (b) is the electric field $\vec{E_{41}}$ and (c) is the related magnetic field $\vec{B_{41}}$.

Results and conclusion

We further modified the modied extended mapping method to discover various kinds of families of traveling wave solutions of three-dimensional Zakharov-Kuznetsov-Burgers equation in a magnetized dusty plasma. The obtained solitary wave solutions are in the form of hyperbolic, exponential, trigonometric and rational functions in which some of them are expressed graphically. Electric fields potential, electric and magnetic fields are also formulated. These solutions can help to better understand the mechanism and provide help to study and comprehend the physical interpretation of the system.

The solutions found in this paper are different from solutions obtained by the researchers using different methods because our proposed method has a different structure from the other methods and has different types of parameters. Choosing different values of d_i's ($i = 0; 1; 2; 3; 4$), equation (4) has many types of special solutions in different forms like trigonometric, hyperbolic, exponential and rational functions. Moreover, the derivative term G' involving in the supposed solution and the simplied form of the solutions make the solutions more different. However some of our solutions have similarities with the solutions obtained before. We showed the results graphically in three-dimension for slight different parameters values, for example one set of parameters values is: $\alpha = 0{:}9$, $\beta_2 = 1{:}2$, $\beta_3 = 2$, $\beta_4 = 0{:}7$, $h_1 = 1{:}5$, $k = 1{:}2$, $\epsilon = 1$. The two-dimentional and contour plots of these graphs are also mentioned which makes the results more clear to understand.

These results show the reliability and effectiveness of the method. Many higher-order nonlinear equations arising in Plasma, Mathematical Physics, Hydrodynamics, Engineering and other areas of applied sciences can also be solved by this in influential, reliable and capable method.

Acknowledgment

This work was partially supported by NSF of China (Grants 11571140, 11371090, 11701228), NSF for Outstanding Young Scholars of Jiangsu Province (BK20160063), NSF of Jiangsu Province (BK20150478), the six big talent peaks project in Jiangsu Province(XYDXX-015).

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