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Inventory Routing Problem: A Problem that is Still Relevant Today

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Abstract

The most important challenge of a supply chain is the control of its logistics costs. The managers who drive the various logistics flows tend to reduce its overall costs in order to achieve overall profits. Among the problems that help them achieve their goals is the inventory routing problem (IRP). In its classical definition, IRP is an integration tool that can provide a joint answer to inventory management and vehicle routing problems. In this review article, we present an overview of supply chain management with a focus on the inventory routing domain. We will start by introducing the concept of IRP and the various works done in this area since its emergence that have contributed to its development. Then, we will discuss recent research and take the most studied model to expose its impact on the development of this research area until today. This paper shows that despite the age of this tool, this field of research has maintained its importance and continues to evolve and produce considerable research articles.

Keywords: Inventory Routing Problem, Exact Methods, Meta-Heuristic, Matheuristic, Replenishment Policies, Optimization.

1. Introduction

Supply chain management is a developing discipline because of its direct impact on business growth. Good supply chain management is a competitive advantage for these companies. They attribute several logistics practices to managing their supply chain network, such as controlling holding and transportation costs. Indeed, logistics can provide a better answer to analyze the different flows (of movement and information) between the different parts of the supply chain and reduce the costs involved. However, managing these costs separately creates problems. Therefore, the implementation of a collaborative system is necessary to achieve a global optimization of the supply chain.

Several resolution models are created to meet the organizational needs of companies. In this domain, the Inventory Routing Problem (IRP) represents a key solution because it is designed to jointly address several logistics problems. In its classic definition, IRP is an integrative tool capable of providing an answer to inventory management and vehicle routing problems. It is a question of rethinking in a combined way two activities of the logistic chain. Moreover, this problem is a variant of the routing problem. It is highly coveted by researchers in transport and logistics because of its impact on the development of industrial economies.

IRP is a decision support tool for the supply planner to provide an optimal distribution plan over a predefined time horizon. Its objective is to minimize the cost of holding inventory at different locations in the network and the total distance traveled over a time horizon. IRP addresses the following issues: inventory management for each customer and supplier, assignment of customers to delivery periods, definition of quantities to be delivered in each period to avoid stock-outs, route design and optimization. Nowadays, IRP has become a very common problem studied by researchers because of its benefits in economic life.

In 1983, [1] gave a first definition of IRP. They considered IRP as a process of routing a single product from a supplier to a set of customers with an infinite and homogeneous fleet of vehicles. Initially, the vehicles are located at the supplier, and the routing of the vehicles is organized over several periods. The consumption of each product by each customer occurs at a constant (deterministic) rate over time. The capacity of the supplier's warehouse is greater than that of the customer's storage sites. Next, IRP was introduced by who defined this problem as an extension of the classical Vehicle Routing Problem (VRP) [2]. They presented the IRP as a set of vehicle routes with minimum cost, starting and ending the tour at the same supplier, while satisfying capacity constraints and customer requirements with simultaneous decision making.

Since its emergence, many authors have invested in the research that concerns IRP due to its logistical advantages in a supply network such as [3,4,5]. Existing studies in this area focus on developing variants of the simplistic routing model to most closely approximate real models, as well as solution methods by developing and applying exact or approximate techniques to efficiently solve these logistic problems. In this work, we present an overview of supply chain management focusing on the area of inventory routing. A bibliographical study of the works which concern the IRP especially since the establishment of a model qualified of complete by most of the researchers will be exposed in this article.

The article is organized as follows. The section 2 is devoted to the presentation of works that have impacted and developed the IRP concept in the literature and present the best converged model. In the section 3, we will present the investigations established for the resolution of the reference model as well as the new variants that have emerged from this model. The last section 4 represents the conclusion of the paper.

2. Overview of the Inventory Routing Problem

Since its introduction, IRP has become a topic of interest for researchers in transportation and logistics. In the literature, this problem has been studied by several authors. In this review, we will present the works that have marked this research axis and that have the highest number of citations in the literature according to Google Scholar. The figure below shows the most cited research articles in IRP. The blue bars represent the number of citations of each article.

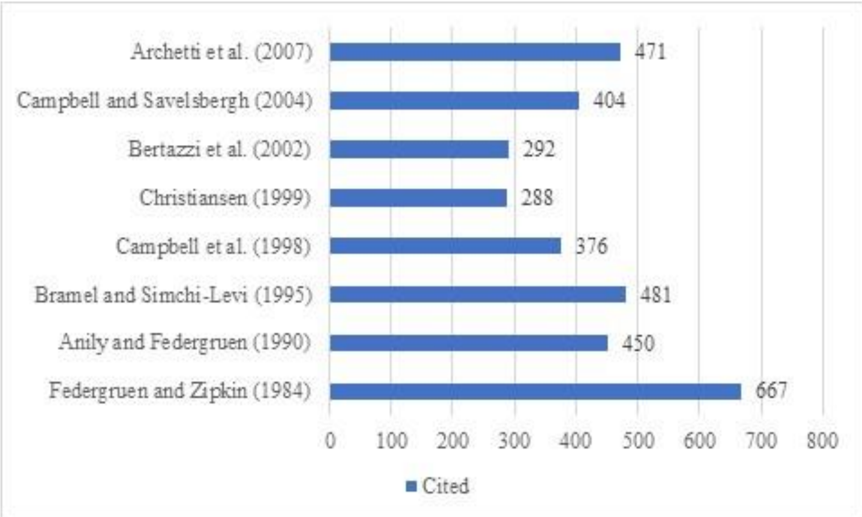

Figure 1: The most cited research articles in IRP.

As mentioned in the introduction, was the first year to announce the first IRP model and their work served as a basis for other researchers to develop this problem [2]. The classic IRP model was extended by [7]. The authors introduced the cost of transporting vehicles into the inventory routing model. Their objective is to determine feasible replenishment strategies that minimize average transportation and inventory costs over an infinite planning horizon. They analyzed fixed partition policies for single-product IRP with an unlimited number of vehicles. Customers are partitioned by region, and each region is served by a vehicle whose capacity could meet the deterministic and independent demand of customers in the region. Customers can belong to multiple clusters, and the visit of a customer in a region implies the visit of the rest of the customers in that region. The authors set lower and upper bounds on the long-run minimum average cost for all fixed partition policies. They proposed the first clustering algorithm for IRP to solve their problem and from their experimental studies they showed the good performance of their method.

Bramel and Simchi-Levi studied an inventory routing problem that considered depot location and vehicle scheduling [8]. Their main objective was to minimize the total distance from customers to their nearest seed, while ensuring that the total demand assigned to a hub does not exceed Q. They used an incremental heuristic in two phases. First, the seeds are determined by solving a capacitive location problem. Then, the remaining vertices are incrementally included in their assigned route. Then, the vehicle routes are constructed by inserting the client assigned to that route seed with the least insertion cost at each step. The authors showed that the algorithm is asymptotically optimal, but its empirical performance is not competitive with other methods in the literature.

Campbell et al invested in an inventory routing problem that dealt with replenishing a set of customers from a single facility with a single product over a given planning period [9]. The customers consume the product at a constant rate and can maintain a local inventory of the product. The objective was to minimize distribution costs during the planning period while avoiding customer stock-outs, as well as to determine the optimal quantities to deliver to each customer on a delivery route. A two-phase heuristic, based on a linear programming model, was proposed to solve this problem. In which, the exact visit period and the quantity to be delivered to each customer were calculated. Then, the customers were included in the vehicle routes.

Christiansen tackled a real-world inventory routing problem in the maritime context with a time window [3]. The objectives addressed in his model are the scheduling of visits by each ship and the management of inventory for each port. The model consists of the periodic distribution of a single product, ammonia, by ships calling at different ports to supply plants around the world owned by a given company. The time windows for the start of service and the range of load quantities achievable at each call are also considered. The author used a DantzigWolfe decomposition approach to solve his main problem while using Dantzig-Wolfe column generation and Branch-and-Bound (B&B) approaches for the subproblems. The computational results indicated that the proposed method is capable of efficiently solving the real planning problem.

In their paper, focused on a single-vehicle IRP with a deterministic demand and a periodic planning horizon [4]. They introduced the Order-up-to Level (OU) policy simplifying the set of possible decisions in the problem. The problem was solved using a two-step heuristic. The first step was to create a feasible solution and the second step was to try to improve this solution by minimizing the total cost function that took into account transportation and storage costs at the customer and supplier. Their method solved the problem quickly and efficiently.

Campbell and Savelsbergh studied a periodic IRP with deterministic customer demand under the constraint that no customer shortages are allowed [5]. They developed a two-phase approach that decomposes the decision set, so that a replenishment schedule is first created using an integer programming method, followed by the construction of a set of delivery routes using routing and scheduling heuristics. They used large instances drawn from the real world for their tests. Computational experiments demonstrated the effectiveness and good potential of their optimization approach.

The last work most cited in the literature and which has kept a positive impact on the development of this research axis until today is that of developed a very simple and efficient model to solve a basic IRP problem [6]. Their model consists of a single-product IRP where a supplier must replenish a set of customers with deterministic and independent demand over a periodic time horizon by a single vehicle. Two replenishment policies were considered, the OU policy and the maximum level (ML) policy.

The objective of model was to assign customers to specific replenishment periods, find the appropriate quantities to transfer from the supplier to the customers, and determine the best delivery routes [6]. The authors' approach to shared management of customer inventories was the Vendor Managed Inventory (VMI) system, in which the supplier manages customer inventories, decides on appropriate inventory levels and the timing of customer replenishment. The authors established an efficient mathematical formulation and proposed an exact Branch-and-Cut (B&C) algorithm to solve it. They also randomly generated sets of benchmark instances to test their model, and the optimal solution of these instances was revealed.

The work of has been a reference for several authors who have invested in solving the same model by different approaches, or who have developed new variants of IRP based on the formulation proposed in or who have simply used their instances to test other IRP models [6]. Even today, the study of is of interest to researchers, which motivated us to make a review of the work of and to write a survey that takes up most of the investigations made from their work [6]. This review will be the subject of the following sections.

3. Analysis of Literature

Archetti et al. studied a simple supply network structure consisting of two echelons [6]. At the top echelon is the supplier who is responsible for supplying all customers with a single vehicle with a single type of product. The customers are at the bottom echelon and have deterministic, independent, and periodic demand. Their IRP can be defined as a mixed integer linear problem (MILP), in which the supplier must make simultaneous decisions to optimally organize inventory levels and inventory transfer from their site to the selected customers under the VMI strategy. Two replenishment policies were used, namely OU and ML.

The main objective of their IRP is to optimize the costs of transport and holding the total stock in the different nodes of the supply chain network. The implemented model deals essentially with the following issues: inventory management of customers and suppliers, avoiding stock-outs for customers, determining the quantities of stock to be delivered at each period, assignment of customers to each delivery period, design, and optimization of vehicle routes. Two different sets of instances were used, the first with a low holding cost and the second with a high holding cost. They developed an exact algorithm (B&C) to obtain optimal results.

This section presents a state of the art of the works developed by researchers representing a continuity to work [6]. We will start with the works that have invested in the production or the use of different resolution methods for the same model. Then, we will discuss the works developed that were inspired by model in order to make new IRP models and variants. Finally, we will present papers that have used the instances developed by in other models [6].

3.1 Resolution Methods for the Referenced Model

In this subsection, we will expose the different works that have proposed resolution methods based on the IRP model of shown their advantages of use [6].

A hybrid heuristic called Hybrid Approach to Inventory Routing (HAIR) was introduced by to solve the IRP proposed by [6,14]. This heuristic uses MILP models embedded in a tabu search scheme to intensify the search in certain promising parts of the solution space. This Meta-heuristic was first tested on the benchmarks of to show that it produces results within the range of accepted solutions established by [6]. The effectiveness of their method was proven on sets of instances with known optimal solutions. They showed that HAIR can find optimal solutions for most small instances and, for the rest of the instances, to give results with very low average error rates. Furthermore, the use of the MILP-based intensification phase is computationally efficient. Indeed, this has been demonstrated by comparing the performance of the Meta-heuristic with and without the intensification phase. HAIR allowed to obtain the optimal solution, on the selected instances. Then, they used HAIR to solve two larger classes of instances with up to 200 clients over a six-period horizon. The results they found became a basis for subsequent research.

Bertazzi and Speranza reviewed the main heuristic approaches for solving IRPs [15]. They presented some interesting ideas for designing a new Meta-heuristic, resorting to the use of mathematical programming models, typically MILPs, inside a heuristic. They presented different ways to incorporate MILPs into a heuristic scheme. They tested their Meta-heuristics on instances of for the case of a single vehicle IRP, compared their results with the optimal solution of computed the average error rates of their results [9]. They designed a new Metaheuristic, which was used to solve the basic IRP using this time the instances of where a tabu search heuristic included, as an improvement step, the solution of two different MILP models [6,14]. The MILP models were run each time a best new solution was obtained by the tabu search. The computational results showed that this hybrid approach was more efficient than the conventional tabu search. Although the IRP models were NP-hard, CPLEX was able to solve the models to optimality in a short computational time for instances up to 200 clients.

Coelho et al. developed a Meta-heuristic called Adaptive Large Neighbourhood Search (ALNS) enhanced by the exact solution of two types of MILPs to initially solve the problem [6,14]. The first is a network flow model used to compute the delivery quantities associated with a given set of routes. The second provides an approximation of the cost of a new solution obtained by applying vertex deletions and re-insertions to a given solution. These requirements are used not only to provide cost-effective solutions to their customers, but also to provide a high-quality service to gain a competitive advantage. To evaluate the performance of their algorithm, they used a multitude of benchmarks among which are the instances of starting with the use of a single vehicle and then multiple vehicles serving the same network structure [9]. First, they compared their computational solutions to the exact results of [6]. Second, they formulated an IRP with a consistency requirement constraint in the form of MILP. They considered six different consistency features in the IRP solutions, namely quantity consistency, vehicle fill rate, OU policy, driver consistency, partial driver consistency, and visit spacing. They also analysed the effect of different replenishment policies, routing decisions, and delivery sizes.

In a supplementary work, Coelho and Laporte provided a formal statement of the IRP problem as well as an exact MIL formulation [11]. They started by presenting the existing inequalities. Then, they introduced new classes of valid inequalities based on the relationship between demand and available capacities, for the single-vehicle IRP based on the model which are then extended to the multi-vehicle case [6]. They outlined the notion of input order for the IRP. They also showed how the order of the inputs could have a major effect on the linear relaxation of the proposed IRP model. The authors solved their problem with the exact B&C algorithm. Finally, they analysed the impact of the change of the order of the input data on the value of the linear relaxation, and thus, on the value of the best lower bound obtained after a given computation time. They have shown how these first two contributions lead to improved lower bounds. They tested their algorithm on the instances to provide new good quality solutions for large IRP benchmark instances [6,14].

Archetti et al. presented and compared the extensional formulations for IRP [6,17,18]. They studied the same objective, which is the minimization of the sum of inventory holding and transportation costs. They thoroughly analysed different formulations of the multi-vehicle IRP, with valid inequalities from previous studies in the same research area, and systematically tested their performance. They examined and compared these formulations and proposed a new one. Computational tests were performed on the instances of by the B&C algorithm [6]. Based on their tests, they selected the best formulation for the ML and OU policies. They also tested the effectiveness of Fractional Capacity Cut (FCCC) and Subtour Elimination (SEC) constraints for all formulations. They imposed the maximum CPU for each test run and each run was performed on a single thread.

Adding cuts to the formulations expanded the models and thus degraded the performance of the CPLEX MIP solver. The most significant findings were that the formulations that used vehicle-indexed variables were superior to the more compact aggregate formulations and reduced the variance at the root node of the B&C.

Desaulniers et al. introduced an innovative formulation for IRP that specifies in which time periods the delivered quantity should be consumed [19]. They developed a Branch-Price-and-Cut algorithm (B&P&C) that incorporates known and new families of valid inequalities, an ad hoc labelling algorithm for solving column generation sub-problems, and several speedup techniques. They proposed an adaptation of capacity inequalities that proved to be a very effective component of their algorithm. To evaluate their algorithm, they used instances in the single vehicle case and then in the multiple vehicle case (between two and five vehicles) [6]. Optimal solutions were only known for a limited number of instances solved with B&C algorithms. They compared the results of their algorithm with those of the B&C algorithm of [6,11]. They proved that the proposed valid inequalities, branching decisions, and other speedup strategies were effective and, in most cases, necessary to solve some instances. Computational results demonstrated the outperformance of their algorithm over existing exact algorithms for instances with more than three vehicles. Their formulation provided much smaller completeness gaps. And finally, they concluded that their B&P&C algorithm clearly outperforms the B&C algorithm on instances with four and five vehicles.

Franco and García presented a column generation algorithm for solving the single-vehicle IRP designed by [9,20]. To solve it, they divided the problem into two steps. The first step was a strategy to find feasible routes and the second step was an optimization step, it aimed to find the optimal routes and optimal delivery quantities. The objective was to find a subset of possible routes by using dual information to find an optimal solution of each sub-problem that is feasible routes. The sub-problem is then formulated as a Shortest Path Problem (SPP) where each node represents a customer, and the depot is represented by the initial and final node. In the second step, the solution provided is designated as the set of best routes of the sub-problems after solving them all as SPPs. They used the dual information to solve the sub-problem using Liner Programming (LP) relaxation of the problem. They used an exact method called the Pulse algorithm to solve the constrained SPP. The algorithm involves sending pulses into the network and using pruning strategies to prevent the pulses from continuing to propagate through the network. If the pulse reaches the end of the network, it contains all the information for a feasible path. The impulse algorithm uses four types of pruning strategies: by cycles, by dominance, by infeasibility and by boundaries. Once they found a subset of feasible routes, the mixed integer problem was solved given the inventory policies. The proposed algorithm was tested on the instances of [6]. Their program resulted in better solutions for all instances at a competitive computation time [6]. The authors proved also that if the OU policy is relaxed, their algorithm can find better solutions than those of the classical OU policy.

Archetti et al. considered the same supply network proposed by a multi-vehicle fleet case [6,21]. They implemented a meta-heuristic, called Model Relaxations, Tabu Search, And Restrictions (MORTAR) that combines a Tabu Search heuristic and mathematical programming formulations. Their problem formulation is the same as that given in with vehicle index and constraints for symmetry breaking [17]. MORTAR was tested on the instances of [6,14]. On the instances of they compared the results of MORTAR with the following three algorithms: the B&C algorithm proposed by the B&P&C algorithm and ALNS developed by [6,11,20,22]. On the instances of they compared MORTAR only with the upper bound found by the B&C algorithm of because ALNS and the B&P&C algorithm were not tested on these instances [14,11]. Preliminary tests were performed to find the most reliable parameter for MORTAR. The results of these preliminary tests provided high quality solutions. For instances where optimal solutions were available, MORTAR achieved the optimum for many instances, and the average percentage error is less than 1%. For instances where no optimal solutions were available, MORTAR substantially improves on the best known upper bound in most instances.

Amri-Sakhri studied the multi-period IRP model of [6,23]. He chose the genetic algorithm to solve his problem and tested in addition to the classical genetic operators of two-point crossover and random mutation, several crossover structures in order to detect the structure that generates a better result with the least execution time. The genetic crossover operators he used are: Flip, Swap, Slid, Sort and Permutation. He used the benchmarks for his experiments and finally found that the best structure to generate good quality results in less time and improve GA performance is Permutation [6,14].

Recently, used the model in a study of a two-echelon multi-site supply network [14,24]. Used two metaheuristics to solve their problem, the first being the classical genetic algorithm (GA) which used the order crossover operator (OX) and random mutation as genetic operators and the second a hybrid meta-heuristic called the memetic algorithm (MA) which took advantage of the benefits of variable neighborhood search (VNS) in GA as a mutation operator and uses the same OX two-point crossover structure [24]. The proposed algorithms were tested on benchmarks of only in the case of the OR policy and for low holding costs [6,14]. Most of the results of MA were optimal solutions for small instances, it improved the quality of solutions for instances of that have not been solved optimally so far [6,14]. The computational results showed that MA is better suited than GA to solve the proposed instances and very competitive with other methods proposed in the literature in terms of solution quality.

Table 1 summarizes the different works mentioned in this subsection and identifies the resolution methods used by each study to solve the problem [6]. From this subsection, we have realized the importance of IRP model, which is a reference for researchers in this research area. We are not going to stop at this level. Indeed, using the model of as a basis for creating new variants of IRPs is still relevant and topical [6]. We will discuss this topic in the next subsection.

Reference	Applied Method
Archetti et al. [14]	Hybrid Approach to Inventory Routing (HAIR)
Bertazzi and Speranza [15]	Tabu Search included two MILP models (MILP-TS)
Coelho et al. [16]	Adaptive Large Neighbourhood Search (ALNS)
Coelho and Laporte [11]	Branch-and-Cut algorithm (B&C)
Archetti et al. [17]	Branch-and-Cut algorithm (B&C)
Desaulniers et al. [19]	Branch-Price-and-Cut algorithm (B&P&C)
Franco and García [20]	Pulse algorithm (PA)
Archetti et al. [21]	Model Relaxations, Tabu Search, And Restrictions (MORTAR)
Amri-Sakhri [23]	Genetic Algorithm (GA)
Amri-Sakhri et al. [24]	Memetic Algorithm (MA)

Table 1: Investigations in the Archetti et al. (2007) problem.

3.2 The Variants Developed from the Reference Model

Based on the problem of many complex IRP variants have been designed by researchers up to now [6]. Figure 2 schematizes the classes of IRPs studied in this subsection, explicitly indicating for each class the optimization problem that arises in combination with the routing problem initially proposed by [6]. In the following paragraphs, we will review the different works that have developed its variants.

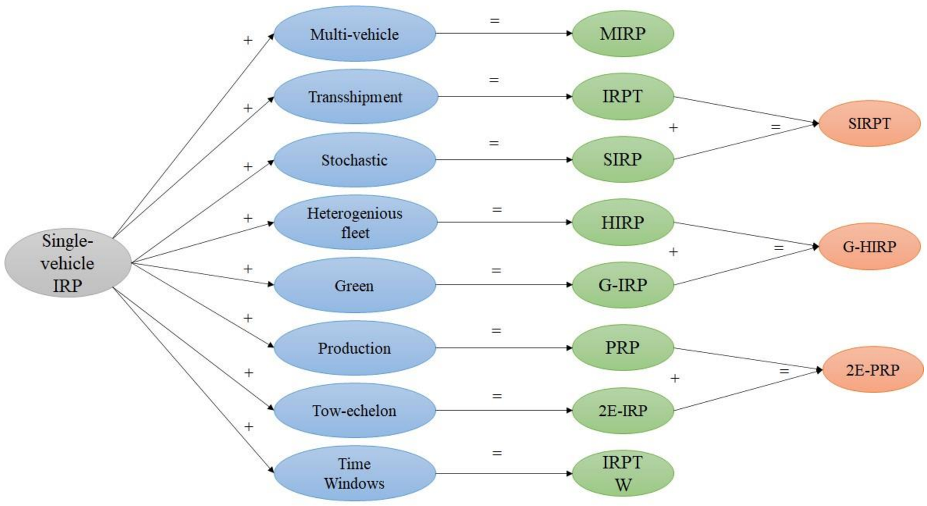

Figure 2: A representation of the classes of IRPs covered in this subsection.

Coelho et al. proposed an Inventory Routing Problem with Transshipment (IRPT), in which transshipment can take place either from the supplier to the customers or between the customers [25]. The transshipment process was performed by outsourcing an external vehicle other than the supplier's single vehicle. The model was modified to fit this new problem [6]. An exact algorithm and an ALNS heuristic were applied to solve this problem. In their model, the quantity transferred was studied in both policy cases: OU and ML. Transshipment was included in the stock transfer in each period if it was necessary and aimed at reducing the total network costs. To test the performance of both algorithms, they used the benchmarks of [6]. The test results showed that their heuristics could produce high quality solutions in reasonable computation times, and that the use of transshipment could reduce the solution cost significantly.

Coelho and Laporte solved several classes of IRPs such as the Multi-vehicle Inventory Routing Problem (MIRP) with homogeneous and heterogeneous fleets, they also considered the transshipment option and introduced consistency constraints [26]. They presented a unified model inspired by implemented a B&C algorithm capable of solving all the above-mentioned classes of IRPs [6]. They opted for the edge formulation because it required far fewer variables, which became a relevant problem for large instances. The tests showed the out-performance of the primal simplex method. To evaluate the performance of the algorithm, they used the single vehicle instance set proposed by evaluated the algorithms for the single vehicle IRP and then in the MIRP case [6]. The computational results confirmed the success of the proposed algorithms.

In another work, Coelho and Laporte studied the same IRP structure introduced by under OU and ML policies, and proposed a tactical policy called Optimized Target Level (OTL) [6,27]. This inventory replenishment policy is such that when the supplier visits a customer, the quantity delivered was such that the final inventory could always be at the same OTL depending on the customer. To implement their models, they used the B&C algorithm applied on the instances of [6]. They also showed the advantages of the OTL policy over the OU policy in terms of increased computation time and confirmed the managerial interest of using this policy both in terms of cost reduction and inventory levels.

Adulyasak et al. introduced MIRP and Production Routing Problem (PRP) formulations with and without vehicle index [28]. To formulate the multi-vehicle PRP (MPRP) with a vehicle index, they extended the single-vehicle IRP formulation used by to the single-vehicle PRP formulation and then to the MPRP formulation [6]. They proposed several valid inequalities, including symmetry breaking constraints to strengthen the formulations. A heuristic based on an Op-ALNS technique is also developed to determine the initial solutions. They also proposed B&C algorithms to solve the different formulations. They adapted the Op-ALNS heuristic previously developed for IRP and PRP to solve the problems under ML and OU replenishment policies. They tested the performance of their algorithms on the instances and other benchmarks [6]. Most of the instances were solved optimally in very competitive computation times. Op-ALNS was able to provide high quality solutions, especially for the PRP instances, in a few seconds. The results showed that the vehicle index formulations were superior in finding optimal solutions. The non-vehicle index formulations provided better lower bounds for larger instances that have not yet been solved optimally.

Chrysochoou and Ziliaskopoulos implemented an Inventory Routing Problem with stochastic demand (SIRP) and the option of transshipment when it was needed [29]. They used both to build their model [6,16]. Both authors proposed a two-stage stochastic programming model. In the first stage, their model was designed to give an answer to the SIRP problem. In the second step, constraints related to transshipment were added to their model which are used when transshipment is necessary. Valid inequalities were proposed to determine the optimal delivery quantities for the ML policy. For solving the problem, they proposed an L-exact algorithm that efficiently solved the stochastic IRP using transshipment as the recourse action tested their program on the stochastic model and its deterministic equivalent [29]. Experiments demonstrated the effectiveness of their model that the L-shaped method converged to good results, and that it had the potential to be applied to more complex real-life problems.

Archetti and Speranza considered an IRP in the VMI and retailer-managed inventory (RMI) cases under the inventory management policy (s, S) [30]. In the first case, a supplier delivers goods to customers according to a delivery schedule imposed by the customer. In the second case, the supplier has access to the customers' inventory levels and knows their demand process. Based on this information, the supplier sets the delivery schedule. The single-vehicle version of the IRP problem performed in their work is derived from for the multi-vehicle version several works such as were the basis for their model design [4,6,14,18,26,17]. They used the state-of-the-art heuristic method presented in [31]. The tests are performed on a subset of the benchmark instances created and tested for the MIRP derived from the instances proposed in for the single vehicle case [6]. They analyzed the results of both approaches and compared the costs and characteristics of the different solutions. The results show that VMI policies provide remarkable savings, both in terms of the cost of the solution and the number of vehicles used. The savings that can be achieved with an integrated policy are relevant in terms of total cost and number of vehicles, even provided that the final inventory levels for a VMI policy are equal to the final inventory levels of the RMI policy.

Cheng et al. studied an IRP that simultaneously considers environmental issues and a heterogeneous fleet (G-HIRP), where fuel consumption and emissions are influenced by loading, distance, speed, and vehicle characteristics [32]. Their model extended the classic IRP model of which consisted of minimizing the sum of inventory holding cost and transportation cost [6,26,27,11]. In addition to the classic IRP objectives, their model also sought to minimize driver salary, vehicle fixed cost, fuel cost, and emissions. They also extended some valid inequality classes from strengthen their model [6,11]. They developed a MIP and then performed numerical tests on instances of to identify the benefits of their model [26]. From a parameter analysis, they proved the difficulty for firms with high inventory costs to control emissions at a lower price. Then, they showed that a higher fuel price does not always mean a better environmental benefit, which can provide suggestions to governments when implementing emission control policies.

Schenekemberg et al. presented a Two-Echelon Production Routing Problem (2E-PRP), inspired by a real case of VMI in the petrochemical industry [33]. They introduced explicit production decisions in a Two-Echelon IRP (2E-IRP), thus defining their PRP model. They presented a set of well-known valid IRP inequalities introduced by for single-vehicle IRP, which are extended to MIRP by multi-depot IRP by [6,11]. They designed a B&C method to solve the problem under different replenishment policies. They also proposed a new exact parallel algorithm, combining MIP-based local searches with B&C, which we call LS-B&C. They generated a set of instances with 1 and 3 vehicles, and 3 and 6 periods, low and high inventory cost at customers for the 2E-PRP adapted from the IRP instances of [6]. Computational experiments have shown that the LS-B&C method is very competitive. LS-B&C outperforms B&C in terms of the number of proven optimal solutions, and the quality of the bounds; Upper Bound (UB) and Lower Bound (LB); without compromising the processing time. The managerial analysis showed that the OU policy applied to the models in the literature resulted in an increase in the total network cost.

Zapata-Cortes et al. proposed an Inventory Routing Problem with Time Windows (IRPTW), which allows simultaneous decision making of inventory allocation and transportation routes to supply a set of customers over a specific time horizon [35]. The formulation of the IRPTW model was based on the work of [6,36,37]. A GA was developed to solve their IRP model using the aggregation strategy. The GA is tested on the fitted instance of C101 and based on the parameters proposed in [6]. The GA was able to find a combination of inventory allocation and distribution routes that meet customer service constraints and reduce total distribution costs. A comparison between the results obtained in the IRPTW case and the VRP with time windows (VRPTW) was made by the researchers which showed that when the solution of the total distribution cost over the whole-time horizon generated higher costs than the IRPTW.

Amri-Sakhri et al. treated the case of a deterministic replenishment demand in a distribution network consisting of a supplier and a set of customers to be served by a single vehicle over the planning horizon [39]. They used the model as the basis for building their models [6]. The authors studied the impact of increasing supplier lead time on network costs. They also introduced the Lateral Transshipment (LT) technique and analyzed the effects of this technique on the overall network cost. Their models were solved by an exact method with the use of the benchmark to test their models [6]. The results showed that varying the replenishment lead time generated additional costs in the supply network. They also concluded that LT was an effective tool for improving total network cost and balancing customer inventory levels.

Table 2 lists the articles reviewed in this subsection, mentioning the IRP variants for each work emerging from the model [6].

Reference	Problem Type
Coelho et al. [25]	IRPT
Coelho and Laporte [26]	MIRPT with Homogeneous and Heterogeneous fleets and Consistency constraints
Coelho and Laporte [27]	IRP under OTL policy
Adulyasak et al. [28]	MIRP and MPRP
Chrysochoou and Zil-iaskopoulos [29]	SIRPT
Archetti et al. [30]	IRP under RMI and VMI
Cheng et al. [32]	G-HIRP
Schenekemberg et al. [33]	2E-IRP and 2E-PRP
Zapata-Cortes et al. [35]	IRPTW
Amri-Sakhri et al. [39]	IRP with variable lead time and IRPLT

Table 2: Emerging variants of the Archetti et al. (2007) model

In the following, we present studies conducted in different IRP models using in the computational tests the benchmarks of [6].

3.3 Survey of Works that Used the Referenced Benchmarks

In the following investigations, the authors used the benchmarks of to test their developed models [6].

The research presented by aimed to extend the IRP formulation developed by based on location heuristics [8,40]. The main objective was to develop a hybrid approach to solve their problem using more advanced methods than simple heuristics. MIP is initially used to determine the partitioning of customers and the dates and quantities of deliveries. Then, they used the 2-opt algorithm to solve the traveling salesman problem, the optimal routes for each partition were determined, under both OU and ML delivery policies. The classical IRP model is extended by additional constraints such as visit spacing, vehicle fill rate, vehicle driver consistency, and heterogeneous vehicle fleet, as well as additional criteria were discussed. The impact of using each of the proposed extensions on the solutions was analyzed. To evaluate their hybrid model, several benchmarks were used. The results of the computational tests, on the instances of confirmed the efficiency of their hybrid approach [6]. They used the benchmarks of to evaluate the performance of their algorithm by comparing their results with those obtained by the HAIR algorithm of and the ALNS algorithm of [14,16].

Darvish et al, studied two integrated systems dealing with production, inventory, and routing decisions [41]. Their models are known as IRP and PRP, in which a commodity produced at the factory is shipped to customers over a finite time horizon. They extended their parameters and contributed to the literature by proposing a loadbased formulation for minimizing emissions in the IRP and PRP settings. They also studied the trade-offs between total cost, distance, and emission minimization. A B&C algorithm is used to solve the problems with objective functions of total cost minimization, inventory, and routing. They also designed a powerful exact algorithm that improved the emission minimization solutions. This combined both a B&B algorithm and an improved exact algorithm called Variable MIP Neighborhood Descent (VMND). To test the performance of their models, they used the benchmark instance sets of for IRP and for PRP [6,14]. They have shown that the measurement of emissions depends on the distance and also on the load of the vehicle. Based on the sensitivity analysis of several performance indicators, they were able to provide guidance on how to manage production and distribution to minimize the cost of the supply chain and the environmental protection.

Archetti et al. studied the Inventory Routing Problem with Logistics Ratio (IRP-LR) a variant of the classical IRP where the logistics ratio is minimized [42]. Indeed, the logistics ratio refers to the ratio of the routing cost to the total quantity of inventory delivered. The only costs of interest in this variant are the delivery costs. In this case, the distribution plan does not affect the total inventory cost, which is a constant. The new objective function makes the problem harder to solve to optimality than the classical IRP. The tested instances are those for the single vehicle case and those of adapted to the multiple vehicle case [6,11,17,19]. An exact iterative algorithm for IRP-LR is proposed and its results are compared to the literature solutions obtained by the B&C algorithm. Experiments show that the proposed algorithm is faster when the number of vehicles is small, solving instances with a larger number of clients and over a longer planning horizon.

He et al. study a real case of IRP proposed by Air Liquide [43]. Their problem deals with continuous-time scheduling of driver activity, different levels of time discretization, continuous trailer quantity management, and a non-linear logistics ratio objective, as well as other activity constraints. Their paper proposes a matheuristic to solve an inventory routing problem. The mat-heuristic integrates a fixed sequence mathematical program, two randomized greedy algorithms, and a heuristic based on column generation. Their experiments are performed on two well-known IRP benchmark datasets, [6,14]. They established a comparison of the performance of their model with that of a single commodity flow model. Then, they evaluated the effectiveness of the valid inequalities they introduced to tighten the root node relaxation of the two-commodity model. Next, they compared the performance of the separation strategies and evaluated the effectiveness of the OU and ML versions of their models. They also compared the results of state-of-the-art exact, Meta-heuristic and mat-heuristic approaches to recognize the performance of their approach. The experiments showed that the proposed algorithm is effective as a post-optimization process and is even able to improve the best solutions obtained in the literature. To conclude, this subsection highlights the research conducted to date using the benchmarks of [6].

4.Conclusion

In this work, we began by establishing a literature review of the main research works that have marked the IRP. In a second phase, we focused this paper on a popular inventory routing model and benchmarks realized by [6]. After presenting the problem, we exposed a survey that covers the various works that have invested in solving the model by different resolution approaches [6]. Subsequently, we presented research that focused on IRP variants related to the one of [6]. Finally, we have presented the research done on IRP variants whose models are different from but which have used their benchmark in the experimental stage [6]. This work can present a state of the art of the works produced during the last decade in relation with the study of [6]. It can also serve as a basis for future investigations in the IRP research axis or be an opportunity to produce new IRP variants.

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