

# How much could gravitational binding energy act as hidden cosmic vacuum energy?

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## Abstract

Confronted with recent microwave background observations by WMAP and with puzzling supernovae locations in the magnitude - redshift  $m$ - $z$ -diagram the present-day cosmology seems to call for cosmic vacuum energy as a necessary, unavoidable cosmological ingredient to get a rhyme for the disjunctive cosmological facts. Most often nowadays this vacuum energy is associated with Einstein's cosmological constant  $\Lambda$  or with the so-called: "dark energy" paradigm, both of which are conceptually not well determined or physically clearly handable. Hereby, a positive value of  $\Lambda$  describes an inflationary action on cosmic scale dynamics which in view of recent cosmological data appears as an absolute need.

In this article, however, we shall at least question the hypothesis of a constant vacuum energy density, since it is not justifiable on physical grounds and inconsistent with the energy conservation principle. Instead we show here that changes in gravitational binding energy of cosmic matter - connected with structure formation during the cosmic expansion - mathematically acts in a way very similar to vacuum energy, since it reduces the effective proper mass density and thus reduces the net cosmological gravitational attraction. Thus one may feel encouraged to believe that actions of cosmic vacuum energy, gravitational binding energy and effective mass reduction - taken by their pure cosmological effects - are closely related to each other, perhaps in some respects even have identical cosmological roots.

Based on the results presented here we propose that the action of vacuum energy on cosmic spacetime dynamics inevitably leads to a decay of vacuum energy density. Connected with this decay is an increase of negative cosmic binding energy and the diminution of effective mass in the universe. If this all is adequately taken into account by the energy-momentum tensor of the GR field equations, one is then led to non-standard cosmologies which for the first time at least can guarantee the conservation of the total, global energy, both in static and expanding universes showing that the action of so-called vacuum energy is nothing else but the increase of gravitational binding energy in an evolved self-structured universe with a correlation coefficient of  $\alpha \geq 1$ .

## Introduction

### Why Should Vacuum Energy Induce Antigravitation?

For fundamental conceptual reasons it may be necessary to first explore why at all a vacuum should gravitate, since, when really - as the word implies: representing "nothing"-, then it should most likely also not do anything, e.g. not induce gravitational actions. At least based on an understanding that the ancient greek atomists like Leukipos or Democritos had, the vacuum is a complete emptiness simply offering disposable places, and thereby allowing the possibility for atoms to freely move from one place to others. One should then really in fact not expect any gravitational action from such a "place-holder"- vacuum.

The Greek philosopher Aristotle, however, brought into this conceptual view his principle of nature's objection against emptiness ("horror vacui"). This is a new aspect realizing that "empty space

around matter particles" is not as "empty" as it would be without those particles, but vacuum when surrounding matter is polarized by the existence or presence of this real matter. This idea did furtheron very much complicate the concept of vacuum making it nowadays a rather lengthy and even not yet finished story (e.g. see Blome and Priester, 1984, Fahr, 1989, 2004, Genz, 1996, Wesson, 1999, Barrow, 2000, Overduin and Wesson, 2002, Fahr, 2003, Peebles and Ratra, 2003, Kragh and Overduin, 2014). In the recent decades it became evident that vacuum must be energy-loaded (see e.g. Steeruwitz, 1975, Zel'dovich, 1981, Birrell and Davies, 1982, Lamoreaux, 1997, Rafelsky and Müller, 1985, Blome et al., 2001) and by its energy it should hence, according to general-relativistic doctrines, also somehow influence or induce gravitational fields, - even, if it is not yet clear till today in which specific form that should take place.

Nowadays the GRT action of the vacuum is taken into account by an appropriately formulated, hydrodynamical energy-momentum tensor  $T_{\mu\nu}^{vac}$ , formulating the metrical source of the energy sitting in the vacuum as described by a special, hydrodynamic-like fluid with vacuum pressure  $P_{vac}$  and with an equivalent vacuum mass energy density  $\rho_{vac}$ . Consequently with its constant vacuum energy density  $\epsilon_{vac} = \rho_{vac} c^2$ , as assumed in the present-day standard cosmology (see Bennet et al., 2003), one logically would obtain this tensor in the form (see e.g. Overduin and Fahr, 2001)

$$T_{\mu\nu}^{vac} = (\rho_{vac} c^2 + p_{vac}) U_\mu U_\nu - p_{vac} g_{\mu\nu} = \rho_{vac} c^2 g_{\mu\nu}$$

where  $U_\lambda = 0$  are the 4-vector components of the vacuum fluid velocity vector which vanish when the system of the vacuum defines the rest frame.

Thus this term, taken together with Einstein's cosmological constant term  $\Lambda$  (Einstein, 1917), and placed on the right-hand side of the GRT field equations then leads to an "effective cosmological constant" given in the form:

$$\Lambda_{eff} = \frac{8\pi G}{c^2} \rho_{vac} - \Lambda$$

The first problem - always seen after Einstein's introduction (1917) of his integration constant - is connected with the free choice concerning the numerical value of Einstein's constant  $\Lambda$ , by its nature: an integration constant! One way to obtain a first answer to that question, at least for the completely empty, i.e. matter-free space, is a rationally pragmatic and aprioristic definition, - namely an answer coming up from an apriori definition of how? empty space should be constituted and should physically manifest itself.

If it was always postulated as an "apriori" that the "completely empty space" should be free of any spacetime-curving sources, and thus free of local or global curvatures.

Consequently one has to require that selfparallelity of 4-vectors at parallel transports along closed worldlines in this empty space should be guaranteed, and that no action of empty space on freely propagating test photons occurs. Then as shown by Overduin and Fahr (2001) or Fahr (2004) the only viable solution is:  $\Lambda_{eff,0} = 0!$  meaning that the cosmological constant should be fixed such that

$$\Lambda_0 = -\frac{8\pi G}{c^2} \rho_{vac,0}$$

where  $\rho_{vac,0}$  denotes the equivalent mass density of the vacuum of empty, i.e. matter-free space. Once fixed in this above form, the cosmological constant cannot be different from this value  $\Lambda_0$  in a matter-filled universe, simply meaning that in a matter-filled universe the effective quantity representing the action of the vacuum energy density is given by:

$$\Lambda_{eff} = \frac{8\pi G}{c^2} (\rho_{vac} - \rho_{vac,0})$$

This, however, expresses the interesting fact that in a matter-filled universe only the difference between the values of the vacuum energy densities  $\rho_{vac,0}$  of empty space and of matter-polarized space  $\rho_{vac}$  induces gravitational effects, i.e. influences the spacetime geometry. That at least could give the ardently looked-for explanation, why obviously that vacuum energy calculated by field theoreticians (see e.g. Goenner, 1994) turns out to be 120 orders of

magnitudes too large compared to what is needed and cosmologically admitted. - Obviously not the full magnitude of this vacuum energy gravitates!

This also points to the perhaps most astonishing fact that the geometrically-, i.e. curvature-relevant vacuum energy density also depends on the matter distributed in space, and in a homogeneous universe this can only mean that:  $\rho_{vac} = \rho_{vac}(\rho)$ , an idea that deeply reminds one to the views already developed by Aristotle at around 400 bC. Though this idea of the vacuum state being influenced by the presence of matter in space appears to be reasonable in view of the analogue of real field sources polarizing space around them by acting on sporadic quantum fluctuations and partly screening off the strength of real field sources (see e.g. Rafelsky and Müller, 1997), it nevertheless, however, remains hard to draw any quantitative conclusions from that context. Amongst others, just for that reason we here shall try another way to find the unknown function:  $\rho_{vac} = \rho_{vac}(\rho)$ .

### What is Comologically to Expect from an Absolutely Empty Space?

The question what means "empty space", or synonymous for that - "vacuum" -, in fact is a very fundamental one and has already been put by mankind since the epoch of the greek natural philosophers till the present epoch of modern quantum field theoreticians. The changing opinions given in answers to this fundamental question over the changing epochs have been reviewed for example by Overduin and Fahr (2001), but we here do not want to repeat all of these different answers that have been given in the past, but only at the begin of this article we want to emphasize a few fundamental aspects of present-day thinking of the physical constitution of empty space. Especially challenging in this respect is the possibility that empty space could nevertheless be "energy-charged". This strange and controversial aspect we shall investigate further below in this article.

In our brief and first definition we want to denote empty space as a spacetime without any topified or localized energy representations, i.e. without energy singularities in form of point masses like baryons, leptons, darkions (i.e. dark matter particles) or photons, even without point-like quantum mechanical vacuum fluctuations. If then nevertheless it should be needed to discuss that such empty spaces could be still energy-loaded, then this energy of empty space has to be seen as a pure volume-energy, somehow connected with the magnitude of the volume or perhaps with a scalar quantity of spacetime metrics, like for instance the global curvature of this space. In a completely empty space of this virtue of course no spacepoints can be distinguished from any others, and thus volume-energy or curvature, if existent, are numerically identical at all space coordinates.

Under these prerequisites it nevertheless would not be the most reasonable assumption, as many people believe, that vacuum energy density  $\epsilon_{vac} = \rho_{vac} c^2$  needs to be considered as a constant quantity whatever spacetime does or is forced to do, i.e. whether it expands, collapses or stagnates. This is simply true because the unit of volume is no cosmologically relevant quantity - and consequently vacuum energy density  $\rho_{vac}$  neither is. If at all, it would probably appear more reasonable to assume that the energy loading of a homologously comoving proper volume does not by its magnitude reflect the time that has passed in the cosmic evolution, i.e. perhaps

one should conclude that this specific quantity has to be a constant. But this then, surprisingly enough, would logically mean that the relevant quantity, instead of the vacuum energy density  $\epsilon_{vac}$ , rather is

$$e_{vac} = \epsilon_{vac} \sqrt{-g_3} d^3V$$

where  $g_3$  is the determinant of the 3d-space metric which in case of a Robertson-Walker geometry is given by

$$g_3 = g_{11}g_{22}g_{33} = -\frac{1}{(1 - Kr^2)} R^6 r^4 \sin^2 \vartheta$$

with  $K$  denoting the curvature parameter, the function  $R = R(t)$  determines the time-dependent scale of the universe, and the differential 3d-space volume element in normalized polar coordinates is given by

$$d^3V = drd\vartheta d\varphi$$

This finally then leads to the following relation

$$e_{vac} = \epsilon_{vac} \sqrt{R^6 r^4 \sin^2 \vartheta / (1 - Kr^2)} drd\vartheta d\varphi = \epsilon_{vac} \frac{R^3}{\sqrt{1 - Kr^2}} r^2 \sin \vartheta drd\vartheta d\varphi$$

which then shows that a postulated invariance of  $e_{vac}$  consequently and logically would lead to a variability of the vacuum energy density in the form

$$\epsilon_{vac} = \rho_{vac} c^2 \sim R(t)^{-3}$$

which for instance would already exclude that Einstein's cosmological constant could ever be treated as an equivalent to a vacuum energy density, since requiring the identity  $\Lambda = 8\pi G \rho_{vac} / c^2$ .

On the other hand the invariance of the vacuum energy per co-moving proper volume,  $e_{vac}$ , can of course only be expected with some physical sense, if this quantity does not do any work on the dynamics of the cosmic geometry, especially by physically or causally influencing the evolution of the scale factor  $R(t)$  of the universe.

If, on the other hand, such a work is done and vacuum energy influences the dynamics of the cosmic spacetime (either by inflation or deflation), as in case of a non-vanishing energy momentum tensor, then automatically thermodynamic requirements need to be fulfilled, for example relating vacuum energy density and vacuum pressure by the standard thermodynamic relation (see Goenner, 1996)

$$\frac{d}{dR}(\epsilon_{vac} R^3) = -p_{vac} \frac{d}{dR} R^3$$

This equation is shown to be fulfilled by an expression of the form

$$p_{vac} = -\frac{3-n}{3} \epsilon_{vac} \quad (9)$$

if the vacuum energy density itself is represented by a scale-dependence  $\epsilon_{vac} \sim R^n$ . Then, however, it turns out that the above thermodynamic condition, besides for the trivial case  $n = 3$  when the vacuum does not at all act as a pressure (since  $p_{vac}(n = 3) = 0$ ), is only non-trivially fulfilled for exponents  $n > 3$ , thus allowing for  $n = 0$ , i.e. a constant vacuum energy density  $\epsilon_{vac} \sim R^0 = const.$

A more rigorous and highly interesting restriction for  $n$  is obtained when one recognizes that the above thermodynamic expression (9) under cosmic conditions needs to be enlarged by a term representing the work that the expanding volume does against the internal gravitational binding of this volume. For mesoscale gas dynamics (aerodynamics, meteorology etc.) this term does usually not play any role, however, on cosmic scales there is a need to take into account this term. Under cosmic perspectives binding energy is an absolutely necessary quantity to be brought into the gravodynamical or thermodynamical energy balance. As worked out in quantitative terms by Fahr and Heyl (2007a/b) this then leads to the following completed relation

$$\frac{d}{dR}(\epsilon_{vac} R^3) = -p_{vac} \frac{d}{dR} R^3 - \frac{8\pi^2 G}{15c^4} \frac{d}{dR} [(\epsilon_{vac} + 3p_{vac})^2 R^5]$$

where the last term accounts for internal gravitational binding energy.

This completed equation, as one can easily show, is also solved by the relation of the form  $p_{vac} = -\frac{3-n}{3} \epsilon_{vac}$  leading to the requirement

$$\frac{-3}{3-n} (3-n) p_{vac} R^2 = -3 p_{vac} R^2 - \frac{8\pi^2 G}{15c^4} \frac{6-3n}{3-n} \frac{d}{dr} (p_{vac} R^5)$$

which, however, now is fulfilled only by:  $n = 2!$  meaning that the corresponding cosmic vacuum energy density must vary like

$$\epsilon_{vac} \sim R^{-2}$$

This consequently means that, if it has to be taken into account that vacuum energy acts upon spacetime in a thermodynamical sense, then the most reasonable assumption for the vacuum energy density would be to assume that it drops off with the expansion inversely proportional to the square of the cosmic scale - instead of being a constant.

## Does Progress in Cosmologic Structure Formation Accelerate Cosmic Expansion?

Cosmic structure formation denotes the phenomenon of growing clumpiness of the cosmic matter distribution in cosmic space during the ongoing evolution of the expanding universe, i.e. the origin of larger and larger mass structures like galaxies, clusters or super-clusters of galaxies. Usually one does start cosmology with the assumption that at the beginning of cosmic time and the evolution of the universe cosmic space has a uniform deposition with matter and energy, justifying the use of the famous Robertson-Walker geometry. The question then may arise whether or not the later cosmic expansion dynamics and the scale evolution  $\dot{R} = dR/dt$  may then be influenced by an ongoing structure formation, as it has to happen in order to create the hierarchically structured present-day universe from its earlier uniformity? May this process of an upcoming structuration perhaps influence the ongoing Hubble expansion of the universe, either accelerating or perhaps decelerating its expansion with respect to the solutions of a normal Friedmann universe (e.g. see Goenner, 1996)? - This,

however, could simply be due to the fact that under the new conditions of selfstructuring matter the effective mass density  $\rho_{eff} = \rho_{eff}(t)$  of the universe is not behaving, like it normally does in a Friedmann universe as  $\rho = \rho_0 \cdot (R_0/R)^3$ , but rather as  $\rho_{eff} = \rho_{eff,0}(t)(R_0/R)^3$ .

First we start an easy-minded exercise showing that gravitational structure formation in the universe may in fact have the quite unexpected tendency to accelerate, like a force would do, the Hubble flow velocity, - a virtue that is nowadays all over in the modern cosmological literature ascribed to the action of a vacuum pressure  $p_{vac}$  (see e.g. Peebles and Ratra, 2003, Perlmutter et al., 1999, Perlmutter, 2003, Riess et al., 1998, Schmidt et al., 1998). We shall start assuming that cosmic structure formation has started to develop at some past epoch of cosmic evolution reaching nowadays some organized state of matter distribution, such that not anymore a homogeneous matter density distribution prevails, but instead a point-related, homogeneous distribution, however, of a hierarchically organized matter distribution. In fact from galactic number count statistics one presently knows that the hierarchical state of the present universe manifests itself by observed local two-point correlation functions  $\xi(l)$  expressing the probability to find another galaxy at a distance  $l$  from any arbitrary local space point. For an unstructured, homogeneous matter distribution the function  $\xi$  would of course be a constant, in cosmic reality, however, this two-point correlation probability over wide ranges of scales has observationally proven to fall off according to the following function:

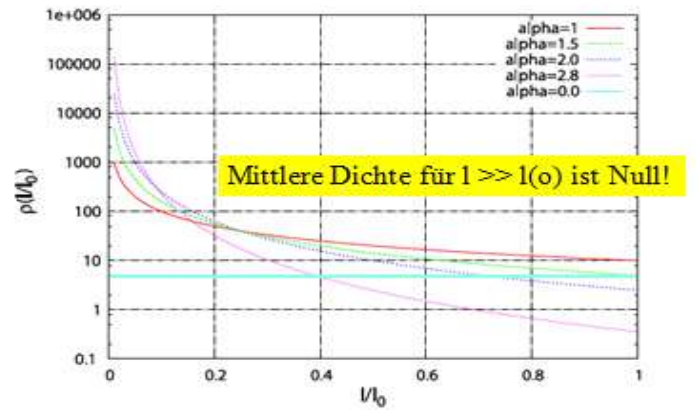
$$\xi(l) = \xi(l_0) \cdot \left(\frac{l_0}{l}\right)^\alpha \quad (34)$$

with the power index  $\alpha \approx 1.8$  and some inner scale  $l_0$  typical for galaxies (see Bahcall and Chokski; 1992). In terms of matter density this expresses an organized cosmic matter distribution, however, so that the average density over cosmic scales  $R$  has not changed compared to the value of the associated homogeneous universe. Nevertheless, a clustering appears at local scales  $l \leq R$  with higher than average densities. This clustering, however, is automatically associated with a more pronounced gravitational binding of this organized matter, i.e. more negative gravitational potential energy has developed during the process of structuring. The important question thus must be posed: How does this affect cosmic expansion dynamics?

To calculate the local potential energy of gravitating matter we start from that local density distribution  $\rho(l)$  corresponding to the probability function  $\xi(l)$  given by Equ.(34) and write the associated density of clustered matter in the form  $\rho(l) = \rho_{0,\alpha} (l/l_0)^{-\alpha}$  there by representing the number of stars at a spherical shell ( $4\pi l^2$ ) by their standard masses  $m_s$ . Then in order to conserve the initial mass at the structuring process, the associated central density  $\rho_{0,\alpha}$  has to be defined by the following value

$$\rho_0 = \frac{3-\alpha}{3} \bar{\rho} \cdot (l_m/l_0)^\alpha$$

with  $l_m$  as an outer integration scale, and  $\bar{\rho}$  denoting the average mass density in the associated homogeneous universe. This obviously limits the structure coefficient to values  $\alpha \leq 3$ .



**Figure 1:** Density in a structured universe as function of the distance  $l/l_0$  for different correlation indices  $\alpha$

Now the total self-gravitation energy of this organized matter within a scale  $l_m$  can then be calculated according to an integration procedure described by Fahr and Heyl (2007) in the following form:

$$\epsilon_{pot}(l_m) = G\rho_0^2 l_0^5 \int_1^{x_m} 4\pi x^2 dx x^{-\alpha} \frac{1}{x} \int_1^x 4\pi x'^2 dx' x'^{-\alpha}$$

where the normalized distance scale has been defined by  $x = l/l_0$ . Hence one obtains

$$\epsilon_{pot}(l_m) = (4\pi)^2 G\rho_0^2 l_0^5 \int_1^{x_m} x dx x^{-\alpha} \left[ \frac{1}{3-\alpha} (x^{3-\alpha} - 1) \right]$$

leading to

$$\epsilon_{pot}(l_m) = \frac{(4\pi)^2}{3-\alpha} G\rho_0^2 l_0^5 \int_1^{x_m} dx [(x^{4-2\alpha} - x^{1-\alpha})]$$

Evaluation of the integral expression yields

$$\epsilon_{pot}(l_m) = \frac{(4\pi)^2}{3-\alpha} G\rho_0^2 l_0^5 \left[ \frac{x^{5-2\alpha}}{5-2\alpha} - \frac{x^{2-\alpha}}{2-\alpha} \right]_1^{x_m}$$

and when taking  $x_m \gg 1$  leads to

$$\epsilon_{pot}(l_m) = \frac{(4\pi)^2}{3-\alpha} G\rho_0^2 l_0^5 x_m^{5-2\alpha} \left( \frac{1}{5-2\alpha} - \frac{x_m^{-3}}{2-\alpha} \right) \approx \frac{(4\pi)^2}{(3-\alpha)(5-2\alpha)} G\rho_0^2 l_0^5 x_m^{5-2\alpha}$$

With the requirement  $\rho_0 = \frac{3-\alpha}{3} \bar{\rho} x_m^\alpha$  this finally delivers

$$\epsilon_{pot}(l_m) = \frac{(4\pi)^2 (3-\alpha)}{9(5-2\alpha)} G\bar{\rho}^2 l_0^5 x_m^5$$

obviously limiting the permissible structure coefficient to values  $\alpha \leq 2.5$ . It is interesting to recognize that for  $\alpha = 0$  (i.e. homogeneous matter distribution) in fact the potential energy of a homogeneously matter-filled sphere with radius  $l_m$  is found, which does not vanish, but has a finite value, namely  $\epsilon_{pot}(\alpha = 0) = \frac{(4\pi)^2}{15} G\bar{\rho}^2 l_m^5$

(see Fahr and Heyl, 2007). This latter binding energy, however, is

fully incorporated by the Friedmann-Lemaître cosmology as the one responsible for the deceleration of the normal Hubble expansion of the universe. If in contrast the cosmic deceleration turns out to be less than that, or it even indicates an acceleration, then in our view this must be ascribed to the increased production of binding energy due to the upcome of structure formation, i.e. what counts is the difference  $\Delta\epsilon_{pot} = \epsilon_{pot}(\alpha) - \epsilon_{pot}(\alpha = 0)$  between a structured and an unstructured universe.

The value  $\epsilon_{pot}(\alpha=0)$  hereby serves as a reference value for that potential energy in the associated, re-homogenized universe. This means what really counts in terms of binding energy of a structured universe causing a deviation from the expansion of the Friedmann-Lemaître universe is the difference  $\Delta\epsilon_{pot}$  between structured and the unstructured universe, since naturally the unstructured universe has its own, nonvanishing amount of binding energy, i.e. in general cases one obtains:

$$\Delta\epsilon_{pot}(\alpha, l_m) = \frac{(4\pi)^2(3-\alpha)}{9(5-2\alpha)}G\bar{\rho}^2l_m^5 - \frac{(4\pi)^2}{15}G\bar{\rho}^2l_m^5 = \frac{(4\pi)^2}{3}\left[\frac{(3-\alpha)}{3(5-2\alpha)} - \frac{1}{5}\right]G\bar{\rho}^2l_m^5$$

Equating now this actually counting potential energy  $\epsilon_{pot}(\alpha, l_m)$  with a corresponding, equivalent, cosmic density reduction one finds:  $\Delta\rho = \bar{\rho}_{eff}(\alpha) - \bar{\rho}(\alpha)$

$$c^2\Delta\rho\frac{4\pi}{3}l_m^3 = \Delta\epsilon_{pot}(\alpha, l_m) = \frac{(4\pi)^2}{3}\left[\frac{(3-\alpha)}{3(5-2\alpha)} - \frac{1}{5}\right]G\bar{\rho}^2l_m^5$$

leading to a density reduction by:

$$\Delta\rho = \bar{\rho} \cdot \left[\frac{(3-\alpha)}{3(5-2\alpha)} - \frac{1}{5}\right] \frac{4\pi G\bar{\rho}l_m^2}{c^2}$$

This expression tells us that the equivalent density reduction  $\Delta\rho$  is proportional to the actual average cosmic density  $\bar{\rho}$ . When looking especially now for the situation that  $\bar{\rho}$  equals  $\Delta\rho$ , i.e. that the upper amount of gravitational binding would completely reduce the effective density to zero, would then indicate a critical density value  $\bar{\rho}_c$  of:

$$\bar{\rho}_c = \frac{1}{\left(\frac{(3-\alpha)}{3(5-2\alpha)} - \frac{1}{5}\right) \frac{4\pi Gl_m^2}{c^2}}$$

The unstructured universe ( $\alpha = 0$ ) would thus have a vanishing effective density  $\rho_c$  associated with a density value:

$$\bar{\rho}_c = \frac{15c^2}{12\pi Gl_m^2}$$

or a total mass within the scale R of the universe of

$$M_u(R) = \frac{4\pi}{3}R^3\bar{\rho}_c = \frac{5c^2R}{3G}$$

This would mean that under these conditions the universe would have an outer scale  $R_c$  smaller than its Schwarzschildradius  $R_{cs}$ , namely:

$$R_c = \frac{3}{10}R_{cs}$$

This by the way does mean that the Big-Bang universe expected

to originate from a vanishing scale  $R \leq R_c$  does at its earliest times in any case sit deeply in its Schwarzschild radius and does consequently not have any gravitationally active matter at this early cosmic phase.

Taking  $l_m$  as the outer scale R of the universe and taking the maximum of this scale to be given by  $R = c/H$  in addition leads one to the expression:

$$\bar{\rho}_c(\alpha = 0) = \frac{5}{3}\left(\frac{9}{4\pi}\right)^2\frac{H^2}{G} = \frac{5}{3}\left(\frac{9}{4\pi}\right)^2\frac{1}{G}\left(\frac{\dot{R}}{R}\right)^2 \simeq \frac{3H^2}{8\pi G}$$

which in fact is astonishingly close to the value for the critical cosmic density value  $\rho_{cr}$  that defines, as one may know, an uncurved Friedman universe, i.e. one with the curvature  $k = 0$ !

### The Evolution of the Hubble Parameter

The above result leads to the question of what Hubble parameter  $H = H(t)$  one may expect to prevail at the different cosmologic evolution times t. For Friedman-Lemaître-Robertson-Walker cosmologies (FLRW) the Hubble parameter  $H(t)=\dot{R}(t)/R(t)$  generally is not a constant, but is given in form of the following differential equation (derived from the 1. Friedman equation; e.g. see Goenner, 1996, Peebles and Ratra, 2003, Fahr, 2016, 2022):

$$H^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}[\rho_B + \rho_D + \rho_v + \rho_\Lambda]$$

where G again is Newton's gravitational constant, and  $\rho_b, \rho_D, \rho_v, \rho_\Lambda$ , denote the relevant equivalent cosmic mass densities  $\rho_b, \rho_D, \rho_v, \rho_\Lambda$  of baryons, of dark matter, of photons, and of the vacuum energy. In case all of these quantities do count for the same cosmologic period, then it is complicated to find a closed solution for  $H(t)$  and  $R(t)$  over these cosmic times, because  $\rho_b$  may vary proportional to  $R^{-3}$ ,  $\rho_D$  most probably also according to  $R^{-3}$ , but  $\rho_v$  is generally thought to vary according to  $R^{-4}$  (see Goenner, 1996, or Fahr and Heyl, 2017, 2018). Amongst these quantities the cosmic vacuum energy density  $\rho_\Lambda$  is perhaps physically the least certain quantity, but - if it is described with Einstein's cosmological constant  $\Lambda$ , then it represents a positive, constant energy density, i.e. its mass equivalent  $\rho_\Lambda$  in connection with a constant and positive vacuum energy density  $\Lambda$ , would consequently as well be a positive, constant quantity. This in fact would offer for the late phases of cosmic expansion, i.e. for  $\rho_\Lambda \gg \rho_B, \rho_D, \rho_v$  an easy and evident solution of the above equation for the late Hubble parameter  $H=H(t)=H(\Lambda)$ :

Namely from recent supernova SN1a observations (Permuter et al., 1999, Perlmutter, 2003, Riess et al., 1998, Schmidt et al., 1998) it has been concluded that at the present cosmic era, most probably already sometimes ago, we were and are in an accelerated expansion phase of the universe. Taking it serious that this is due to the term  $\Lambda$  connected with cosmologic vacuum energy density, this then expresses the fact that  $\rho_\Lambda$  is the dominant quantity in the universe amongst the other upper ingredients  $\rho_B, \rho_D, \rho_v$ .

Then in fact we can assume that the above differential equation

for the Hubble parameter  $H=H(t)$  can be written in a much more simplified form:

$$H_{\Lambda} = \frac{\dot{R}}{R} = \sqrt{\frac{8\pi G}{3} [\rho_B + \rho_D + \rho_v + \rho_{\Lambda}]} \simeq \sqrt{\frac{8\pi G}{3} [\rho_{\Lambda}]} = const$$

Under these auspices the expansion of the universe is described by the expression:

$$R(t) = R_0 \exp\left[\sqrt{\frac{8\pi G}{3} [\rho_{\Lambda}]} (t - t_0)\right] = R_0 \exp[H_{\Lambda}(t - t_0)] \quad (43)$$

with  $R_0$  and  $t_0$  denoting the present-day scale of the universe and the present cosmic time.

### The Kinetic Transport Equation for Cosmic Gases in an Expanding Hubble Universe

Here we may start out from the generally accepted assumption in modern cosmology, that during the collision-dominated phase of the cosmic evolution, just before the time of matter recombination, matter and radiation, due to frequent energy exchange processes, are in complete thermodynamic equilibrium, i.e. matter and radiation temperatures are identical  $T_m = T_r = T_0$ . In the following cosmic evolution this equilibrium, however, will experience perturbations. Even if a Maxwellian distribution would actually prevail at the entrance to the collision-free cosmic expansion phase, it would not have persisted at times then after. Just after the recombination phase when electrons and protons recombine to H-atoms, and photons start propagating through cosmic space practically without further interaction with matter, the thermodynamic contact between matter and radiation furtheron is abolished or completely switched off (see e.g. Partridge, 1965, Fahr and Loch, 1991, Fahr and Zoennchen, 2009, Fahr, 2021).

This expresses the need to care for a kinetic description of cosmic gases at times after matter recombination. To elucidate this point a little deeper, let us consider here a collision-free particle population in an expanding, spatially symmetric Robertson-Walker universe. Hereby it is clear that due to the cosmological principle and the requirement of spatial homogeneity, the velocity distribution function  $f(v,t)$  of the particles must be isotropic in velocity space  $v$  and independent on the local cosmic place  $x$ . Thus it must be of the following general form

$$f(v,t) = n(t) \cdot \tilde{f}(v,t)$$

Where  $n(t)$  denotes the time-variable, cosmic density, only depending on the worldtime  $t$ , and  $\tilde{f}(v,t)$  is the normalized, time-dependent, isotropic velocity distribution function with the property:

$$\int_0^{\infty} \tilde{f}(v,t) d^3v = 1.$$

If we now face the fact that particles, moving freely with their velocity  $v$  into their  $v^{\rightarrow}$ -associated direction over a distance  $l$ , at their new place have to reconstitute the actual cosmic distribution there, despite the differential Hubble flow and the explicit time-dependence

of  $f$ , then a locally prevailing co-variant distribution function  $f(v^{\prime}, t^{\prime})$  must exist with the property that the two associated functions  $f(v^{\prime}, t^{\prime})$  and  $f(v, t)$  are related to each other in a Liouville-conform way (see e.g. Cercigniani, 1988, Landau-Lifshitz, 1990). To fix this required relation needs some special care, since particles that are freely moving in a homologically expanding Hubble universe, do in this specific case at their motions not conserve their associated phase space volumes  $d^6\Phi = d^3v d^3x$ , as they usually do in classical gas dynamics. This is because in a homologically expanding cosmic space no particle Lagrangian  $L(v,x)$  exists as usually in gas dynamics, and thus no Hamiltonian canonical relations of their dynamical coordinates  $v$  and  $x$  are valid. Consequently Liouville's theorem (see e.g. Chapman and Cowling, 1952) does not require that the differential  $6D$ -phase space volumes  $d^6\Phi$  are identical, but that the conjugated differential phase space densities are identical to guarantee particle conservation. This is expressed by the following relation:

$$f^{\prime}(v^{\prime}, t^{\prime}) d^3v^{\prime} d^3x^{\prime} = f(v, t) d^3v d^3x$$

When arriving at the place  $x^{\prime}$  these particles, after passage over a distance  $l$  are incorporated into a particle population which has a relative Hubble drift with respect to the origin of the particle given by  $v_H = l \cdot H$  co-aligned with  $v^{\rightarrow}$ . Thus the original particle velocity  $v$  registered at the new place  $x^{\prime}$  appears as locally tuned down to  $v^{\prime} = v - l \cdot H$  since at the present place  $x^{\prime}$ , displaced from the original place  $x$  by the increment  $l$ , all velocities have to be judged with respect to the new local reference frame (standard of rest) with its differential Hubble drift of  $(l \cdot H)$  relative to the particle's origin.

If all of that is taken into account, it can be shown (see Fahr, 2021a) that one finally is led to the following kinetic transport equation:

$$\frac{\partial f}{\partial t} = vH \cdot \left(\frac{\partial f}{\partial v}\right) - H \cdot f \quad (40)$$

This partial differential equation should allow to derive the resulting distribution function as function of the velocity  $v$  and of the cosmic time  $t$ .

As it was shown already by Fahr (2021a), the above kinetic transport equation does not allow for a solution in the form of a separation of variables, i.e. putting  $f(v, t) = f_t(t) \cdot f_v(v)$  but one rather needs a different, non-straightforward method of finding a kinetic solution of this above transport equation Equ.(40). Under the assumption a) that at time  $t = t_0$  still a Maxwellian distribution is valid, and b) that since that time a constant Hubble parameter  $H_0 = H_{\Lambda}$  prevails, one can then write the actual distribution function derived with the above partial differential equation in the following form (Fahr, 2021b):

$$f(v,t) = n_0 \exp[-3H_{\Lambda}(t - t_0)] \cdot \frac{(1 - H_{\Lambda}(t - t_0))^3}{\pi^{3/2} v_0^3} \exp[-x^2 \cdot (1 - H_{\Lambda}(t - t_0))^2]$$

where  $v_0$  denotes the thermal velocity by  $v_0^2 = kT_0/m$  at the time  $t_0$ , when a temperature  $T(t_0) = T_0$  prevails. Hereby the normalized velocity coordinate  $x$  was introduced by  $x = v/v_0$ . Furthermore it turns out that one can interpret the actually prevailing distribution

function  $f(v, t)$  as an actual Maxwellian with the time-dependent temperature  $T(t)$  given by:

$$T(t) = \frac{T_0}{(1 - H_\Lambda(t - t_0))^2}$$

and a time-dependent density

$$n(t) = n_0 \exp[-3H_\Lambda(t - t_0)]$$

Hence one finds that under the given cosmologic auspices of a Hubble expansion with the constant Hubble parameter  $H_\Lambda$  the thermal energy of matter in this universe increases like:

$$\epsilon_{therm} = \frac{4\pi}{3} R^3 \cdot n(t) \cdot \left(\frac{3}{2} kT(t)\right) = \frac{4\pi}{3} \frac{(3/2)n_0 kT_0 R_0^3}{(1 - H_\Lambda(t - t_0))^2}$$

meaning that the thermal energy of the matter in the whole Hubble universe of this type increases - violating normal, thermodynamical principles, since here the temperature of matter increases with the increase of cosmic space. This would, however, indicate a total energy increase occurring in this universe with ongoing time, - somehow giving an alarm to the physics operating here.

$$\frac{d}{dt} \left[ \frac{(4\pi)^2}{3} \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] G \bar{\rho}^2 R^5 \right] = - \frac{d}{dt} \left[ \frac{4\pi}{3} \frac{(3/2)n_0 kT_0 R_0^3}{(1 - H_\Lambda(t - t_0))^2} \right]$$

leading to:

$$\frac{d}{dt} \left[ 4\pi \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] G \bar{\rho}_0^2 \left(\frac{R_0}{R}\right)^3 R^5 \right] = - \frac{d}{dt} \left[ \frac{(3/2)\bar{n}_0 kT_0 R_0^3}{(1 - H_\Lambda(t - t_0))^2} \right]$$

or furtheron:

$$4\pi \frac{d}{dt} \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] G \bar{\rho}_0^2 R_0^3 R^2 + \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] 8\pi G \bar{\rho}_0^2 R_0^3 R \dot{R} = - (3/2)\bar{n}_0 kT_0 R_0^3 \frac{d}{dt} \left[ \frac{1}{(1 - H_\Lambda(t - t_0))^2} \right]$$

or, when assuming the correlation coefficient  $\alpha = \alpha(t)$  as a time-dependent quantity:

$$4\pi \frac{d}{dt} \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] G \bar{\rho}_0^2 R_0^3 R^2 + \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] 8\pi G \rho_0^2 R_0^3 R^2 H_\Lambda = (3/2)\bar{n}_0 kT_0 R_0^3 \left[ \frac{-2H_\Lambda}{(1 - H_\Lambda(t - t_0))^3} \right]$$

and furthermore:

$$\frac{d}{dt} \frac{1}{2} \left[ \frac{(3 - \alpha)}{6(5 - 2\alpha)} \right] + \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] H_\Lambda = \frac{(3/2)n_0 kT_0 R_0^3}{8\pi G \bar{\rho}_0^2 R_0^3 R^2} \left[ \frac{-2H_\Lambda}{(1 - H_\Lambda(t - t_0))^3} \right]$$

Now one can express  $R$  as function of  $t$  by putting:  $R(t) = R_0 \exp[H_\Lambda(t - t_0)]$  (see Equ.(43) and one obtains:

Now, at this place of the argumentation, an interesting idea may be brought into this mysterious game; namely that the increase in thermal energy of cosmic matter is just compensated by the increase in negatively valued, cosmic binding energy  $\Delta\epsilon_{pot}(\alpha, l_m)$  due to ongoing structure formation, i.e. increase of the correlation coefficient  $\alpha$ , the latter being suspected here as the true reason for the operation of a so-called "vacuum pressure" corresponding to an equivalent mass density of  $\rho_\Lambda = \Lambda c^2 / 8\pi G$ .

To pursue a little more this idea, we start from the two competing quantities, i.e. the potential binding energy on one hand:

$$\Delta\epsilon_{pot}(\alpha, l_m) = \frac{(4\pi)^2}{3} \left[ \frac{(3 - \alpha)}{3(5 - 2\alpha)} - \frac{1}{5} \right] G \bar{\rho}^2 R^5$$

and the thermal energy difference of cosmic matter on the other hand:

Now, in order to guarantee energy conservation, we shall require that the change with cosmic time  $t$  of the first quantity  $\Delta\epsilon_{pot}(\alpha, l_m)$  is equal to the negative change of the second quantity  $\epsilon_{therm}$  which leads us to the following request:

$$\frac{d}{dt} \left[ \frac{(3-\alpha)}{6(5-2\alpha)} \right] + \left[ \frac{(3-\alpha)}{3(5-2\alpha)} - \frac{1}{5} \right] H_\Lambda = \frac{(3/2)n_0kT_0}{8\pi G\bar{\rho}_0^2 R_0^2 \exp[2H_\Lambda(t-t_0)]} \frac{-2H_\Lambda}{(1-H_\Lambda(t-t_0))^3}$$

Next we obtain from:

$$\frac{d}{dt} \left[ \frac{(3-\alpha)}{6(5-2\alpha)} \right] = \frac{(-\dot{\alpha})}{6(5-2\alpha)} - \frac{(3-\alpha)(-2\dot{\alpha})}{6(5-2\alpha)^2} = \dot{\alpha} \left[ \frac{-1}{6(5-2\alpha)} + \frac{(3-\alpha)}{3(5-2\alpha)^2} \right]$$

and with the following selfsuggestive guess for the time-dependence of the correlation coefficient :  $\alpha = \alpha_0 \exp[H_\Lambda(t-t_0)]$  with  $\alpha_0 = 1.8$  one finds:

$$\left[ \frac{-\alpha}{6(5-2\alpha)} + \frac{\alpha(3-\alpha)}{3(5-2\alpha)^2} + \frac{(3-\alpha)}{3(5-2\alpha)} - \frac{1}{5} \right] = \frac{(3/2)n_0kT_0}{8\pi G\bar{\rho}_0^2 R_0^2 \exp[2H_\Lambda(t-t_0)]} \frac{-2}{(1-H_\Lambda(t-t_0))^3}$$

yielding furthermore:

$$\frac{30-21\alpha}{6(5-2\alpha)^2} - \frac{1}{5} = \frac{(3/2)n_0kT_0}{8\pi G\bar{\rho}_0^2 R_0^2 \exp[2H_\Lambda(t-t_0)]} \cdot \frac{-2}{(1-H_\Lambda(t-t_0))^3}$$

Approximating the exponential function for small arguments (i.e.  $2H_\Lambda(t-t_0) \ll 1$ ) then yields:

$$\frac{30-21\alpha}{6(5-2\alpha)^2} - \frac{1}{5} = \frac{(3/2)n_0kT_0}{8\pi G\bar{\rho}_0^2 R_0^2} \cdot \frac{-2(1-2H_\Lambda(t-t_0))}{(1-3H_\Lambda(t-t_0))} = -\frac{3n_0kT_0}{8\pi G\bar{\rho}_0^2 R_0^2} (1+H_\Lambda(t-t_0))$$

Setting  $\alpha = 1.8$  for times  $t \simeq t_0$  then brings us to;

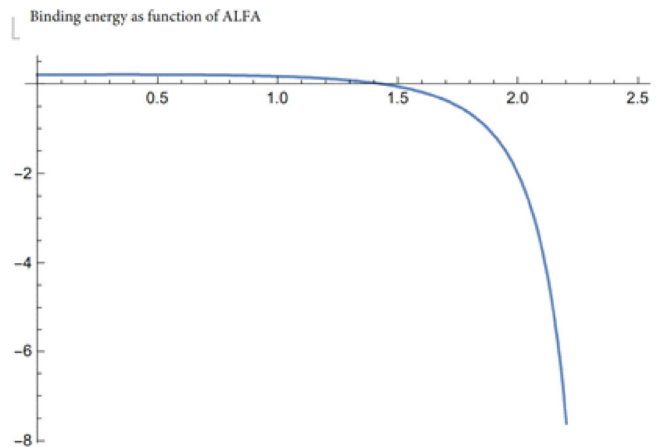
$$\Delta\epsilon_{pot}(\alpha) = \frac{-7.8}{6(1.4)^2} - \frac{1}{5} = -0.863 = -\frac{3n_0kT_0}{8\pi G\bar{\rho}_0^2 R_0^2} (1+H_\Lambda(t-t_0))$$

For the times  $t \simeq t_0$  this relation would require:

$$\Delta\epsilon_{pot}(\alpha) = 0.863 = \frac{3n_0kT_0}{8\pi G\bar{\rho}_0^2 R_0^2} = \frac{kT_0 R_0}{2(4\pi/3)Gm\bar{\rho}_0 R_0^3}$$

## Conclusion

To state the above result in short: The required condition can in fact be fulfilled, if at the time  $t = t_0$  the gravitational binding energy of the mass  $m$ , i.e.  $G_m(4\pi/3)\rho_0 R_0^3/R_0$  equals the actual thermal energy of the particles  $kT_0$ . What concerns the needed and necessary correlation coefficient  $\alpha$ , one can find, however, that only when this coefficient has attained a value of  $\alpha \geq \alpha_c = 1.5$ , then the resulting mathematical sign allows a physical solution in the expected form (see our Figure 2). This means that only when the structure formation process in the universe has progressed far enough, then the above required equality can be really achieved. But then, at times after that, when an accelerated expansion of the universe with a Hubble parameter  $H = H_\Lambda$  prevails, then in fact the increase in negative potential energy of cosmic matter  $\Delta\epsilon_{pot}(\alpha, R)$  is exactly balanced by the increase of thermal cosmic energy  $\Delta\epsilon_{therm}(R)$ . During this phase of the expansion of the universe one is obviously justified to assume that the creation of negative binding energy is the reason for the accelerated expansion of the universe, -normally in present-day cosmology ascribed to the action of vacuum energy.



**Figure 2:** The quantity  $\Delta\epsilon_{pot}(\alpha)$  as a function of the correlation coefficient  $\alpha$



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